Stochastic Dynamic Programming Models:
An Application to Agricultural Investment

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1 INTRODUCTION

The nature of farm firms has special implications for the choice of risk management strategies. Most farms are small and have a noncorporate ownership structure which prevents spreading of risks among many individuals. Benefits derived from specialization in production and resource limitations restrict opportunities for enterprise diversification. This has spurred an interest in finding alternative methods of reducing the variability of cash flows and introducing greater diversity into the asset and liability structure of farm firms (Young and Barry). One method which has received limited attention is investment in financial assets. By allocating resources outside of the farm, cash flows may be stabilized without the inherent losses in productive efficiency which often accompany diversification into other agricultural enterprises. Young and Barry approached the problem of holding financial assets as part of a long term portfolio. Their results suggest that investment in financial assets may stabilize long term cash flows.

To date, very few studies have explicitly considered the stochastic, dynamic nature of investment returns in models of firm growth and investment (Schnitkey and Taylor; Larson, Stauber and Burt). Theoretical studies (Robison and Barry; Robison and Brake) have outlined the limitations of conventional portfolio theory as a farm planning and decision making tool. Since conventional portfolio analysis is static, it does not result in an operational investment strategy and ignores some key determinants of financial performance. Portfolio models fail to incorporate the implications of asset indivisibility, the liquidity characteristics of assets, tax impacts, and the costs of altering investment portfolios. Farm managers are concerned with the distinction between cash returns and asset appreciation and how these factors influence the firm's operating structure and growth. In a dynamic world, decisions made in one time period affect decisions in later periods by altering financial structure and the nature of productive assets. The dynamics of asset accumulation are thus a major point of interest as well. The analysis of investment decisions should incorporate these dynamic interrelationships in a multiperiod model.

The objective of this paper is to address the issue of dynamic investment problems in agriculture. Specifically, the potential effects of stock investment outside of an agricultural enterprise on a firm's financial structure are analyzed. The study also considers the influence of financial structure and returns to the agricultural enterprise on optimal investment decision rules. This is accomplished by specifying and numerically solving a stochastic dynamic programming (SDP) model for an Illinois hog finishing operation.

The paper also addresses some of the inherent problems associated with firm level financial models. Specifically, the issues of modelling bankruptcy and dealing with multiple stochastic state variables are considered. The calculation and use of conditional probability methods which have been adapted to the dynamic programming framework are also discussed and illustrated.

2 THE DYNAMIC PROGRAMMING MODEL

A monthly stochastic dynamic programming model was specified and solved to determine the optimal stock investment decision rule for an Illinois hog finishing operation. It was assumed that the decision maker could invest his funds into either stocks (S) or other financial instruments (OF) such as a money market fund. The stock investment could be easily achieved through a mutual fund. Returns to stock investment were calculated as changes in the price of the fund from period to period plus any dividend income. The model contained three stochastic state variables: hog returns (HR),defined as revenue minus variable costs, stock prices (PS), and return on other financial instruments (ROF), and two deterministic state variables: holdings of other financial instruments (OF), and stock holdings (S). Investments in other financial instruments represented an asset when held in positive amounts or a liability when negative (ie. operating credit). The decision in each period was the level of stocks to buy or sell (DS).

The stock investment model was formulated as a terminal wealth maximization problem rather than the standard present value maximization. Denoting the terminal year as T, terminal wealth can be written as a function of the state variables:

$$V_T(HR_T,\ PS_T,\ ROF_T,\ OF_T,\ S_T) =$$
$$(PS_T * S_T) + OF_T + FarmAssets_T - TermDebt_T$$

where $V_T(\cdot)$ is the recursive objective function for year T and *FarmAssets* were those assets devoted to the production of finished hogs and *TermDebt* is the level of long term debt. This function leads to the following general recursive equation:

$$V_{t-1}(HR_{t-1}, PS_{t-1}, ROF_{t-1}, OF_{t-1}, S_{t-1}) =$$

$$\max_{DS_t} E[V_t(HR_t, PS_t, ROF_t, OF_t, S_t)]$$

where $E[\cdot]$is the expectations operator and $V_t(\cdot)$ is the value of wealth assuming that optimal decisions are made.

This maximization is subject to the following state transition equations:

$HR_t = f_1(HR_{t-1})$

$PS_t = f_2(PS_{t-1}, ROF_{t-1})$

$ROF_t = f_3(ROF_{t-1})$

$S_t = S_{t-1} + DS_t$

$OF_t = OF_{t-1} - With - DS_t * PS_t + DIV_t + HogRet_t$

where the terms in the state transition equations are:

$DS_t = \{200, 0, -200\}$

$Borrowing/Lending\ Differential(BLD) = 3\%$

$If\ OF_t \leq 0\ then\ ROF_t = ROF_t + BLD$

$HogReturn_t\ (HogRet_t) = HR_t * MHP - FC * MHP$

$Withdrawals(With) = MCW + Payment\ on\ Term\ Debt$

$0 \leq S_t \leq 2000$

$100 \leq PS_t \leq 310$

$-20 \leq RH_t \leq 40$

$-350000 \leq OF_t \leq 350000$

$.06 \leq ROF_t \leq .16$

$Monthly\ Hog\ Production\ (MHP) = 750$

$Monthly\ Consumption\ Withdrawal\ (MCW) = 2{,}000$

$Fixed\ Cost\ Per\ Hog\ (FC) = 5.00$

$StockDividend\ (DIV_t) = .083 * S_{t-1}$

$Farm\ Assets_1 = 450{,}000$

$Bankruptcy \equiv \{Wealth \leq 0\}$

$Beginning\ TermDebt = 100{,}000$

Rather than discounting returns, as in a present value maximization, returns in the terminal wealth maximization problem are compounded. Within the stock investment model, compounding is achieved through the other financial instruments holding variable (OF).

3 ESTIMATION OF TRANSITION PROBABILITIES

Numerical solution of the investment model required state transition probabilities which were derived from estimated state transition equations. This section describes the data and estimation procedures for the hog return (HR), stock price (PS) and interest rate (ROF) state transition equations.

Monthly hog returns were based on budgets reported in the Livestock Meat Situation and Outlook Report published by USDA. Data from the Illinois Farm Business Farm Management Association (FBFM) were used to adjust the return series to reflect Illinois costs of production as closely as possible.

Monthly stock prices (S&P 500 index) were collected from the Standard and Poor's Statistical Reporting Service and dividend data were based on information provided by Ibbotson and Associates. Short term interest rate data were from the Economic Report of the President. All series covered the period from the beginning of 1974 to the third quarter of 1987.

Modelling of multiple stochastic state variables provides a special problem in that transition relationships originate from multivariate stochastic processes. Thus, a state variable's transition relationship may include not only its own lagged variables but other lagged state variables as well.

The nature of the economic variables in this model provided an additional problem. An index was used to represent stock prices. The index showed a continual upward trend which was the result of economic growth. Likewise, interest rates, expressed in nominal terms, have also trended upwards over the sample period. In terms of time series analysis, the existence of trends results in non-stationary data. Stationarity is required to ensure that the estimated transition probabilities are derived from a process which is time invariant. Stationarity was achieved by differencing the stock price and interest rate series once.

Tentative dynamic interrelationships between hog returns, stock prices and interest rates were originally identified through time series techniques (Granger and Newbold). Sample autocorrelations and cross-correlations suggested that hog returns were not correlated with either interest rates or stock prices. Hog returns showed evidence of lower order autoregressive structure. Autocorrelations and partial autocorrelations for stock price and interest rate suggested that these variables were interrelated and that lower level autoregressive models would adequately capture their Markovian relationships.

3.1 Hog Return Transition Relationship

Examination of autocorrelations and partial autocorrelations suggested that the hog return relationship could be modelled with a second order autoregressive process, AR(2). A goal of reducing the number of state variables prompted the estimation of a first order process (AR(1)) as well as the AR(2) model.

Estimation of the AR(1) model produced the following equation (t-statistics in parentheses):

$$HR_t = \underset{(2.58)}{1.895} + \underset{(18.70)}{.811}\, HR_{t-1} \qquad R^2 = .684 \qquad \sigma_e = 8.003$$

where σ_e is the standard error of the estimate. This formulation resulted in autocorrelated errors indicating that an AR(1) model did not adequately capture the series' time dependent nature.

Estimation of the AR(2) model resulted in the equation:

$$HR_t = \underset{(3.60)}{2.430} + \underset{(16.54)}{1.177}\, HR_{t-1} - \underset{(-6.29)}{.439}\, HR_{t-2} \qquad R^2 = .736 \qquad \sigma_e = 7.239$$

which showed no sign of autocorrelation and yielded normally distributed errors as judged by the Jarque-Bera test statistic.

Based upon these results, and analyses of higher order models, the AR(2) model was judged to adequately describe the series' Markovian nature. To reduce the dimension of the DP model only one hog return variable was included. The reduction was accomplished using Burt and Taylor's method of reducing the order of an autoregressive process. This procedure resulted in the following form:

$$HR_t = 1.688 + .8177\, HR_{t-1} \qquad \sigma = 8.058$$

From this equation, transition probabilities were estimated using a hyperbolic tangent method (Taylor).

Table 1 shows the resulting transition probability matrix for hog returns as well as the limiting distribution which is approached within one year of any beginning state level. This table illustrates the extreme variability of returns in hog feeding enterprises. This variability provided some of the rationale for the choice of a monthly rather than an annual model.[1]

1 Quarterly and annual models were investigated for all three stochastic state variables but no Markovian structure could be identified.

Table 1: Transition Probabilities for Hog Returns

HR_{t-1}	-20	-5	+10	+25	+40
HR_t					
-20	.6063	.3760	.0176	.0001	.0000
-5	.1064	.6217	.2644	.0075	.0000
+10	.0032	.1778	.6462	.1699	.0029
+25	.0000	.0082	.2744	.6167	.1007
+40	.0000	.0001	.0190	.3869	.5940
Limit	.0719	.2544	.3786	.2342	.0608

3.2 Interest and Stock Price Transition Relationships

Autocorrelations, partial-correlations and cross-correlations were examined to identify autoregressive relationships within the first differenced stock price and interest rate series. These plots suggested a first order autoregressive structure for each variable and across variables.

Estimation of the AR(1) model for stock price resulted in the following parameters (t-statistics in parentheses; all variables in log form):

$$PS_t - PS_{t-1} = \underset{(3.13)}{.124}(PS_{t-1} - PS_{t-2}) - \underset{(-3.63)}{.126}I_{t-1} + \underset{(3.27)}{.118}I_{t-2}$$
$$\sigma_e = .035$$

which reduced to:

$$PS_t = 1.124PS_{t-1} - .124PS_{t-2} - .126I_{t-1} + .118I_{t-2}$$

Estimation of the AR(1) model for first differenced interest rates resulted in the following parameters:

$$I_t - I_{t-1} = \underset{(4.95)}{.328}(I_{t-1} - I_{t-2})$$
$$\sigma_e = .0636$$

which reduced to:

$$I_t = 1.328I_{t-1} - .328I_{t-2}$$

Residuals from both of the above equations were normally distributed as judged by the Jarque-Bera statistic and were independent across equations based on cross-correlations.

As was the case with the hog return transitions, a reduction in the number of state variables was preferred to lower the dimension of the DP model. Burt and Taylor's method for reducing the order of interdependent autoregressive equations was employed to produce the following equations:

$S_t = S_{t-1} - .0064 I_{t-1} \quad \sigma_e = .0381$

$I_t = I_{t-1} \sigma_e = .0717$

$\sigma_{SI} = -.0035 \quad \Rightarrow \rho_{SI} = -.1267$

From these equations transition probabilities for stock prices and interest rates were estimated using a numerical integration routine (Gerald and Wheatley).

4 OPTIMAL STOCK INVESTMENT DECISION RULE

The optimal stock investment decision rule was derived using a value-iteration dynamic programming algorithm. Numerical solution required specification of discrete state and decision variable levels. Four hog return intervals ranging from -$20 to $40 produced state levels of -20, 0, 20, and 40 dollars respectively. Stock prices covered 15 intervals ranging from 100 to 310 and stock holdings ranged from 0 to 2,000 units in increments of 200. Financial instrument holdings covered the range -$350,000 to $350,000 in $70,000 increments and return on financial instruments ranged from 6 percent to 16 percent in two percent increments. This formulation resulted in 43,560 states. The stock purchase decision was allowed to take on values of -200 (sell), 0, or 200 (buy).[2]

The optimal investment rule was obtained by backward recursion beginning at the final year of the planning horizon. Linear interpolation of the objective function was used to increase the convergence rate and reduce biases resulting from discretizing the state variables. Interpolation was used on the financial holdings variable because the ending values for this variable did not necessarily match the state interval midpoints. Optimal decisions were found for all state intervals except those combinations which defined technical bankruptcy (i.e. negative wealth). In the case of bankruptcy, the farming operation was presumed to be liquidated.

Optimal decision rules were generated until the optimal decision rules converged; which occurred by month six of year three. Thus, the converged decision rule was applicable to all periods up to the thirty months before the end of the planning horizon. For example, if the planning horizon is ten years long, the converged decision rule would be applicable from year one through to month six of year seven.

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2 The model was also solved using a finer grid with 6 hog return states and 11 interest rate states for a total of 127,776 states. The results were essentially the same as for this smaller version.

The large size of the optimal decision rule prevents a complete description within this paper. A graphical presentation of a portion of the decision rule follows in figures 1 and 2.

Figure 1 presents the optimal decision rule when hog returns are at the $20 level and stock holdings are 400 units. Panels A through C illustrate the effects of changing stock price levels for the complete range of interest rates and financial holdings.

For given levels of stock price and financial holdings, the graphs illustrate the dampening effects of higher interest rates on the desirability of stock purchases. For example, at a stock price of $115 (Panel A) and financial holdings of $0, stock purchases occur up to an interest rate of 12 percent. Over a range of 12 to 14 percent, existing stocks are held, and stocks are sold at rates above 14 percent. This interest rate effect exists at all stock price levels although the absolute values of interest rates at which decisions change vary with stock price.

For fixed levels of stock price and interest rates, the graphs illustrate the effects of higher financial holdings levels on stock purchases. As financial holdings increase, the firm has more funds to purchase stocks. At a 12 percent interest rate and stock price of 115, for example, financial holdings of less than -$70,000 are associated with stock sales. The range from -$70,000 to $70,000 are associated with a decision not to purchase or sell stocks, and levels above $70,000 are associated with stock purchases. As was the case with interest rates, this wealth effect is consistent across stock price levels although the breakpoints differ for each stock price.

Figure 2 illustrates the effects of three different hog return levels on stock purchases. At constant stock prices of $190 and stock holdings of 400 units, panels A through C illustrate the the effects of three different hog return levels across the range of interest rates and financial holdings. Note that the interest rate and financial holdings effects discussed above occur for different hog return levels. The effect of changes in hog returns is seen in the positioning of buy and sell breakpoints. For example, at an interest rate of ten percent,a hog return of -$20 (panel A) implies almost no purchases at lower levels of financial holdings. A return level of $0 (panel B) implies purchases at lower levels of financial holdings up to an interest rate of 12 percent. A $20 return also allows purchases up to an interest rate of 12 percent at most levels of financial holdings and infers no stock sales until 12 percent even at the lowest level of financial holdings.

Figure 1 Portions of the Optimal Decision Rule for Hog Returns of $20 and Stock Holdings of 400.

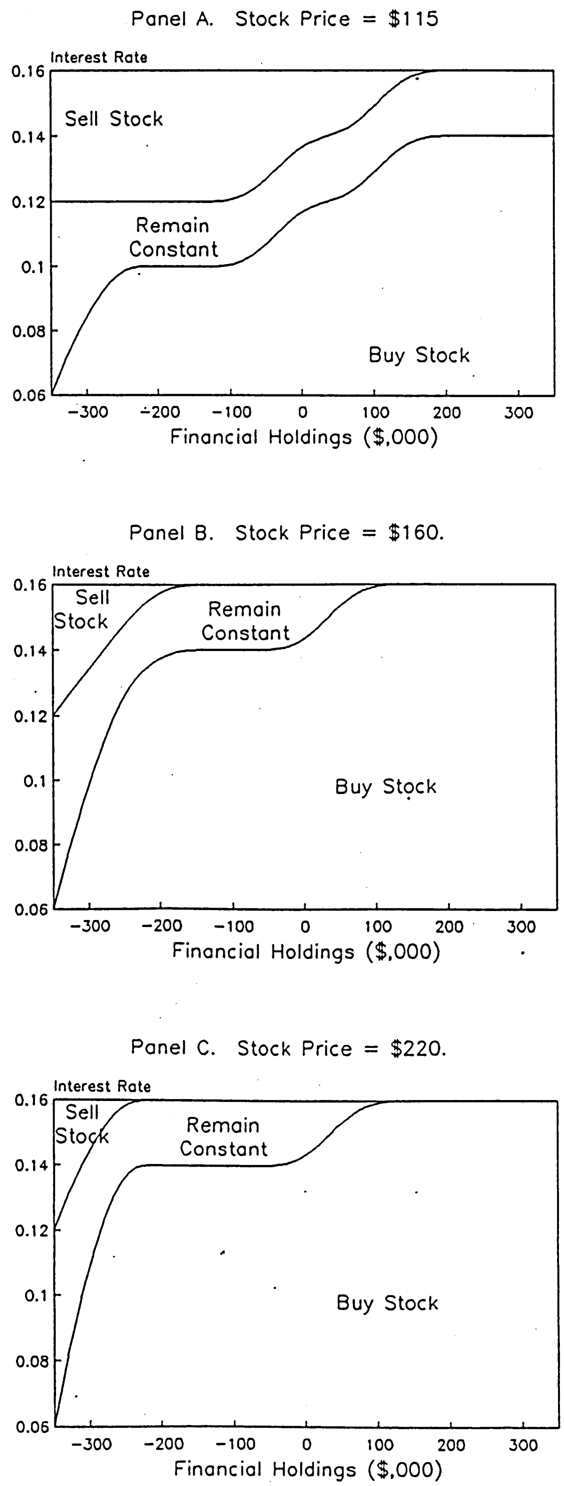
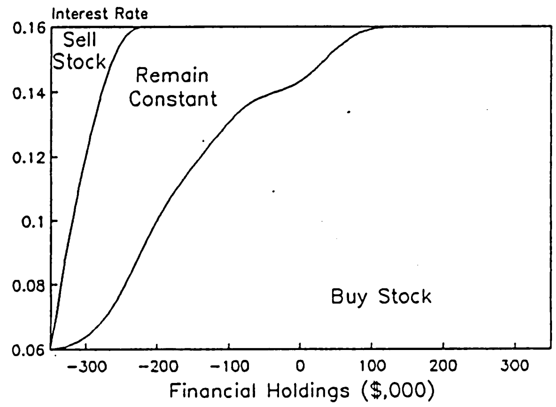
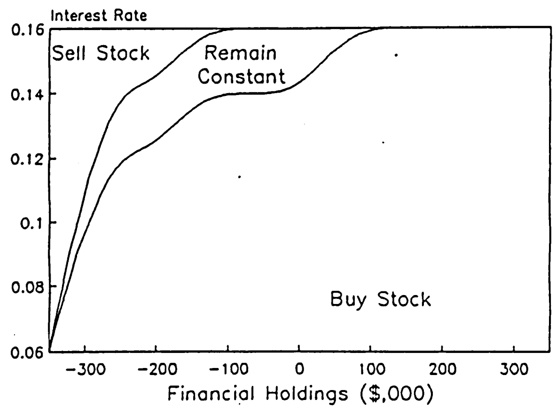


Figure 2 Portions of the Optimal Decision Rule for Stock Holdings of 400 and Stock price of 190.

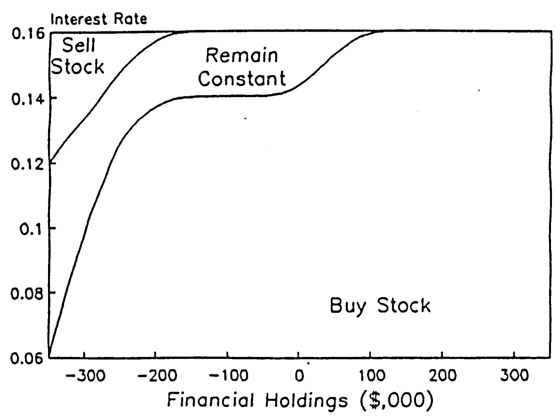
Panel A. Hog Return = −$20



Panel B. Hog Return = $0



Panel C. Hog Return = $20



5 CALCULATION OF CONDITIONAL PROBABILITIES

Discrete conditional probability methods were used to determine ex ante distributional forecasts of financial holdings and stock holdings. The general approach for calculating conditional probabilities requires constructing an N dimensioned state probability vector denoted as Π_t, where N is the number of state intervals and t equals the number of the stage. The i'th element of this vector ($\Pi_t(i)$) gives the probability of being in the i'th state at time t. To satisfy the basic properties of probabilities, each element must be between zero and one and the sum of all elements must equal one. The state probability vector for the initial state Π_0 contains one element that is one while the rest are zero, indicating that the beginning state is known with certainty.

The movements of the system between points in time are given by an N by N transition matrix (P). The rows of P correspond to states at the current point in time while the columns correspond to states at the next point in time. The elements of the i'th row give the probability of moving from state i to any of the N possible states at the end of the period.

Multiplication of the initial state probability vector by the transition matrix yields the state probability vector for the next time period:

$$\Pi_L = \Pi_{L-1} P$$

The i'th element of the Π_1 vector gives the probability of being in state i in period one conditional on the probabilities in the Π_0 vector. In general, the transition during time period L is given by:

$$\begin{aligned}\Pi_L &= \Pi_{L-1} P \\ &= \Pi_{L-2} P P \\ &= \Pi_0 P^L\end{aligned}$$

where P^L indicates that the P matrix is postmultiplied L times. The Π_L vector gives the probabilities of being in each of the states at time L. These probabilities represent an ex ante forecast given the initial state.

The investment model's state probability vector contains five dimensions with each element represented as:

$$\pi_t(hr, ps, rof, of, s)$$

where hr,ps,rof,of,and s are state interval indices for hog returns, stock price, interest rate, financial holdings and stock holdings respectively.

The same state variable discretation was used in solving the conditional probabilities as was used in solving the dynamic programming model. In addition, a bankruptcy state was added. This state accumulated the probability associated with state intervals resulting in bankruptcy. Thus, each state probability vector contained 43,561 elements

(4 hog return states x 15 stock price states x 6 interest rate states x 11 financial holdings states x 11 stock holdings level states + 1 bankruptcy state). Each element of the state probability vector representing a solvent state was referenced as:

$$\pi_t(hr, ps, rof, of, s)$$

where hr, ps, rof, of, and s were state interval indices for the hog return, stock price, return on other financial assets, financial holdings and stock holdings level respectively. The final element represented bankruptcy.

The transition matrix was constructed using the state transition equations and the optimal stock investment decision rule. This matrix was a square matrix of dimension 43,561. The 43,561st row represented bankruptcy. The remaining rows of the transition matrix were decomposed as follows:

$$[HR] \otimes [PSR] \otimes [OFS_{hr,ps,rof}]$$

where [HR] was the hog returns state transition matrix, [PSR] was the stock price--interest rate state transition matrix, and $[OFS_{hr,ps,rof}]$ were matrices giving other financial holdings and stock holdings. This partitioning was possible because the [HR] matrix depended only on the hog returns state transition equation and the [PSR] matrix depended only on the stock price and interest rate state transition equations. The [HR] matrix was a square matrix of dimension 4. The [PSR] matrix was a square matrix of dimension 90 (15 stock price states x 6 interest rate states). The $[OFS_{hr,ps,rof}]$ matrices were square matrices with dimension 121 (11 financial holding states x 11 stock holding states). To calculate these matrices, the hog return, stock price, and return on other financial asset states had to be known. Thus, there were 360 matrices, with one matrix corresponding to each hog return, stock price, and return on other financial asset combination. These matrices were calculated using all state transition equations and the optimal decision rule.

Within the computer program which calculated conditional probabilities, the [HR] and [PSR] matrices were calculated once and stored in RAM. Elements within the $[OFS_{hr,ps,rof}]$ were calculated as needed. By repeating the above process, conditional probabilities were found for a five year horizon.

Resulting state probability vectors represented a joint probability density function, conditional on the beginning state variable levels, presuming that the optimal decision rule was followed. Standard discrete probability techniques (see, for example, Hogg and Craig) were used to find a state variable's marginal distribution. This required finding the probability of being in each of the state variables' intervals. For example, the probability of occupying the of_i financial holdings state interval equaled:

$$Pr(of_i) = \sum_{hr} \sum_{ps} \sum_{rof} \sum_{s} \pi_t \ (hr,\ ps,\ rof,\ of_i,\ s)$$

Repeating the above operation for all financial holdings state intervals resulted in the marginal distribution. Marginal distributions were found for financial holdings and stock holdings levels.

Application of conditional probability theory (Hogg and Craig) to the marginal distributions developed above yields important information on the linkages between the state variables. The probability of being in the of_i finanical holdings state interval given a particular stock holding, s_i is calculated as:

$$Pr(of_i \mid s_i) \quad = \quad \frac{\sum_{hr} \sum_{ps} \sum_{rof} \Pi_t(hr, ps, rof, of_i, s_i)}{Pr(of_i)}$$

Completing this calculation for all financial holdings states gives the financial holdings distribution conditional on the stock holdings state s_i.

The effects of bankruptcy were investigated using this method. Financial holdings and stock holdings distributions conditional on being solvent were calculated. For example, the conditional probability of being in the of_i state equaled:

$$Pr(of_i \mid solvent) = Pr(of_i) \ / \ (1 \ - \ Pr(Bankrupt))$$

where Pr(bankrupt) equals the probability in the bankruptcy state. Repeating the above operation for all state intervals resulted in the conditional distribution. A similar process was conducted for the stock holdings interval.

Also, expected wealth levels were calculated using the state probability vector. This was accomplished by summing the result of each state interval's probability times each state interval's wealth.

6 CONDITIONAL PROBABILITY RESULTS

Conditional probabilities were calculated for three different initial financial holdings levels: -\$70,000, \$0, and \$70,000. The initial values for the remaining state variables were the same for the above three conditional probability calculations. These state variables were a \$10 hog return, a 130 stock price, a 10 percent interest rate, and a 0 unit stock holding level.

Panel A of figure 3 shows marginal financial holdings distributions at the end of years 1, 3, and 5 for an initial financial holdings level of \$0. At the end of year 1, the majority of the probability is in the negative financial holdings region, indicating that debt has been accumulated. Most of this debt results from stock purchases, as illustrated in panel B. This panel shows the probability associated with each stock holdings level. As can be seen, the majority of the probability is associated with stock holding levels greater than 1,600 units.

Over time, probabilities associated with financial holdings and stock holdings become more evenly distributed across the respective states. This is illustrated by the general flattening of both the financial holdings and stock holdings distributions in years 3 and 5 (Figure 3). More evenly distributed probability results because of wider possible ranges of stock prices and returns on other financial instruments. As adverse stock prices result, stock holdings will be reduced, resulting in higher financial holdings.

A correlation exists between stock holdings and other financial holdings. This is illustrated in figure 4, which shows financial holdings distributions conditional on stock holdings levels of 0, 600, and 1,800. As can be seen, higher stock holdings are associated with lower financial holdings. Higher stock holdings require debt purchases, yielding the resulting skewed distributions. Note also that the 0 stock holding level has considerable probability associated with financial holdings levels above $200,000. This high probability, along with the .23 marginal probability of having 0 stock holdings, suggest that stock price and interest rate combinations exist in which stock is not a wise investment. This result is supported by the decision rule presented earlier.

The bankruptcy probability in year 5 is .3343 for the initial financial holdings level of $0. As initial financial holdings increase, the probability of bankruptcy decreases. For example, initial financial holdings of -$70,000, $0, and $70,000 result in year 5 bankruptcy probabilities of .4564, .3097, and .2441, respectively. Initial financial holdings levels also have large impacts on wealth in year 5. Expected wealth levels of $292,467, $380,210, and $467,016 respectively result from initial financial holdings levels of -$70,000, $0, and $70,000.

The interaction between initial financial holdings, bankruptcy and ending financial and stock holdings distributions was of great interest. The results above indicated that beginning financial holdings affect both bankruptcy and ending stock and financial holdings distributions. In order to isolate the effect of bankruptcy on these distributions, we calculated these distributions conditional on solvency. The resulting distributions (figure 5) indicate that initial financial holdings have little impact on the distributions once bankruptcy is taken into account.

Higher initial wealth results in more probability being in the higher financial levels (panel A). This difference in probability, however, is relatively small when compared to the $140,000 difference in initial financial holdings. Stock holdings are much more closely distributed (panel B). The financial holdings and stock holdings distributions tend to converge towards some particular distribution, given that the firm is solvent and that the optimal stock investment rule is followed.

Figure 3 Yearly Conditional Distributions
$0 Beginning Financial Holdings

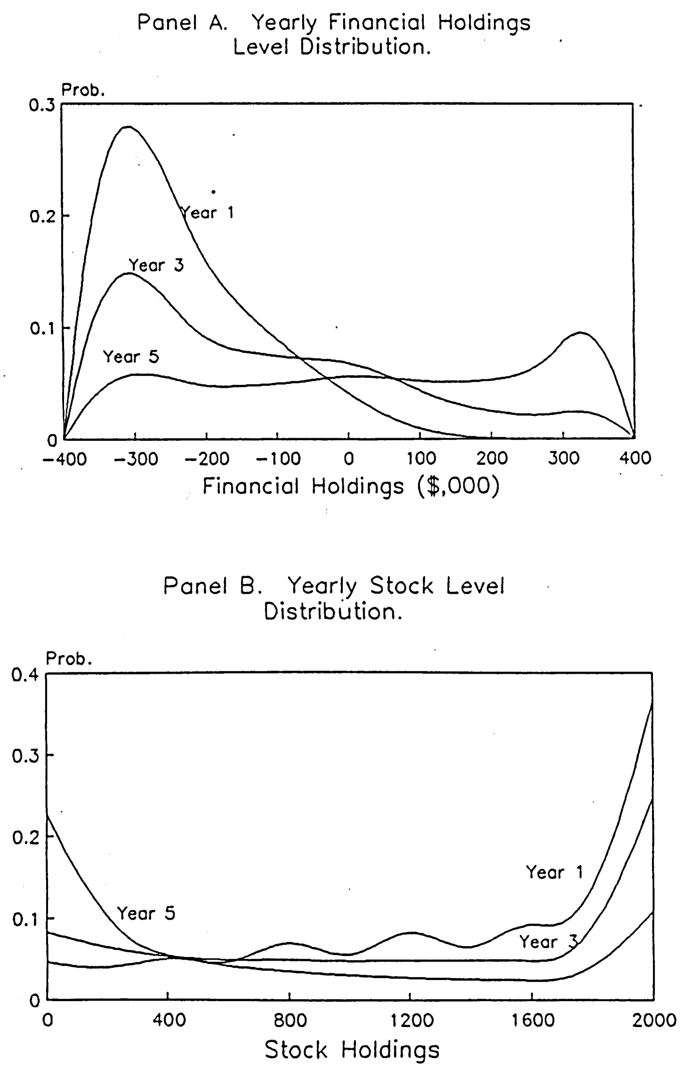


Figure 4 Financial Holdings Given Differing Stock Levels
Year 5

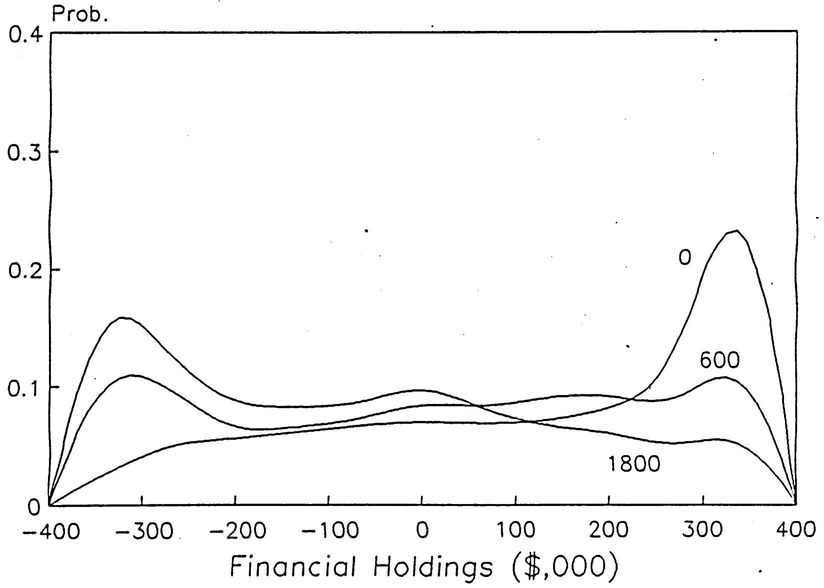
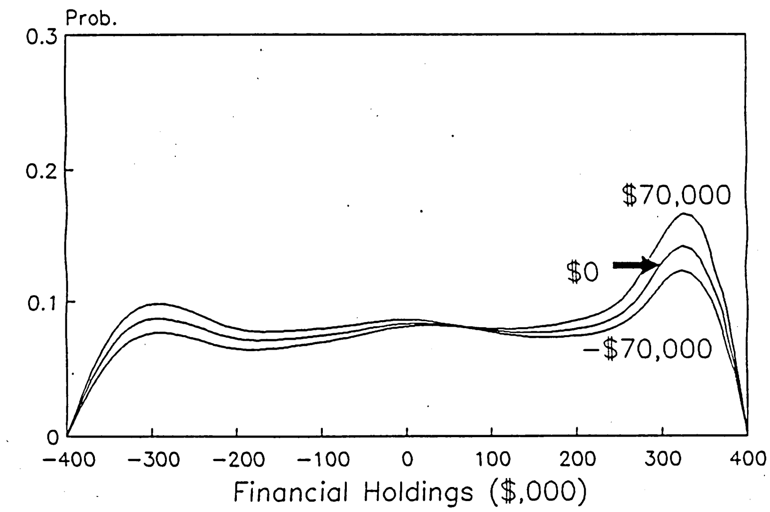
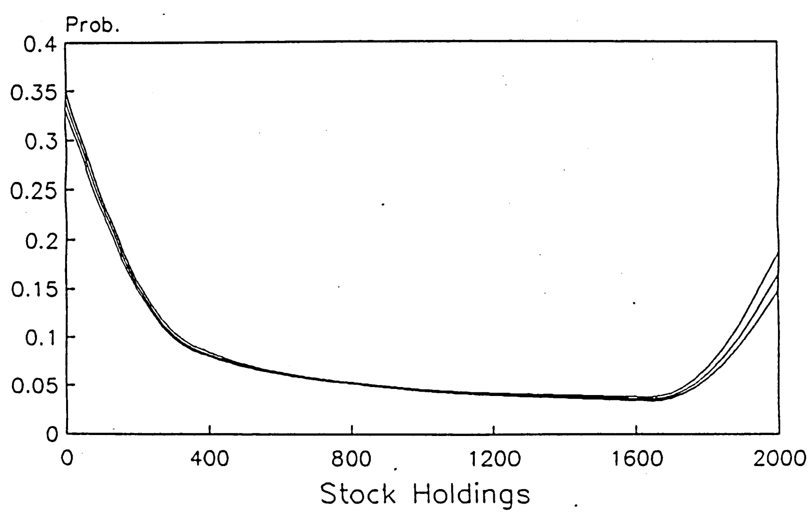


Figure 5 Conditional Distributions for Stock and Financial Holdings - Year 5

Panel A. Conditional Financial Holdings Distribution.



Panel B. Conditional Stock Holdings Distribution.



7 SUMMARY AND CONCLUSIONS

The first objective of this paper was to address the issue of off-farm investment in a dynamic framework, incorporating the effects of farm financial structure and market conditions. A second objective was to illustrate some of the important issues and techniques which arise when developing and solving stochastic dynamic firm level decision models.

Methods were presented for estimating transition probabilities for interrelated stochastic state variables and reducing model dimensions. Conditional probability methods which enhance the information available from stochastic dynamic programming models were discussed and illustrated.

The dynamic programming model of a hog finishing operation identified the effects of interest rate levels, stock prices and financial structure on optimal stock purchases. Higher interest rates were shown to dampen stock purchases over all ranges of financial holdings and stock prices. The absolute levels for buy/sell breakpoints were dependent upon both financial holdings and stock prices. The model also identified the effects of different hog return levels on investment decisions. Greater profits in hog production implied greater stock purchases for given levels of interest rates or financial holdings.

Conditional probability methods were used to project future financial structure and investment holdings for different beginning states. A correlation between stock holdings and financial holdings was identified and illustrated. Greater stock holdings implied the use of larger amounts of short term debt. Initial financial holdings were shown to affect the probability of bankruptcy and expected terminal wealth but had little influence on distributions of financial and stock holdings.

It is worth noting that static investment models would only provide information on the mix of investments given some static objective function. The dynamic investment model provides an operational investment strategy as well as providing ex ante forecasts of future financial structure and investment mix when the optimal decision rule is followed. The extra information obtained from the dynamic model should be of interest to decision makers.

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