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INEQUALITY CONSTRAINED ESTIMATION OF CONSUMER SURPLUS

W. L. Adamowicz

T. Graham-Tomasi

J. J. Fletcher

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This paper represents cooperative research among the authors; senior authorship is not assigned. The authors are Assistant Professor, Department of Rural Economy, University of Alberta, Edmonton, Alberta, Canada, Assistant Professor, Department of Agricultural and Applied Economics and Department of Forest Resources, University of Minnesota, St. Paul, Minnesota, and Associate Professor, Division of Resource Management, West Virginia University, Morgantown, West Virginia respectively. Partial support was provided by the Alberta Recreation, Parks and Wildlife Foundation.

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INEQUALITY CONSTRAINED ESTIMATION OF CONSUMER SURPLUS

Abstract

[Welfare measures derived from statistical demand functions are random variables. If the purpose of developing demand estimates is to derive welfare measures, estimation procedures should extend beyond consideration of demand relationships and concentrate on the measurement of welfare in an accurate and theoretically consistent fashion. In this paper we employ an explicitly Bayesian framework to estimate welfare measures under theoretical restrictions. An empirical example is presented using the travel cost recreation demand model. The result is a welfare measure that satisfies theoretical restrictions, is readily estimable and exhibits a lower variance than the corresponding unrestricted measure. The estimation process also provides a mechanism for evaluation of prior information and some insight into the choice of functional form for the demand function.]

Inequality Constrained Estimation of Consumer Surplus

Introduction

Welfare measurement is an integral component of economic analysis and the associated policy recommendations. Considerable effort has been devoted to the measurement of exact welfare measures and consumer surplus (Bockstael, Hanemann and Strand, 1984; Hanemann, 1982a; Hausman, 1981; Vartia, 1983). In practice these measures are derived from statistical demand functions and thus they are random variables. We are often interested in obtaining accurate estimates of economic welfare and not necessarily in estimates of demand *per se*. Hence we should concern ourselves directly with the estimation and statistical properties of the welfare measures. Estimation procedures should reflect this concern and concentrate on the measurement of welfare in an accurate and theoretically consistent fashion.

In this paper we address the implications of statistical demand estimation for welfare analysis. In particular, we examine the estimates of consumer surplus from demand functions estimated by ordinary least squares with a normally distributed error term. The random surplus so generated has properties which are not reasonable for welfare measures: First, it may be negative, and second, for some common functional forms it becomes unbounded as the price coefficient approaches zero. Since consumer surplus estimates approximate willingness to pay, and the latter is bounded by zero and income, these properties are theoretically undesirable (e.g. Just, *et al.*, 1982, p.86). The second effect also may cause the variance of the welfare measure to be quite high relative to its mean (Adamowicz, *et al.* 1989). As well, there may be other theoretical constraints which should be imposed prior to

welfare measurement¹. For example, integrability conditions, as described in Bockstael, *et al.* (1984), are required for consistency between the empirical demand relation and consumer theory.

We employ an explicitly Bayesian framework to examine the effects of imposing such prior restrictions on the welfare measure. Inequality constraints on the magnitude of consumer surplus are imposed using the approach developed by Geweke (1986) and Griffiths (1988). The result is a welfare measure that satisfies theoretical restrictions, is readily estimated, and exhibits a lower variance than the corresponding unrestricted measure. The estimation process also provides a mechanism for evaluation of our prior information and some insight into the choice of functional form for the demand function.

The Model

We examine here the linear and semilog forms of demand curves². Equations (1) and (2) are the linear demand and corresponding consumer surplus functions while equations (3) and (4) are the semilog demand and its welfare measure. Here, Y is the quantity variable and X is the price variable.

$$Y = \alpha_L + \beta_L X \quad (1)$$

$$CS_L = \frac{Y^2}{(-2\beta_L)} \quad (2)$$

¹ Preliminary work by Kling and Sexton (1989) addresses similar issues.

² These forms are chosen since they can be derived from a utility maximizing framework, see Bockstael, Hanemann and Strand, 1984.

$$\ln(Y) = \alpha_{SL} + \beta_{SL} X \quad (3)$$

$$CS_{SL} = \frac{Y}{(-\beta_{SL})} \quad (4)$$

Given an error term that is distributed normally, OLS estimation results in an estimate of β with a distribution that also is normal and thus ranges from minus to plus infinity. Using a Classical repeated sampling framework, there will be some draws of β^* which result in negative consumer surplus estimates (where β^* is the estimate of β). Similarly, the estimate of CS will approach infinity for those draws of β^* which are near zero, yielding surplus measures greater than income. In Figure 1 the OLS distribution of the estimated price coefficient in such a model is illustrated along with the implied consumer surplus. One method of eliminating the unboundedness feature is truncation of the demand curve; however, this approach is not theoretically consistent (see Adamowicz, *et al.*, 1989 or Bockstael, *et al.*, 1984).

In a Bayesian approach the distribution of β would not be examined in a repeated sampling sense but as the posterior resulting from the combination of a prior distribution and a likelihood function. Our task is to define a prior for the model which excludes the possibility of negative or infinite consumer surplus.

The constraint in this case is an inequality restriction on consumer surplus rather than on β but it is β that is constrained in the estimation procedure. The prior we suggest is that consumer surplus be between zero and income. A more restrictive prior may be imposed if one considers a branch

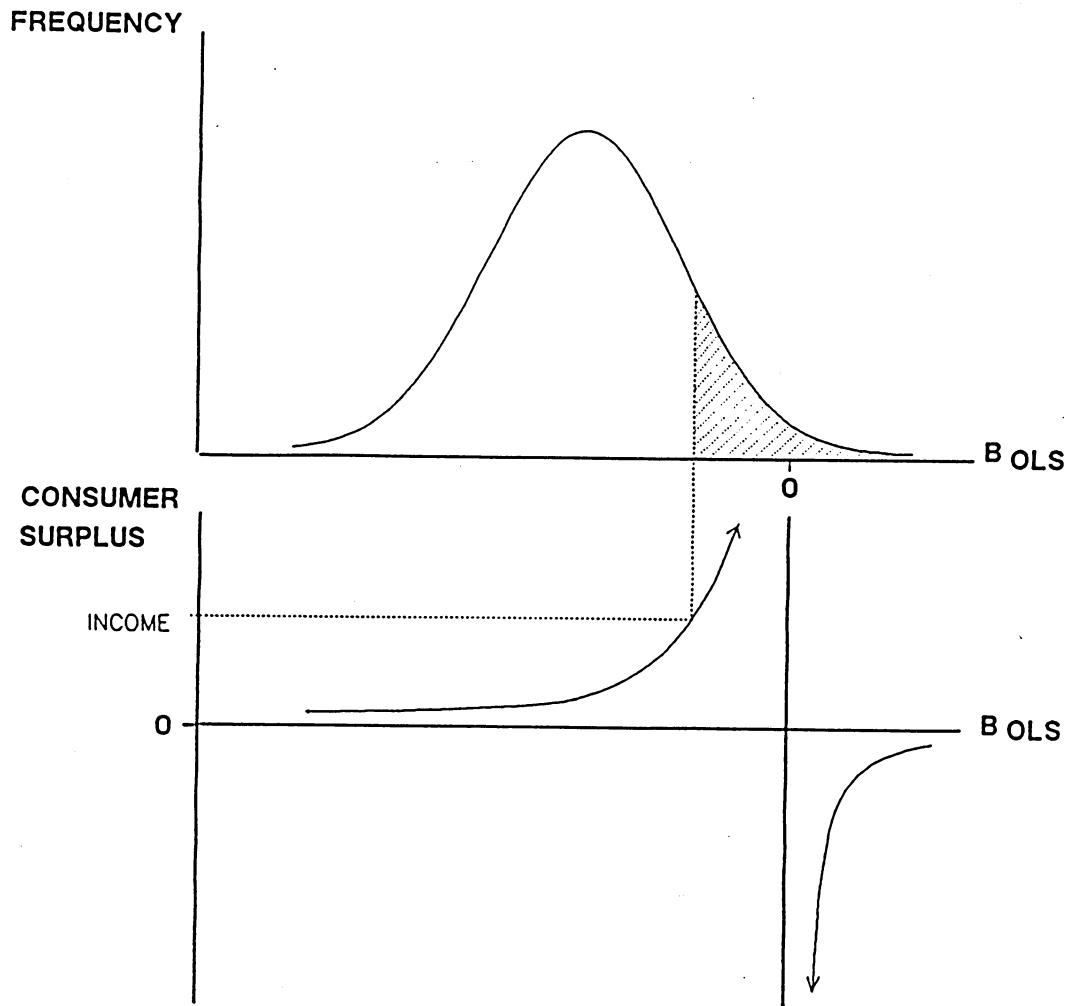


FIGURE 1: CONSUMER SURPLUS AND THE PRICE COEFFICIENT

budgeting process where the appropriate income measure within a branch is the expenditures on that branch (Hanemann, 1982b). In this case the welfare measure is bounded by zero and expenditures. Of course, other prior information could be incorporated as well.

The computation of the constraint is relatively simple in an individual setting. For any individual the relation

$$0 \leq CS \leq I$$

must hold, where I is income. Since CS is defined for the models above in terms of β and Y (the quantity demanded), we can represent the constraint for the linear model as

$$\beta_L \leq \frac{Y^2}{-2I} \quad (5)$$

and the constraint for the semilog model as

$$\beta_{SL} \leq \frac{Y}{-I} \quad (6).$$

However, some complexity is introduced in estimation of the parameters since this is based on a sample of individuals rather than a single individual. As income and quantities demanded vary over individuals the constraints in (5) and (6) must be imposed at the most general level. In the empirical section below we examine three options for this bound.

In estimating the constrained model we use an approach to Bayesian estimation under inequality constraints suggested by Geweke (1986). The prior for this estimation is defined as noninformative if the constraint is

satisfied and assigns zero probability to obtaining estimates outside of the constraint. Thus, the prior for the linear model, $g(\beta_L)$, is defined as

$$g(\beta_L) = \begin{cases} 1 & \text{if } \beta_L \leq \frac{Y^2}{-2I} \\ 0 & \text{otherwise.} \end{cases}$$

The prior for the semilog is defined similarly as

$$g(\beta_{SL}) = \begin{cases} 1 & \text{if } \beta_{SL} \leq \frac{Y}{-I} \\ 0 & \text{otherwise.} \end{cases}$$

Estimation of such a model is described in Geweke (1986) and by Chalfant and White (1988). The model is estimated using Monte Carlo methods employing the appropriate distributions. The theoretically correct distribution to use for the OLS model is a t-distribution; however in cases with larger samples the normal distribution provides a reasonable approximation. We provide the results from both the t and the normal distributions.

An interesting outcome of the constrained estimation procedure is the ability to evaluate the prior. The proportion of draws satisfying the prior can be interpreted as a measure of the support provided for the prior by the data.

Empirical Model

In our application of inequality constrained estimation of consumer surplus we employ a travel cost recreation demand model (see McConnell, 1985 for a detailed description of this model). The approach uses the number of visits to a recreation site as the quantity demanded, while the "price" is

the round trip travel cost (travel time is not included). The data employed are from a sample of recreational hunters collected in Alberta in 1983. The income variable is annual household income, given by the midpoint of the categorical income variable used in the survey instrument. We employ these data and a simple travel cost model merely to illustrate our approach; a more earnest attempt to value recreation would likely consider substitute sites (and prices), a more sophisticated treatment of time, and may use expenditures on recreational activities rather than household income.

The OLS results for the demand functions and the point estimates of CS are presented in Table 1. Both equations satisfy traditional statistical criteria for goodness of fit (F-statistics). Also presented in Table 1 are the point estimates of consumer surplus (as calculated using equations (2) and (4)). The point estimate is a biased (though consistent) estimator of expected surplus. In small samples, one can employ an approximation to the expected value of consumer surplus as suggested by Bockstael and Strand (1987). The approximation is the second order expansion of the expected value of a ratio of random variables given by ;

$$E\left(\frac{x}{y}\right) = \frac{E(x)}{E(y)} - \frac{\text{cov}(x,y)}{E(y)^2} + \frac{E(x) \text{ var}(y)}{E(y)^3} \quad (7).$$

In most cases the calculation of this approximation results in a zero covariance between the β coefficient and quantity demanded thereby eliminating the second term of (7). We also follow this procedure, while noting that this covariance may be non-zero in some cases. Also, this approximation may not be accurate for some models (see Graham-Tomasi, *et al.*, 1988). We examine the approximation for both the semilog and linear models and compare the results with the expected values of the empirically

generated distributions of consumer surplus.

The Bayesian procedure is carried out as follows. First, the OLS model is estimated. The independent variables are then used as the design matrix and repeated samples from the normal (and t) distribution are used to generate an empirical distribution of the estimated β coefficients. In this repeated sampling procedure we use antithetic replications; i.e. each randomly generated variable is used twice, once in its original form and once as the negative of the original. This aids in developing a symmetric distribution (see Geweke, 1986). We employ 5000 replications to form the empirical distributions of the OLS coefficients.

In our estimation procedure we employ three constraints to bound consumer surplus: Constraint 1 uses the lowest income in the sample and the maximum quantity demanded as the elements of the right hand sides of equations 5 and 6, Constraint 2 uses the lowest income and the mean quantity demanded, and Constraint 3 uses the minimum of the observed values of the right hand sides of equations (5) and (6).

On each draw in the sampling process the coefficient is checked to determine if it satisfies the constraints. If it does not, it is removed from the development of the constrained distribution. The expected value of the distribution is calculated using the mean of the retained draws. For the OLS (unrestricted) case the mean of the empirical distribution should equal the OLS coefficient. The means of the constrained distributions are the Bayesian coefficients. During each replication the consumer surplus is also computed: here, we report the mean consumer surplus over the sample of individuals³. Repeating this for each replication provides a distribution of

³ The results are similar if obtained for the surplus evaluated at sample

consumer surplus for each estimator. The means of these distributions provide the expected values of the surplus estimators. The point estimates, approximations of expected value given by (7) and variances of the empirical distributions are also calculated.

Results

When compared to the Bayesian estimates, the unrestricted or OLS estimates provide higher expected values of surplus and large variances, relative to their means, particularly in the linear model (see Table 2). The standard deviation is approximately ten times the mean for the expected values of the unrestricted distribution of surplus. The standard deviation of the surplus measure for the semilog unrestricted model also exceeds the mean. The constrained estimators provide expected values of surplus which are somewhat lower than the unconstrained estimates. The variances of the constrained estimates are considerably lower than those of the unconstrained estimates. For example, the expected value of the linear surplus measure with constraint 3 imposed is 322.92 with a standard deviation of 163.65. This measure of surplus is 25% lower than the unconstrained measure and the variance is many times smaller. In general, the tighter the bounds on consumer surplus (i.e., the smaller the right hand side terms for equations (5) and (6)), the smaller the consumer surplus and the smaller the variance. However, the expected values and variances across the constrained estimators do not differ as much as do the constrained and unconstrained estimators.

The comparison between the t-distribution method for estimation and the normal distribution suggests that either approach works well in determining

means. Once again the problem of aggregation over individuals arises, as the theory is based on individual welfare measures.

the truncated distribution of consumer surplus. In particular, the constrained estimates of surplus are very similar for either t or normal distribution Monte Carlo procedures.

The second order approximation (provided in Table 1) does not provide an exceedingly accurate measure of the expected value of the unconstrained welfare measure. However, the approximation performs reasonably well as a measure of the expected value of the truncated distribution. The approximation underestimates the expected value of surplus by only about 1 percent in the case of the linear model with the tightest constraint and 5 percent in the semilog model with the tightest constraint (using either the t-distribution results or the normal distribution results). This suggests that use of this approximation is valid if one considers a constrained estimation procedure or if the constraint is not violated by the OLS estimator, a point that is useful for applied work as the second order approximation is more easily computed than the Monte Carlo estimate.

The proportion of draws satisfying the constraints in either the semilog or linear model is very high. At least 97 percent of the draws satisfy the constraints in each case. Nevertheless, it is apparent that the semilog, with a minimum proportion of satisfactory draws of .9968, satisfies the constraint more often than the linear, which has a minimum proportion of draws of .9796. This fact may be useful in the choice of a functional form. Similar tests may be applied to integrability constraints or other priors suggested by theory.

Conclusions

The application of explicit Bayesian procedures to estimation of welfare

measures is useful theoretically and empirically. Bayesian estimation may provide a welfare measure that is consistent with theory. Our application resulted in inequality constrained estimation procedures that are readily available (e.g. Bayesian inequality constrained estimation is now an option in SHAZAM, see White *et al.*, 1988). The availability of other prior information may imply different empirical procedures.

The Bayesian estimates of the expected value of consumer surplus we obtained are lower in variance than their unconstrained counterparts. The support provided by the data for the prior information can easily be calculated and interpreted. In the case of welfare measures from these models, Bayesian estimation can provide an additional criterion for judging between models. Finally, this example provides some support for the use of the approximation of expected value of surplus as long as the approximation is considered in terms of a Bayesian inequality constrained estimator.

Table 1: Results of OLS Regressions and Estimates of Consumer Surplus

LINEAR MODEL				
Var	Coef	Std. Error	t-Stat	P-Value
CONST	2.832428	0.513356	5.52	0.000
TRAVEL COST	-0.015445	0.005960	-2.59	0.011
INCOME	0.000011	0.000013	0.91	0.367

Observations:	96	Degrees of freedom:	93
R-squared :	0.071	Rbar-squared :	0.051
Residual SS :	219.099	Std error of est :	1.535
Total SS :	235.958	F(3 , 93)=3.57	P-value=0.02

POINT ESTIMATE OF CONSUMER SURPLUS (EQUATION 2): 278.54

EXPECTED VALUE OF CONSUMER SURPLUS USING SECOND ORDER APPROXIMATION: 320.02

SEMILOG MODEL				
Var	Coef	Std. Error	t-Stat	P-Value
CONST	0.877335	0.198184	4.43	0.000
TRAVEL COST	-0.007641	0.002301	-3.32	0.001
INCOME	0.000006	0.000005	1.24	0.216

Observations:	96	Degrees of freedom:	93
R-squared :	0.113	Rbar-squared :	0.094
Residual SS :	32.654	Std error of est :	0.593
Total SS :	36.833	F(3 , 93)=5.95	P-value=0.00

POINT ESTIMATE OF CONSUMER SURPLUS (EQUATION 4): 324.46

EXPECTED VALUE OF CONSUMER SURPLUS USING SECOND ORDER APPROXIMATION: 353.88

Table 2: Results of the Inequality Constrained Estimation Procedure
Coefficient Estimates and Consumer Surplus Estimates

MODEL	PRICE COEFFICIENTS			
	NORMAL DISTRIBUTION		T-DISTRIBUTION	
	LINEAR	SEMILOG	LINEAR	SEMILOG
Unrestricted				
Expected Value	-.015475	-.007587	-.015522	-.007630
Std. Dev.	.005985	.002311	.005927	.002244
Draws Satisfying	5000	5000	5000	5000
Constraint 1 ^a				
Expected Value	-.015782	-.007609	-.015819	-.007645
Std. Dev.	.005643	.002280	.005599	.002221
Draws Satisfying	4898	4984	4900	4990
Constraint 2 ^b				
Expected Value	-.015596	-.007593	-.015640	-.007636
Std. Dev.	.005827	.002302	.005776	.002234
Draws Satisfying	4965	4996	4965	4996
Constraint 3 ^c				
Expected Value	-.015636	-.007598	-.015658	-.007641
Std. Dev.	.005748	.002294	.005756	.002227
Draws Satisfying	4952	4992	4959	4993

MODEL	CONSUMER SURPLUS ESTIMATES			
	NORMAL DISTRIBUTION		T-DISTRIBUTION	
	LINEAR	SEMILOG	LINEAR	SEMILOG
Unrestricted				
Expected Value	433.20	389.04	356.37	365.37
Std. Dev.	4826.51	492.08	3167.16	519.24
Draws Satisfying	5000	5000	5000	5000
Constraint 1 ^a				
Expected Value	322.92	368.79	319.53	360.96
Std. Dev.	163.65	173.08	159.99	148.56
Draws Satisfying	4898	4984	4900	4990
Constraint 2 ^b				
Expected Value	356.92	377.63	346.88	365.95
Std. Dev.	408.34	259.91	332.15	210.60
Draws Satisfying	4965	4996	4965	4996
Constraint 3 ^c				
Expected Value	340.44	373.63	339.60	362.79
Std. Dev.	242.18	206.29	252.55	166.42
Draws Satisfying	4952	4992	4959	4993

^a Constraint 1: maximum quantity demanded and minimum income.

^b Constraint 2: average quantity demanded and minimum income.

^c Constraint 3: minimum observed value of equations (5) and (6).

References

- Adamowicz, W.L., J.J. Fletcher and T. Graham-Tomasi. 1989. Functional Form and the Statistical Properties of Welfare Measures. *American Journal of Agricultural Economics*. 71:(in press).
- Bockstael, N.E., W.M. Hanemann and I.E. Strand. 1984. Measuring the Benefits of Water Quality Improvements Using Recreation Demand Models. Draft Report EPA No. CR-811043-01-0, Department of Agricultural and Resource Economics, University of Maryland. College Park, Maryland.
- Bockstael, N.E. and I.E. Strand. 1987. The Effect of Common Sources of Measurement Error on Benefit Estimates. *Land Economics*. 63:11-20.
- Chalfant, J. and K.J. White. 1988. Estimation of Demand Systems with Concavity and Monotonicity Constraints. University of British Columbia Discussion Paper, Vancouver, B.C.
- Geweke, J. 1986. Exact Inference in the Inequality Constrained Normal Linear Regression Model. *Journal of Applied Econometrics*. 1: 127-141.
- Graham-Tomasi, T., W.L. Adamowicz and J.J. Fletcher. 1988. On Approximations to Expected Consumer Surplus. University of Minnesota, Department of Agricultural and Applied Economics Working Paper # P88-44. St. Paul Minnesota.
- Griffiths, W.E. 1988. Bayesian Econometrics and How to Get Rid of Those Wrong Signs. *Review of Marketing and Agricultural Economics*. 56:36-56.
- Hanemann, W.M. 1982a. Some Further Results on Exact Consumer's Surplus. University of California Working Paper. Berkeley, California.
- Hanemann, W.M. 1982b. Welfare Analysis Based on Systems of Partial Demand Functions. University of California Working paper No. 243. Berkeley, California.
- Hausman, J.A. 1981. Exact Consumer's Surplus and Deadweight Loss. *American Economic Review*. 71:662-676.
- Just, R.E., D.L. Hueth and A. Schmitz. 1982. *Applied Welfare Economics and Public Policy*. Englewood Cliffs New Jersey: Prentice Hall Inc.
- Kling, C.L. and R.J. Sexton. 1989. Estimating the Statistical Properties of Consumer Welfare Measures Using Bootstrapping Methods. Paper presented at the W-133 Western Regional Research Meeting, San Diego, California. February.
- McConnell, K.E. 1985. The Economics of Outdoor Recreation. in A.V. Kneese and J.L. Sweeney editors, *Handbook of Natural Resource and Energy Economics*. Vol II, pp. 677-722. New York: Elsevier Science Publishers Ltd.
- Vartia, Y.O. 1983. Efficient Methods of Measuring Welfare Change and Compensated Income in Terms of Ordinary Demand Functions. *Econometrica*. 51: 79-98.

White, K.J. S.A. Haun, N.G. Horsman and S. D. Wong. 1988. SHAZAM Econometrics Computer Program. New York: McGraw Hill Book Company.

APPENDIX: DATA AND SUMMARY STATISTICS

VISITS	TRAVEL COST	INCOME	VISITS	TRAVEL COST	INCOME
6.0	66.80	22500.00	7.0	32.00	32500.00
3.0	40.40	12500.00	1.0	81.60	37500.00
2.0	59.20	42500.00	6.0	18.40	32500.00
1.0	66.80	17500.00	2.0	68.00	50000.00
6.0	40.40	27500.00	1.0	44.00	27500.00
1.0	66.80	42500.00	2.0	46.40	12500.00
7.0	40.40	32500.00	1.0	38.00	7500.00
5.0	46.40	32500.00	1.0	81.60	32500.00
2.0	39.60	22500.00	1.0	119.20	17500.00
1.0	40.40	17500.00	2.0	56.40	50000.00
4.0	40.40	50000.00	3.0	66.80	50000.00
4.0	40.40	12500.00	2.0	18.40	32500.00
3.0	40.40	37500.00	3.0	2.00	27500.00
1.0	40.40	17500.00	2.0	46.40	50000.00
1.0	66.80	32500.00	4.0	40.40	27500.00
2.0	52.40	7500.00	3.0	40.40	27500.00
1.0	2.00	27500.00	2.0	27.20	12500.00
4.0	46.40	7500.00	2.0	18.40	27500.00
4.0	46.40	27500.00	2.0	50.00	27500.00
4.0	66.80	50000.00	5.0	40.40	50000.00
5.0	18.40	50000.00	1.0	66.80	42500.00
1.0	52.40	50000.00	1.0	46.40	22500.00
2.0	40.40	50000.00	5.0	66.80	37500.00
3.0	40.40	37500.00	1.0	81.60	32500.00
1.0	52.40	50000.00	2.0	39.60	50000.00
1.0	85.20	17500.00	2.0	33.60	50000.00
4.0	40.40	27500.00	4.0	39.60	22500.00
3.0	83.60	27500.00	2.0	105.60	37500.00
6.0	46.00	32500.00	5.0	40.40	50000.00
2.0	2.00	22500.00	1.0	66.80	50000.00
4.0	2.00	32500.00	3.0	18.40	37500.00
1.0	66.80	27500.00	2.0	18.40	37500.00
1.0	4.00	37500.00	2.0	14.00	12500.00
2.0	40.40	22500.00	4.0	40.40	50000.00
1.0	18.40	42500.00	2.0	52.40	50000.00
1.0	100.00	50000.00	2.0	40.40	32500.00
2.0	40.40	50000.00	2.0	18.40	32500.00
2.0	40.40	32500.00	1.0	40.40	42500.00
1.0	66.80	32500.00	2.0	2.00	27500.00
1.0	66.80	27500.00	1.0	40.40	32500.00
3.0	18.40	50000.00	1.0	48.80	12500.00
3.0	18.40	42500.00	4.0	27.20	50000.00
4.0	66.80	32500.00	4.0	40.40	27500.00
1.0	100.00	27500.00	3.0	80.40	50000.00
3.0	18.40	50000.00	1.0	26.80	27500.00
3.0	66.80	50000.00	1.0	40.40	32500.00
1.0	52.80	22500.00	1.0	144.00	37500.00
1.0	111.20	50000.00	1.0	67.20	50000.00

SUMMARY STATISTICS

	MEAN	STD. DEV.	MIN.	MAX.
VISITS	2.48	1.57	1.00	7.00
TRAVEL COST	47.84	26.54	2.00	144.00
INCOME	33906.25	12616.89	7500.00	50000.00