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# AGRICULTURAL ECONOMICS

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THE LONG TERM PEAK LOAD PROBLEM  
IN GRAIN STORAGE\*

D. BRENNAN and R. LINDNER

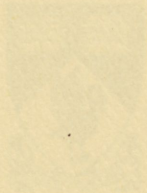
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Discussion Paper 7/88

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


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**THE LONG TERM PEAK LOAD PROBLEM  
IN GRAIN STORAGE\***

D. BRENNAN and R. LINDNER

 Agricultural Economics  
Discussion Paper 7/88

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## THE LONG TERM PEAK LOAD PROBLEM IN GRAIN STORAGE

### Abstract

Fluctuating annual harvest volumes create a peak load problem in the provision of grain storage capacity. There are a number of technologies for handling and storing grain, ranging from capital intensive to labour intensive. Optimal provision of grain storage capacity can therefore be analysed in the framework of the conventional peak load pricing model.

A revised version of the peak load pricing model, with specific application to investment in centralised grain storage capacity, is presented. The implications of economies of scale in the capital intensive storage technologies, and of the availability of low cost options to central storage are discussed.

### Introduction

There are three peak load problems that can be identified in the provision of grain handling facilities. The first is due to intra-daily cycles in the ability of farmers to harvest grain, the second is imposed by biological constraints on the time of the year when grain crops ripen giving rise to a short annual harvest period, and the third relates to varying annual crop production levels. Respective manifestations of these three problems may include early morning and/or late evening queuing at centralised storage receipt depots, inefficient utilisation of grain transport facilities, and the provision of excessive grain storage infrastructure. Each of these peak load problems raises interesting issues for the grain handling industry. Apart from transport, handling and storage capacity investment decisions, other issues which should be of concern to the industry include opening hours for grain receipt terminals, optimal mode mixes for grain transport, and peak load pricing for various forms of grain handling. Many of these issues are currently before Royal Commission on Grain Handling, and a number have been the subject of previous research. However most of the research to date has bypassed the peak load dimension of many of these issues, so a study funded by Wheat Industry Research Committee of Western Australia has been initiated to provide such a focus for economic research on grain handling in WA. The daily and seasonal peak load problems referred to above are the subject of separate papers still under preparation.

This paper is restricted to an analysis of the impact of peak load problems in grain handling arising from year to year fluctuations in crop size on the optimal mix of different types of grain storage. Grain handling authorities can utilise many different types of grain storage facility, ranging from capital intensive structures such as large concrete vertical silos with automated control of grain storage conditions to very basic forms involving little more than piling grain up in heaps on the ground.

However, the essential nature of the choices to be made can be analysed quite simply if it is recognised that each type of storage differs with respect to capital (construction) and/or operating costs. Hence the essence of the choice between different types of grain storage can be represented as a tradeoff between high capital costs and low operating costs at one extreme and low capital, but high operating costs at the other extreme.

### Peak Load Investment Criteria for Grain Storage

The optimal provision of storage capacity to meet annual fluctuations in grain production can be considered in the peak load pricing and investment framework originally developed in the literature dealing with similar problems facing public utilities such as electricity generating companies (eg. Steiner 1957, Brown and Johnston 1969, Crew and Kleindorfer 1976). It has been shown that if such utilities wish to maximise welfare, then the appropriate investment rule is to provide capacity up to the point where the welfare gains from satisfying a marginal unit of demand for storage in the peak period are just equal to the cost of providing the extra unit of capacity. Furthermore, peak load pricing policies should be followed which maximise utilisation of capacity in the short run subject to the condition that operating costs are recovered. The extra price that needs to be charged to ration demand in the peak period is used to recover the cost of capital investment. In stochastic models price must be set before demand is known so it cannot perform a rationing role, and full cost recovery isn't guaranteed.

One of the main practical problems with the peak load pricing model is that application of the model requires knowledge of demand functions (Malko et al 1977). This is required in order to determine the willingness to pay for capacity at the margin, and to set appropriate pricing rules that will ration demand in the peak periods. In the following sections, a simple peak load model of grain storage is discussed which assumes away the need to know demand elasticity. The investment and pricing criteria are discussed, then the model is refined to allow for a more realistic representation of grain storage costs which allows for economies of scale. The paper concludes with a discussion of some of the assumptions of the model, including the implications of less than perfectly inelastic demand for grain storage capacity at a particular site.

#### A Peak Load Model for Grain Storage

In order to analyse investment in centralised grain storage capacity, the peak load pricing model may be simplified by interpreting the term "storage" liberally to encompass holding grain for market by any possible means, including in the paddock and/or in on-farm storage, as well as in centralised facilities, and by assuming a totally inelastic demand for such storage which will be satisfied by one means or another. In such a model, the level of demand for storage in toto can be treated as a stochastic variable equal to the annual volume of grain produced which fluctuates independently of grain storage capacity and/or charges.<sup>1</sup> Thus the demand for centralised grain storage capacity can be represented as a derived demand determined by the total demand for storage (grain production) and the cost of alternatives to central storage.

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1. The assumption that the annual volume of grain production is independent of the annual price of storing the grain is not wildly unrealistic given that planting decisions are made long before the size of total annual harvest, and hence the cost of storing grain is known. For the moment we are assuming that the annual demand for centralised storage is equal to the annual volume of grain produced because there are no substitutes.

The costs of alternatives to centralised storage will be referred to as the costs of 'supply failure', and the 'demand' for alternatives to central storage, (production minus the demand for central storage) will generally be referred to as 'excess demand' in the following discussion. If the cost of alternatives are independent of the level of demand, then the derived demand for centralised storage may be considered inelastic up to a level equal to these costs, after which it becomes perfectly elastic because the farmer will not pay any more for centralised grain storage than the cost of alternatives. For the sake of analytical simplicity, we initially also assume that the cost of alternatives is equal across farmers and is always greater than the cost of central storage. This implies that all of the harvested grain will flow into CBH silos, up to the point where capacity is reached. Given these assumptions, we can treat the central grain storage investment problem as minimising all expected grain storage costs, including the cost of 'supply failure'. Most of these simplifying assumptions are relaxed later in the paper. To formulate the cost minimisation problem we need to know how production varies, the costs of storing the grain in the central system, and the cost involved in having inadequate storage facilities.

### Production Variation

Let the annual volume of grain delivered to a particular site be denoted by  $q_i$ , and its density function by  $\phi(q_i)$ . Hence expected annual production is

$$E(q) = \int_0^{\infty} q_i \phi(q_i) dq \quad (1)$$

and variance of annual production is

$$V(q) = \int_0^{\infty} [q_i - E(q_i)]^2 \cdot \phi(q_i) dq \quad (2)$$

The cumulative density function which describes the probability that grain production will be greater than some amount  $q^*$  will be denoted by  $\Phi(q^*)$  where:

$$\Phi(q^*) = \int_{q^*}^{\infty} \phi(q_i) dq \quad (3)$$

so

$$0 \leq \Phi(q^*) \leq 1$$

It is assumed that the maximum throughput of grain through a storage in a particular year is equal to the physical storage capacity--that is, no grain is railed out while deliveries are still being made. While this is an unrealistic assumption for the system as a whole, it is relevant for many individual sites. Because of the physical capacity limits of the rail system, only a small amount of the grain can be transported out during the harvest period (which is about 40 days long at individual sites), and economies of scale in the transport of grain indicate that the allocation of rail capacity in the harvest period should be concentrated at a few sites, rather than be spread throughout the system. The implications of using rail to increase throughput are discussed later in the paper.

Having set the annual turnover of grain in country sites (throughput relative to capacity) equal to one, we can define the marginal utilisation of storage in terms of production variation. That is,  $\Phi(q^*)$  of equation 3 also describes the probability that the marginal unit of storage capacity (at  $q^*$ ) will be used. The (expected) total volume of grain that will be stored in a storage capacity of size  $q^*$  is given by

$$\mu(q^*) = \int_0^{q^*} q_i \phi(q_i) dq + q^* \int_{q^*}^{\infty} \sigma(q_i) dq \quad (4)$$

### Grain Storage Costs

As in standard peak load pricing models with diverse technology (eg. Crew and Kleindorfer 1976, Wenders 1976) there are a number of methods of grain storage that have inversely related capital and operating costs. For simplicity, in this paper we will only consider three plant types, which in order of decreasing capital costs and increasing operating costs will be referred to as vertical, horizontal, and temporary (also called bunker) storage. In the conventional peak load pricing literature, it is common to assume that marginal and average cost for both operating and capital costs are constant. This is not likely to be an accurate reflection of the costs in the grain handling industry. There is evidence to suggest that marginal construction costs for permanent storage facilities may be declining (Kerin 1985). In addition, there may be declining average construction costs due to the fixed set up costs of the inloading and outloading machinery. On the other hand, while operating costs also might have a fixed component, most if not all of it can be attributed to the fixed labour requirement for receiving grain, and so should not influence the choice of type of storage as it is site, rather than storage type specific.

For simplicity then, we assume that average and marginal operating costs for each type of storage are independent both of quantity stored and of storage capacity, and denote operating cost per tonne for vertical, horizontal, and temporary storage respectively by  $b_v$ ,  $b_h$ , and  $b_t$ . Likewise, for the time being we let the constant parameters  $B_v$ ,  $B_h$ , and  $B_t$  be the annuitised average cost<sup>2</sup> of investing in a tonne of vertical, horizontal and temporary storage capacity respectively; even though there may well be economies of scale in the construction of at least vertical and horizontal storage (any equivalent economies for temporary storages are likely to be insignificant). As in the conventional literature, we also assume that  $B_v > B_h > B_t$ ; while  $b_v < b_h < b_t$ .

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2. Alternatively, the problem may be formulated in terms of costs over the life of the plant, where capital costs are in present dollars, and operating costs are in terms of the discounted stream of costs over the life of the plant; the former is adopted here for simplicity in dealing with plant types of different lifespans.

In addition it is necessary that  $B_v + b_v < B_h + b_h < B_t + b_t$  (i.e. as long as capacity is fully utilised, the most capital intensive storage will be the least costly, and at very low utilisation levels the marginal cost of the least capital intensive storage is the lowest.). Preliminary evidence suggests however, that vertical storage is not an efficient option under our assumption that maximum turnover is equal to 1. It can be seen in Table 1 that the marginal cost of vertical storage exceeds that of horizontal storage at full utilisation. Therefore, for country storage sites where it is either infeasible or uneconomic to turnover storage capacity during the harvest season, it can be seen vertical storage is not an efficient option. While the effect of increased turnover rates on the efficiency of vertical storage will be discussed later, the main part of the following discussion will assess only the optimum combinations of horizontal and temporary storage. It should be noted that while the analysis is made simpler by examining only two plant types, it can easily be extended to analyse a larger number of efficient alternatives.

Table 1: Grain Storage Costs

Costs \$/t	Vertical (L and SI)	Horizontal (A Type)	Temporary (Bunker)
Capital	83.32	38.21	19.04
Life	40	30	20
Annual Cost (5%) B	4.86	2.49	1.45
Operating b	1.18	1.69	3.59
B + b	6.04	4.17	5.04
Horizontal vs Temporary	$(B_h - B_t)/(b_t - b_h)$		= 0.54
Efficient Turnover	$= (B_v - B_h)/(b_h - b_v)$		= 4.7
	$= B_v/(B_h + b_h - b_v)$		= 1.6

Sources: Cooperative Bulk Handling Ltd.  
Royal Commission into Grain Handling.

Examining the marginal costs of our efficient alternatives,  $B_h + b_h < B_t + b_t$ , it can be seen that if demand for storage were known with certainty, then the investment decision with respect to storage type mix would be trivial. Quite simply, sufficient horizontal storage should be built to fully cater for the known level of demand, and no other type of storage would be needed. On the other hand, when storage demand is stochastic, and therefore unknown in any particular year, the optimal mix of horizontal and temporary storage will depend, inter alia, on expected throughput of grain.

To see why, let  $K_h$  and  $K_t$  denote the capacity (in tonnes) to store grain in horizontal and temporary facilities. Because horizontal facilities have lower operating costs we know that these facilities will always be utilised first in the short run. Hence we can write the actual cost of grain storage per se for horizontal storage as :

$$C_h = K_h \cdot B_h + b_h \cdot q_i \quad \text{if } q_i < K_h \quad (5a)$$

$$= K_h \cdot B_h + b_h \cdot K_h \quad \text{if } q_i \geq K_h \quad (5b)$$

Likewise, for temporary storage costs,  $C_t$  :

$$C_t = K_t \cdot B_t \quad \text{if } q_i < K_h \quad (6a)$$

$$= K_t B_t + b_t (q_i - K_h) \quad \text{if } K_h \leq q_i < K_h + K_t \quad (6b)$$

$$= K_t B_t + b_t K_t \quad \text{if } q_i \geq K_h + K_t \quad (6c)$$

So far the cost specification resembles Crew and Kleindorfer's (1976) peak load pricing model. However, the above storage costs do not include any costs of failure to meet demand for storage, which must be included as a substitute for a demand curve. We have assumed for the time being that supply failure costs are linearly related to the amount of excess demand<sup>3</sup>. By definition, such failure costs do not include any fixed cost component, and will be denoted by  $C_f$ , where:

$$C_f = 0 \quad \text{if } q_i < K_h + K_t \quad (7a)$$

$$= b_f (q_i - K_h - K_t) \quad \text{if } q_i \geq K_h + K_t \quad (7b)$$

where  $b$  = average (and marginal) failure cost (measured in dollars per tonne).

Thus the cost minimisation problem involves determining the right balance of infrastructure so that more permanent facilities are used to cope with "more certain" demand, while temporary facilities are used to store the extra grain that is produced less frequently thus resulting in lower utilisation (throughput relative to capacity) of any additional capacity. Furthermore, at some level it will not pay to

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3. This supply failure cost should not be compared with Crew and Kleindorfer's (1978) rationing cost, which represents any welfare losses that may arise from non-price rationing, as well as the administrative costs of doing so. Rather, our supply failure cost is analogous to the welfare losses arising from unsatisfied demand, which is all those unsatisfied customers who would have been prepared to cover the operating cost of their purchase had more capacity been available.

In our model, rationing costs are assumed to be minimal even when there are different supply failure costs among farmers, because it is possible to accurately predict total demand before the price is set. This compares to the public utilities case where price must be set before the level of excess demand is known.

construct additional centralised storage capital to cope with the very infrequent demand arising from bumper harvests.

Consequently, we can write the total cost function to be minimised as:

$$\begin{aligned}
 \text{TEC} = & K_h \cdot B_h + b_h \int_0^{K_h} q_i \phi(q_i) dq + b_h \cdot K_h \cdot \int_{K_h}^{\infty} \phi(q_i) dq \\
 & + K_t \cdot B_t \cdot b_t \cdot \int_{K_h}^{K_h+K_t} (q_i - K_h) \cdot \phi(q_i) dsq + b_t \cdot K_t \int_{K_h+K_t}^{\infty} \phi(q_i) dq \\
 & + b_f \cdot \int_{K_h+K_t}^{\infty} (q_i - K_h - K_t) \cdot \phi(q_i) dq \quad (8a)
 \end{aligned}$$

which can be rewritten as:

$$\text{TEC} = K_h B_h + b_h \mu(K_h) + K_t B_t + b_t \mu(K_t) + b_f [E(q) - \mu(K_h) - \mu(K_t)] \quad (8b)$$

where:

$$\mu(K_h) = \int_0^{K_h} q_i f(q_i) dq + K_h \int_{K_h}^{\infty} \phi(q_i) dq$$

= expected throughput of grain through horizontal storage

and:

$$\mu(K_t) = \int_{K_h}^{K_h+K_t} (q_i - K_h) \phi(q_i) dq + K_t \int_{K_h+K_t}^{\infty} \phi(q_i) dq$$

= expected throughput of grain through temporary storage.

Also note that  $[E(q) - \mu(K_h) - \mu(K_t)]$  is total expected supply failure (ie, the amount of grain that is produced in excess of storage capacity).

#### Investment Rules

In the literature, first order conditions for cost minimisation are obtained by differentiating the total cost function with respect to the level of capacity of each type of storage, and setting each partial derivative equal to zero. Setting  $(\partial \text{TEC} / \partial K_h) = 0$ , we obtain

the following first order condition:<sup>4</sup>

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4. See Appendix A for derivation.

$$B_h = (b_t - b_h)[\Phi(K_h) - \Phi(K_h + K_t)] + (b_f - b_h)\Phi(K_h + K_t) \quad (9)$$

which says, that capacity  $K_h$  should be added up to the point where the marginal capital cost of permanent capacity is just equal to the savings arising from having an extra unit of the cheaper capacity. In this case, savings arise both from substitution of horizontal for temporary storage at the margin, plus avoidance of supply failure at the margin. Likewise, setting  $(\partial \text{TEC} / \partial K_t) = 0$ , we obtain:

$$B_t = (b_f - b_t) \cdot \Phi(K_h + K_t) \quad (10)$$

which says we should continue to invest in temporary capacity until the marginal capital cost of adding temporary capacity is just equal to the expected savings that arise (net of temporary storage operating costs) from avoiding an extra unit of supply failure at the margin.

By simultaneous solution of these two conditions, we obtain the standard criterion that investment in horizontal storage capacity,  $K_h$ , should continue as long as:

$$\Phi(K_h) > (B_h - B_t) / (b_t - b_h) \quad (11)$$

In other words, optimal horizontal capacity is reached where the probability that the marginal unit will be used is just equal to the net extra capital cost of substituting horizontal capacity for temporary capacity divided by the corresponding savings in operating costs at the margin (ie, the saving in capital cost must equal the expected saving in operating cost at the margin). Having determined horizontal capacity we can solve for optimal temporary capacity by substitution back into equation (10).

In Table 1 we present some data on storage costs provided by CBH to the RCGH. This data consists of estimated average costs, and for reasons discussed below is not strictly suitable as a measure of the variables in the conditions derived above. Consequently only speculative conclusions can be derived from this data. It can be seen that as long as horizontal storage capacity is sufficient to handle total grain deliveries in a median production year, the fact that the ratio  $(B_h - B_t) / (b_t - b_h) = 0.54$  suggests that extra production in bumper years should only be catered for, if at all, by temporary storage. At this stage, we have no evidence on the cost of "supply failure", and so are not able to determine optimal investment in temporary storage capacity.

If we assume that the annual volume of grain production is normally distributed with mean,  $\mu$ , and variance,  $\sigma^2$ , we can derive rather more general propositions about the criterion for investment in horizontal storage capacity. Define  $\theta_h$  as the value of the standard normal variable such that  $\Phi(\theta_h) = (B_h - B_t) / (b_t - b_h)$ . Likewise, let  $\theta_t$  denote the value of the standard normal variable such that  $\Phi(\theta_t) = (B_t) / (b_f - b_t)$ . Given these assumptions, the optimal level of  $K_h (=K^*) = \mu + \sigma \cdot \theta_h$ , while optimal  $K_t (=K_t^*) = \mu + \sigma \cdot \theta_t$ .

In other words, the larger the difference in capital cost relative to the operating cost savings, the lower the optimal capacity of storage, and vice versa. Also, the larger the variance relative to

the mean production level, the lower the optimal capacity when capital costs are relatively high. Likewise, when it is operating cost savings that are relatively more important then a larger variance will lead to a relatively higher optimal capacity.

These conclusions are subject to several caveats arising from simplifying assumptions referred to above. The implications of relaxing some of these assumptions will now be discussed.

#### Turnover of Storage Capacity

As already noted, it often is possible for the handling authority to use rail to transport some of the grain out during the harvest period. This increases the 'effective capacity' of the storage beyond its physical limits. In order to arrive at optimum investment criteria for those sites where it is possible to rail grain during the harvest period, we need to respecify storage capability limits to equal physical capacity multiplied by turnover level. However, the co-determination of the optimal mix of storage capacity, and of the best allocation of a finite amount of transport capacity between different types of storage so as to increase turnover is a quite complex problem which is beyond the scope of this paper. Instead, in order to illustrate the principles involved, let us examine the effect of a higher turnover rate on our efficiency criterion. Let us assume that some amount of rail capacity is allocated to a site so that the turnover rate is now greater than 1, and defined by  $\phi$ . We also assume that average operating costs are independent of storage turnover, the principal effect of which is to reduce capital cost per tonne of grain stored. That is, capital costs relative to throughput may be written  $B_v \cdot \phi^{-1}$  per tonne. If we also assume that the turnover of grain is independent of the type of storage and depends only on the allocation of rail capacity, our relevant efficiency criteria for vertical storage becomes  $B_v \cdot \phi^{-1} + b_v < B_h \cdot \phi^{-1} + b_h$ .

Looking at Table 1, this implies that there must be a turnover level of 4.7 in order for vertical storage to become an efficient option. It might be pointed out that the faster unloading facilities of vertical storage means that higher turnover rates can be achieved from a given amount of rail capacity (because of faster rail turnaround times). Consequently, a fixed amount of rail capacity will lead to a higher turnover of vertical storage relative to horizontal storage. However, even at the extreme, if we assume that outloading times for horizontal storage are so slow that the turnover level is almost as low as 1; the level of turnover  $\phi_v$  required to enable vertical storage to be an efficient option is defined by the formula  $B_v \phi_v + b_v < B_h + b_h$ . According to available cost estimates (Table 1), this implies that the turnover level required to justify vertical storage must at least be in excess of 1.6.

#### Variable Average and Marginal Costs of Storage Construction

So far the discussion has used the simple constant average and marginal cost assumption that has been applied in conventional peak load pricing literature. We now assume that for construction of

horizontal storage<sup>5</sup>, there are both fixed costs as well as declining marginal costs, so total capital costs of horizontal storage become:

$$C_k(K_h) = B_{0h} + B_{1h} K_h - B_{2h} K_h^2 \quad (15)$$

The new investment rule for horizontal storage becomes:<sup>6</sup>

$$B_{1h} - 2B_{2h} K_h - B_t = (b_t - b_h) \int_{K_h}^{\infty} \phi(q_i) dq \quad (16)$$

Thus the effect of the declining marginal cost (the extra term  $-2B_{2h} K_h$ ) is to reduce the value of the LHS, as the capacity of horizontal storage increases. This is analogous to increasing capital cost of temporary storage as horizontal capacity increases. Consequently, declining marginal costs of horizontal storage will mean that the optimal solution is likely to include a larger amount of horizontal storage, ceteris paribus.

Another implication of declining average construction costs, due to fixed costs and declining marginal construction costs, is that it is important to specify the cost function correctly. Simple use of average costs (rather than reference to marginal costs) will overestimate the costs of expanding capacity. Because fixed costs are likely to be higher for horizontal facilities, use of average cost data will bias the investment decision away from horizontal (or capital intensive) facilities.

#### Supply Failure Costs

To date it has been assumed that average and marginal supply failure costs are constant and always exceed the marginal operating cost of central storage. In practice, there are a number of alternatives to central storage that will affect the shape of the 'supply failure' cost function. We will now discuss these alternatives as well as the effects of the implied elastic demand for central grain storage.

The alternatives are:

(a) Store some of the grain on the farm. Even if central storage is the cheaper long run alternative due to economies of scale, utilisation of existing farm storage will provide a cheap short run alternative, because the capital cost of the farm storage is sunk. Most farmers have limited on-farm storage. The average level in Western Australia is 186t (Howard and Lawrence 1986). It is possible that some of this storage may be available as an 'emergency grain storage measure'.

(b) Because variations in the production of grain will not be perfectly correlated between regions, it is possible that localised

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5. As already noted, economies of scale in temporary storages are likely to be insignificant so are ignored.

6. See Appendix B.

shortages of storage space may be alleviated by getting farmers to transport the excess grain to neighbouring sites.

(c) CBH may alleviate storage pressures by using railing some of the grain during the harvest. This has a high opportunity cost in that the orderly least cost allocation of rail cars during the harvest will be interrupted.

(d) CBH have developed a system of mobile storage structures, which are a particular type of bunker storage that can be transported between sites prior to the opening of the season.

(e) at the extreme, excess production of grain may mean that the grain cannot be stored, and so it is not harvested. The costs associated with this are limited by the market value of the grain minus the costs of harvesting and marketing the grain, minus the value of the unharvested grain in the field. (There may be some stock feeding benefits.) The cost to the farmer will depend on his sunk costs of production, and his valuation of the stock feeding benefit of the unmarketed grain, which depends on the extent of his livestock enterprise.

(f) Another factor that may affect the demand for central storage is the attitude of farmers to risk associated with cost variation. They may be willing to pay more to guarantee supply rather than accept infrequently high grain storage costs. This aspect is not treated in the following discussion.

Because there are substitutes to centralised storage, the cost of supply failure almost certainly will vary between farmers (and between regions). Indeed, it is likely that the cost of supply failure will be an increasing function of the amount of 'excess demand', and at low levels, the cost of alternatives may actually be lower than the cost of centralised storage (where temporary storage is being used at the margin). This 'elasticity' in the demand for central storage has implications for the model presented in this paper.

#### Pricing Rules

An essential element of our cost minimisation model is that price is set equal to short run marginal cost. We have assumed that grain is allocated into the cheapest available storage option, so it is necessary that farmers are aware of the marginal cost of centralised storage, so they can compare it with the cost of alternatives. Also, in peak years CBH will need to use price signals to ensure that those farmers with the highest willingness to pay for central storage (due to a higher supply failure cost) deliver their grain in preference to those with a lower willingness to pay. In Figure 1, we illustrate the relevant pricing policies given predetermined levels of capacity for the various types of grain storage. Note that:

(a) In off peak years, where total production is less than the amount of horizontal storage, CBH need to encourage full utilisation of horizontal storage in the short run, by setting price equal to the operating cost of horizontal facilities in these years. This will ensure that farmers do not adopt the more costly grain storage alternatives in these years.

(b) In 'intermediate' years, where total production is greater than the amount of horizontal capacity, but less than total storage capacity the appropriate price will be the operating cost of temporary storage, as this is the one being used at the margin. This will ensure

that if some growers have 'supply failure' costs less than the cost of temporary storage, they will use these options first.

(c) In very high harvest years (every  $\Phi(K_h + K_t)$  years) when total production exceeds available central storage capacity, cost minimisation requires that available storage space is rationed according to the willingness to pay (ie. to those farmers with a higher supply failure cost). Setting price equal to the operating cost of temporary storage will not be sufficient to ration demand, because supply failure costs are likely to be higher than temporary operating costs. In order to ensure cost minimisation, CBH have to set a higher price that will ensure that the farmers with the highest supply failure cost use the centralised storage. Determination of the appropriate price will require an estimation of the magnitude of supply failure costs. Just how this might be achieved is not immediately apparent, and will be the subject of further study.

In our model, demand is specified as a function of stochastic production and the cost of alternatives. Consequently, the model differs from the conventional stochastic peak load model (eg Brown and Johnson 1969) because it will be possible to predict the actual demand (to a reasonable degree of accuracy) before the price is set. Thus unlike the stochastic models of the public utility industries, it is possible to use price as a rationing tool for grain storage. This has two implications. Firstly, it implies a greater level of efficiency, as the resource costs of non-price rationing will be avoided. Secondly, it implies that a greater level of cost recovery will be achieved.

#### Further Investment Rules

The implications of a non-constant supply failure cost depend on the exact nature of the supply failure cost function. For example, it is possible over some range, the costs of alternatives to temporary storage are less than the cost of temporary storage. For example, the use of on farm storage may have lower or comparable operating costs to temporary storage (it is assumed that capital costs of on farm storage are zero, because of multiple use). If this is the case, the optimal level of horizontal storage will be lower than in the model outlined above, because the marginal operating cost savings from investing in an extra unit of horizontal storage will be less.

On the other hand, if the main alternative to central storage involves late harvesting, or even total 'supply failure' (not harvesting), the costs of supply failure  $b_f$  will always exceed the cost of temporary storage. If this is the case, the optimal investment in permanent facilities will not be effected, because the cost of the least cost alternative to horizontal storage remains  $b_t$ . This is because the investment decision for a certain storage type only depends on the cost of the next best alternative, and production variation. In either case, the optimal level of total storage will be affected. This is because the savings from investing in temporary storage at the margin depend on the expected savings in 'supply failure' costs. A steeper supply failure cost result in a higher level of temporary storage ceteris paribus.<sup>7</sup>

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7. These results can be shown analytically.

### Conclusion

The optimal investment and pricing rules for centralised grain storage require consideration of the variability of production, and can be analysed in the peak load pricing framework. The model outlined is a revised version of Crew and Kleindorfer's stochastic peak load pricing model, where the demand for the service is specified in terms of the costs of supply failure. Investment and pricing rules depend on the total and marginal cost curves for a particular site, and it was shown that declining marginal construction costs and fixed cost can effect the outcome of the model. After consideration of the possible alternatives to central storage, it becomes obvious that the assumption of constant marginal supply failure cost is not appropriate. Generally, it will be necessary to use the assumption that supply failure cost is an increasing function of the amount of supply failure. In this case the model becomes almost synonymous with the conventional stochastic peak load pricing models (eg. Crew and Kleindorfer), because there is a less than perfectly elastic demand for central storage. The only difference is in the specification of the demand for the service. In our model 'demand' is made more explicit, in that it is a function of total production and the cost of alternatives. In contrast, the conventional model assumes that demand is a stochastic variable, and the total uncertainty of actual demand in the short run means that pricing rules cannot be used to ration demand (eg. Brown and Johnson 1969). In contrast, the stochastic element of our model is the value of production is a particular year. Because it is possible for CBH to forecast production (hence demand) before the harvest commences, they will be able to use price as a rationing tool. In theory therefore, we should be able to achieve a comparatively more efficient outcome, than in the public utility industry, because there will be no subsequent rationing costs. In addition, in the public utility model, demand has to be explicitly measured, and it is often difficult because there is no data (the price for services has not ranged much in the past). In our model we only need to estimate the cost of supply failure, which should be more tangible.

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## Appendix A

The total cost function to be minimised is :

$$\begin{aligned} \text{TEC} = & K_h \cdot B_h + b_h \int_0^{K_h} q_i \phi(q_i) dq + b_h \cdot K_h \cdot \int_{K_h}^{\infty} \phi(q_i) dq \\ & + K_t \cdot B_t + b_t \cdot \int_{K_h}^{K_h + K_t} (q_i - K_h) \cdot \phi(q_i) dq + b_t \cdot K_t \cdot \int_{K_h + K_t}^{\infty} \phi(q_i) dq \\ & + b_f \cdot \int_{K_h + K_t}^{\infty} (q_i - K_h - K_t) \cdot \phi(q_i) dq \end{aligned}$$

which can be rewritten as :

$$\text{TEC} = K_h B_h + b_h \mu(K_h) + K_t B_t + b_t \mu(K_t) + b_f [E(q) - \mu(K_h) - \mu(K_t)]$$

$$= K_h B_h + (b_h - b_f) \mu(K_h) + K_t B_t + (b_t - b_f) \mu(K_t) + b_f [E(q)]$$

$$\text{where : } \mu(K_h) = \int_0^{K_h} q_i \phi(q_i) dq + K_h \int_{K_h}^{\infty} \phi(q_i) dq$$

$$\text{and : } \mu(K_t) = \int_{K_h}^{K_h + K_t} (q_i - K_h) \phi(q_i) dq + K_t \int_{K_h + K_t}^{\infty} \phi(q_i) dq$$

Taking partial derivatives, we obtain :

$$\partial \mu(K_h) / \partial K_h = \int_{K_h}^{\infty} \phi(q_i) dq; \quad \partial \mu(K_t) / \partial K_t = \int_{K_h + K_t}^{\infty} \phi(q_i) dq;$$

$$\partial \mu(K_t) / \partial K_h = - \int_{K_h}^{K_h + K_t} \phi(q_i) dq ;$$

and differentiating TEC we get :

$$\partial \text{TEC} / \partial K_h = B_h + (b_h - b_f) \partial \mu(K_h) / \partial K_h + (b_t - b_f) \partial \mu(K_t) / \partial K_h$$

$$= B_h + (b_h - b_f) \int_{K_h}^{\infty} \phi(q_i) dq - (b_t - b_f) \int_{K_h}^{K_h + K_t} \phi(q_i) dq$$

$$= B_h + b_h \int_{K_h}^{\infty} \phi(q_i) dq - b_t \int_{K_h}^{K_h + K_t} \phi(q_i) dq - b_f \int_{K_h + K_t}^{\infty} \phi(q_i) dq$$

$$\partial \text{TEC} / \partial K_t = B_t + (b_t - b_f) \partial \mu(K_t) / \partial K_t$$

$$= B_t + (b_t - b_f) \int_{K_h + K_t}^{\infty} \phi(q_i) dq;$$

Solving simultaneously, we get the investment rules :

Invest in horizontal storage capacity as long as :

$$\Phi(K_h) > (B_h - B_t) / (b_t - b_h)$$

and thereafter invest in temporary storage capacity as long as :

$$\Phi(K_h + K_t) > (B_t) / (b_f - b_t)$$

### Appendix B

The total cost function to be minimised when there are decreasing marginal construction costs becomes:

$$\begin{aligned} \text{TEC} = & B_{0h} + B_{1h} \cdot K_h - B_{2h} K_h^2 + b_h \int_0^{K_h} q_i \phi(q_i) dq + b_h \cdot K_h \cdot \int_{K_h}^{\infty} \phi(q_i) dq \\ & + K_t \cdot B_t + b_t \cdot \int_{K_h}^{K_h + K_t} (q_i - K_h) \cdot \phi(q_i) dq + b_t \cdot K_t \int_{K_h + K_t}^{\infty} \phi(q_i) dq \\ & + b_f \cdot \int_{K_h + K_t}^{\infty} (q_i - K_h - K_t) \cdot \phi(q_i) dq \end{aligned}$$

which may be rewritten as :

$$\begin{aligned} \text{TEC} = & B_{0h} + B_{1h} \cdot K_h - B_{2h} \cdot K_h^2 + b_h \mu(K_h) + K_t B_t + b_t \mu(K_t) + b_f (E(q) - \mu(K_h) - \mu(K_t)) \\ = & B_{0h} + B_{1h} \cdot K_h - B_{2h} \cdot K_h^2 + (b_h - b_f) \mu(K_h) + K_t B_t + (b_t - b_f) \mu(K_t) + b_f E(q) \end{aligned}$$

$$\text{where : } \mu(K_h) = \int_0^{K_h} q_i \phi(q_i) dq + K_h \cdot \int_{K_h}^{\infty} \phi(q_i) dq$$

$$\text{and : } \mu(K_t) = \int_{K_h}^{K_h + K_t} (q_i - K_h) \phi(q_i) dq + K_t \int_{K_h + K_t}^{\infty} \phi(q_i) dq$$

Taking partial derivatives, we obtain :

$$\partial \mu(K_h) / \partial K_h = \int_{K_h}^{\infty} \phi(q_i) dq; \quad \partial \mu(K_t) / \partial K_t = \int_{K_h + K_t}^{\infty} \phi(q_i) dq;$$

$$\partial \mu(K_t) / \partial K_h = - \int_{K_h}^{K_h + K_t} \phi(q_i) dq$$

and differentiating TEC we get :

$$\begin{aligned} \frac{\partial \text{TEC}}{\partial K_h} = & B_{1h} - 2B_{2h} \cdot K_h + (b_h - b_f) \partial \mu(K_h) / \partial K_h + (b_t - b_f) \partial \mu(K_t) / \partial K_h \\ = & B_{1h} - 2B_{2h} \cdot K_h + (b_h - b_f) \int_{K_h}^{\infty} \phi(q_i) dq - (b_t - b_f) \int_{K_h}^{K_h + K_t} \phi(q_i) dq \end{aligned}$$

$$= B_{1h} - 2B_{2h} \cdot K_h + b_h \int_{K_h}^{\infty} \phi(q_i) dq - b_t \int_{K_h}^{K_h + K_t} \phi(q_i) dq - b_f \int_{K_h + K_t}^{\infty} \phi(q_i) dq$$

$$\frac{\partial \text{TEC}}{\partial K_t} = B_t + (b_t - b_f) \partial \mu(K_t) / \partial K_t$$

$$= B_t + (b_t - b_f) \int_{K_h + K_t}^{\infty} \phi(q_i) dq$$

Solving simultaneously, we get the investment rule,

$$B_{1h} - 2B_{2h} K_h - B_t = (b_t - b_h) \int_{K_h}^{\infty} \phi(q_i) dq$$

