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# RURAL ECONOMY



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Department of Rural Economy  
Faculty of Agriculture and Forestry  
University of Alberta  
Edmonton, Canada



**An Opportunity Cost Criterion for Scheduling Timber Harvest:  
A Comparison of Area and Volume Based Approaches**

Glen W. Armstrong, William E. Phillips, and James A. Beck, Jr.

Rural Economy Staff Paper 91-05  
FEPA Working Paper 160

Authors are Research Associate, Department of Rural Economy, Professor and Chairman, Department of Rural Economy, and Professor, Department of Forest Science, University of Alberta, respectively.

The authors acknowledge the contributions of Rick Melvin, Frank Novak, and Luis Constantino. Funding for model development and empirical analysis was provided by Forestry Canada through the Canada-Saskatchewan Forest Resource Development Agreement and supplemented by funding from the Forest Economics and Policy Analysis Research Unit at the University of British Columbia. The cooperation and interest of Forestry Canada, Saskatchewan Parks, Recreation, and Culture, and Weyerhaeuser Canada Ltd. (Saskatchewan Division) is gratefully acknowledged.

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# **An Opportunity Cost Criterion for Scheduling Timber Harvest: A Comparison of Area and Volume Based Approaches**

## **Abstract**

This paper develops an economic criterion for scheduling timber stands for harvest when there are upper limits on the timber volume harvested in a period. The opportunity cost of delayed harvest is used to rank the harvest priority for stands: those with the highest opportunity cost are to be harvested first until the upper limit on volume is reached or there is no available wood with a positive opportunity cost of delay.

The paper compares the effect of ranking stands for harvest by opportunity cost per cubic metre ( $m^3$ ) and by opportunity cost per hectare (ha). The opportunity cost per ha ranking is clearly inappropriate when periodic harvest volume constraints are used. However, because of the spatial orientation of forest management, it is tempting to use the opportunity cost per ha measure.

This paper shows that when stands are ranked for harvest by opportunity cost per  $m^3$ , a wider variety of timber types and site classes are scheduled for harvest than when the opportunity cost per ha criterion is used. This mix of timber types and site classes corresponds with the observed harvest behaviour of logging firms.

Other authors have argued that the wide mix of timber actually harvested by logging firms is driven by non-economic considerations. It is argued here that this harvest variety can be attributed to economically rational behaviour.

## Introduction

The determination of the forest rotation that maximizes the value of forest land is perhaps the most discussed topic in the forest economics literature. Pearse (1967) and Samuelson (1976) provide clear descriptions of the optimum forest rotation problem and its analysis. Both are based on the forest land valuation method of Faustmann (1968). The central result of optimal rotation theory is, if the objective of stand management is profit maximization, that the optimal harvest age for a timber stand is the age where the rate of value growth of the stand is just offset by the interest costs of holding timber inventory and timber land. The land holding costs are a consequence of not replacing the existing stand with a new, faster growing stand. Using this result, any stand for which the marginal benefit of delaying harvest is less than the marginal cost should be harvested immediately. The difference between this marginal cost and marginal benefit will be referred to here as the opportunity cost of delay.

This principle is directly applicable to stand level problems, or to forest level problems where there is no constraint on periodic volume or area harvested. Davis and Johnson (1987) discuss the differences between stand level and forest level optimization. In many areas, a forest products company owns the harvesting rights to a large number of stands and must choose which stands to harvest in a particular time period. If the firm's periodic harvest volume is constrained by mill capacity or policy (e.g. annual allowable cut regulations), the simple decision rule (cut all stands where the marginal costs exceed the marginal benefits of delayed harvest) is no longer appropriate. However, the opportunity cost of delay derived from the single stand model is still useful in establishing harvest priorities for individual stands.

Armstrong *et al.* (1990) develop a multiperiod forest planning simulation model in which the opportunity cost of delayed harvest is used to set harvest priority for the stands on the land base considered. In each period, the model calculates the opportunity cost of a one period delay in harvest for each stand (actually each aggregate of similar stands). Stands with a positive opportunity cost of delay are eligible for harvest in the current period. These are sorted by decreasing opportunity cost. Stands are selected for harvest from this priority list until the harvest volume capacity is met or all eligible stands are harvested.

In the standard exposition of the forest rotation problem, these costs and benefits are expressed on a per unit area basis (e.g. \$/ha). Forest management is inherently a spatial proposition. It is therefore tempting to use area-based opportunity cost calculations to set harvest priority. However, it is important that the opportunity cost of delay measurement is compatible with harvest constraint units. If the periodic harvest constraint is specified in  $m^3$ , the opportunity cost of delay should be measured in  $\$/m^3$ . Similarly, if the periodic harvest constraint is specified in ha, the net marginal benefit of delay of a stand should be measured in  $\$/ha$ .<sup>1</sup>

This paper demonstrates the importance of consistency between measurement units for the harvest constraint and opportunity cost. The results are nearly trivial, but the implications of a misunderstanding can be very significant.

## Model

The objective of the model developed here, from the perspective of industrial forest management, is to maximize the present net worth of the financial contribution of a forest area to the managing firm. The firm faces a positive discount rate reflecting the firm's cost of capital, a fixed forest land base, and mill capacity constraints. The opportunity costs of a delay in harvest are developed below.

For now, assume that the entire forest is accessible, that the optimal silvicultural regime (except for the final harvest age) is known, and that there is no harvest volume or area constraint to consider. The value of bare land to be used for a perpetual series of timber rotations can be expressed as

$$[1] \quad F(T) = \frac{H(T)e^{-rT} - E}{1 - e^{-rT}}$$

where  $H(T)$  is a function expressing the value of timber ( $\$/ha$ ) at different stand ages ( $T$ ),  $r$  is the appropriate discount rate, and  $E$  represents the establishment costs for regenerated stands including the present value of all silvicultural costs.

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<sup>1</sup> In a fully regulated forest where the annual volume harvested is precisely equal to the annual volume growth, this distinction is unimportant.

Given a strictly concave function for bare land value ( $F''(T) < 0$ ), the optimal harvest age  $T^*$  is the age where the first order condition for maximization

$$[2] \quad F'(T^*) = H'(T^*) - \left( rH(T^*) + r \frac{H(T^*)e^{-rT^*} - E}{1 - e^{-rT^*}} \right) = 0$$

or

$$[3] \quad H'(T^*) - r[H(T^*) + F(T^*)] = 0$$

is satisfied. The optimal harvest age is the age where the marginal rate of value growth is just offset by the interest costs incurred by not liquidating the existing forest inventory and starting a new timber stand. The decision rule is to choose  $T^*$  such that

$$[4] \quad H'(T^*) = r[H(T^*) + F(T^*)]$$

The opportunity cost of delay in the harvest of a ha of forest land,  $D_a(T)$ , is

$$[5] \quad D_a(T) = rH(T) + rF(T^*) - H'(T)$$

The term  $rH(T)$  is the interest cost of holding forest inventory;  $rF(T^*)$  is the interest cost of holding land; and  $H'(T)$  is the net value growth rate of the timber.

The opportunity cost of delay in the harvest of a  $m^3$  of timber from the same ha,  $D_v(T)$ , is

$$[6] \quad D_v(T) = \frac{rH(T) + rF(T^*) - H'(T)}{V(T)}$$

where  $V(T)$  is the stand volume ( $m^3/ha$ ) at  $T$  years of age.

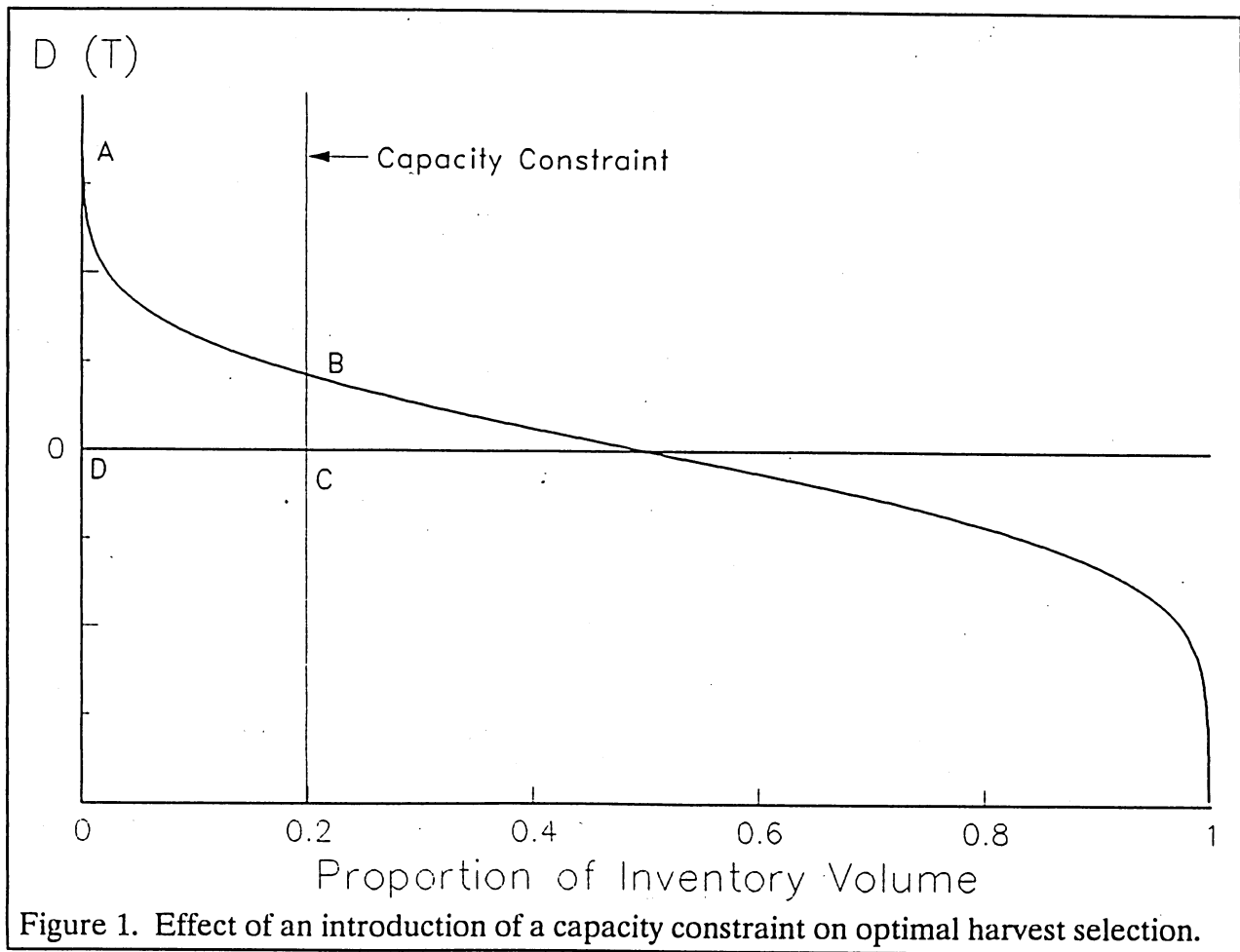
A profit maximizing forest manager facing fixed product prices and no harvest volume or area constraint would choose to harvest every stand where  $D_a(T)$  is positive. If  $D_a(T)$  is negative, the forest manager would be better off to delay the harvest; if  $D_a(T)$  is 0, the



forest manager would be indifferent. The manager is choosing stands to harvest so as to minimize the net opportunity cost of not harvesting. Under these circumstances, the same criteria can be used with  $D_v(T)$ .

If the forest manager faces harvest volume constraints, the decision to harvest now or later becomes more complicated. The harvest of any stand where  $D_v(T) < 0$  should, unquestionably, be delayed. If the total volume of stands where  $D_v(T) \geq 0$  exceeds the harvest volume constraint, some choices must be made as to which stands to harvest. The harvest rule consistent with opportunity cost minimization is to harvest the stands with the greatest  $D_v(T)$  first, until the harvest volume constraint is met. With harvest volume constraints, the formula for  $D_v(T)$  presented above is not precisely correct but should be a close approximation to the opportunity cost of delaying the harvest of a stand. Because it is unlikely that future stands will be harvested at the optimal rotation age, land holding costs will be overestimated. However, the effect of this overestimate will usually be small because inventory holding costs are typically much larger than land holding costs.

In the example presented in Figure 1, the costs of delaying harvest outweigh the benefits (i.e.  $D_v(T) > 0$ ) for half of the total forest inventory volume. However, the mill can process only 20 percent of the total inventory in the current period. The opportunity cost of delayed harvest is minimized by harvesting the stands to the left of the capacity constraint. The sum of costs avoided by harvesting this period is the area  $ABCD$ . It is this numerical integration of the supply curve which makes consistency between the units used for constraints and costs critical.



## Analysis and Results

The results presented here are for one period of the multiperiod simulation analysis presented in Armstrong *et al.* (1990). The data were developed for the Weyerhaeuser Canada Ltd. forest management license area (FMLA) near Prince Albert, Saskatchewan. The data used for this study included the spatial distribution of timber classes<sup>2</sup>, area of timber classes, yield functions, harvest, transportation, and silvicultural costs, timber values,

<sup>2</sup> Individual stands were aggregated into timber classes on the basis of planning unit, species association, site class, year of stand origin, and management regime. Each timber class was treated as a record in the inventory list.

and a discount rate chosen for the analysis. These data were used to develop estimates of opportunity cost of a one-period delay in the harvest of each timber class in the FMLA using the equations presented earlier.

Timber values were developed using the conversion return approach discussed in Davis and Johnson (1987): the value of standing timber is the selling price of the final product (bleached kraft pulp, in our analysis), less all costs incurred in the process of converting standing timber to baled pulp in a Chicago warehouse. The details of these calculations are tangential to this paper. The interested reader is referred to Armstrong *et al.* (1990).

The optimal harvest priority for inventory records can be determined by sorting the inventory by descending opportunity costs per  $m^3$ . The model selects stands to be harvested from this sorted inventory list until the volume request is met or the opportunity cost becomes negative. This approach minimizes the total opportunity cost incurred in a period when maximum harvest volume constraints must be satisfied. However, forest managers make decisions as to which stands or parts of stands to harvest: real decisions are area based decisions.

Because most harvest decisions are area based, it is tempting to determine the harvest priority using the opportunity cost per ha. This approach is incorrect, because the harvest constraint is a volume constraint<sup>3</sup>, but leads to some interesting results. Figures 2 and 3 show the net opportunity cost curves for the harvested inventory sorted by opportunity cost per ha and opportunity cost per  $m^3$  respectively. Each graph also shows the opportunity cost measured in the other units calculated by multiplying or dividing by the stand volume

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<sup>3</sup> The example that follows should help to illustrate this result. Suppose you have a mill that will process  $1000 m^3$  of timber per year and two stands from which you can take this year's harvest. Your objective is to minimize the opportunity cost for this year. Stand A is 4 ha in area and supports  $200 m^3/ha$  with an opportunity cost of  $\$400/ha/year$  ( $\$2/m^3/year$ ). Stand B is also 4 ha in area and supports  $100 m^3/ha$  with an opportunity cost of  $\$250/ha/year$  ( $\$2.5/m^3/year$ ). If an area based sort was used to determine harvest priority, all 4 ha of stand A and 2 ha of stand B would be harvested. The opportunity cost reduction would be  $\$2,100$ . If a volume based sort was used, all 4 ha of stand B and 3 ha of stand A would be harvested. The opportunity cost reduction would be  $\$2,200$ . The volume based sort clearly allows for a greater reduction in opportunity cost when there are constraints on maximum harvest volume.

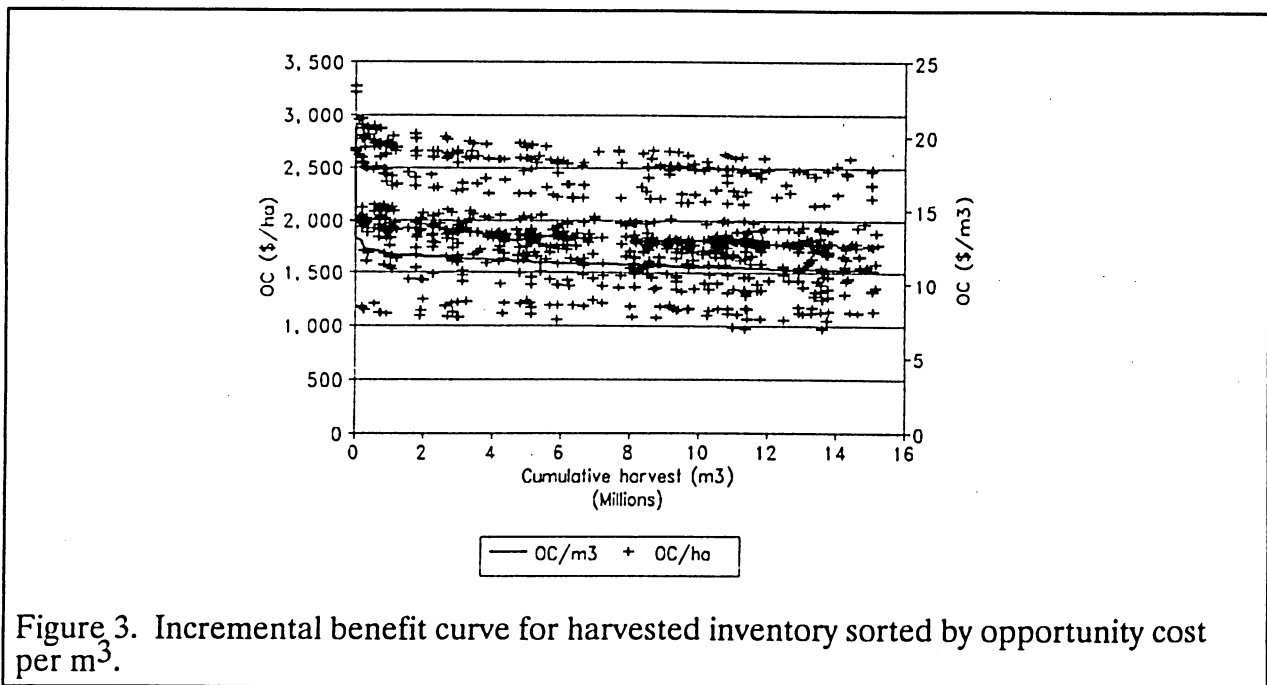
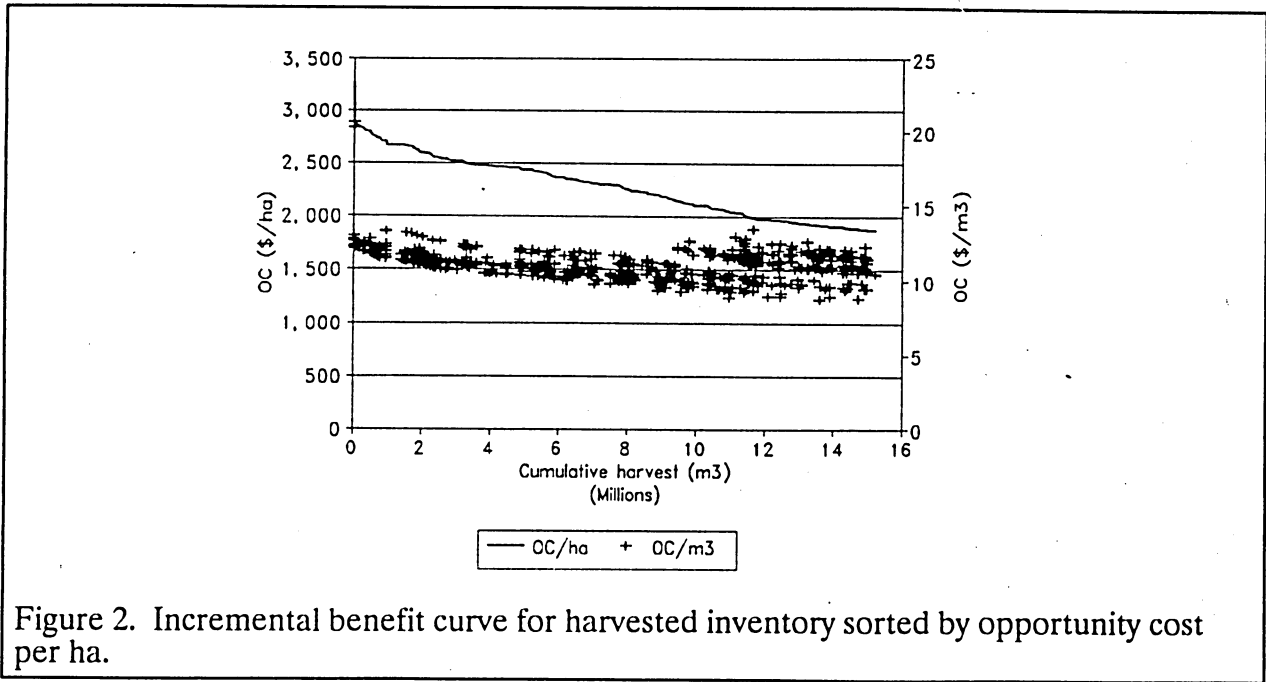
(m<sup>3</sup>/ha) as appropriate. These curves display the opportunity cost (\$/m<sup>3</sup> and \$/ha) that are avoided with each incremental m<sup>3</sup> harvested in a period. These curves are essentially inverted supply curves.

If the inventory is sorted by opportunity cost per ha, the opportunity cost per m<sup>3</sup> is much more tightly distributed than the opportunity cost per ha when the inventory is sorted by the opportunity cost per m<sup>3</sup> (Figures 2 and 3). This difference indicates that there is much more variation in the type of stands scheduled for harvest under the opportunity cost per m<sup>3</sup> sorting scheme. This result is confirmed by the distribution of harvested area by site class (Figures 4 and 5) and distribution of harvested area by species association (Figures 6 and 7).

Under the area based sorting scheme, 80 percent of the harvested area is from site class 1, the remaining 20 percent is from site class 2. When inventory is sorted by opportunity cost per m<sup>3</sup>, 56 percent of the harvested area is from site class 1, 41 percent from site class 2, and 2 percent from site class 3. The area harvested is also much more widely distributed across species associations when the inventory is sorted by opportunity cost per m<sup>3</sup>. With the area based sort, nearly half the harvested area is from the HSSP species association<sup>4</sup>. With the volume based sort, the harvest area is fairly evenly distributed across species associations.

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<sup>4</sup> Nine different species associations were recognized for this study. The first letter or first two letters of the species association code indicate whether the stand aggregate is considered to be softwood (S), hardwood (H), softwood-hardwood mixedwood (SH) or hardwood-softwood mixedwood (HS). With the exception of the HTA species association, the remaining letters indicate the primary softwood species (JP for jack pine; BS for black spruce; WS for white spruce; JPBS for jack pine and black spruce; and SP for undifferentiated spruce). The HTA code is used for nearly pure aspen stand aggregates.



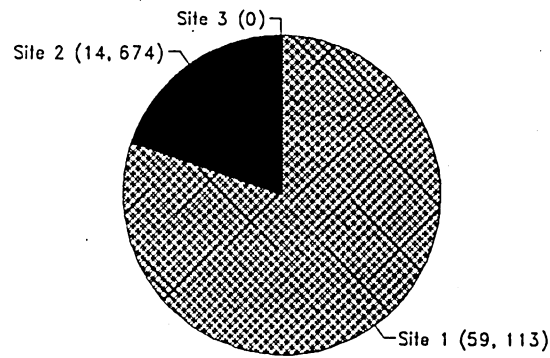


Figure 4. Harvest area distribution by site class for inventory sorted by opportunity cost per ha.

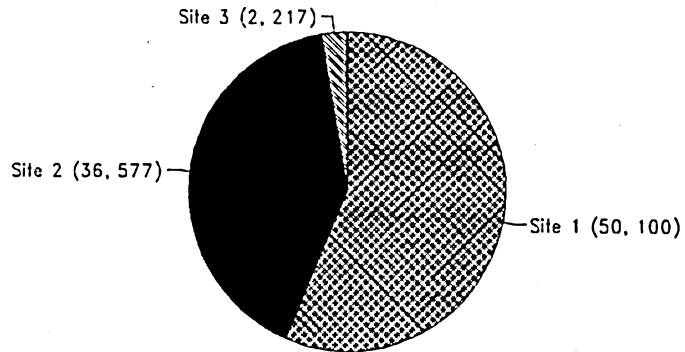


Figure 5. Harvest area distribution by site class for inventory sorted by opportunity cost per m<sup>3</sup>.

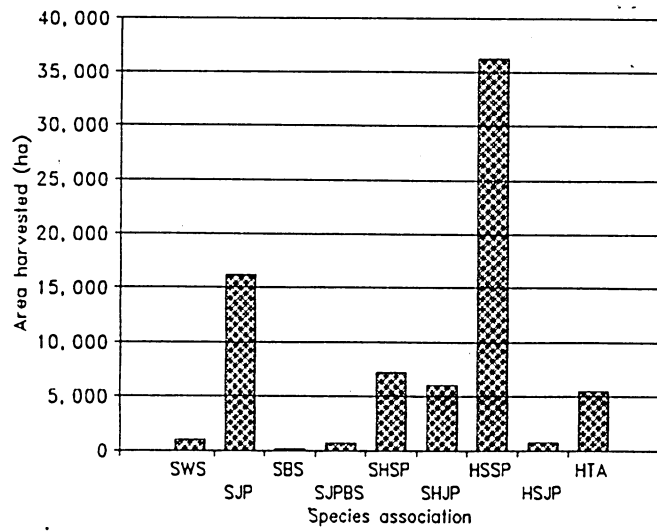


Figure 6. Harvest area distribution by species association for inventory sorted by opportunity cost per ha.

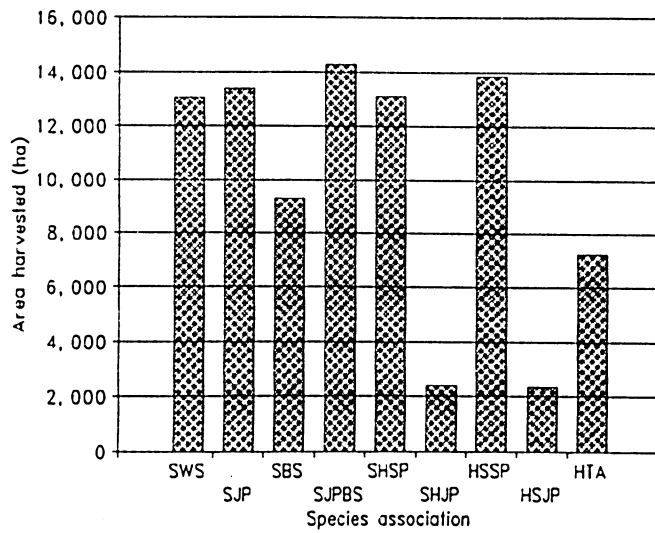


Figure 7. Harvest area distribution by species association for inventory sorted by opportunity cost per m<sup>3</sup>.

Harvests are more evenly distributed across site classes and species associations when the inventory is sorted by opportunity cost per  $m^3$  because of the dominance of the inventory holding cost in harvest priority determination. Because the conversion return for a  $m^3$  of timber is nearly constant across species associations and site classes, the inventory holding cost per  $m^3$  is nearly constant across species associations and site classes. The inventory holding cost per ha is greatest for areas with a large volume per ha. Sorting by opportunity cost per ha would tend to schedule large volume, site class 1 stands for harvest first.

With the area based sorting scheme, many timber classes with opportunity costs less than  $\$10/m^3$  are harvested. None are under  $\$10/m^3$  with the volume based sorting scheme. This means that the total opportunity cost avoided should be greater under the volume based sorting scheme. In fact, harvest with the volume based sorting scheme reduces the net marginal cost of delayed harvest by  $\$173.3$  million while harvest with the area based sorting scheme reduces net marginal cost of delayed harvest by only  $\$165.9$  million for this one period.

## Discussion and Conclusions

It is clear from these results that opportunity cost expressed on a  $\$/m^3$  basis is the relevant economic criterion to use for scheduling timber harvest when a firm is constrained by policies limiting periodic harvest volume. Making harvest decisions using opportunity costs expressed in  $\$/ha$  is incorrect and will lead to suboptimal harvest schedules.

Economic theory suggests that the best natural resources should be extracted first (Pearse, 1990). It has been observed that forest products companies in Canada harvest from a wide variety of sites and cover types (e.g. Beck et al. 1988) which, at first glance, would seem to contradict this principle. They argue that this behaviour makes marginal economic analysis inappropriate for economic timber supply studies, and that average revenues and costs are more relevant.

Why would a firm harvest a low volume black spruce stand when a high volume white spruce stand is available? It has been suggested that this mix of harvested cover types occurs because of government requirements that firms harvest "the bad with the good". The results presented here suggest that this decision is simply rational economic behaviour when a company is facing harvest volume constraints. The problem comes with the definition of



good: in our case, "good" cubic metres are those which, when harvested, will reduce the opportunity cost of delayed harvest the most. Good, in this context, is not the same as large timber volume or high site productivity.

Beck *et al.* (1988) notice that a wide variety of site classes and timber quality is cut in Alberta. Their hypothesis was that this was due to government regulation requiring that companies harvest the good with the bad. They went on to suggest that because of this behaviour, using an average cost criterion for determining economic timber supply may be more appropriate than marginal cost under such regulation.

The model and results presented here suggest that marginal economic analysis can be appropriate for timber supply studies and that a company facing harvest volume constraints may be acting rationally when it harvests low volume, apparently low value stands. A mix of harvested timber types is appropriate.

In a sense, this paper documents what some might consider obvious. However, much discussion about optimization centers around area based cut block selection. Others have explained away the wide variety of timber classes actually harvested by forest products companies facing harvest volume constraint as a response to government regulation or other non-economic factors. If one is determining the harvest priority for a stand using an opportunity cost per ha basis, harvesting a wide variety of timber classes seems economically irrational. However, the results here show that the area-based criterion is incorrect under these circumstances. The correct volume-based opportunity cost criterion results in a much wider variety of timber classes being harvested in a period. This corresponds better with the observed behaviour of logging firms.

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