Assessment of Simulation Behavior of Different Mathematical Programming Approaches

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Abstract

This paper investigates the simulation behaviour of different PMP methods being developed in the past. About 800 of identical farms for eight years from the German FADN[1] were used to aggregate 45 farm groups. The aggregated farms were calibrated for 1996/97 and the observed prices, direct payments and yields for 2002/03 were applied. The ex-post simulation results underline that the simulation behaviour is mainly controlled by the considered PMP methodology. Even the Maximum Entropy approach proposed by Paris and Howitt (1998) for one observation on base year allocation did not improve the findings. In addition first experiences with an alternative model to PMP where the Q matrix is recovered using multiple observation proposed by Heckelei and Wolff (2003) are illustrated for one particular farm.

Key words: PMP, ex-post evaluation, FADN.

1. Introduction

For the analysis of the multi-output, multi-input supply behaviour in agriculture, either programming models or dual systems of supply and input demand equations are commonly employed. Econometric approaches inferring the structure or the model behaviour from observed decisions of the agents (positive approach). In contrast, the normative programming model approaches set up the structure of the economic agent's decision process.

In this context, the methodology of Positive Mathematical Programming (PMP), originally introduced to a wider range of economists by Richard Howitt (1995), plays an important role

[1] Farm accounting data network.

in bridging the gap between the econometric approach and the mathematical programming framework. The ordinary programming model is not able to make use of the information, which is based on the decisions of farmers. Hence, the information content of observed cropping patterns, which is used in an econometric approach for the estimation of the structural relationship, can not be included in normal programming approach. PMP uses this information through calibration constraints and derives a non-linear object function. A distinguished outcome of PMP is the calibration of the model on the observed base year. The origins of implementing PMP in programming models was seen as overcoming the calibration problem. This property of the PMP approach was very attractive to applied modelling and has lead to an extensive use of PMP in the past in many applications on farm, regional and sector modelling.

Nevertheless PMP is criticized, mainly related to the simulation behaviour (Heckelei and Wolff (2003) and Heckelei and Britz (2000)), missing theoretical foundation of the employed non linear object function and the self selection problem. Therefore, testing the predicative capacity and further developments in PMP are necessary.

Recent developments in PMP have focused on using formal econometric estimation procedures to obtain the non-linear object function and aiming at a further bridge of the positive econometric approach to the programming approach.

Paris and Howitt (1998) have shown how to use the Maximum Entropy Criterion for the estimation of the nonlinear object function even under a negative degree of freedom. Recently an 'up-to-date' PMP approach called Symmetric Positive Equilibrium Problem[2] (SPEP) was introduced by Paris (2001). Heckelei and Wolff (2003) proposed a general alternative to PMP in calibrating and estimating agricultural programming models based on the first order condition.

The object of this paper is to evaluate different calibration approaches developed in the past based on an ex-post analysis for selected arable farms in Germany. The paper is structured as follows: Section one briefly reviews the PMP as a model calibration approach. In section two the different PMP calibration scenarios and the corresponding ex-post simulation run is presented. Further different parameters and the farm data used for the simulation will be introduced. In the last section results for the ex-post simulation are discussed. In line with this, an alternative PMP approach by Heckelei and Wolff (2003) is illustrated for one particular farm. Finally specific consideration is given to problems and direction for further research.

2. Review of the PMP Approach

The general idea of PMP is to use information contained in dual variables of a linear programming model (LP), which are bound to the observed activity levels applied through calibration constraints. A non linear object function is derived in such a way, that the optimal solution will exactly reproduce the observed activity levels without employing any

[2] This Calibration approach is not considered in this paper.

additional constraints. Hence the use of a non-linear object function helps to prevent the model from generating overspecialized solutions. In the literature this approach is called the three stage PMP approach. The **first step** considers the following linear programming approach, where all observed variables are denoted by the superscript "o".

$$\max_{x} Z = p'x - c'x \text{ subject to } \quad Ax \le b[\lambda], \quad x \ge 0 \tag{1}$$

Z denotes the objective function value, **p** is a $(N \times 1)$ vector of product prices, **x** is a $(N \times 1)$ vector of production activity levels, **c** is a $(N \times 1)$ vector of cost per unit of activity, **A** denotes a $(M \times N)$ matrix of coefficients in resource constraints, **b** is a $(M \times 1)$ vector of available resource quantities and λ is a $(M \times 1)$ vector of dual variables associated with the resource constraints.

Applying the calibration constraints, the solution will be forced to the observed activity level.

$$\max_{x} Z = p'x - c'x \text{ subject to } Ax \le b[\lambda], \quad x \le (x^{o} + \varepsilon) \quad [\rho], \; x \ge (0) \tag{1.1}$$

The $(N \times 1)$ vector x^{o} denotes the observed activity levels, the $(N \times 1)$ ε is a vector of small positive numbers, which guarantee that all resource constrains remain binding, and ρ are the dual variables associated with the calibration constraints. The dual values will certainly be smaller than those being obtained in model (1), because the marginal instead of preferable activities determine the dual values of the resource (Heckelei 2002: 7).

Let us now consider an example of wheat and corn with a gross margin of 300 Euro/ha and 100 Euro/ha and land resources of 30 hectares. Without any additional calibration constraints, wheat would be the preferable activity and the dual of land would be 300 Euro/ha. If calibration constraints of 20 hectares for wheat and 10 hectares for corn are included, the preferable activity would still be wheat and the marginal activity would be corn. Hence the vector **x** can be divided into two subsets, a vector of preferred activities x^{p}, which is constrained by the calibration constraint and a vector x^{m} of marginal activities which is bounded by the resource constraint.

In the **second step,** the non-linear object function will be calculated such that the final model will calibrate (under the assumption of decreasing marginal returns in activity level) exactly to the observe activity levels.

The idea of PMP can therefore be understood as detecting the hidden costs for each crop in the farming pattern, in order to get a solution to the programming problem which calibrates and involves the "true" costs of farming. Hence the farm's production structure is assumed to already be at an economic optimum. Because the hidden costs are unobservable to the modeller, the nature of the cost is unknown, and hidden costs are viewed as a consequence of any factors that could contribute to increasing marginal costs. Decreasing marginal returns can be caused by increasing marginal costs whereas marginal revenue was kept constant. Alternatively, the PMP approach can also be specified for decreasing marginal return based on decreasing marginal crop yields and constant marginal costs.

Both approaches can be implemented by taking either cost or production functions for the parameter estimation into consideration. In the following the general PMP in the form of increasing marginal costs is shown. Quadratic functions are often used in the literature. Paris and Howitt (1998) used other functional forms. For simplicity a quadratic object function will be used. In principle, any type of non-linear function convex in activities can be applied. The following 'variable cost function' can be taken as the non-linear part of the object function.

$$C^{v} = d'x + \frac{1}{2}x'Qx \qquad (1.2)$$

d denotes the $(N \times 1)$ vector of parameters associated with the linear term. The $(N \times N)$symmetric, positive (semi-)definite matrix **Q** are parameters associated with the quadratic term. To reconstruct the parameters of the Q Matrix and the d vector the 'marginal variable cost' has to fulfill:

$$MC^{v} = \frac{\partial C^{v}(x^{o})}{\partial x} = d + Qx^{o} = c + \rho \qquad (1.3)$$

Providing the PMP coefficients are recovered, the final non-linear programming problem can be specified as:

$$\max_{x} Z = p'x - d'x - \frac{1}{2}x'Qx \quad \text{subject to} \quad Ax \leq b[\lambda], \quad x \geq 0 \qquad (1.4)$$

Since the beginning of PMP, different 'versions' were developed which can either be differentiated by the type of function (cost or production function) or by the estimating procedure applied to recover the matrix coefficients. For the ex-post scenarios, a quadratic cost function is applied, whereas different approaches to recover the Q matrix are employed.

3. Description of the Ex-Post Calibration of Scenarios

The ex-post evaluation of different calibration methods refers to the period from 1996 to 2003. All scenarios are calibrated to the observed land allocation 1996/97 using four different PMP calibration models. After calculation of the parameters of the marginal cost function the observed yields, direct payments from 2002/03 and the expected prices from

2001/02 are applied to 45 farmgroups. The calculated land allocations are then compared to the observed allocation in 2002/2003.

The following four PMP calibration scenarios are included: a) the "original PMP version", b) the Paris (1998) procedure, c) use exogenous elasticities to recover the Q-Matrix, whereby two different vectors of own gross margin elasticities are applied, d) recovering the full Q-Matrix by Maximum Entropy using two formulations for the support points. In addition another investigation deals with more than one observation for the estimation of the Q-Matrix. Therefore a particular farm group was selected and the traditionally three stage PMP procedure is replaced by the "First Order Condition Calibration Model" (FOC) proposed by Heckelei and Wolff (2003).

3.1 Original PMP Version (Original PMP)

Here the estimation of the non-linear cost function was solved by letting $d_i = c_i$ and setting all off-diagonal elements of **Q** to zero (Howitt and Mean, 1983; Arfini and Paris, 1995; Bauer and Kasnakoglu, 1990).

$$p_{ii} = \frac{\rho_i}{x_i^o} \qquad (2.1)$$

This specification gives a linear cost function for the "marginal" activities, caused by the zero dual value of the marginal activities. Beside the general misspecification of that approach, the resulting simulation behavior is determined through the still linear cost function of the marginal activity.

3.2 PARIS (1988)

Paris (1988) tried to overcome the additional needs for priori information, which arose when the original PMP approach was improved, and developed a modified version. He uses duality in order to derive the coefficients of the non-linear object function,

$$q_{ii} = \frac{c_i + \rho_i}{x_i^o} \qquad (2.2)$$

which achieves positive diagonal elements of **Q** as well for the marginal activities. The vector ρ denotes the dual values, x_i^o the crop allocation and c is a vector of costs from the linear formulation.

3.3 Exogenous Elasticities

In this scenario exogenous elasticities are used to recover the parameter of the marginal cost function (Helming et al., 2001; Osterburg et al., 2001). The off-diagonal elements of Q are set to zero. In the ex post analysis, land allocation elasticities with respect to own gross margins (ε) elasticities are considered. Because the partial derivative $\frac{\partial x_i}{\partial \varepsilon_i}$ is equal to q_{ii}^{-1} the exogenous land allocation elasticity can be used to calculate Q as:

$$q_{ii} = \frac{1}{\varepsilon_{ii}} \frac{rev^o{}_i}{x^o{}_i} \tag{2.3}$$

In order to satisfy the calibration condition in equation 1.3, the linear parameter of the variable cost function is set to:

$$d_i = c_i + \rho_i - q_{ii} x^o{}_i \tag{2.4}$$

3.4 Calibration with maximum entropy

Paris and Howitt (1998) addressed the potentially arbitrary parameter specification problem by suggesting a maximum entropy (ME) procedure to generalize and objectify the calibration phase. In this approach the maximum entropy approach is used for the case of one observation to recover the Q matrix. The information for estimating the full Q Matrix is given by the marginal costs from the first stage (equation 1.1) and the observed output levels. If each farm realizes N products with $i = 1, \ldots, N$, (N(N+1)/2)) parameters must be estimated and the problem is ill posed. Using this information the marginal cost function can be stated as

$$MC = \rho + c = Q * x^o \tag{2.5}$$

The corresponding formulation in matrix notation of the maximum entropy problem (Paris and Howitt, 1998) is:

$$\max_{P_l, P_D} H(P_L, P_D) = -\sum_{i,i',k} P_L(i,i',k) \log[P_L(i,i',k)] - \sum_{i,i',k} P_D(i,i',k) \log[P_D(i,i',k)] \tag{2.6}$$

subject to

$$MC = Qx^o = LDL'x^o = (Z_L P_L)(Z_D P_D)(Z_L P_L)'x^o \tag{2.7}$$

$$1 = \sum_k P_L(i,i',k) \qquad i,i' = 1,\ldots,n \tag{2.8}$$

$$1 = \sum_k P_D(i,i',k) \qquad i,i' = 1,\ldots,n \tag{2.9}$$

$$P_L(i,i',k) > 0 \text{ and } P_D(i,i',k) > 0 \tag{2.10}$$

where H denotes the Entropy, MC denotes the marginal cost vector, x is the land allocation vector, Z_L and Z_D are the support matrices and P_L and P_D are the individual probabilities.

The formulation of the Q matrix in equation 2.7 satisfies the theoretical requirement of a symmetric positive semi-definite matrix, which ensures the Cholesky factorisation of Q. For the ex-post scenario the support matrices are calculated as suggested in Paris and Howitt (1998)[3] where the vector of suitable weights W_L with k=5 were set to (-2; -1, 0; 1; 2) and W_D were set to (0; 1; 2; 3; 4). In addition, alternative weights W_L are considered in the ex-post simulation were the vector of suitable weights W_L is set to (-1; -.5, 0; .5; 1) and W_D is set to (0; .66; 1.33; 2; 2.66) to investigate the possible impact of different support point.

3.5 A "general alternative" to PMP with multiple observations

The aforementioned maximum entropy approach introduced the idea of using econometric criteria for the calibration of programming models. In this context Paris and Howitt 1998 suggested the use of multiple observations. However empirical applications with more than one observation are limited to cross sectional estimation by Heckelei and Britz (2000) and Paris (2001). Heckelei (2002) argued for a "general and theoretical consistent" alternative to PMP. His approach is based on the first order condition of the model.

This paper aims to apply this approach with more than one observation to real farm data from the FADN. In fact one particular farm group is selected to illustrate the methodology on real farm data.

The model will now be explained briefly. For a detailed discussion please see Heckelei and Wolff 2003 Section 4.1 Model 16.

Assuming that the optimal land allocation satisfies the land constraints and that the price vector in equation 1.4 is replaced by gross margins, the first order condition of the problem for observation T with $t = 1, \ldots, T$, is

$$gm_t^o - \lambda_t A - d - Q(x_t^o - e_t) = 0 \qquad \forall t \tag{2.11}$$

$$A'(x_t^o - e_t) = b_t^o \qquad \forall t \tag{2.12}$$

where e_t is an (Nx1) vector of stochastic error terms with standard deviation σ_i added to the observed land allocation x_t^o to obtain the optimal land allocation. The (N*(2N)) V matrix with two support points for each error term bounds the support to $-/+5$ standard deviations.

$$\mathrm{e}_t = \mathrm{V}\mathrm{w}_t \tag{2.13}$$

For the simulation 11 crops (N) are included. Hence the error term will be calculated by the multiplication of V with a ((N*2)*1) vector of probabilities w_t.

[3] Please see Equation 29-33 in Paris and Howitt (1998).

Heckelei and Wolff (2003) showed that in the case of small sample, the use of external information is necessary to avoid poor estimates. Because the number of observations in the sample is small (T=5), priori information on supply elasticities becomes an important point to obtain a sensible model specification. For the used model a (Nx1) vector of land allocation elasticities with respect to own gross margins ε are employed. The elasticities are based on the marginal effects on activity levels, which can be obtained by using the first order condition of equation 1.4, where prices are replaced by gross margins, land is considered as the only constraint and d is set to zero. Using the Lagrangian formulation we obtain:

$$\frac{\partial L}{\partial \mathrm{x}} = gm - Qx - A'\lambda = 0 \quad (2.14) \qquad \text{and} \qquad \frac{\partial L}{\partial \lambda} = b - Ax = 0 \quad (2.15)$$

Solving 2.14 for x we obtain

$$x = Q^{-1}(gm - A'\lambda) \tag{2.16}$$

than replacing the right hand side of 2.16 into 2.15 and solve for

$$\lambda = \left(AQ^{-1}A'\right)^{-1}(AQ^{-1}gm - b) \tag{2.17}$$

The optimal values of x can than be expressed as

$$x = Q^{-1}(gm) - Q^{-1}A'\left(AQ^{-1}A'\right)^{-1}(AQ^{-1}gm - b) \tag{2.18}$$

and the marginal effect on activity level is

$$\frac{\partial x}{\partial gm} = Q^{-1} - Q^{-1}A'\left(AQ^{-1}A'\right)^{-1}AQ^{-1} \tag{2.19}$$

Hence the own gross margins can be represented as[4]

$$\varepsilon = \mathrm{diag}\left(\frac{\partial \mathrm{x}}{\partial \mathrm{gm}} \;\square\; \left[\frac{\overline{\mathrm{gm}}}{\mathrm{x}^{o}}\right]'\right) \tag{2.20}$$

where $\frac{\partial \mathrm{x}}{\partial \mathrm{gm}}$ represents the land demand function in the case of a single land constraint.

[4] The symbol $\square$ represents the element wise product of two matrixes.

The i, j-th element of the (N*N) matrix $\left[\frac{\overline{gm}}{\overline{x^o}}\right]$ is calculated as the sample mean of the gross margins i, $\overline{gm_i}$, divided by the sample mean of observed land allocation to crop j, $\overline{x_j^o}$.

The reparameterisation of the elasticities is done analogously to the specification of the error term, where the (N*(2N)) $V^å$ Matrix with 2 support points for each prior information on elasticity bounds the support to +/-2.

$$V^å w^å = diag\left(\left(Q^{-1} - Q^{-1}A'\left(A'Q^{-1}A\right)^{-1} AQ^{-1}\right) \square \left[\frac{\overline{gm}}{\overline{l^o}}\right]'\right) \tag{2.21}$$

with

$$V^å = \begin{bmatrix} v_{i1}^å & v_{i2}^å & 0 & 0 & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 0 & v_{n1}^å & v_{n2}^å \end{bmatrix} \text{ and } w^å = \begin{bmatrix} w_{i1}^å \\ w_{i2}^å \\ \vdots \\ \vdots \\ w_{n1}^å \\ w_{n2}^å \end{bmatrix} \tag{2.22}$$

Heckelei and Wolff (2003) used the Generalized Maximum Entropy (GME) approach, which was introduced to a wider range of economists by Golan et. al (1996). The complete GME formulation is

$$\max_{w_t w^\varepsilon, Q, L, \lambda} H\left(w_t, w^\varepsilon\right) = -\sum_t^T w_t' \ln w_t - w^{\varepsilon}{}' \ln w^\varepsilon \tag{2.23}$$

subject to

$$gm_t^o - \lambda_t A - d - Q(x_t^o - Vw_t) = 0$$

$$A'(x_t^o - Vw_t) = b_t^o \qquad \forall t .$$

$$Q = LL' \text{ with } L_{ij} = 0 \, \forall j > i \tag{2.24}$$

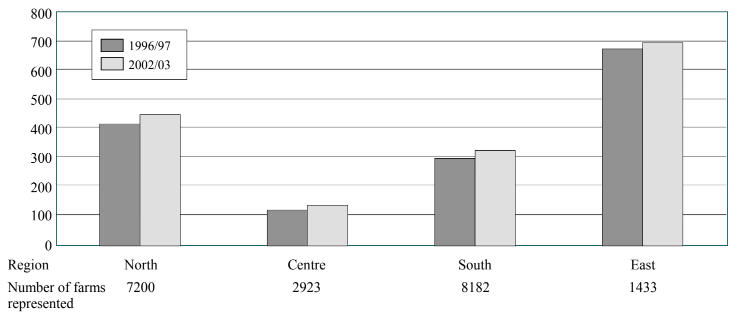
$$\sum_{s=1}^{S} w_{its} = 1 \;\; \forall i, t \tag{2.25}$$

$$\sum_{s=1}^{S} w^{\varepsilon}{}_{is} = 1 \;\; \forall i \tag{2.26}$$

λ_t denotes the shadow price vector for land, which is estimated endogenously in the model, $H\left(w_t, w^\varepsilon\right)$ denotes Entropy and equation 2.17 guarantees the positive (semi-) definiteness of Q, based on the Cholesky factorisation. Equation 2.18 and 2.19 ensures that the probabilities add up to one. Hence the direct use of the first order condition of the assumed behavioural optimisation model makes the use of the PMP-approach obsolete (Heckelei, 2002).

4. Data

The aforementioned calibration methods are evaluated using farm data from the German FADN[5]. In order to aggregate the farm group, identical arable farms between 1996 and 2003 are selected. From about 6000 existing identical farms over eight years, 845 arable farms were used to obtain 45 farm groups for the ex-post run. To aggregate the single farm accounts to farm groups, an aggregation program developed by Gocht (2004) at the FAL[6] was used. The single farm accounts were aggregated for the 45 farm groups, crop specific costs and prices, yields and premiums were calculated using generation modules for input output coefficients of the sector consistent farm model "FARMIS"[7]. For presentation, farms are aggregated by four regions. Furthermore the first year and the target year were calculated as the average of two years. Figure 1 depicts the total of arable land for the 45 aggregated farm groups from 1996 to 2003. The use of arable land increased in the north by around 7 percent, in the centre of Germany by 11 % and in the south by 9 percent. In the eastern part the arable area increased only by three percent, due to the restructuring process after the reunification of Germany.



Source: FARMIS 2004, FADN Germany.

Figure 1. Sum of arable land 1996/97 and 2002/03 for farm groups and regions

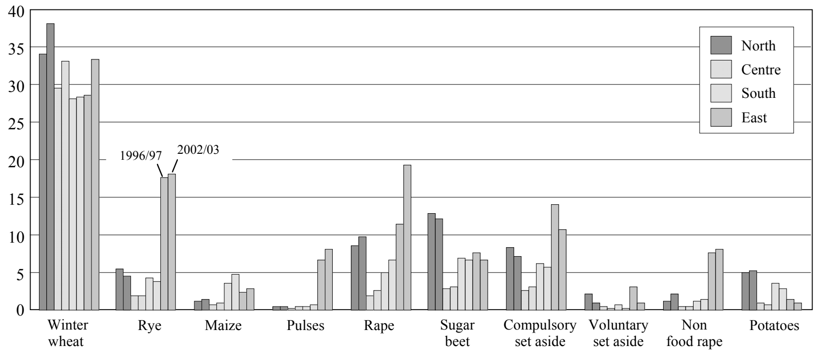
Figure 2 shows the share of the crop allocation on arable land in 1996/97 and 2002/03. Beside the southern region, the share of winter wheat increased. Rye increased only in the eastern part of Germany, whereas rape was expanded the most in all regions. Compulsory

[5] Farm Accounting Data Network.

[6] Federal Research Institute for Agriculture in Braunschweig.

[7] FARMIS sector consistent farm group supply model developed at the FAL Braunschweig

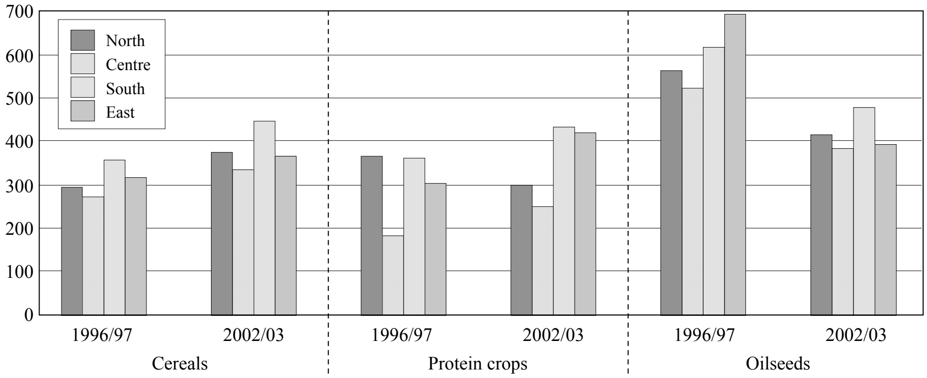
set-aside was reduced, while in the North, Centre and South, the specific regulation for small farms has to be taken into account.



Source: FARMIS 2004, FADN Germany.

Figure 2. Land allocation in 1996/97 and 2002/03 for farm groups and regions

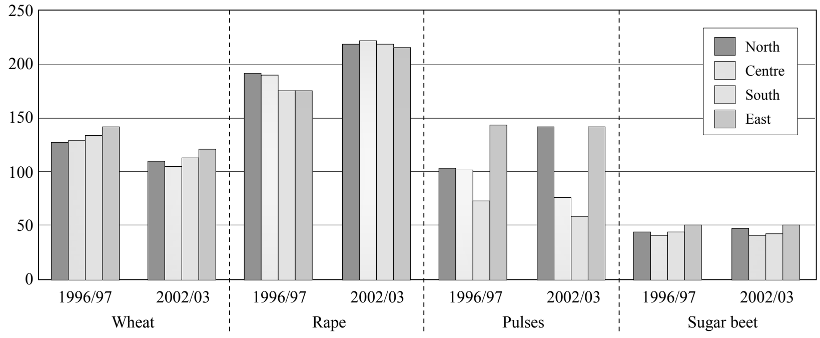
Under Agenda 2000 the levels of direct payments for cereals, oilseeds and protein crops was harmonized. It becomes clear that the relative advantage of oilseed premiums declined to the level of cereals in 2002/03. The direct payments for protein crops are disturbed by vegetable peas, which do not obtain any payments.



Source: FARMIS 2004, FADN Germany.

Figure 3. Direct payments 1996/97 and 2002/03

Figure 4 shows the price change of selected crops. The price for wheat decreased whereas the price for rape increased compared to the first year 1996/97.



Source: FARMIS 2004, FADN Germany.

Figure 4. Prices change 1996/97 and 2002/03 for farm groups and regions

5. Results

The results are evaluated using the percentage absolute deviation (PAD), whereby the observed land use is compared with the calculated land allocation for each calibration scenario. The percentage absolute deviation is calculated as,

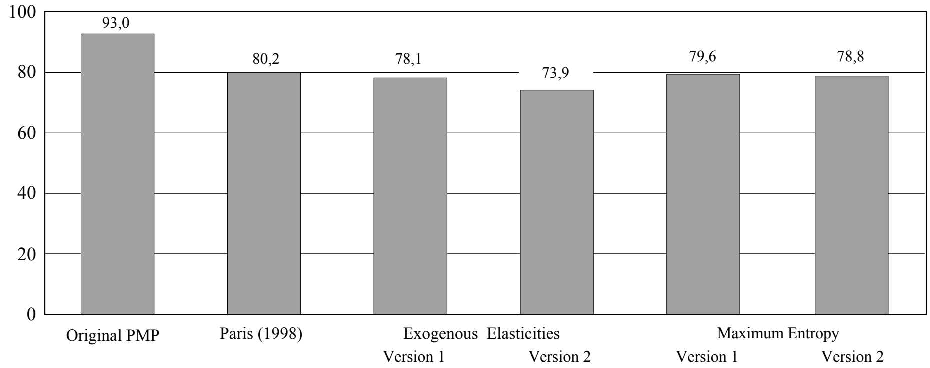
$$PAD = \frac{100}{N}\sum_i ABS\,|\,(\hat{x}_i - x_i)/x_i\,| \qquad (4.1)$$

where N denotes the number of crops, x_i the observed land use in 2002/03 and $\hat{x}_i$ the calculated crop allocation. For the calibration scenario with exogenous elasticities and with maximum entropy, two versions are considered as described in Section 2. Figure 5 depicts the mean of PAD for all farms for each single ex-post scenario.[8]

It is outstanding that the overall PAD is relatively high for all scenarios. One explanation could be the low crop yield in 2002 caused by the strong winter and the flood after august 2002. In addition we have to take into account that the PAD was obtained only for crops which were observed in the base year 1996/97. Therefore the absolute value of the PDA must be interpreted with caution. Nevertheless, the relative differences of the percentage absolute deviation can be used to interpret the simulation behaviour among the calibration scenarios. The "Original PMP" scenario has the highest PAD value. Here for the "marginal" activities (crops with zero dual value on the calibration constraint) the cost function is linear. A price increase of the preferable production activity leads to a substitution of marginal activities, but leaves the other preferable activities unchanged until the

[8] Table A1 in the appendix shows the PAD for each farm.

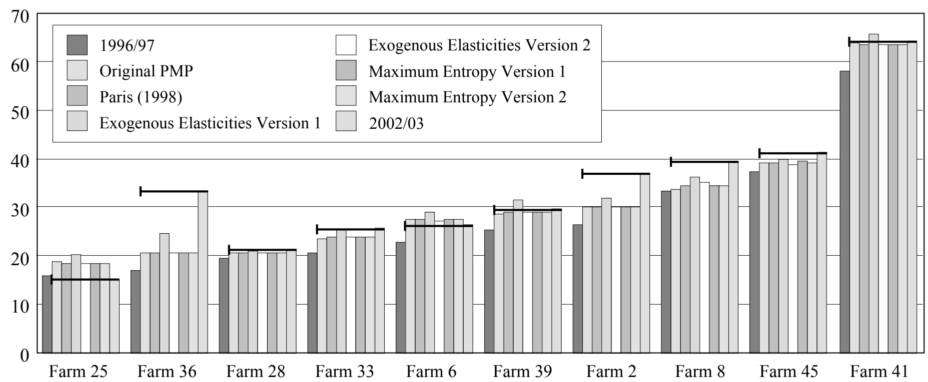
first marginal activity is replaced. This characteristic was responsible for the relatively poor PAD to the other scenarios



Source: FARMIS 2004, FADN Germany.

Figure 5. Mean of the percentage absolute deviation for all farms

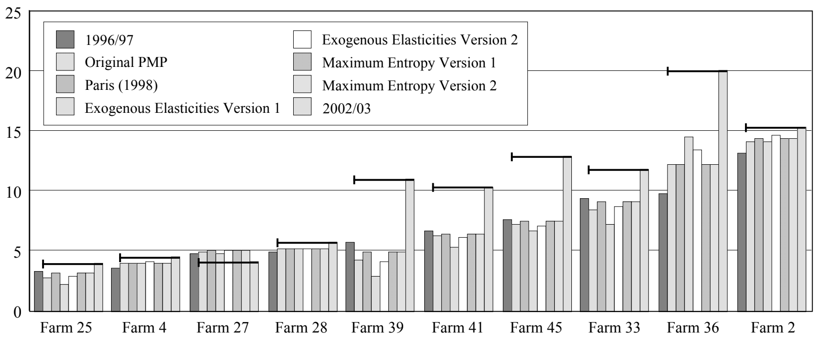
Two alternative exogenous own gross margin elasticities for rape and wheat were considered for the calibration scenario with exogenous elasticities. The results in Figure 6 and 7 show how sensitively this calibration approach reacts in respect to the simulation behaviour. The second version benefits from the increase of the own gross margin elasticity for wheat, shown in Figure 6. The exogenous elasticity scenario reduced the role of PMP to all that it really is, a calibration method Heckelei (2002). The specification of the underdetermined Q matrix and therefore the resulting simulation behaviour is controlled by the elasticities.



Source: FARMIS 2004, FADN Germany.

Figure 6. Allocation of wheat for large farms (>10 000 hectares)

The advantage of Maximum Entropy principles is the possibility of fully using any amount of sample information, no matter how scarce. The recovery of a fully specified Q matrix for the cost function, and hence dealing with ill-posed problems (more parameters than observation) was no longer impossible. However the results for the ME approach are very similar to the calibration approach presented by Paris (1988) (see Figures 6 and 7). This behaviour can be explained, if Equation 2.2 and Equation 2.5 are compared. For both approaches the linear part d was set to zero, whereas the ME approach recovering the full Q Matrix and the Paris (1988) approach calculated the diagonal elements of the Q Matrix. Furthermore the differences of ME Version 1 and 2 are very small, which implies that the different support points for the simulation have only a marginal impact. The fully specified Q Matrix for one observation does not seem to contain more information on how the marginal incentives change if one moves away from the observed land allocation.



Source: FARMIS 2004, FADN Germany.

Figure 7. Allocation of rape for large farms (> 2000 hectares)

Nevertheless the ME approach allows the flexible introduction of more information by either using priori information on elasticities or incorporating multiple observations. Hence it was the first step toward bringing econometric models and programming models closer together.

The ex-post scenarios illustrated that, as long as the conditions in Equation 1.3 are satisfied, the calibration of the resulting model is guaranteed, but the different specification of d and Q results in different simulation behaviour. These results were found also by Cypris (2000) with the German regional sector "RAUMIS" for the original PMP approach. An infinite number of possible specifications of the cost function could be solved when the maximum entropy method was applied. Nevertheless the support for the ME specification was defined without any valuable priori information on the cost function, which causes a uniform distribution of probabilities, since the centre of the support ranges are already satisfied by the data constraints and therefore the resulting parameters from the ME approach are exactly the ones implied by the Paris (1988) formulation. These results coincide with findings from Heckelei and Britz (2000).

The previous results point to the fact that the infinite number of possible specifications to recover the N+(N(N+1)/2) parameters was solved with the ME approach, but the used condition for the ME approach seemed unfavourable. A meaningful and consistent alternative to the ME - PMP formulation has to be found, which brings the general alternative to PMP already presented in Section 2 into the discussion. In this context the PMP with multiple cross sectional data points has to be mentioned, which was applied by Heckelei and Britz (2000). They extended the ME formulation to multiple observation but still used the PMP procedure. The authors themselves pointed out that the direct use of the corresponding first order condition of the desired model avoids the fundamental problems of PMP[9] which leads to inconsistent parameter estimates. Therefore the use of a ME-PMP model with more than one observation was not a option in this paper, but the recovery of the cost function's parameter with multiple data is applied using "a general alternative to PMP." From the beginning of this study, the author's intention was to introduce this approach in the ex-post simulation. Unfortunately problems in finding a stable optimum for models with more than five crops for all considered farms limited this estimation to a pure illustration for one particular farm.

Illustration of the "general alternative to PMP" with multiple observations

The approach is based in the "First Order Condition" and omits the first stage of the normal PMP approach. The dual values for land and the full Q matrix are estimated using a data set of gross margins and observed land allocation over five years. As far as the author knows, this model approach with time series was never applied to farm data from FADN. It has to be mentioned that preliminary tests[10] were realized by Heckelei (2002) where a regional quadratic cost function for France was performed with the regional programming model "CAPRI" with six observations in the time domain.

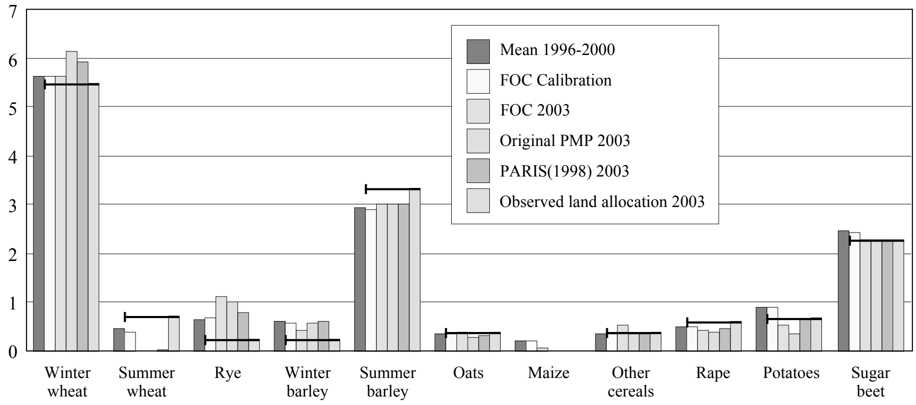
Unlike cross sectional data employed by Heckelei and Britz (2000) and Paris (2001), FADN time series data can probably give a more direct observation on adjustments to changing economic incentives and probably contain more information for the supply response in time. Whether cross sectional data can provide a valid supply response over time is questionable and the use is rather driven by the lack of other data. The model illustrated here was solved with CONOPT, which did not seem to be the best device for such problems. Therefore the author's intention is to illustrate that the proposed model can be applied to real farms using observations in the time domain.

Figure 8 shows the simulation results for Farm 17, whereas the mean of the land allocation from 1996-2000 is depicted in the first bar for each crop, the calibrated land allocation with the recovered Q Matrix is denoted as "FOC Calibration" and shown in the second bar. Furthermore, the observed land allocation in 2003 and the different simulation scenarios are presented

[9] For a detailed discussion about the inconsistencies of the PMP-approach see Heckelei (2002)

[10] The author is not aware of any results of theses tests.

in the remaining pillars. The corresponding gross margins, recovered Q matrix, the matrix of estimated elasticities and the crop allocation for Farm 17 can be found in the annex. The original PMP and the Paris (1988) calibration method were used to compare the results. The ME-PMP formulation was not considered, because of the resulting similar simulation behaviour. For the other two scenarios, namely Paris (1998) and the "original" PMP calibration method, the mean over time was taken to calibrate the model. For the simulation the gross margins in 2003 are used.



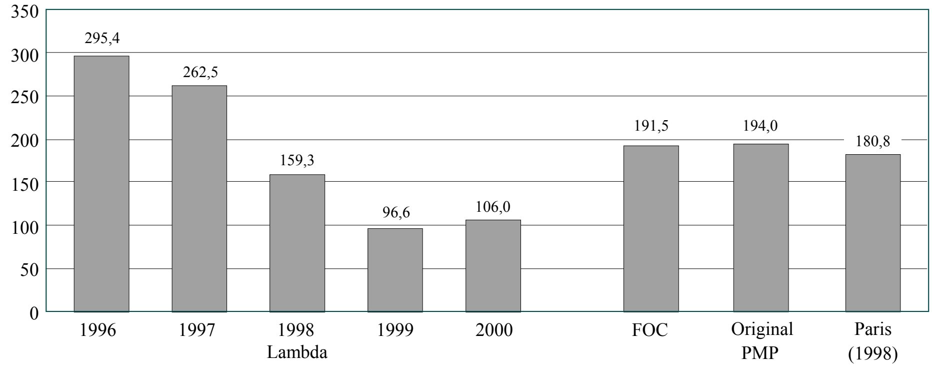
Source: FARMIS 2004, FADN Germany.

Figure 8. Crop allocation for Farm 17 for different calibration models

The discussion of the results for Farm 17 with respect to the simulation behaviour and the adjusted crop allocation for the target year will be restricted to a pure description of the obtained allocation and the estimated shadow prices. No final answer can be made at this stage, whether in real situations the first order condition approach with multiple data points outperforms the other considered approaches with respect to the quality of the supply response. However, the principle procedure was already proven in Heckelei and Wolff (2003).

The adjusted crop allocation, when the gross margins from 2003 are applied, indicates that the recovered Q matrix for the FOC approach behave differently than the original PMP and the Paris (1988) PMP calibration methods. The recovered Q Matrix of the FOC approach calibrates for the mean of land allocation over time, even in the case that crops were not observed for one year.

Because the first phase is avoided, the dual values of calibrations constraints are estimated endogenously. Lambda (see Figure 8) denotes the estimated shadow prices obtained from the FOC model over the five years. Alternatively the shadow prices for the final model using the recovered Q Matrixes are presented. The estimated shadow price Lambda decreased from 19996 to 2000, due to the gross margins, which can be seen in the annex. However the resulting shadow prices for all three final models have only a small deviation.



Source: FARMIS 2004, FADN Germany.

Figure 9. Dual values of land for farm 17

The principle procedure and its functionality for a real farm could be demonstrated. Nevertheless, during execution it became clear that technical problems have to be sorted out in order to extend this alternative to complex farm supply models. The increased numbers of observations combined with the differentiated set of crop activities cause considerable computational demand. In addition, initial numerical difficulties have to be overcome. Further a suitable solver routine adjusted for the underlying problem has to found.

Conclusions

The paper investigates the ex-post simulation behaviour of different standard PMP approaches. The results show that the simulation behaviour is determined by the estimation routine which recovers the parameters of the non linear cost function of the desired model. The underdetermined problem was solved using different approaches. The most promising PMP approach, where Maximum Entropy (Paris and Howitt, 1998) was applied as estimation technique, could not improve the supply response in time. The percentage absolute deviation from of the ex-post simulation is relatively high, which can be attributed on the one hand to the bad harvest in 2002/03 and on the other to the common problem of the standard PMP approaches, which only consider cropping activities observed in the base year.

Nevertheless the ME approach as an estimation method allows the flexible introduction of more information. Hence it was the first step toward bringing econometric models and programming models closer together. In line with this it was demonstrated, that the general alternative suggested by Heckelei and Wolff (2003), which is based on the first order condition of the model can be applied to time series obtained from the FADN data to recover the parameters of the cost function. Further work has to be done to exploit the data available through the Farm Accounting Data Network to build more reliable and consistent supply models in respect to the response behaviour. Of particular interest is the extension of the first order condition approach to the full sample data. Here reformulation of the complementary constrains will probably help the solver. In addition to the specification problem of the non linear objective function, the general structure of the model and the resulting effects on the supply response have to be investigated upon in the future.

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Annex

Table A1. Percentage Absolute Deviation ex-post Scenario 1996/97 to 2002/03

		Original PMP	Paris (1998)	Exogenous Elasticities		Maximum Entropy	
				Version 1	Version 2	Version 1	Version 2
Farm	1	82,2	69,1	76,9	82,9	69,1	69,1
Farm	2	101,6	99,2	99,9	85,5	99,0	99,3
Farm	3	134,2	119,0	116,4	119,5	118,9	118,4
Farm	4	72,8	49,5	47,5	40,9	49,5	49,6
Farm	5	29,0	27,3	26,1	24,6	27,3	27,3
Farm	6	85,8	76,1	43,7	38,6	60,9	44,9
Farm	7	90,4	55,3	55,7	51,1	56,5	57,7
Farm	8	91,2	78,3	72,7	65,3	78,4	78,4
Farm	10	36,4	32,0	35,7	39,4	32,0	32,0
Farm	11	99,1	65,8	67,5	63,6	66,8	65,8
Farm	12	32,3	27,1	31,4	33,2	27,1	27,1
Farm	13	84,1	80,6	56,8	56,5	80,6	80,6
Farm	14	16,8	15,7	15,1	16,3	15,7	15,7
Farm	15	73,7	73,1	77,6	75,6	73,1	73,1
Farm	16	83,0	70,2	63,5	58,9	70,2	70,2
Farm	17	74,6	56,4	60,0	60,7	56,4	56,4
Farm	18	39,5	33,8	32,2	30,8	33,9	33,9
Farm	19	169,0	104,7	103,7	101,0	104,7	104,7
Farm	20	108,6	108,5	106,0	100,9	108,5	108,4
Farm	21	93,4	67,2	63,0	53,5	67,2	66,9
Farm	22	93,1	30,5	30,5	28,7	30,5	30,5
Farm	23	109,5	96,1	93,8	91,7	96,1	96,1
Farm	24	110,5	35,3	33,7	31,8	35,3	35,3
Farm	25	141,4	83,1	83,4	82,1	83,1	83,1
Farm	26	58,7	31,8	29,3	30,8	31,7	31,7
Farm	27	96,8	95,8	77,5	69,7	95,9	95,8
Farm	28	160,3	152,3	143,3	136,5	152,3	152,3
Farm	29	26,4	24,6	26,8	22,7	24,6	24,6
Farm	30	51,7	46,7	51,5	53,1	46,7	46,7
Farm	31	146,3	138,0	144,9	114,4	128,4	113,4
Farm	32	100,5	103,3	53,7	39,4	103,3	103,3
Farm	33	156,1	129,0	137,8	120,0	129,1	129,1
Farm	34	137,4	141,3	141,9	125,4	141,3	141,3
Farm	35	78,1	82,1	75,8	65,2	82,1	82,1
Farm	36	183,0	196,0	186,2	184,4	196,0	196,0
Farm	37	170,2	174,2	174,0	167,1	174,2	174,2
Farm	38	44,2	43,3	35,6	43,5	43,3	43,3
Farm	39	72,5	74,0	72,2	63,7	74,0	74,0
Farm	40	126,9	114,8	129,2	138,0	114,7	114,4
Farm	41	109,6	126,7	154,8	155,4	126,7	126,7
Farm	42	82,2	96,9	93,0	79,2	96,9	96,9
Farm	43	61,6	49,8	46,2	48,4	46,0	38,6
Farm	44	97,4	95,5	97,8	92,6	95,6	95,6
Farm	45	47,2	43,1	45,7	39,1	42,7	43,1
Farm	46	124,7	96,5	102,7	103,4	96,5	96,5
Mean		93,0	80,2	78,1	73,9	79,6	78,8

Source: FADN, FARMIS

Table A2. Gross Margins for farm 17 (€ per hectare)

	1996	1997	1998	1999	2000	2001	2002	2003
Winter wheat	788,1	535,5	457,1	479,5	485,1	805,2	915,0	750,2
Summer wheat	412,1	267,8	584,0	257,2	264,5	553,8	622,4	
Rye	562,4	428,3	452,6	328,3	473,4	850,1	822,7	773,6
Winter barley	782,8	487,2	341,2	365,7	462,1	968,5	845,7	646,9
Summer barley	759,1	613,0	502,9	477,9	485,9	893,3	814,4	748,0
Oats	825,9	554,7	967,1	677,5	341,9	849,0	1401,6	754,0
Maize	296,1	557,7	-79,5	1,9		901,3	879,7	718,8
Other cereals	769,8	607,2	576,1		247,7	1111,4	1319,7	927,1
Rape	1070,1	1133,7	862,3	801,7	978,9	356,6		859,4
Potatoes	641,0	2122,1	2509,4	1414,5	1568,6	1111,1	1881,2	954,9
Sugar beet	1877,3	2478,2	2276,4	2154,4	2160,4	2503,3	2153,3	2342,1

Source: FADN, FARMIS

Table A3. Observed land allocation farm 17 (hectare)

	1996	1997	1998	1999	2000	2001	2002	2003
Winter wheat	5,78	5,55	5,81	5,53	5,57	5,46	5,08	5,49
Summer wheat	0,25	0,26	0,39	0,65	0,68	0,75	0,77	0,72
Rye	0,88	0,83	0,52	0,55	0,39	0,32	0,24	0,24
Winter barley	0,44	0,58	0,70	0,74	0,52	0,58	0,54	0,23
Summer barley	2,61	2,97	2,89	2,88	3,30	2,76	3,60	3,34
Oats	0,34	0,40	0,30	0,30	0,36	0,24	0,31	0,38
Maize	0,37	0,21	0,19	0,10		0,45	0,48	0,39
Other cereals	0,38	0,31	0,33		0,34	0,55	0,34	0,61
Rape	0,49	0,45	0,62	0,56	0,39	0,11		0,09
Potatoes	0,80	0,94	0,93	0,80	0,92	0,79	0,85	0,68
Sugar beet	2,41	2,30	2,50	2,47	2,55	2,38	2,29	2,26

Source: FADN, FARMIS

Table A4. Recovered Q Matrix for Farm 17 with the "First Order Condition" approach with multiple data points

	Winter wheat	Summer wheat	Rye	Winter barley	Summer barley	Oats	Maize	Other cereals	Rape	Potatoes	Sugar beet
Winter wheat	114	-169	-77	87	43	-79	-149	-153	-4	-121	-200
Summer wheat	-169	1148	409	-84	-140	485	-161	572	-200	-82	152
Rye	-77	409	853	136	-200	-200	-200	-58	-139	-90	166
Winter barley	87	-84	136	1193	-200	-200	199	35	-200	-112	-191
Summer barley	43	-140	-200	-200	261	-101	-159	77	-200	-106	-181
Oats	-79	485	-200	-200	-101	2000	-200	170	-200	-74	167
Maize	-149	-161	-200	199	-159	-200	882	94	123	-121	396
Other cereals	-153	572	-58	35	77	170	94	2000	56	-87	-90
Rape	-4	-200	-139	-200	-200	-200	123	56	1478	-65	287
Potatoes	-121	-82	-90	-112	-106	-74	-121	-87	-65	2000	75
Sugar beet	-200	152	166	-191	-181	167	396	-90	287	75	1203

Source: Own calculation.

Table A5. Recovered Elasticity Matrix for Farm 17 with the "First Order Condition" approach with multiple data points

	Winter wheat	Summer wheat	Rye	Winter barley	Summer barley	Oats	Maize	Other cereals	Rape	Potatoes	Sugar beet
Winter wheat	1,317	0,011	-0,119	-0,255	-0,857	-0,117	0,005	0,131	-0,392	-0,034	0,444
Summer wheat	0,217	1,325	-0,984	0,123	-0,027	-0,687	-0,006	-0,593	0,173	-0,070	-0,284
Rye	-1,299	-0,552	1,326	-0,196	0,484	0,436	0,114	0,052	0,225	-0,052	-2,233
Winter barley	-2,714	0,068	-0,192	1,327	1,707	0,100	-0,210	-0,323	0,814	-0,101	0,428
Summer barley	-1,596	-0,003	0,083	0,298	1,330	0,085	-0,029	-0,210	0,428	-0,073	-0,481
Oats	-1,579	-0,479	0,541	0,127	0,615	1,330	0,062	-0,082	0,417	-0,058	-1,909
Maize	0,351	-0,022	0,764	-1,440	-1,130	0,337	1,399	-0,337	-0,717	0,096	-8,215
Other cereals	2,169	-0,507	0,079	-0,503	-1,869	-0,101	-0,076	1,333	-0,935	-0,038	1,584
Rape	-2,490	0,057	0,131	0,486	1,462	0,196	-0,062	-0,359	1,988	-0,074	-1,777
Potatoes	-0,074	-0,008	-0,010	-0,020	-0,084	-0,009	0,003	-0,005	-0,025	0,866	-0,271
Sugar beet	0,257	-0,008	-0,119	0,023	-0,150	-0,082	-0,065	0,055	-0,162	-0,073	1,320

Source: Own calculation.