DYNAMICAL SYSTEMS AND LIMIT CYCLES FOR MODELLING SUSTAINABLE AGRICULTURE AND COOPERATION

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Abstract:

Concern about sustaining agriculture stems from the growing realization that deficiencies in meeting the social, economic and ecospheric purposes of agriculture may jeopardize its role in provisioning future generations of humans. The problem arises within the complexity of the agricultural system. This complex human system is difficult to model using the strong causality principle so successfully applied to disciplinary parts of the system. Almost twenty years ago, Samuelson addressed this issue with modifications to the Lotka Voltera predator-prey model. More recently, Mandelbrot's discovery of fractal geometry and independent work on the persistence and stability behaviour of nonlinear dynamical systems have generated new hope for modelling the holism of complex systems. This paper examines these developments in the context of sustainable agriculture and the role of cooperative processes. Sustainability emerges as a matter of seeking flexibility and solving problems at the boundaries of systems rather than seeking the correct trajectory or arriving at an equilibrium. The conclusions are that sustention of agriculture is a purpose-related concept, that the domain of attraction about an equilibrium is more important than the equilibrium itself, and that the bifurcation and adjoining of sets of trajectories of system variables at system boundaries is at the centre of development processes for sustainable agriculture and cooperation.

Key Words:

Sustainable agriculture; dynamical systems; predator-prey; fractals

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I

Over many years sustainable agriculture and cooperation have continued to command attention. Two reasons are farm income problems worldwide and the globalization of ecospheric side effects associated with the intensification of agriculture (Schultz, 1979; Mellor, 1988). At the same time development of production capacity has been remarkably successful. This success stems from a prolonged preoccupation with the single objective of production and with the success of strong causality modelling.

Now that the ideological content of the debate over administered and market based allocation and distribution is waning, with the collapse of the Marxian experiment, attention is focussing on the logical and empirical economics of sustaining agricultural development (Pouliquen, 1990). Agricultural economists and economists are exploring problems of economic behaviour which remain unyielding to the mechanical paradigm (Anderson, Arrow and Pines, 1988; Arthur, 1990; Brossier, 1987; Schilizzi, 1987). Systems science, which in the 1970s faltered in its attempts to model the complexity of agricultural provisioning systems, is being revisited (Cox, 1984; Martens, 1988; Holt and Schoorl, 1990). Holism is increasingly accepted as an essential quality in systems modelling (Le Moigne, 1989). Multiobjective behaviour and public choice issues are emerging from the older welfare economics (Love, Rausser and Burton, 1990; Simon, 1978).

Sustainability in this paper applies to achievement of the multiple purposes of agriculture; provisioning, incomes and long run integrity of the ecosphere. Provisioning may be viewed as the production of a mix of private goods, represented by factor incomes, and of public goods including among others management of the ecosphere. One could model the sustention problem using feedbacks from the ecosphere and farm factor returns to the provisioning objective. Cooperation plays a part in sustention, related to modelling the process of aggregation and disaggregation of activities, functions and milieux of agriculture.

In our ongoing research, the process is explored using predator-prey models founded on the Lotka Voltera type starting with the innovative work by Samuelson (1971). These models offer the advantage of inclusion of feedbacks, modifications to reflect cooperation and of nonlinear dynamics. It is proposed that persistence of such systems (Freedman and Waltman, 1985) facing exogenously generated impulses (Bainov and Simeonov, 1989), is an oscillating phenomenon with varying periodicity according to the aggregation and disaggregation of parts of the food system (Cook, 1986; Thompson and Stewart, 1986). The relationships at the boundaries of the parts constituting an aggregation, no matter how small or how large, are hypothesized to obey the emerging laws of fractal geometry (Falconer, 1990).

II

The predator-prey model used by Samuelson envisages three possible outcomes. The predator-prey relationship in each case would tend to an equilibrium characterized by the possibility of repeating nonidentical movements, by dampening and by limit cycle oscillations. He was particularly interested in the effect of diminishing returns attributed to limited land and inorganic elements in the environment to these systems. In the more recent context of fractal geometry and persistence of dynamical systems, however, the stable or limit cycle outcome is most interesting.

Samuelson introduces a nonlinear term in the form of a function of the prey population which reverses its direction at limit values above and below the equilibrium value. The stable limit concept represents a means of modelling turning points in periodic movements corresponding to a wide range of observed phenomena. These include the increasing problems of cooperation as population densities increase for fixed habitat, as new predators emerge, as availability of prey declines, or as parasites, disease or starvation jeopardize erstwhile predators (Gove, 1976; Takeuchi, 1990; Freedman, 1990). The same lines of reasoning may apply to the ebb and flow of economic discipline associated with affluence and hardship respectively in human populations.

The stable limit outcome recognizes periods of increasing returns to size in the evolution of dynamic systems. In development terms for human systems, increasing returns are due to learning and the consequent technological change, at rates exceeding the rates of change elsewhere in the system promoting diminishing returns. This circumstance could be viewed as enabling system aggregation leading to opening and enhanced control over the environment. The converse situation corresponds to system closing and an increasing role for entropy-type frictions impeding productivity and efficiency. Alternating aggregation and disaggregation, possibly part of a bifurcation process, would produce a situation not unlike the mechanical stick-slip phenomenon (Cook, 1986, p 91).

The time based function for the intersection of sets of predators and prey may display fractal properties over a long enough period of time (Falconer, 1990; Ch 11). Fractals are geometric representations of the boundaries of sets. Since a dynamical human system such as agriculture is a set of activities, it is interesting to explore the fractal behaviour of this system as it pulsates with time. The fractal structure of a system boundary is the encoded data for that system compressed into an appropriate functional form, iterated with transformations until an invariant boundary behaviour emerges. Each fractal possesses a unique measure or dimension allowing it to be identified and compared with others using the concepts of similarity and affinity (Mandelbrot, 1986).

The meaning of invariance is quite distinct from that of no change. A fractal may exhibit considerable variation of the location of its coordinates in N dimensional space over time. From a behavioural point of view this variation can be associated with a flexible response by the system to impulses from its environment. For evidence of the importance of flexibility in system performance, see the work reported recently by Goldberger, Rigney and West (1990) on the functioning of the human heart. Invariance in this context refers to the nonrepeating fluctuation or oscillation of the descriptive variables within limits unique to that dynamical system.

An invariant set is one of the properties of fractal forms. Pfeifer explored the consequences of two or more fractals interacting (Pfeifer, 1986; p 47). He concluded from experiments with adsorption of polymer chains on a porous surface, that there may be a general tendency for the geometry of the adsorbed polymer to be of similar dimension to that of the chain in solution. Equilibrium would not be altered in either set.

Relating this result to the global asymptotic stability properties of invariant sets, suggests that merging sets may be a way to capture the starting points for the same trajectories or for others in different state spaces. Such a process of joining sets is analogous to aggregation and suggests the possibility that cooperative processes may be associated with changing trajectories particularly with respect to the form of nonlinear dampening in systems. In economic terms cooperation seeks to reduce the play of factors contributing diseconomies, and/or increase the play of factors generating economies, and/or both.

In the relatively short time of recorded agricultural history, the actual true limits to fluctuation of an agricultural system, consistent with the principle of invariance may not have been experienced. However persistence theory suggests that such limits exist and that systems perform within them (Freedman and Waltman, 1985). These pulsations appearing in the form of oscillations or cycles projected over a relevant period such as fifty years could exhibit the effects of overall diminishing or of increasing returns depending on the extent to which system aggregation approaches true globality to begin closing the system.

The boundaries of agricultural systems may be considered to be characterized by energy flows. The fractal dimension of these boundaries would represent the way energy flows respond to impulses from the environment. Impulses could include a range of actions and feedbacks from the predator behaviour of weeds on soil nutrients and light, to price signals from capital and commodity markets. The actual fractal dimension could be considered to reflect the manner in which the milieu of the system governed the tradeoffs which underly the response to environmental impulses.

In subsistence agricultural systems, this dimension would be invariant because of the dominance of empirical memory-based rules in determining tradeoffs. In industrialized agriculture, with ongoing integration of rural and urban milieux, the invariance principle might not be observable because the generator functions would be experiencing a longer series of transformations. It is even possible that invariance does not emerge at the outer boundaries of aggregating systems until they begin to close. Consequently the modelling task would involve selection/definition of the generator functions and of the transformation functions analogous to the task of identifying the nonlinear term representing diminishing returns introduced by Samuelson into the Lotka Voltera equation.

III

There are many unresolved issues in studying agricultural systems using these new lines of reasoning. One is that for dynamical systems, the pattern of evolution is sensitive to small variations in the initial generator function. This ergodic property is troublesome for human systems because of the principle of intentionality (Ackoff and Emery, 1982; Le Moigne, 1983). The combination of predator-prey modelling with fractal geometry places considerable weight on system memory to explain the development process for human systems. This weight is becoming more manageable with advances in information technology strengthening institutional memory, but denies roles for vision, independent learning and adaptive preferences in development.

A second issue is the possibility of cooperation between predator and prey explored in general terms by Hirshleifer (1078) for 'natural economies'. In human systems, collusion between capitalist and worker, buyer and seller, among sellers and among buyers is common place. Economic integration is the superimposition of collaborative processes over predator-prey relationships based on reciprocation, mutualism and enforcement. Examples are corporate mergers, producer marketing boards, cooperative rice mills and plantation/out-grower contracting.

The modelling solution to this issue appears to be to treat these relationships as unique forms of aggregation of smaller systems. In principle each should be definable in terms of its unique fractal dimension or 'fingerprint' in the form of a dimension print (Falconer, 1990, p 51). The intentioned restructuring of system relationships in agriculture could be expected to behave similarly to the iterative transformation process which settles into an invariant fractal dimension. Sustainability in this context is represented by the invariance property of the system.

A third issue is the time dimension of the evolution/development of agriculture. At the heart of human concern about agricultural sustainability is the possibility of dampening factors, whether internally or externally generated, affecting the performance of agricultural systems in the future. To be sustainable, agricultural activities and functions must constitute an invariant set. An invariant set contains the future states, that is to say the future parts of trajectories (Cook, 1986; p 108). If that set also contains an equilibrium point with the property that trajectories beginning within the set approach that point as time approaches infinity, the set is called a domain of attraction, or basin of attraction (Becker and Dorfler, 1989).

In the absence of monotonic dampening or decay from within or from externally driven impulses, the invariant set should persist to infinity, that is be sustainable. However, human perceptions of time in the future are much less than infinite, ranging from a few hours, to a human generation or the useful life of a capital good. The effective range is shortened by discounting for economic decisions. These varying perceptions of time affect the extent to which opportunism and shirking require modifications to institutional arrangements governing the behaviour of the agricultural system (Hayami, 1989). Markets, nepotism, patronage, rules of justice and property rights embodied in the system milieu appear to govern the economic behaviour of system aggregation in a manner loosely analogous to the governing of cell multiplication by DNA.

Thus the definition of time introduces at least three problems. A problem arises when the time required to reach an invariant state is different from the time perceived to be relevant to human initiative. A second problem arises when independent human initiative changes trajectories for state variables so that an invariant set with its domain of attraction becomes disassociated from the purpose of agriculture. A third problem arises for modelling the complex agriculture system in determining how to treat independent human behaviour and the dynamics of institutional change referred to by Hayami. Are they an integral part of the dynamical agricultural system or do they constitute *un tier exclu* (Le Moigne, 1983)? These problems suggest that sustainability, as a means-related concept tied to the actions, functions and institutions of agriculture, is not workable and cannot constitute a basis on which to rely for the guidance of human interventions in agriculture.

Returning to limit cycles produces a cautionary note for cooperation aiming to achieve sustainability by the joining of sets to evade system decay in the short human view of time. Limit cycles may possess either stable or unstable manifolds. A manifold is the set of all trajectories tending to the equilibrium point as time approaches infinity (Cook, 1986; p 121). The intersection of unstable with stable manifolds of limit cycles yields potential chaos properties. Resulting unstable equilibria are associated with the merging of boundaries with different behavioural properties. Different feedback and action rules determined by the milieu of agricultural systems which drive the trajectories for the state variables may be the source of such instability. An example is the social choice problem concerning the role of agriculture in managing European rural space (Bodiguel, 1990). Thus the task for cooperation could be defined as being to identify and isolate those systems with nonconverging trajectories for the same state variables so as to prevent merging.

A further issue is the possibility for extinction of a population within an agricultural system. Any biological population may face extinction. Sustainable agriculture at its most basic level involves the possibility of extinction of the human race, or on a more limited level the extinction of biological species within the system. Deterministic multidimensional systems under given conditions demonstrate limit cycle behaviour and persistence. Persistence exists if each component population exists initially and over time to infinity. Persistence is a stability concept in that it involves a limit cycle. The boundaries of the values taken on by the state variables remain in the positive quadrant of two dimensional Euclidean space (Freedman and Waltman, 1984, p 214).

The same authors noted that persistence of a population within a deterministic system may give way to extinction for stochastic reasons when population size becomes periodically small (Freedman and Waltman, 1985; p 100). They term this outcome a violation of the deterministic hypothesis. This may not be so however. It may be instructive to examine boundary relationships between deterministic invariant systems, each constituting the environment to the other.

In predator-prey modelling, the initial populations, in the absence of competitors, may be considered to grow to the carrying capacity of the ecosphere, even drawing on environing systems for nutrients (Freedman and Waltman, 1984; p 214). The so-called stochastic reasons for extinction may be the clash of different behaviours in adjoining manifolds, when one is unstable. Alternatively, a holistic problem definition would acknowledge that a system with several populations could be in a predator-prey relationship with neighbouring deterministic nonlinear dynamical systems triggered only by as yet undefined circumstances leading to aggregation of the systems. Presumably the reverse should be possible through disaggregation, bifurcation or disengagement.

In these ways the extinction of a population need not be stochastic but rather could be deterministic. The process of adjoining may be fractal enabling measurement of unique fractal dimensions characteristic of extinction processes. Thus pursuit of sustainability of agriculture has to do with 'managing' processes of aggregation and bifurcation such that invariant sets are created with strong attraction domains. The point is not to achieve equilibria but to design boundaries within which trajectories of important state variables can pulsate about an equilibrium. Since each perception of the appropriate aggregation of systems to make up the agricultural system derives from a perception of control over activites and outcomes, the design problem involves gaining control over 'neighbourhoods' outside the boundaries so that intersections may be stable.

The sustainability problem in this context is linked to understanding system aggregation processes and measures identifying jeopardy or to understanding the meaning of the information conveyed by the measures. A lack of intentionality due to lack of vision or conflict over the means of reaching consensus such as the democracy movement in China, may also have atavistic consequences for a system.

Cooperation as a means to adjoining systems relates to more than determining a common vision. The Freedman and Waltman work, like that of Hirshleifer, suggests that cooperative systems exist as alternative to competitive systems. The result in terms of sustainability could be stronger than persistence, in the absence of impulses from environing systems, in that all solutions are eventually bounded away from all coordinate planes in multidimensional space (Hofbauer, 1981 quoted by Freedman and Waltman, 1985; p 98). This stronger persistence may involve Hofbauer's cooperativity and be analogous to the concept of uniform persistence and global stability addressed by Mukherjee and Roy (1990) for two predator-prey pairs.

Thus the underlying behaviour of the system in question is a precondition to cooperation and results in a stronger form of persistence. It becomes evident therefore that the juxtaposition of cooperative and competitive systems requires both to exhibit stable manifolds to preclude the possibility of chaotic processes upon joining. It is not clear whether two such systems, each exhibiting global asymptotic stability, would result in a new aggregate also with global asymptotic stability, especially with increasing economies of size. It also appears to be important to consider the nature of the aggregation in terms of joining different trophic levels or remaining within the same trophic level.

The last issue is that nonlinear dynamical systems modelling is premised on determinism and invariance. Determinism connotes purpose. Invariance presumes that trajectories 'settle down' in state space about an attractor. For human systems, such as agricultural systems, these qualities work counter to the diversity characteristic of human behaviour and vision, and to the flexibility of response required to handle impulses from an environment containing unstable systems. Furthermore, nonlinear differential equations of higher orders with discontinuities, needed to model complex agricultural systems, are generally intractable. Considerable work is required to bring these aspects of systems theory and method closer together for agricultural applications.

IV

The main context of sustainability is the persistence of human populations, for which reason they intervene in their agricultural systems. Sustainability does not apply to the activities, functions and actions of humans in this process, that is to say to the means of attaining the purposes of the system. Sustainability is attained through generic persistence in the face of impulses generated outside the system. The internal longevity of the system may be expressed as limit cycles within invariant sets of state variables. Invariant sets are usefully modelled as iterative transformations of generator functions to produce trajectories attracted to an equilibrium. These trajectories may have fractal structures which could shed light upon the role of an equilibrium in the economic behaviour of an agricultural system.

An equilibrium is important as an attractor for trajectories of state variables over time. This role may not involve the system arriving at equilibrium or indeed ever being in equilibrium. Furthermore, equilibria are not the only form of attractors in nonlinear dynamical systems. Other strange attractors cannot be ruled out as alternative anchors for system performance. Thus accounting identities premised on achieving an equilibrium state, may be counterproductive or at best incidental in modelling the complexity of agricultural systems.

International cooperation is a restricted form of cooperative behaviour which seeks to capture economies of size and complementarity among constituencies constituted as states. Cooperation reinforces sustainability when the resulting aggregations of systems do not result in, or prevent, closing a global system. Agricultural pollution, as a spillover effect of the intensive industrialization of agriculture, is an example of system closing through lack of cooperation. Cooperation, as an alternative form of discipline, substitutes common interest, for individual fear of extinction. Cooperation would jeopardize sustention when an unstable set of trajectories is involved, an attractor is weakened or when cooperation closes a system boundary.

The application of dynamical systems theory and fractals to agricultural systems raises several issues. The evolution of a set toward invariance is sensitive to the starting point and like mechanical modelling, learning though feedbacks is limited to knowing the new starting point in an iteration. This ergodicity is incompatible with both learning and intentionality. A second issue is the need to allow for cooperative elements in predator-prey models. A third issue is the noncongruence of time between the perceptions of humans for their interventions and the time involved in the transformations of initial states in the process of achieving invariance. The distinction between renewability and depletion of resources is part of this incongruity.

A fourth issue is the periodic vulnerability of systems characterized by limit cycles, to aggregation involving unstable or predatory manifold sets in neighbouring systems. Vulnerability in these cases extends beyond disturbances caused by impulses from the environment to the agricultural system, to population extinctions. However it should be noted that dynamical systems theory treats this issue as a deterministic rather than as a stochastic problem. Fractal attractors replace stochastic forces to explain system behaviour, raising questions about the treatment of unexplained residuals in applications of econometric techniques to time series used to estimate such system dynamic behaviour as technological change and comparative advantage.

A fifth issue is the empirical nature of these mathematical approaches. In their still early stages of development, trial and error remains an important element in determining the dimension of a fractal structure. The sixth and last issue is the many contradictions between the reliance on generator functions and correct identification of starting points for trajectories, and the qualitative notions in systems methods concerning the purposeful nature of systems and their properties of autoregulation, equifinality, flexibility and diversity. Considerable work is required to bring these two aspects of systems theory and method closer together.

The meaning of all this for agricultural economists is that attention is needed to modelling and labeling the various persistence characteristics of economic variables in agricultural systems over time. The estimation of the fractal dimension of variables may reveal characteristic fingerprints for structural change. Economists could reconsider variations on the theme of predator-prey models for testing three phenomena relating to sustainability; one, cooperative feedback behaviour within systems; two, the oscillation of the dominance of economies relative to diseconomies over time; and three, the modelling of deterministic impulse generators in the environment to an agricultural system. Some work suggests these three modelling initiatives when linked could explain periodicity in the oscillation of the system (Ruud, 1988). International cooperation could be modelled for example by treating the GATT as an impulse generator for simulating the links between international cooperation and sustainability.

This work suggests that taxonomic approaches to agricultural systems analysis, especially according to institutional and technical criteria, may be misplaced. Not only are there large numbers of agricultural systems, each specific to groups of humans and their intentions and endowments, but in the context of sustainability, functions, activities and institutions are transitory and at the centre of the dynamics of evolution and development.

Progress toward sustainable agriculture requires a holistic understanding gained by modelling the boundary behaviour of the system. Complex human systems, of which agriculture is an example, persist over time according to the extent and strength of their attraction domains including attractors other than equilibria. Preoccupation with sustaining an equilibrium may impoverish agricultural economic modelling in the light of limit cycles, persistence, need for system flexibility, and emerging evidence of the behaviour and role of attractors in agriculture system development.

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