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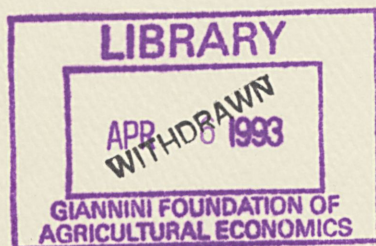
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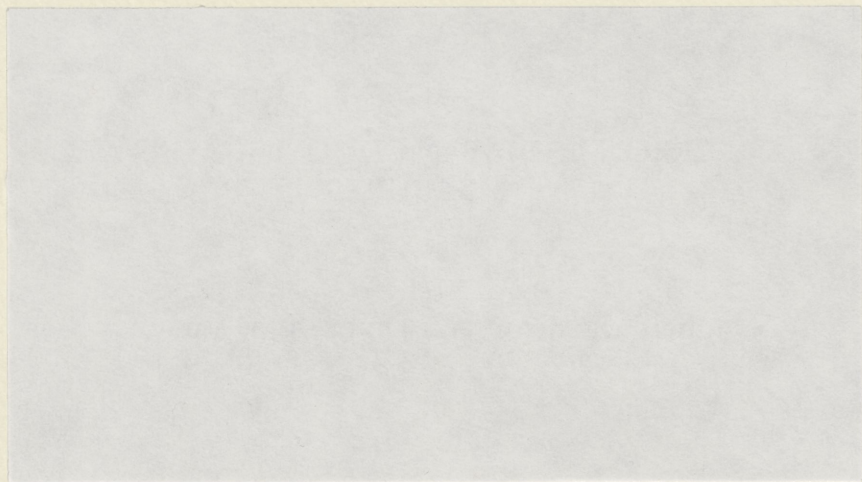
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MEASURES

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**Working Paper 92-14**

**Department of Agricultural Economics**

**University of California, Davis**

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## Introduction

Agricultural economists and environmental economists have much in common, yet the approaches used in welfare economics applied to markets for natural resources differ from those used in commodity market applications. In particular, environmental economists often use Hicksian "exact" welfare measures while agricultural economists typically use Marshallian surplus measures. Why this is so is not clear. One hypothesis is that agricultural economists have been slow to adopt the alternative approaches because they have an established tradition that rests on a belief that Marshallian measures provide an acceptable approximation, while environmental economics has no such tradition and there has been no reason not to adopt the measures that have greater theoretical support. An alternative explanation might be that each field is using the appropriate measures for its applications. Then the underlying explanations for the differences in approaches may include differences in the types of markets being analyzed, differences in the types of questions being addressed, or differences in the types of data that are available, that could mean that the Marshallian measures are adequate for agricultural policy but not for environmental policy.

The main purpose of this paper is to explore these issues and to develop arguments about whether agricultural and environmental economists ought to change their ways. With this in mind we review some recent developments in applications of welfare economics methods in those fields, in particular the emerging interest in precision, as well as bias, of welfare measures. Perhaps the most important contribution of the paper is to explore the implications of the analyst's uncertainty about parameters for the choice of welfare measure. We show that a mean squared error criterion can be used to evaluate the bias-variance tradeoff in specific empirical studies, and simulate mean-squared errors of consumer's surplus and Hicksian

willingness-to-pay measures of welfare change in a wide range of circumstances likely to be encountered in practice. For a demand specification widely used in empirical studies, the semilog functional form, the mean squared error criterion gives intuitively-sensible advice about which welfare measure is preferred, depending on the magnitudes and precision with which consumer surplus and the income coefficient are measured, as well as their covariance. A particularly interesting finding is that, counter to expectations *a priori*, great precision in measuring the income coefficient is not required for the Hicksian measure to emerge as the preferred measure for this functional form, though as one would expect the Marshallian measure is preferred if either the consumer's surplus estimate is very precise or the income coefficient is very imprecise.

We begin, as is common, with a treatment of the issues based on the linear model. This is useful for raising several important, but neglected, issues that arise when the question of Hicksian versus Marshallian welfare measurement is applied to more general settings. We then turn to a more specific analysis, using the mean squared error criterion to evaluate the choice between exact Hicksian and Marshallian measures based on the semilog functional form.

### Theoretically "Correct" Welfare Measures

Figure 1 represents a commodity market with an initial equilibrium price and quantity of  $P_0$  and  $Q_0$  determined by the intersection of supply ( $S$ ) and ordinary (Marshallian) demand ( $D$ ) and, when an output subsidy of  $t$  per unit is imposed, the price and quantity are  $P_1$  and  $Q_1$ . The standard Marshallian welfare measures are the change in *consumer's surplus* ( $CS = \text{area } P_0abP_1$ ), the change in *producer's surplus* ( $PS = \text{area } P_0acd$ ), the change in *taxpayer's surplus*



or government revenue ( $TS = - \text{area } P_1bcd$ ), and the net change in national surplus ( $NS = CS+PS+TS = - \text{area } abc$ ). The latter is the Harberger triangle of deadweight loss.

Assuming (approximately) linear supply and demand, defining the elasticities of supply and demand evaluated at the new equilibrium as  $\epsilon$  and  $\eta$ , respectively ( $\eta < 0$ ), and defining  $\tau = t/P_1$ , the changes in Marshallian surplus are measured, using simple geometry, as:

$$\begin{aligned} CS &= P_1 Q_1 \frac{\tau \epsilon}{\epsilon - \eta} \left[ 1 + \frac{1}{2} \frac{\tau \epsilon \eta}{\epsilon - \eta} \right]; & PS &= -P_1 Q_1 \frac{\tau \eta}{\epsilon - \eta} \left[ 1 + \frac{1}{2} \frac{\tau \epsilon \eta}{\epsilon - \eta} \right]; \\ TS &= -\tau P_1 Q_1; & NS &= \frac{1}{2} P_1 Q_1 \frac{\tau^2 \epsilon \eta}{\epsilon - \eta}. \end{aligned} \quad (1)$$

The corresponding Hicksian measures differ only in the measure of consumer welfare change. The *compensating variation* measure of consumer welfare change for the price change is measured off the Hicksian demand  $h(u_0)$  holding utility at  $u_0$  (the value associated with  $P_0, Q_0$ ):  $CV = \text{area } P_0ab'P_1$ . The *equivalent variation* measure is taken off  $h(u_1)$ :  $EV = \text{area } P_0a'bP_1$ .

It is convenient for our purposes to compare the estimates of welfare change from linear approximations to the Marshallian and Hicksian demand equations. Hausman argued that such approximations are unnecessary, at least in the case of a single price change.<sup>1</sup> However, the linear approximation is probably very good for small changes (errors in approximations of curvature probably don't matter much) and it is helpful for illustrating the issues.

Taking a linear approximation to  $h(u_1)$  in figure 1 between  $a'$  and  $b$ , and defining the elasticity of this demand curve as  $\eta^H$  at the equilibrium, the  $EV$  measure of consumer benefit is

$$EV = P_1 Q_1 \frac{\tau \epsilon}{\epsilon - \eta} \left[ 1 + \frac{1}{2} \frac{\tau \epsilon \eta^H}{\epsilon - \eta} \right]. \quad (2)$$

The only difference between this equation and that for consumer's surplus is the use, in the last term, of the Hicksian demand elasticity ( $\eta^H$ ) rather than the Marshallian elasticity ( $\eta$ ), thus

$$EV - CS = \frac{1}{2} P_1 Q_1 \left[ \frac{\tau \epsilon}{\epsilon - \eta} \right]^2 (\eta^H - \eta) = \frac{1}{2} P_1 Q_1 k \eta_Y \left[ \frac{\tau \epsilon}{\epsilon - \eta} \right]^2, \quad (3)$$

where  $\eta_Y$  is the income elasticity of demand for the good,  $k = P_1 Q_1 / Y$  is the fraction of total income ( $Y$ ), spent on the commodity, and we have used the Slutsky equation in elasticity form for  $k \eta_Y = \eta^H - \eta$ .<sup>2</sup> Because we are taking linear approximations,  $CS - CV = EV - CS$ . Willig showed that Marshallian consumer's surplus will provide a good approximation to the exact Hicksian measures when a small value is taken by the expenditure share,  $k$ , or the expenditure elasticity,  $\eta_Y$ .<sup>3</sup> Hausman pointed out that the same error would be relatively big as a fraction of the deadweight loss (e.g., see figure 1), and might be too great to be tolerated in some cases, depending on the parameters. The difference in deadweight loss between the approaches is:

$$NS^{CV} \approx NS^{CS} \left[ 1 + \left[ \frac{\epsilon \eta}{\epsilon - \eta} \right] k \eta_Y \right]; \quad NS^{EV} \approx NS^{CS} \left[ 1 - \left[ \frac{\epsilon \eta}{\epsilon - \eta} \right] k \eta_Y \right]. \quad (4)$$

These results and observations that apply in the analysis of tax or subsidy interventions, might not apply with the same force in the analysis of the welfare impacts of a research-induced supply shift. The impacts of the parallel supply shift from  $S$  to  $S-t$  on consumer welfare and producer's surplus are identical to those from the subsidy ( $t$ ). However, there are no taxpayer welfare effects and thus the total welfare change is very different from that of the subsidy.

$$NS = P_1 Q_1 \tau \left[ 1 + \frac{1}{2} \frac{\tau \epsilon \eta}{\epsilon - \eta} \right]. \quad (5)$$

The Hicksian measure is equal to this Marshallian measure plus or minus the same error as before, which will be much smaller as a percentage of the net gain in this case.

Table 1 shows the error in consumer's surplus (also as a percentage of consumer welfare and total welfare) for a subsidy and for an equivalent research-induced supply shift. The

normalization that  $P_1 Q_1 = 100$  expresses the error, effectively, as a percentage of industry revenue. For low values of either the income share ( $k = 5\%$ ) or the income elasticity ( $\eta_Y = 0.2$ ) the error is negligible (much less than 0.1%), compared with both the consumer welfare change and the net research benefits, and tolerable compared with the deadweight loss from a subsidy (less than 6.7%). The error is significant as a share of the deadweight cost of the subsidy but tolerable relative to consumer or national research benefits, even when  $\eta_Y = 1$  and  $k = 25\%$ . The size of  $\tau$  does not affect the error as a fraction of deadweight loss but it does affect the size of the error and the error as a fraction of consumer benefits.<sup>4</sup>

For many agricultural goods in richer, industrialized countries such as the United States, the income elasticities are less than one and the expenditure shares are less than 5% (the very low end of the range shown) so that the Marshallian measures are likely to provide an adequate approximation. The small expenditure share is likely to be the most important factor, here, for most food commodities. However, in applications to less-developed countries, where income elasticities are likely to be larger, and where staple food commodities account for a substantial fraction of income, the approximation to deadweight loss could be very poor.

Such analysis has led to the conclusion in the literature that Hicksian measures are theoretically more accurate but the importance of the difference is an empirical matter, depending on income elasticities and expenditure shares. Clearly in some cases the benefits of improved accuracy, from correcting a Marshallian welfare measure for income effects, are negligible. Some economists have argued that, nevertheless, the corrections ought to be made because there is little justification for using a biased measure (e.g., Hausman; McKenzie and Pierce) and the corrections are easy and inexpensive to make.



## Current Practice in Agricultural and Resource Economics

Table 2 provides an informal, and somewhat stylized, description of differences in problems that agricultural and resource economists analyze. Often agricultural economists are concerned about inefficiencies accompanying policies that introduce or modify market distortions; thus relatively small price changes and welfare triangles are important. Environmental economists, on the other hand, have often been interested in policies that may preclude, or make possible, certain types of uses of natural resources or the environment; thus large price changes, to represent changes in value associated with provision or removal of a good, have also been of interest.

There are striking differences also in the nature of the data used by each group. Agricultural economists often analyze time series data aggregated over individuals, whereas environmental economists most often use cross-section data. Goodness of fit is often very high in the former and very low in the latter. This has led to differences in the emphasis placed on problems encountered in the practice of applied welfare economics. For example, the issue of aggregation has received relatively more attention in the agricultural economics literature, whereas the precision of welfare estimates has concerned environmental economists much more than agricultural economists. Similarly, the fact that agricultural economists study commodity markets has led to more concern and inquiry into the implications of multi-market welfare measures and the empirical significance of partial versus general equilibrium measures. In contrast, environmental economists study non-marketed commodities for which prices are non-existent and for which data are not systematically collected on likely substitutes for any given environmental amenity. Thus, single-equation demand models are much more commonly estimated and multiple-market considerations are, for the most part, beyond reach empirically.

Another interesting area of both similarity and difference is in the study of non-price changes, which for agricultural economists often represents the effects of commodity promotion on demand or research-induced supply shifts. Another occasion is measuring welfare changes in related markets, say for the case of measuring the change in welfare due to an output supply shift in an input market. All of these changes involve measuring areas between curves as they shift due to a change in a non-price argument, and the condition that permits these areas to be taken as the full benefits of the parameter change is essentiality of the relevant input or output to production (see, e.g., Just, Hueth, and Schmitz, Appendix D, pp. 446-451). The analogous condition in environmental economics is "weak complementarity" of the non-price argument (e.g., environmental quality) with a set of private market goods; if this condition holds, it proves possible to measure the benefits of the quality changes from the areas between the Hicksian demands for the private goods as they shift with the quality change.

A major difference also arises between the fields in the analysis of non-price changes. While the input or output essentiality condition is reasonable almost always in agricultural economics applications, an emerging area of concern in environmental economics is "nonuse value" (e.g., McConnell 1983; Freeman), the willingness to pay of individuals for improvements in environmental amenities they do not themselves directly use. Examples include valuing visibility improvements in the Grand Canyon, or willingness to pay for old growth forest protection, by people who have no expectation of visiting such areas. To the extent that this category of benefits is large, as many argue is the case for prominent irreplaceable environmental amenities with few or no substitutes, the weak complementarity assumption about preferences is not suitable because one of its implications is that nonuse value is zero.

### *Environmental Economics Applications*

Three main types of applications seem to dominate welfare analyses in environmental economics. They are (a) analysis of large price changes to represent provision or removal of a valued private good related to an environmental amenity, (b) changes in the quality of a private good which corresponds to changes in environmental quality, and (c) changes in the quantity of a public good which is available for consumption.

The situation which often characterizes (a) is recreational use of public lands. A question frequently asked concerns the value of such lands when committed to recreational purposes, and sometimes an all-or-nothing value is needed, for example when a choice must be made to commit the lands in question between use for recreation and another, incompatible use (e.g., clearcutting). Given a demand curve for the recreational activity, conditioned suitably for substitutes, raising the price to the level which chokes off demand is a convenient way of measuring the total value of the recreational activity.

Situation (b) arises in a wide variety of circumstances when environmental quality affects one or more market goods. Cases which have been considered include demands for housing stock, medical care, recreation, and labor. Typically the value of environmental quality is inferred as the difference in areas between Hicksian demands conditioned on the initial and subsequent levels of the quality variable, provided preferences for the private goods and the public good (environmental quality) are characterized by weak complementarity.<sup>5</sup>

Case (c) is similar to (b), but the welfare measurement problem is more direct since it is expressed as a change in the public good directly, rather than as a difference in two integrals over price. This is convenient, pedagogically, when describing the analytical framework used



when data are collected through direct questioning, or contingent valuation (see, e.g., Randall and Stoll or Mitchell and Carson). It is typically less useful for the approach of attempting to make inferences from observable behavior with respect to public goods, since there are not many mechanisms for observing demands for public goods directly. At least two significant problems arise: first, since the quantity of the public good is typically exogenous, there is not a meaningful quantity choice problem; and second, since there is no market, there is no direct observation of the "price" variable.

#### *Agricultural (Commodity) Market Applications*

Three major types of applications of welfare economics in relation to commodity markets are (a) the analysis of commodity market distortions, especially those associated with domestic farm programs or border distortions for traded commodities, (b) the analysis of the benefits and costs of research-induced technological change, and (c) political economy models of commodity programs or investments in agricultural R&D (in which measures of welfare impacts are data). In the literature on these three types of applications of welfare economics the analysis has almost exclusively used Marshallian surplus measures obtained from comparative statics conducted in partial equilibrium (single-market) models. This approach to analyse welfare impacts in agricultural commodity markets is so well entrenched that basic references on policy (e.g., Gardner 1988; McCalla and Josling) do not entertain the alternatives to any significant extent.

Various rationales are possible. First, it may be thought that the error in using Marshallian rather than Hicksian measures is *comparatively* small. In the evaluation of returns to research, the dominant sources of uncertainty about the measures of the size and distribution

of benefits are uncertainty about the size and the nature of the research-induced supply shift and the elasticities of supply and demand (e.g., Lindner and Jarrett). For instance, producers necessarily are made worse off by a pivotal research-induced supply shift against an inelastic demand function, whereas the total benefit is roughly doubled and producers necessarily gain when the supply shift is parallel rather than pivotal. Given the virtual impossibility of identifying the nature of a research-induced supply shift *ex ante*, the virtue in correcting for the relatively small error in Marshallian surplus measures is questionable.

Similarly, in the evaluation of the impact of commodity programs, other sources of inaccuracy may be more important by orders of magnitude than the error from using producer and consumer's surplus (except perhaps in relation to deadweight loss measures). Examples of other issues to consider, that are left out from the typical analysis, include (a) cross-commodity effects when there are distortions (due to either policies, imperfect competition, or externalities) in related commodity markets, (b) technological or pecuniary distortions in factor markets, and (c) dynamics, imperfect information, incomplete markets, and risk. An important further complication introduced in some of the recent literature concerns the appropriate valuation of government revenues. The conventional analysis assumes a dollar of government spending has a social opportunity cost of a dollar but several studies of commodity policy (e.g., Gardner 1983, and Alston and Hurd) have argued for charging the marginal social opportunity cost of taxation, which has been evaluated as significantly greater than one dollar per dollar of expenditure (see Fullerton). Assumptions about the opportunity cost of government revenues are likely to have a much more serious impact on the calculation of welfare impacts and deadweight costs than any corrections for income effects in the consumer's surplus calculation.

On the whole, in many cases it seems likely that the increased precision from using Hicksian measures could be relatively unimportant compared with the other matters. In addition, there are pedagogic and computing advantages from using the measures of changes in quantities and prices (from the analysis of the price and quantity effects of the program) for evaluating the welfare impacts. A counter-argument is that it in most cases of commodity policy analyses it would be very cheap to adjust the Hicksian measures for income effects (at least as a first approximation) using data on income elasticities and shares and there is little justification for using a measure that is known to be biased.

One story that can be told is that for a majority of commodity market studies there are "good" observations of consumption, production, and prices that can be assumed to lie upon relevant Marshallian supply and demand functions. Partial equilibrium comparative statics conducted in such a context (e.g., tax incidence) yield measures of changes in prices and quantities that can be drawn on a simple diagram and used, with simple transformations, to depict welfare areas. Similarly, quantitative measures of welfare change are readily derived as simple transformations of the supply and demand equations. In such a context, invoking Harberger's three postulates, welfare measures follow from a market equilibrium model virtually at no extra cost.<sup>6</sup> Contrast this with a typical willingness-to-pay study of a nonmarket good. In contingent valuation studies, for example, the raw data are often, by definition, observations of individual Hicksian welfare measures. Marshallian or Hicksian demand functions might be derived subsequently but are not required as such for the welfare evaluation. The contrast between these two situations involves differences in the types of questions being asked, the types of data available for the analysis, and the types of models being used, as illustrated in table 2.

## Emerging Issues in Agricultural Economics Applications

While in many cases (including most applications to agricultural commodity markets in richer countries), both the costs and benefits from making the corrections may be small there would seem to be little excuse for not using the unbiased Hicksian measure. However, this may overstate the benefits (or understate the costs) of making such corrections in many applications.

The conventional argument has been derived in a relatively simple setting. What happens when we expand the problem to a more general setting? Are other issues more important than income effects? Are our measures so flawed that the returns to fine-tuning are negligible? There are a large number of specific situations that might be discussed in an exploration of such issues. Here we consider some issues that arise when the analysis is extended beyond the simple case of a single (often exogenous) price change in a single market for a final consumer good, with no (other) distortions, and with no uncertainty about the parametric form for the demand equation. We discuss briefly the implications of intermediate goods and general equilibrium adjustments with distorted markets.

*Intermediate Goods* — In most agricultural economics applications, the commodity in question is not a final consumer good. Indeed, most applications of welfare economics by agricultural economists pertain to commodities that are not even recognizable as final consumer goods (e.g., markets for wheat or other grains, livestock, or fibers such as wool and cotton) or to goods used as inputs by farmers (e.g., land, irrigation water, machinery, or agricultural chemicals). Thus the relevant demands being studied are not final consumer demands but, rather, intermediate derived demands. In such studies, what is called "consumer's surplus" often is really some vague aggregate of final consumer welfare and benefits to suppliers of

intermediate inputs. The same issue arises even in studies of retail demand for food, because most retail food products (e.g., raw meat, fresh fruit and vegetables) are not final consumer goods; rather, they are properly regarded as inputs into a household production process (see Bockstael and McConnell). A further complication, in many applications, is the role of international trade. Demand (or supply) at the total market level often involves demand (or supply) from overseas and it is often desirable to distinguish between domestic and foreign welfare impacts.

*General Equilibrium Effects* — The theory is reasonably clear on the analysis of welfare changes in the context of markets for intermediate goods and for traded goods (e.g., as described by Just, Hueth and Schmitz and discussed more recently by Thurman and by Hueth and Just). While, in theory, exact measures of total welfare change may be obtained from a single-market analysis, Thurman has shown that it will not in general be possible to disaggregate the total change into meaningful measures of consumer and producer welfare change. The problems that arise are associated with multiple sources of feedback of induced price changes into shifts of supply and/or demand in the market of primary interest. To obtain the full set of disaggregated welfare impacts in such circumstances requires a more complete, explicit, multi-market model.

The difficulty in practice, if one begins with the typical single-market model for an intermediate, traded commodity, is to know what adjustments to make to correct the Marshallian measures and to obtain the Hicksian counterparts. This is certainly a more serious challenge than the cases considered by those who have advocated making such corrections as a matter of course. Typically, we want one set of curves to describe the market equilibrium changes and another set for the welfare analysis. Usually agricultural economics applications begin with

Marshallian supply and demand curves. The transition to a Hicksian model may be quite difficult, and demanding of data and effort, when the market is for an intermediate commodity that is traded on international markets. In such a situation, compared with a case of a single market for a non-traded final consumer good, it is less clear that attempting to correct for income effects is either desirable in principle or worth the effort in practice.

*Second-Best Problems* — To make matters worse, the typical commodity market application of welfare economics is one in which there are significant cross-linkages to markets for other commodities that are distorted in important ways. Second-best problems abound in agricultural commodity markets, but the typical study ignores the issue.

In principle it is possible to obtain exact Hicksian welfare measures in such a situation, and at the same time to overcome the problems raised by Thurman. However, to do so it probably will be necessary to go well beyond the typical single-market model and to begin with an explicit general equilibrium model. For example, Martin (also Martin and Alston) have illustrated the application of a *modified trade expenditure function* approach to evaluate the disaggregated welfare impacts of both changes in trade distorting policies, and technical change, in a model with international trade in intermediate goods and a variety of pre-existing distortions. This is a promising development for some studies, but for many studies it will not be worthwhile to construct a full general equilibrium model, especially if the sole justification is to improve the accuracy of the measure of welfare change.

In such circumstances, where the only information that is readily available is the (Marshallian) supply and demand model of the market of interest, it is not clear what corrections can or should be made to the Marshallian welfare measures. In some cases the benefit—in terms

of greater (apparent) accuracy—from using the Hicksian measures may not be worth the cost in terms of more work and loss of transparency. The correction might be expected to involve a reduction in bias in exchange for an increase in variance (if we are relatively unsure about our estimates of income effects). Then the optimal choice may well be an empirical question.<sup>7</sup>

### **Emerging Welfare Measurement Issues in Environmental Economics**

Welfare measurement in environmental economics has been shaped by the structure and features of the issues which have come to dominate policy discussions. In this section we briefly discuss two emerging issues of interest, one based predominantly on theoretical considerations, and the other rooted in the empirical realities often faced in environmental economics research.

The environment generally, and environmental quality particularly, have been prominent in the public consciousness over the last decade. As a result, many welfare measurement problems in environmental economics involve the valuation of quality changes, or, more broadly, of changes in exogenously-provided public goods. This has led to the development of a body of theoretical work based originally on analogy with the Willig bounds for price changes, and subsequently amended based on both empirical evidence and recognition of the differences in the structure of price versus non-price changes in the measurement of welfare change. At the same time, the data which environmental economists work with are typically cross-sectional and frequently result in very low goodness of fit. This has led to recognition that the precision of welfare estimates is of substantial relevance to policy, and to the development of procedures for measuring it and taking it into account in estimation.



### *Applicability of Willig Bounds for Nonprice Changes*

An increasingly common issue in environmental economics is the change in welfare resulting from nonprice changes, arising typically in cases where the level of a public good changes or where the quality of a private good changes. Mäler was the first to show that Hicksian measures of welfare change, such as compensating variation, can be readily extended to these cases. As the policy importance of problems involving the valuation of changes in air or water quality has grown over the last fifteen years, so too has interest in the question of what can be said about the compensating or equivalent variations for quality changes from what is observed, namely the ordinary demand and Marshallian consumer's surplus. This naturally led investigators to ask whether Willig-type bounds are applicable for changes in non-price arguments of the preference function. Randall and Stoll showed that in fact Willig-type bounds could be developed for consumer's surplus as an approximation to compensating or equivalent variation. While their analysis was widely cited as a justification that willingness to pay and willingness to accept measures should not differ greatly for changes in environmental amenities, a substantial body of experimental evidence suggested otherwise (Knetsch and Sinden; Coursey *et al.*). This discrepancy between theoretical results and empirical findings was very troubling, until Hanemann provided the explanation which reconciled the two.

It turns out that the Randall-Stoll bounds depend not just on the magnitude of the income effect, as is the case in the original Willig analysis, but also on the extent to which there are substitutes for the public good being valued. Hanemann showed that while the Randall-Stoll bounds are indeed correct, they have been largely misconstrued because the substitutability of private goods for the public good being changed was not taken into account. The upper and

lower bounds on the error resulting from use of consumer's surplus are, themselves, dependent not just on the income elasticity of the demand for the public good (as is the case in Willig's analysis), but also on the Allen-Uzawa elasticity of substitution between the public good and the Hicksian composite commodity constructed from the private goods in the demand system. Because the elasticity of substitution between public and private goods is inversely related to the "price flexibility of income," whose upper and lower values over the range of the public good change comprise the Randall-Stoll bounds, the magnitudes of those bounds can vary greatly for public goods with low elasticity of substitution.

The implication is that Willig-type bounds are of less relevance for public good changes than for private good price changes, especially when the public good is unique or highly specialized (e.g., amenities associated with prominent national parks). These are the types of public resources for which one might suspect substantial "nonuse" value. The Exxon Valdez oil spill and subsequent legislation (e.g., the Oil Pollution Act of 1990), as well as recent court decisions (e.g., the Ohio case) have focused enormous interest on methods for measuring nonuse value.<sup>8</sup> Hanemann's results were derived under weak complementarity (or no nonuse value), and the presence of substantial nonuse value for an environmental amenity only strengthens his basic point about usefulness of Willig bounds as a rationale for Marshallian measures.

#### *Estimating the Precision of Welfare Measures*

A second area of emerging interest and increasing importance has been stimulated by empirical realities in environmental economics. Since data on environmental commodities are not collected with nearly the regularity nor consistency of data collected for commodities, quite often the data

used in nonmarket valuation studies of commodities such as recreation are cross-sectional, representing a sample of users or nonusers of a particular environmental amenity at a point in time. Goodness of fit in such studies is often very low, as variables suggested by demand theory account for a relatively small fraction of variation in the dependent variable, which is typically recreation trips or some other measure of the frequency of use. As a result, environmental economists have begun exploring the question of how precise welfare measures are.

While traditional discussion of the choice between Hicksian and Marshallian welfare measures (Willig; Hausman; LaFrance) focused on the likely magnitude of the difference between Hicksian and Marshallian measures, the environmental economics literature has begun exploring the question of precision of welfare measures. One likely motivation for this is the fact that goodness of fit is often poor in the cross-sectional data sets usually estimated in recreation demand studies. Thus, how one handles the estimated residual can make a very large difference to the magnitude of estimated consumer's surplus. The idea of using actual quantity instead of predicted quantity to compute consumer's surplus first appeared in Gum and Martin, and the implications of the choice of quantity measure received its first formal treatment by Bockstael and Strand. They showed how the researcher's judgement about the primary source of regression error can lead to selection of the actual or estimated quantity demanded as the basis for measuring consumer's surplus.<sup>9</sup>

Bockstael and Strand's work also led to widespread recognition that while demand functions are estimated because they are observable, it is often a measure of welfare change (such as consumer's surplus) and its precision that are of primary interest, not the demand coefficients themselves. Since the welfare measure is a nonlinear function of the demand

estimates, its standard error is not directly available but must be bootstrapped or otherwise approximated, for example by Taylor's series (Kling 1991). Smith argued that this should be taken into account directly in estimation, proposing an estimator designed explicitly to minimize mean squared error of consumer's surplus.<sup>10</sup>

Kling (1992) extends the Bockstael-Strand analysis of consumer's surplus to measures of its precision. She shows that the estimated variance of consumer's surplus depends substantially on the choice of whether to use actual or predicted quantity demanded. Also, not surprisingly, the precision of consumer's surplus (i.e., its coefficient of variation) varies inversely with goodness of fit. Using a simulated data set on prices and incomes and specific parameterizations of both linear and semilog demand functions, she uses the Willig bounds to show that the magnitude of the difference between compensating variation and consumer's surplus is also likely to vary inversely with goodness of fit.

While these contributions have aided our understanding of precision and bias taken separately as criteria for evaluating welfare measures, no one to our knowledge has yet put both dimensions together in a systematic way to help answer the question we asked above. In what follows, we briefly describe an approach to selecting a welfare measure that is based on the relative mean squared errors of the Marshallian and the Hicksian measures. One can make the argument, as LaFrance, Hueth and Just, McKenzie and Pierce, and Morey do, that the correct welfare measure for the Kaldor-Hicks criterion is the utility-constant measure and it should always be used. While we are sympathetic to the spirit of this contention, it does ignore an important aspect of the reality of empirical estimation, particularly in the environmental economics field: income coefficients are often measured very imprecisely. Thus we view the

practical contention that more error or noise may be introduced through the use of the Hicksian measures as valid; the difficulty is that both of these contentions have been subjected to little systematic scrutiny.

### **Towards Resolving What We Should Do in Empirical Analysis**

As discussed earlier, practices for welfare measurement in agricultural and in environmental economics have developed somewhat differently because of the nature of the problems investigated, the types of data encountered, and the like. Despite these differences, a question which we often encounter in casual discussions with applied researchers in both fields is

*"Given my estimated function(s), which welfare measure should be calculated, the Hicksian or Marshallian measure?"*

There seems to be little disagreement that in principle Hicksian measures should be used, as they are the defensible measures based on the Kaldor-Hicks compensation criterion. Nevertheless, despite Hausman's demonstration that compensating variation can be calculated easily for several functional forms commonly used in single equation demand analysis, the Hicksian measures are rarely used, and when they are it is typically in the environmental economics applications. The rationale for consumer's surplus often heard in informal discussions is twofold. First, Willig's results suggest that, for price changes, errors in approximating compensating variation by consumer's surplus are likely to be small in many empirical situations.<sup>11</sup> Second, since the divergence of Hicksian from Marshallian demands is due to an income effect which is measured imprecisely, more "error" (i.e., statistical noise) is introduced into the Hicksian measure and this may outweigh the advantages of using the

theoretically correct measure. In addition to these considerations that arise in the single-market context, with exogenous price or quantity changes, we can add the various problems (discussed briefly above) that can arise when we go beyond the simple case.

Our point of departure is different from previous analyses: rather than starting with a specific parameterization of the demand function, we begin at the point where the analyst has calculated both consumer's surplus and a measure of its precision, through methods described above.<sup>12</sup> It is at this point that the tradeoffs involved with undertaking the modest additional effort to calculate compensating variation can be illuminated. Since the difference between compensating variation and consumer's surplus depends on the magnitude of the income effect, which is a random variable, the compensating variation might be expected to have a larger variance but no bias. The Marshallian measure, on the other hand, admits some bias but would be expected, in many circumstances, to have a lower variance. Thus it is natural to think of the selection of a welfare measure in terms of its mean squared error.

This approach recognizes that it may be desirable in some contexts not to be a Hicksian purist, because of difficulties with measuring income, and the income slope, with adequate precision.<sup>13</sup> It also has the added advantage of somewhat greater generality than previous approaches. Definitive comparisons can be made based only on three random variables: consumer's surplus, the change in quantity demanded implied by a given policy, and the income coefficient. Correct functional form is a maintained hypothesis (with this as well as previous work). However, it is not essential to the analysis to assume specific parameterizations of other demand covariates, as has been necessary in previous simulation work.

This comparison is straightforward with the simple functional forms that are most commonly used in policy analysis, as compensating variation can be written as a function of random consumer's surplus and random income slope. For reasons of space, we focus on the semilog demand function, which is widely used in the environmental economics literature. For instance, it is probably the most commonly used functional form for recreation demand (McConnell 1985; Smith).

Consider the semilog demand, written for individual  $i$  as

$$x_i = e^{\alpha z_i + \beta p_i + \delta m_i}, \quad (6)$$

where  $p_i$  is own price,  $m_i$  is income, and  $z_i$  is a vector of nonprice shifters that includes the intercept, prices of substitutes, and the regression error, while  $\alpha$ ,  $\beta$ , and  $\delta$  are corresponding parameters. It is well known that for either a price change or a change in  $z_i$ , the consumer's surplus measure can be written as

$$CS_i = \frac{\Delta x_i}{-\beta}, \quad (7)$$

where  $\Delta x_i \equiv x_{i1} - x_{i0}$  is the change in quantity induced by the policy being analyzed; it is presumed that  $x$  is non-Giffen, so that the welfare measure is positive for policies that imply an increase in quantity demanded (and a corresponding increase in area under the demand function above the price line). Employing the techniques of Hausman, one can also integrate back to recover the weakly complementary quasi-indirect utility function

$$v(p_i, z_i, m_i) = -\frac{1}{\beta} e^{\alpha z_i + \beta p_i} - \frac{1}{\delta} e^{\delta m_i}, \quad (8)$$

(e.g., Bockstael *et al.*) and, initializing utility on initial prices  $p_i^0$ , shifters  $z_i^0$ , and income  $m_i^0$ , the compensating variation for a change in  $p$  and  $z$  can be written



$$CV_i = m_i^0 - \frac{1}{\delta} \ln \left[ e^{-\delta m_i} \left( 1 + \frac{\delta}{\beta} e^{\alpha z_i^0 + \beta p_i^0 + \delta m_i} - \frac{\delta}{\beta} e^{\alpha z_i^1 + \beta p_i^1 + \delta m_i} \right) \right] \quad (9)$$

Noting that the last two terms in parentheses are initial and subsequent quantities along the Marshallian demand function, respectively, and factoring out the income exponential in the logarithm, compensating variation for changes in  $p$  or  $z$  can be written more simply as

$$\begin{aligned} CV_i &= \frac{1}{\delta} \ln \left[ 1 + \frac{\delta}{\beta} x_i^0 - \frac{\delta}{\beta} x_i^1 \right] \\ &= \frac{1}{\delta} \ln (1 + \delta CS_i); \end{aligned} \quad (10)$$

that is, as a nonlinear function of the random variables  $\delta$  and  $CS_i$ . Equation (10) can be used to estimate the both the point estimate and precision of  $CV_i$ , given knowledge of the first two moments of  $\delta$  and  $CS_i$  and their covariance.

#### *Mean Squared Errors for the Total Welfare Measures*

The researcher has estimated  $CS_i$  and its variance,  $\sigma_{CS}^2$ , through methods discussed by Kling (1988, 1991) or Smith. This is the point at which the question arises: is it a good idea to calculate and report the compensating variation measure in (10), or is the consumer's surplus and its precision adequate? From (7) and (10), one can calculate  $Bias \equiv CS - CV$ , and obtain the mean squared error of the Marshallian measure as

$$MSE_M \equiv \sigma_{CS}^2 + Bias^2. \quad (11)$$

If the compensating variation is written as  $CV = f(CS, \delta)$ , a first-order Taylor's series approximation to  $var(CV)$  is given by (e.g., Mood *et al.*)

$$\sigma_{CV}^2 \doteq (f_{CS})^2 \sigma_{CS}^2 + 2f_{CS}f_{\delta} \sigma_{CS,\delta} + (f_{\delta})^2 \sigma_{\delta}^2, \quad (12)$$

where the subscripts on  $f$  refer to partial derivatives. Using this with (10), the mean squared error of the Hicksian measure can be written as

$$MSE_H \equiv \sigma_{CV}^2 \doteq \frac{\sigma_{CS}^2}{(1+\delta CS)^2} + 2\sigma_{CS,\delta} \left[ \frac{CS}{\delta(1+\delta CS)^2} - \frac{CV}{\delta(1+\delta CS)} \right] + \sigma_{\delta}^2 \left[ \frac{CS}{\delta(1+\delta CS)} - \frac{CV}{\delta} \right]^2,$$

where (10) has been used to simplify. Writing  $\sigma_{CS,\delta} \equiv \rho\sigma_{CS}\sigma_{\delta}$ , where  $\rho$  is the correlation between  $CS$  and  $\delta$  and  $\sigma_{CS}$  and  $\sigma_{\delta}$  are the standard deviations of  $CS$  and  $\delta$ , yields

$$MSE_H \doteq \frac{\sigma_{CS}^2}{(1+\delta CS)^2} + \frac{2\rho\sigma_{CS}}{(1+\delta CS)} \left[ \frac{Bias^*}{t_{\delta}} \right] + \left[ \frac{Bias^*}{t_{\delta}} \right]^2, \quad (13)$$

where  $Bias^* \equiv CS/(1+\delta CS) - CV$  is an adjusted bias term and  $t_{\delta} \equiv \delta/\sigma_{\delta}$  is the Student's  $t$ -statistic for the hypothesis of no association for the income coefficient, reported as part of standard regression output.

Equation (13) is an expression for the precision of the Hicksian compensating variation based on the random variables  $\delta$  and  $CS$ , measures of their precision ( $\sigma_{CS}$  and  $t_{\delta}$ ), and their correlation  $\rho$ . This can be compared with (11), which gives the  $MSE$  for the Marshallian measure in terms of the same variables. The first four of these are readily available from computations already performed ( $CS$ ,  $\sigma_{CS}^2$ ) or from the regression printout ( $\delta$ ,  $t_{\delta}$ ). The only thing requiring more information is  $\rho$ . Inspection of (11) and (13) suggests it is likely that  $\rho > 0$ , as Kling (1992, p. 324) argues, though it could be negative. Because our goal is to present results that are as free as possible from specific parameterizations of the demand model, we allow  $\rho$  to take on the extreme values -1, 0, and +1 in the comparisons we make.

While the  $t$ -statistic on the income coefficient is intuitive and a range of plausible values is easily identified (we chose 0.5, 1.0, and 2.0), it is less clear what constitutes reasonable ranges for  $CS$  and for  $\delta$ . Fortunately, it is possible to motivate the choice of values for each of these two parameters in terms of unitless quantities about which we have some ideas of the plausible range. In the semilog demand model, the income elasticity  $\eta_Y$  is the product of the income slope  $\delta$  and income  $m^0$ . Thus by knowing the reference income level and plausible values of income elasticity, a range of plausible values for income slope are obtained from  $\delta = \eta_Y/m^0$ . We chose income elasticities of  $\eta_Y = 0.5, 1.0, \text{ and } 1.5$  for this illustration, with reference income  $m^0 = \$25,000/\text{year}$ , resulting in values of  $\delta = 2 \cdot 10^{-5}, 4 \cdot 10^{-5}, \text{ and } 6 \cdot 10^{-5}$ . Similarly, a choice of  $CS$  can be motivated in terms analogous to Willig's by selecting a range for the fraction of income that consumer's surplus represents. Denoting this fraction by  $\phi$ , we considered values of  $\phi = .001, .01, .1, \text{ and } .5$ , which implies corresponding values of \$25, \$250, \$2,500, and \$12,500 for the person with reference income  $m^0 = \$25,000$  per year. The range of  $\sigma_{CS}^2$  is easily generated from  $CS$  by considering values for the coefficient of variation for  $CS$  ( $V_{CS}$ ) as a measure of the precision with which it is measured. We chose values 0.1, 0.8, and 4.0 for  $V_{CS}$ , which spans the range of coefficients of variation Kling (1992) found.

Tables 3-5 present the comparisons of  $MSE_M$  and  $MSE_H$  for the ranges of  $\phi, \eta_Y, t_\delta, V_{CS}$ , and  $\rho$  just noted. Each table corresponds to a different level of  $CS$  for the consumer with  $m^0 = \$25,000/\text{year}$ , ranging from  $CS = \$250/\text{year}$  to  $CS = \$12,500/\text{year}$ . Three cases are presented in each table, for each of the values of income elasticity; the bias  $CV - CS$  is also given as it is constant for each case. Reading across the table, the three values for  $V_{CS}$  (.1, .8, and 4.0) with corresponding value of  $\sigma_{CS}^2$  are given, along with the root mean squared error

(*RMSE*) for the Marshallian consumer's surplus. Then, for each of the values of  $t_\delta$ , the *RMSE* for the Hicksian measure is calculated, along with the percentage change in *MSE* that comes from using the Hicksian measure instead of the Marshallian measure. The three Hicksian measures compared with each Marshallian *CS* correspond to the values  $\rho = -1, 0, \text{ and } +1$ . This gives upper and lower bounds on the *MSE* comparisons.

The patterns in tables 3-5 are broadly consistent with intuition, yet contain some surprises. For a *CS* change relatively small in relation to income ( $\phi \leq .01$ , say, as in table 3), the *MSE*'s are quite comparable in most cases. Because the bias is fairly small, the *RMSE* for the Marshallian measure is virtually indistinguishable from the variance of *CS*, while the *RMSE* is slightly smaller. (The reason for this is discussed below.) The differences between the Hicksian and Marshallian measures are more pronounced as the welfare area increases.

The answer to the question of which *MSE* is smaller depends on both  $\rho$  and how precisely *CS* and the income slope  $\delta$  are measured. Not surprisingly, the Marshallian measure has smaller *MSE* (the % Difference is positive) nearly always when *CS* is measured very precisely ( $V_{CS} = 0.1$ ) and  $\delta$  is measured very imprecisely ( $t_\delta = 0.5$ ). This is also the case where the impact of  $\rho$  is greatest on the magnitude (and, in some cases, the sign) of the difference in *MSE*'s. However, the Hicksian measure consistently has smaller *MSE*, regardless of  $\rho$ , when either the precision of *CS* decreases or the precision of  $\delta$  increases relative to this extreme case. As an example, when  $V_{CS}$  is 0.8, the Hicksian *MSE* is smaller, even when the Student's  $t$  on the income coefficient is insignificant (e.g.,  $t_\delta = 0.5$  or 1.0), for virtually all values of  $\rho$ . When  $t_\delta$  is 1.0 or better and  $V_{CS}$  is 0.8 or higher, the Hicksian *MSE* is smaller regardless of  $\rho$ . The magnitude of the difference increases with income elasticity and the magnitude of *CS* relative to income.

The implication is that one of the standard rationales for using Marshallian welfare measures may be weaker than generally expected: calculating the Hicksian measure does not necessarily introduce a lot of noise into the welfare estimate. On the contrary, unless  $CS$  is very tightly measured, or the income coefficient is very poorly measured, one gains in terms of *both reduced bias and reduced variance* by using the Hicksian measure, at least for the case of willingness to pay measures using the common semilog demand form.

Why does this occur? The reason is straightforward, in light of (13). Consider the separate effects of the terms on the right side of (13) on  $MSE_H$ . The first term is the result of the nonlinear transform (10) operating on the variance of  $CS$ , while the second and third terms are additions to the variance as a result of the introduction of the income slope. Since by (10)  $CV$  is a concave transformation of  $CS$  when the income slope is positive, for willingness to pay measures (i.e., for positive  $CS$  and  $CV$ )<sup>14</sup> the first term in (13) is less than  $\sigma_{CS}$  since the denominator (the Jacobian of the  $CV$ - $CS$  transformation) is less than unity. Thus, for fixed  $\delta$ , using  $CV$  instead of  $CS$  reduces variance.

The income slope in fact is not fixed, and the second and third terms on the right side of (13) account for the additions to variance due to random  $\delta$ , which depends on the correlation between  $\delta$  and  $CS$ . On balance, however, the first term dominates across a fairly wide range of the parameter values we simulated, so the Hicksian variance is commonly less than the Marshallian variance. This is reinforced by the fact that  $\rho > 0$  is likely in many empirical settings, and the *Bias\** term is negative across all our simulations; this moderates the increase to variance by introduction of the noisy income slope.

It is interesting to note that these findings for willingness to pay measures are essentially reversed when willingness to accept measures (*EV* for a price decline or *CV* for a price increase, for example) are considered. The reason again is straightforward: from (13), the denominator of the first term is less than unity, so that even for fixed  $\delta$ , the transformation (10) from *CS* to *CV* is variance-increasing. However, willingness-to-pay measures are often preferred in the environmental economics literature on both practical grounds and in light of the implied property rights for private goods or access conditions for public goods (e.g., Mitchell and Carson, pp. 30-41).

#### *Mean Squared Errors for Deadweight Loss Measures*

The deadweight loss associated with willingness to pay welfare measures can be developed easily from the expressions (7) and (10). We concern ourselves only with the demand-side deadweight loss because it is on the demand side that Hicksian and Marshallian deadweight loss measures differ. A convenient visual interpretation is given in figure 2, for the case of a price decrease from  $p_0$  to  $p_1$ . The consumer's surplus change, a positive number, is given by the area  $p_0abp_1$ , while Marshallian deadweight loss is this area less  $p_0dp_1$ , or area  $abd$ . Similarly, compensating variation is area  $p_0acp_1$ , and the Hicksian deadweight loss is area  $acd$ . Mathematically, these are written:

$$DWL_M = CS + x_0\Delta p, \quad (14)$$

where  $\Delta p \equiv p_1 - p_0 < 0$ ; and

$$DWL_H = CV + x_0\Delta p, \quad (15)$$

for the Marshallian and Hicksian measures, respectively.

Two points are interesting to note from equations (14) and (15). First, they show that the bias in measuring deadweight loss is the same as the bias in measuring the total welfare change. Second, since  $\Delta p$  is taken to be an exogenous price change, if  $x$  is treated as a known quantity measured without error, the variance of the deadweight loss measures are the same as the variances of the respective consumer welfare measures. Thus, the tables on mean squared error for consumer's surplus and Hicksian willingness to pay (tables 3-5) are limiting cases of mean squared errors for deadweight loss measures, as the error in measuring  $x_0$  goes to zero.

In light of (6), however,  $x_0$  is more appropriately treated as a random variable like  $\delta$  and  $CS$  in the analysis of total welfare change measures in the previous section. This complicates comparisons of the mean squared errors somewhat, as the analysis needs to account for three variances and three covariances. The line of analysis remains straightforward, but definitive conclusions are more elusive. To illustrate briefly the possibilities for comparisons of mean squared errors of deadweight loss measures, note that the variance of  $DWL_M$  can be approximated to first order from (14) as

$$\sigma_M^2 \doteq \sigma_{CS}^2 + 2(\Delta p)\sigma_x\sigma_{CS}\rho_{x,CS} + (\Delta p)^2\sigma_x^2, \quad (16)$$

where  $\rho_{x,CS}$  is the correlation of  $x$  and  $CS$ . Since  $\Delta p$  is negative, the likely positive correlation of  $x$  and  $CS$  means that the variance of Marshallian deadweight loss is less than the sum of the variances of  $x$  and  $CS$ . Adding squared bias, the mean squared error of the Marshallian deadweight loss measure is  $MSED_M = Bias^2 + \sigma_M^2$ .

Substituting (10) into (15), and defining  $\rho_{x,\delta}$  as the correlation between  $x$  and  $\delta$  and  $\rho_{\delta,CS}$  as the correlation between  $\delta$  and  $CS$ , referred to previously as simply  $\rho$ , the variance (and mean squared error) of the Hicksian deadweight loss measure is approximately



$$\begin{aligned}
MSED_H \doteq & \frac{\sigma_{CS}^2}{1+\delta CS} + (\Delta p)^2 \sigma_x^2 + \left[ \frac{Bias^*}{\delta} \right]^2 \sigma_\delta^2 + \\
& + 2 \left[ (\Delta p) \frac{\sigma_x \sigma_{CS} \rho_{x,CS}}{1+\delta CS} + (\Delta p) \left[ \frac{Bias^*}{\delta} \right] \sigma_x \sigma_\delta \rho_{x,\delta} + \left[ \frac{Bias^*}{\delta} \right] \frac{\sigma_\delta \sigma_{CS} \rho_{\delta,CS}}{1+\delta CS} \right]. \quad (17)
\end{aligned}$$

Equations (16) and (17) suggest a useful way of comparing *MSE*'s for the deadweight loss measures, given that, in general, the correlations  $\rho_{x,\delta}$ ,  $\rho_{\delta,CS}$ , and  $\rho_{x,CS}$  are unknown. In (16),  $MSED_M$  depends only on the correlation  $\rho_{x,CS}$ , with which it varies inversely since  $\Delta p < 0$ ;  $MSED_H$  in (17) depends on all three correlations. We compare the smallest  $MSE_M$  (i.e., when  $\rho_{x,CS} = 1$ ) with the largest value that  $MSE_H$  can take given  $\rho_{x,CS} = 1$  as the other correlations vary; this is the upper bound on the difference  $MSE_H - MSE_M$ .<sup>15</sup> Similarly, the lower bound on the value of the difference is obtained by comparing the  $MSED_M$  (given  $\rho_{x,CS} = -1$ ) to the smallest value  $MSED_H$  can take (which occurs when  $\rho_{x,\delta} = -1$  and  $\rho_{x,CS} = 1$ ).

Results of these comparisons are given in table 6, for an individual with characteristics similar to those in table 5 (i.e., the consumer's surplus change is large relative to budget with  $\phi = 0.5$ ). More information is needed for the deadweight loss calculation than for the total welfare area comparison; in particular, the rectangle  $x_0 \Delta p$  and the coefficient of variation on  $x_0$ ,  $V_x$ , are required. These were initialized by assuming the budget share of  $x$  was  $(\phi + .1)^2 = .36$ , and that subsequent price is  $p_1 = \$20/\text{unit}$ . This is sufficient to solve for  $x_0 = 75$ ,  $x_1 = 450$ , and  $p_0 = \$79.73/\text{unit}$ . We chose  $V_x = 0.8$ , which would allow comparisons when  $x$  was both more- and less-precisely measured than *CS*.

The triplets of comparisons in table 6, from top to bottom, are for high  $MSED_M$  and low  $MSED_H$ ; for all correlations equal to zero; and for low  $MSED_M$  and high  $MSED_H$ . The results

are less clear-cut than for the total welfare measures, not surprisingly since additional variation is introduced through random  $x_0$  and no restrictions were placed on the relationships between correlations. The influence of the correlations on the percentage difference is especially strong when both  $x$  and  $CS$  are measured with equal precision ( $V_{CS} = V_x = 0.8$ ); depending on the values of the correlations and other parameters in a specific empirical study, using the Hicksian deadweight loss measure could be somewhat beneficial (reductions in  $MSE$  of up to 45% in this example) to very costly (increases in  $MSE$  of up to 237%). The bounds narrow when either  $CS$  is measured more precisely than  $x$ , or vice versa. They also narrow as income elasticity ( $\eta_Y$ ) decreases, and as the Student's  $t$  on income increases. As was the case with the total welfare change measures, there is some evidence in table 6 that as precision on  $\delta$  increases and on  $CS$  decreases, the Hicksian measure has smaller  $MSE$  regardless of correlations; however, in the case of deadweight losses it only appears for high precision on  $\delta$  and low precision on  $CS$ .

These results on deadweight loss, particularly, must be taken only as suggestive, as they are sensitive to many factors. For example, the precision in measuring  $x$  ( $V_x$ , which we held constant at 0.8) has a major impact on how much additional noise is introduced when calculating deadweight loss instead of a total welfare change measure. It seems likely that unless additional information is introduced about the correlations among  $x$ ,  $CS$ , and  $\delta$ , they will continue to dominate any conclusions that can be drawn about whether the Hicksian or Marshallian measure has smaller mean squared error unless  $x$  is measured with very great precision.

### Some Concluding Remarks

The problems that have come to dominate discussion in the agricultural and environmental economics fields are to a great extent shaped by the nature of the policy problems and the data that are available in empirical studies. In applied welfare analysis, the question at a practical level remains one of whether it is a good idea to use a Marshallian or Hicksian measure of welfare change. Some argue that on principle the only good welfare measure is a Hicksian one, whereas others make an equally convincing case that in practice so much additional noise may be introduced into the welfare estimate by using an imprecisely measured income coefficient that the correction isn't worthwhile. Thus the question appears to be one of trading bias for variance: when the (small) extra effort is taken to calculate the Hicksian measure from the Marshallian one, the bias is reduced but variance may increase. We attempt to shed some light on how these arguments net out in practice.

We draw on the developing literature in environmental economics on precision of consumer's surplus measures, and on the literature on integrating back to recover preferences from Marshallian demands, to provide some evidence on the relative mean squared errors of Hicksian versus Marshallian welfare measures. The situation is one often faced by a practitioner who has estimated a Marshallian demand function and turns to the question of welfare calculation.

Using the popular semilog demand function and the maintained hypothesis of correct functional form, the mean squared errors of both consumer's surplus and the Hicksian willingness to pay measure (compensating variation for a price decrease, equivalent variation for a price increase) are compared for a fairly wide range of the magnitude and precision of

consumer's surplus and magnitude and precision of the income coefficient. When consumer's surplus is a small fraction of the total budget (e.g., 1% or less), there is little difference in mean squared errors. However, a surprising result of this comparison is that the Hicksian measure of consumer welfare has a smaller mean squared error even when the income coefficient is not measured with high precision (e.g., with a Student's  $t$  of 1.0). That is, the Hicksian measure has both smaller bias and smaller variance. This improvement in mean squared error changes in intuitive ways with particulars of the problem, increasing as the precision of the income coefficient increases and as the precision of consumer's surplus decreases. The reason for this gain in mean squared error turns out to be straightforward: because the Hicksian willingness to pay measure is a concave transformation of consumer's surplus, with a Jacobian less than unity, the transformation itself is variance-reducing and this outweighs the additional variance introduced through use of the income term.

The question of which measure to use, Hicksian or Marshallian, for deadweight loss calculations in practice is less clear. This is due primarily to the fact that an additional source of variance and covariance (in the measurement of initial quantity consumed) is introduced, and without introducing study-specific information on the relationship between the covariances the comparison is largely indeterminate. However, as the measurement of quantity becomes very precise, the comparisons between total welfare measures (consumer's surplus and Hicksian willingness to pay) are the limiting case of the deadweight loss comparisons as precision of quantity consumed becomes perfect.

It is difficult to generalize too far beyond this specific functional form at present, but similar comparisons using the other common, simple single-equation functional forms are

possible. The insights about the effects of curvature on the variance of the welfare measure probably generalize readily, but whether the variance reduction from a nonlinear transformation of consumer's surplus, when it occurs, dominates the additional noise from an imprecisely-measured income coefficient remains to be seen. Nevertheless, results for the semilog model suggest that the small additional effort needed to calculate Hicksian welfare measures in single market analyses is probably worthwhile in many cases.

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**Table 1: Percentage Errors Relative to Equivalent Variation in Marshallian Measures of Welfare Impacts of a Subsidy and a Research-induced Supply Shift**

Subsidy Rate ( $\tau$ )	Income Elasticity ( $\eta_Y$ )	Income Share (k)	Error EV-CS	Error as a Percentage of		
				NS(a)	CS(a)	NS(b)
0.100	0.200	0.050	0.002	-1.333	0.034	0.023
		0.150	0.007	-4.000	0.102	0.068
		0.250	0.011	-6.667	0.169	0.113
	0.600	0.050	0.007	-4.000	0.102	0.068
		0.150	0.020	-12.000	0.305	0.203
		0.250	0.033	-20.000	0.508	0.339
	1.000	0.050	0.011	-6.667	0.169	0.113
		0.150	0.033	-20.000	0.508	0.339
		0.250	0.056	-33.333	0.847	0.565
0.200	0.200	0.050	0.009	-1.333	0.069	0.043
		0.150	0.027	-4.000	0.207	0.129
		0.250	0.044	-6.667	0.345	0.215
	0.600	0.050	0.027	-4.000	0.207	0.129
		0.150	0.080	-12.000	0.621	0.387
		0.250	0.133	-20.000	1.034	0.645
	1.000	0.050	0.044	-6.667	0.345	0.215
		0.150	0.133	-20.000	1.034	0.645
		0.250	0.222	-33.333	1.724	1.075

(a) denotes the analysis of a per unit subsidy at a fraction,  $\tau$ , of the final price; (b) denotes the analysis of a research induced supply shift down in parallel by an amount equal to a fraction,  $\tau$ , of the final price. CS represents the change in Marshallian consumer's surplus, NS represents the change in total national surplus, and EV represents the equivalent variation measure of the consumer welfare effect. In each case the error is expressed relative to the Marshallian surplus measure of welfare change. If the errors were calculated relative to the compensating variation measure, under our linear approximation the errors, and percentage errors, would be of the same sizes but opposite signs to those in the table.

**Table 2. Some Key Differences That Frequently Arise in the Problems We Analyze.**

Feature of the Problem	Agricultural Economics	Environmental Economics
Types of Data	Time Series Aggregated Over Agents Market Prices	Cross-Section Data on Individual Agents Prices Often Constructed
Policy Problem	Market Distortions Small price changes Demands and Supplies Single or Multiple Markets	Provision or Removal Large Price Changes Usually Demand-Side Usually Single Markets
Goodness of Fit	Often High	Often Low
Welfare Measure of Primary Interest	Deadweight Loss or Changes in Surplus Distributional Effects	Entire Surplus Area
Type of Nonprice Changes Analyzed	Research Benefits Technological Change Advertising	Environmental Quality Changes
Preference Restriction For Nonprice Change	Input/Output Essentiality	Weak Complementarity
Occurrence of "Nonuse Value"	Rarely if Ever	Plausible for Unique Environmental Amenities

**Table 3. RMSEs of Hicksian (CV) and Marshallian (CS) Welfare Measures for an Exogenous Price Change, Semilog Demand With Varying Precision in Measuring CS and  $\delta$ . (m=\$25,000/year, and CS=\$250/year).**

Precision of Measuring CS		Precision in Measuring the Income Slope						
		$t_\delta=0.5$			$t_\delta=1.0$		$t_\delta=2.0$	
$V_{CS}$	$\sigma_{CS}^2$	RMSE Marsh	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE
<i>Case I: <math>\eta_Y=.5</math>; bias=0.62292</i>								
.10	25.	25.	23.6	-10.6	24.3	-5.9	24.6	-3.5
.10	25.	25.	24.9	-.8	24.9	-1.0	24.9	-1.0
.10	25.	25.	26.1	9.1	25.5	3.9	25.2	1.4
.80	200.	200.	197.8	-2.2	198.4	-1.6	198.7	-1.3
.80	200.	200.	199.0	-1.0	199.0	-1.0	199.0	-1.0
.80	200.	200.	200.3	.2	199.6	-.4	199.3	-.7
4.0	1000.	1000.	993.8	-1.2	994.4	-1.1	994.7	-1.0
4.0	1000.	1000.	995.0	-1.0	995.0	-1.0	995.0	-1.0
4.0	1000.	1000.	996.3	-.7	995.7	-.9	995.3	-.9
<i>Case II: <math>\eta_Y=1.0</math>; bias=1.2417</i>								
.10	25.	25.	22.3	-20.7	23.5	-11.7	24.1	-7.0
.10	25.	25.	24.9	-1.2	24.8	-2.0	24.8	-2.2
.10	25.	25.	27.2	18.2	26.0	7.8	25.4	2.7
.80	200.	200.	195.6	-4.4	196.8	-3.2	197.4	-2.6
.80	200.	200.	198.0	-2.0	198.0	-2.0	198.0	-2.0
.80	200.	200.	200.5	.5	199.3	-.7	198.6	-1.4
4.0	1000.	1000.	987.6	-2.5	988.9	-2.2	989.5	-2.1
4.0	1000.	1000.	990.1	-2.0	990.1	-2.0	990.1	-2.0
4.0	1000.	1000.	992.6	-1.5	991.3	-1.7	990.7	-1.8
<i>Case III: <math>\eta_Y=1.5</math>; bias=1.8565</i>								
.10	25.	25.	21.0	-30.1	22.8	-17.3	23.7	-10.5
.10	25.	25.	24.9	-1.3	24.7	-2.9	24.6	-3.3
.10	25.	25.	28.3	27.5	26.5	11.4	25.6	3.9
.80	200.	200.	193.4	-6.5	195.2	-4.7	196.1	-3.8
.80	200.	200.	197.1	-2.9	197.1	-2.9	197.1	-2.9
.80	200.	200.	200.7	.7	198.9	-1.1	198.0	-2.0
4.0	1000.	1000.	981.6	-3.7	983.4	-3.3	984.3	-3.1
4.0	1000.	1000.	985.2	-2.9	985.2	-2.9	985.2	-2.9
4.0	1000.	1000.	988.9	-2.2	987.1	-2.6	986.1	-2.8

Table 4. RMSEs of Hicksian (CV) and Marshallian (CS) Welfare Measures for an Exogenous Price Change, Semilog Demand With Varying Precision in Measuring CS and  $\delta$ .  
( $m = \$25,000/\text{year}$ , and  $CS = \$2,500/\text{year}$ ).

Precision of Measuring CS			Precision in Measuring the Income Slope					
			$t_\delta = 0.5$			$t_\delta = 1.0$		
$V_{CS}$	$\sigma_{CS}^2$	RMSE Marsh	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE
<i>Case I: <math>\eta_Y = .5</math>; bias = 60.492</i>								
.10	250.	257.	121.0	-77.8	179.5	-51.2	208.8	-34.0
.10	250.	257.	265.3	6.4	245.2	-9.1	239.9	-13.0
.10	250.	257.	355.2	90.7	296.7	33.0	267.4	8.1
.80	2000.	2001.	1787.7	-20.1	1846.2	-14.8	1875.5	-12.1
.80	2000.	2001.	1908.4	-9.0	1905.7	-9.3	1905.0	-9.4
.80	2000.	2001.	2021.9	2.1	1963.3	-3.7	1934.0	-6.6
4.0	10000.	10000.	9406.7	-11.5	9465.3	-10.4	9494.5	-9.9
4.0	10000.	10000.	9524.5	-9.3	9524.0	-9.3	9523.9	-9.3
4.0	10000.	10000.	9640.9	-7.1	9582.4	-8.2	9553.1	-8.7
<i>Case II: <math>\eta_Y = 1.0</math>; bias = 117.25</i>								
.10	250.	276.	7.2	-99.9	117.3	-81.9	172.3	-61.0
.10	250.	276.	316.4	31.2	252.5	-16.3	233.8	-28.2
.10	250.	276.	447.3	162.0	337.3	49.2	282.3	4.5
.80	2000.	2003.	1598.1	-36.3	1708.2	-27.3	1763.2	-22.5
.80	2000.	2003.	1831.4	-16.4	1821.5	-17.3	1819.0	-17.5
.80	2000.	2003.	2038.2	3.5	1928.2	-7.4	1873.2	-12.5
4.0	10000.	10001.	8870.9	-21.3	8980.9	-19.3	9035.9	-18.3
4.0	10000.	10001.	9093.6	-17.3	9091.6	-17.3	9091.1	-17.3
4.0	10000.	10001.	9311.0	-13.3	9200.9	-15.3	9145.9	-16.3
<i>Case III: <math>\eta_Y = 1.5</math>; bias = 170.63</i>								
.10	250.	303.	93.5	-90.4	61.9	-95.8	139.7	-78.7
.10	250.	303.	379.4	57.0	267.3	-22.0	230.9	-41.8
.10	250.	303.	528.3	204.0	372.8	51.7	295.1	-4.9
.80	2000.	2007.	1428.2	-49.3	1583.7	-37.7	1661.4	-31.4
.80	2000.	2007.	1766.7	-22.5	1746.1	-24.3	1740.9	-24.7
.80	2000.	2007.	2050.0	4.3	1894.6	-10.9	1816.9	-18.0
4.0	10000.	10001.	8384.7	-29.7	8540.2	-27.0	8617.9	-25.7
4.0	10000.	10001.	8701.2	-24.3	8697.0	-24.3	8696.0	-24.4
4.0	10000.	10001.	9006.6	-18.9	8851.1	-21.6	8773.4	-23.0

Table 5. RMSEs of Hicksian (CV) and Marshallian (CS) Welfare Measures for an Exogenous Price Change, Semilog Demand With Varying Precision in Measuring CS and  $\delta$ .  
( $m = \$25,000/\text{year}$ , and  $CS = \$12,500/\text{year}$ ).

Precision of Measuring CS			Precision in Measuring the Income Slope					
			$t_\delta = 0.5$			$t_\delta = 1.0$		
$V_{CS}$	$\sigma_{CS}^2$	RMSE Marsh	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE
<i>Case I: <math>\eta_Y = .5</math>; bias = 1342.8</i>								
.10	1250.	1835.	1314.	-48.6	157.	-99.2	421.41	-94.7
.10	1250.	1835.	2521.	88.8	1529.	-30.5	1155.3	-60.3
.10	1250.	1835.	3314.	226.0	2157.	38.2	1578.6	-25.9
.80	10000.	10090.	5686.	-68.2	6843.	-54.0	7421.	-45.8
.80	10000.	10090.	8328.	-31.8	8083.	-35.8	8021.	-36.8
.80	10000.	10090.	10314.	4.5	9157.	-17.6	8579.	-27.7
4.0	50000.	50018.	37686.	-43.2	38843.	-39.6	39421.	-37.8
4.0	50000.	50018.	40067.	-35.8	40017.	-35.9	40004.	-36.0
4.0	50000.	50018.	42314.	-28.4	41157.	-32.2	40579.	-34.1
<i>Case II: <math>\eta_Y = 1.0</math>; bias = 2363.4</i>								
.10	1250.	2674.	2773.	7.6	970.	-86.8	68.	-99.9
.10	1250.	2674.	3702.	91.6	1987.	-44.7	1228.	-78.9
.10	1250.	2674.	4440.	175.0	2637.	-2.7	1735.	-57.8
.80	10000.	10275.	3060.	-91.1	4863.	-77.5	5765.	-68.5
.80	10000.	10275.	7580.	-45.5	6906.	-54.8	6727.	-57.1
.80	10000.	10275.	10273.	-0	8470.	-32.0	7568.	-45.7
4.0	50000.	50056.	29727.	-64.7	31530.	-60.3	32432.	-58.0
4.0	50000.	50056.	33528.	-55.1	33382.	-55.5	33346.	-55.6
4.0	50000.	50056.	36940.	-45.5	35137.	-50.7	34235.	-53.2
<i>Case III: <math>\eta_Y = 1.5</math>; bias = 3173.1</i>								
.10	1250.	3410.	3654.	14.7	1470.	-81.4	378.	-98.7
.10	1250.	3410.	4426.	68.4	2298.	-54.6	1305.	-85.3
.10	1250.	3410.	5082.	122.0	2898.	-27.7	1806.	-71.9
.80	10000.	10491.	1346.	-98.3	3530.	-88.6	4622.	-80.5
.80	10000.	10491.	7193.	-52.9	6118.	-66.0	5818.	-69.2
.80	10000.	10491.	10082.	-7.6	7898.	-43.3	6806.	-57.9
4.0	50000.	50101.	24203.	-76.6	26387.	-72.2	27479.	-69.9
4.0	50000.	50101.	28903.	-66.7	28655.	-67.2	28592.	-67.4
4.0	50000.	50101.	32940.	-56.7	30756.	-62.3	29663.	-64.9

Table 6. RMSEs of Hicksian (CV) and Marshallian (CS) Measures of Deadweight Loss, Semilog Demand with Varying Precision in Measuring CS and  $\delta$ .  
( $m = \$25,000/\text{year}$ ,  $CS = \$12,500/\text{year}$ , and  $DWL_M = \$8021/\text{year}$ ).

Precision of Measuring CS			Precision in Measuring the Income Slope					
			$t_\delta = 0.5$			$t_\delta = 1.0$		
$V_{CS}$	$\sigma_{CS}^2$	RMSE Marsh	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE	RMSE Hicks	% diff. in MSE
<i>Case I: <math>\eta_y = .5</math>; bias = 1342.8</i>								
.10	1250.	22791.	20193.	-21.4	21350.	-12.2	21928.	-7.4
.10	1250.	21579.	21654.	.7	21561.	-.2	21538.	-.4
.10	1250.	20296.	22922.	27.5	21717.	14.5	21113.	8.2
.80	10000.	31530.	27479.	-24.0	28625.	-17.5	29198.	-14.2
.80	10000.	23751.	23402.	-2.9	23316.	-3.6	23294.	-3.8
.80	10000.	11579.	17411.	126.0	15792.	85.9	14950.	66.6
4.0	50000.	71514.	62475.	-23.6	63572.	-20.9	64121.	-19.6
4.0	50000.	54444.	49675.	-16.7	49635.	-16.8	49625.	-16.9
4.0	50000.	28531.	30540.	14.5	28916.	2.7	28086.	-3.1
<i>Case II: <math>\eta_y = 1.0</math>; bias = 2363.4</i>								
.10	1250.	22874.	18737.	-32.8	20540.	-19.3	21441.	-12.1
.10	1250.	21667.	21825.	1.5	21601.	-.6	21544.	-1.1
.10	1250.	20389.	24405.	43.2	22546.	22.2	21612.	12.3
.80	10000.	31590.	25009.	-37.3	26783.	-28.1	27671.	-23.2
.80	10000.	23830.	23280.	-4.6	23070.	-6.3	23017.	-6.7
.80	10000.	11741.	20258.	197.0	17974.	134.0	16789.	104.0
4.0	50000.	71540.	56390.	-37.8	58033.	-34.1	58858.	-32.3
4.0	50000.	54478.	46281.	-27.8	46176.	-28.1	46150.	-28.2
4.0	50000.	28597.	30806.	16.0	28619.	.2	27504.	-7.5
<i>Case III: <math>\eta_y = 1.5</math>; bias = 3173.1</i>								
.10	1250.	22971.	17858.	-39.5	20041.	-23.8	21132.	-15.3
.10	1250.	21770.	21961.	1.8	21632.	-1.3	21550.	-2.0
.10	1250.	20498.	25286.	52.1	23047.	26.4	21923.	14.3
.80	10000.	31661.	23377.	-45.4	25516.	-35.0	26588.	-29.4
.80	10000.	23924.	23206.	-5.9	22896.	-8.4	22817.	-9.0
.80	10000.	11931.	21923.	237.0	19298.	161.0	17941.	126.0
4.0	50000.	71571.	51972.	-47.2	53903.	-43.2	54876.	-41.2
4.0	50000.	54519.	43703.	-35.7	43539.	-36.2	43498.	-36.3
4.0	50000.	28675.	30511.	13.2	28139.	-3.7	26941.	-11.7

## Footnotes

The authors are Associate Professor and Assistant Professor, respectively, in the Department of Agricultural Economics, University of California, Davis. Senior authorship is not assigned.

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1. As argued and illustrated by Hausman, if we know the true functional form of the Marshallian demand curve ( $D$ ) in certain cases we can solve explicitly for the corresponding quasi-indirect utility function or expenditure function, and from there we can obtain the exact Hicksian welfare measures. In other cases when we know the true Marshallian demand equation, but cannot solve back, we can nonetheless obtain an arbitrarily close approximation for the exact Hicksian welfare measures (e.g., Vartia). A more common case, however, in applied work is where we don't know the true functional form of the Marshallian or Hicksian demand curves of interest. Often a functional form is assumed arbitrarily and combined with elasticities to obtain measures of price, quantity and welfare changes (as above). Even when equations are estimated econometrically, the functional form is rarely tested and, in the exceptional cases when tests are applied, they usually only validate the performance of the model as a local approximation. Against this background of uncertainty about functional form the emphasis that some authors have placed on *exact* welfare measures may be unwarranted, except as an appeal for internal consistency in an analysis.
2. For some purposes it may be desired to normalize these differences as a fraction of total income, total expenditure on the good, or the Marshallian welfare measure (e.g., as by Willig and by Kling 1992, for instance).
3. The results above are very similar to those of Willig. One difference is the allowance for upwards sloping supply, here, whereas Willig had exogenous price changes. In the limit as  $\epsilon \rightarrow \infty$ ,  $\epsilon/(\epsilon-\eta) \rightarrow 1$ , and the results become more clearly comparable.
4. Algebraically  $(DWL^H - DWL^M)/DWL^M = -[\epsilon\eta/(\epsilon-\eta)]k\eta$ .
5. While plausible in many situations, weak complementarity is not a suitable assumption when analyzing goods for which substantial willingness to pay by nonusers is suspected, since it implies nonuse value is zero. An alternative to weak complementarity is discussed by Neill and by Larson.
6. One consideration is that we do not necessarily need to define supply and demand globally in order to conduct an analysis of the policy-induced equilibrium displacement—local approximations might be adequate for the evaluations of the impacts on prices and quantities.



7. Then the question becomes: how is the "optimal" choice of welfare measure affected by the purpose of the analysis (and the applicable rules of evidence), the data available for the analysis, the type of model being used, and the type of market being analyzed?
8. To assist with its responsibilities under the Oil Pollution Act of 1990, the National Oceanic and Atmospheric Administration in 1992 convened a blue-ribbon panel, headed by Professors Kenneth Arrow and Robert Solow, whose purpose was to evaluate whether contingent valuation could be used for measuring nonuse value and, if not, what alternatives may exist.
9. They argue that if the regression error is due largely to random preferences or measurement error in the dependent variable, the estimated quantity demanded should be used, while the actual quantity should be used if the source is omitted variables uncorrelated with the regression error.
10. His illustrative simulations for representative parameterizations of the semilog demand model showed that it may be necessary to accept a large degree of bias in consumer's surplus to achieve minimum MSE, at least for that functional form and parameterization.
11. A difficulty with this rationale, as both Hausman and LaFrance demonstrated, is that it is more likely to be true when estimating the entire area under a demand curve than for measuring deadweight losses or excess burdens which are often the focus of policy analyses.
12. Like previous authors, we condition our analysis on the functional form chosen for estimation, and evaluate the Hicksian-Marshallian tradeoff within that context. We also take as a given that the calculation of money measures of welfare change is meaningful for policy analysis.
13. This is a generally recognized problem in recreation demand studies (see, e.g., McConnell 1985), which form the bulk of the environmental economics applications in which the Hicksian/Marshallian tradeoff has been discussed.
14. In addition to holding for CV for an improvement in welfare due to a reduction in  $p$  or  $z$ , this holds for equivalent variation for a welfare decline due to an increase in  $p$  or  $z$ , by the symmetry of compensating and equivalent variations. The equivalent variation is also a willingness to pay measure, interpreted as the willingness to pay to avoid the adverse change in  $p$  and  $z$ .
15. This can be verified from (16) and (17). Since it is not possible for two of the three correlations to be negative with the third positive, given the signs on the covariance terms in (17) the maximum  $MSED_H$  occurs either when  $\rho_x = 1$  and  $\rho_{\delta,CS} = 0$ , or when  $\rho_{x,\delta} = 0$  and  $\rho_{\delta,CS} = -1$ .

Figure 1: Price, Quantity and Welfare Impacts of an Output Subsidy (or R&D) in a Non-traded Commodity Market

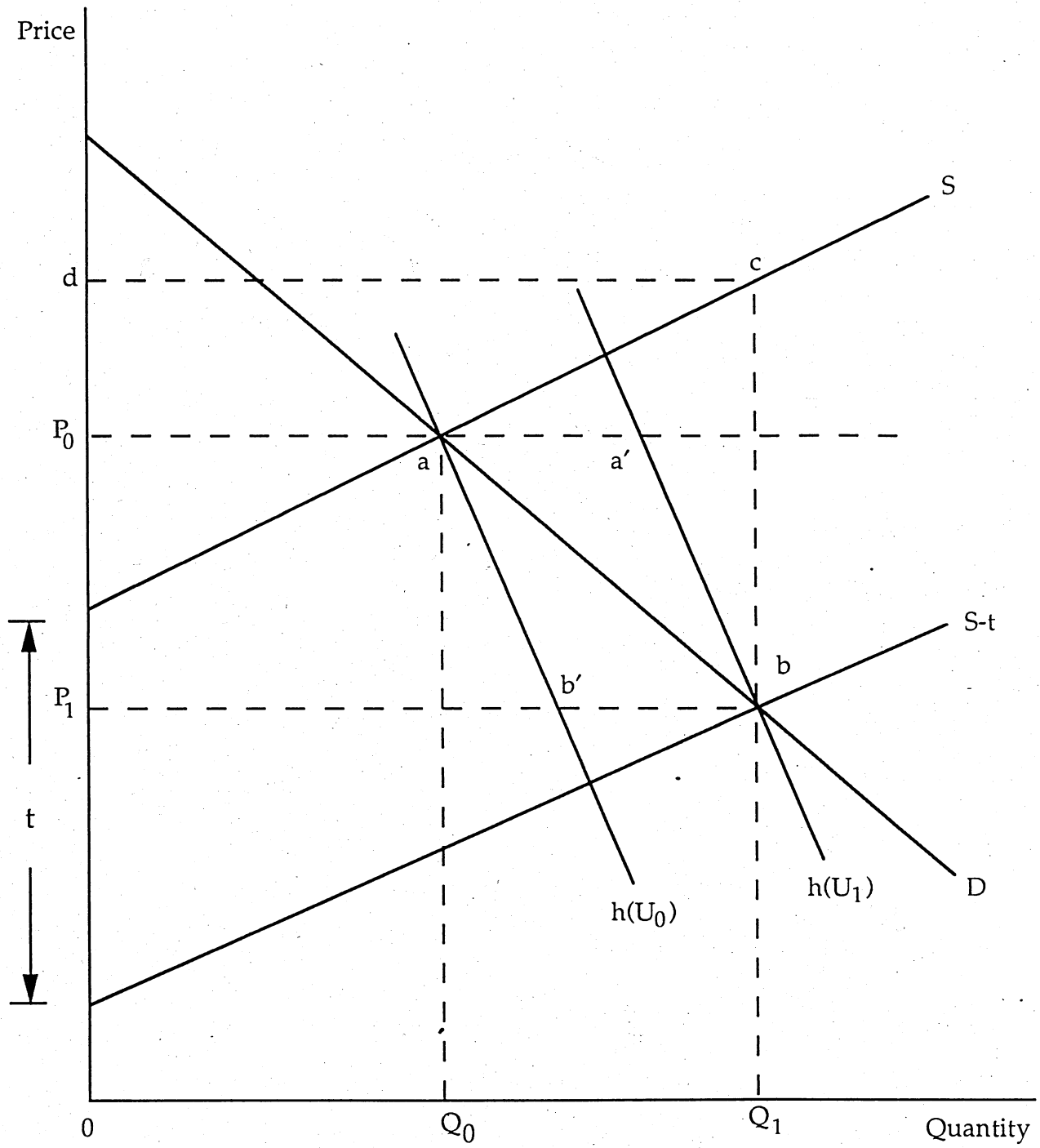


Figure 2: Hicksian and Marshallian Measures of Willingness to Pay and Deadweight Loss

