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DYNAMICALLY OPTIMAL ADOPTION OF ~~FRAMING~~ FARMING PRACTICES WHICH DEGRADE OR RENEW THE LAND

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Abstract

Many farming decisions are choices among discrete practices. This is particularly true for decisions that degrade or renew the land over time. Most models of soil erosion, acidity, or salinity, however, try to approximate discrete choices by a static model or by a dynamic model with continuous control. This study constructs a dynamic model for the optimal adoption of discrete practices. The model is an extension of free-time optimal control. Each discrete practice has its own optimal control problem and these control problems are linked over time to study optimal switching among practices. Land is often considered an exhaustible resource, but several practices renew rather than degrade the land. It is found that an optimal time path typically has an initial phase of degradation followed by a steady-state rotation between degrading and renewing practices. The initial phase could be one of renewal and the final phase one of abandoning or selling the land. The renewal of soil acidity, salinity and, perhaps, erosion makes agriculture sustainable into the indefinite future and shifts the focus of public policy from conserving an exhaustible land resource to attaining the optimal steady-state. It also shown that a discrete but static model or a dynamic but continuous model has no optimality properties whatever. This is unfortunate because the dynamic discrete-choice model is more difficult to solve. A few special cases may be easily solved but the general model requires large-scale mathematical programming with special gradient calculations.

DYNAMICALLY OPTIMAL ADOPTION OF FARMING PRACTICES WHICH DEGRADE OR RENEW THE LAND

Land degradation and renewal are dynamic processes, controlled by the adoption of farming practices. Should a farmer plant an erosive but profitable crop?--adopt conservation tillage?--plant a nitrogen-fixing legume?--rotate crops for weed and disease control?--establish trees?--install contour banks?--or just sell the farm? Farming practices can be managed at greater or lesser intensity but, fundamentally, they are discrete alternatives.

In 1942, Bunce proposed a dynamic model for the adoption of soil-conserving practices. With the exception of Walker (1982; see also Walker and Young, 1986) and Miranowski (1984), more recent authors either do not include the dynamics or assume that discrete practices can be approximated by a continuous control variable.

For example, many empirical studies, too many to list, are an extension of traditional farm-planning. Usually, a large number of practices are included in a static model which is solved by linear programming. But to ignore the dynamics is to ignore the cost of degrading future productivity. A practice may be profitable now but unprofitable over the long term. The static solution may not be optimal.

Other empirical and some strictly theoretical studies (examples are Burt, 1981; and McConnell, 1983) are based on the well-developed literature in natural resource economics. Continuous control of degradation is assumed and medium-sized problems are sometimes solved by either dynamic programming or mathematical programming. No harm is done if continuous control is simply a convenient approximation. Unfortunately, it is not. As will be shown, discrete practices may be adopted in ways that continuous control cannot model.

This study adds to the literature by constructing and applying a dynamic model for the optimal adoption of discrete farming practices. The model is an extension of free-time optimal-control and is analyzed, not in real time, but in what might be called artificial time. In real time, a switch from one practice to another occurs instantaneously. But in artificial time, a switch happens in "slow-motion". This makes the model almost as simple to analyze as an ordinary control model and solvable by dynamic programming or mathematical programming.

Speeding back up to real time, an optimal time-path has an initial degradation phase and a final steady-state phase. During the initial phase, degradation declines as practices which degrade less and less are adopted in succession. During the final phase, degradation is balanced by renewal as practices which degrade and renew are rotated. The initial phase might be absent or could be one of renewal rather than degradation. The final phase could be one of abandoning rather than sustaining the land.

The models of Walker (1982) and of Miranowski (1984) initially degrade and then abandon the land. Models of crop rotations have only a steady-state of sustained farming (El-Nazer and McCarl, 1986; Lazarus and Swanson, 1983). In related literature, forests, machinery and livestock are managed by rotations of discrete alternatives (Clark 1976; Perrin, 1972; Karp et al., 1986; Chavas et al., 1985). As will be shown, these and other special cases are relatively easy to solve. The general model, however, is not so easy. It can be large. It is highly nonlinear and requires unusual gradients to optimize for discrete practices. Therefore it wise to understand the theory of optimal adoption and examine special cases before attempting to solve a more general problem.

The model for the dynamically optimal adoption of discrete farming practices is formulated in the next section. A discussion of the theory follows. Then the model is applied to three important types of land degradation: erosion, acidity and salinity. In the applications, land is degraded by erosion and then abandoned, degraded by acidity and sustained in a steady-state and renewed from a saline state and sustained.

Dynamic Discrete-Choice Model

If a farmer had only one farming practice available but could operate that practice at any intensity, the value of the farm would be maximized by solving an ordinary control problem with continuous control.

1) $$J_0(X_{t_0}) = \text{Max} \int_{t_0}^{t_1} e^{-\delta(t-t_0)}\ \Pi_0(X_t,z_0)dt + e^{-\delta(t_1-t_0)}J_1(X_{t_1})$$

subject to:

$\dot{X}_t = g_0(X_t,z_0)$; and

X_{t_0} given.

The value of the farm, J_0, would depend upon the stock of the land resource, X, at initial time t_0. It would equal the net present value of annual profits, Π_0, where the subscript, 0, is to be explained later. The net present value is maximized by choosing continuous control variable, z_0, over the farmer's working life until retirement at time t_1. Then the farm might be sold for price J_1 which depends on the remaining land resource. The discounted sale price adds to the initial value of the farm. The maximization is dynamic because the choice of intensity can speed degradation or renewal of the land at rate g_0.

The farmer may not only choose how intensely to farm but also when to retire from farming. There would be two very different alternatives to consider: continue farming as before or sell out. Problem (1) for maximizing the value of the farm becomes a free-time control problem. This is the simplest possible discrete-choice model.

Now suppose the farmer could adopt a second farming practice at time t_1. Retirement would be a third alternative at time t_2. The farm's value at t_1 would be the maximized net present value from the second farming practice plus the discounted sale price.

2) $$J_1(X_{t_1}) = \text{Max} \int_{t_1}^{t_2} e^{-\delta(t-t_1)}\ \Pi_1(X_t,z_1)dt + e^{-\delta(t_2-t_1)}J_2(X_{t_2})$$

subject to:

$$\dot{X}_t = g_1(X_t,z_1).$$

Where the first practice had annual profit Π_0 and changed the land resource at rate g_0, depending on intensity z_0, the second practice has profit Π_1 and rate g_1 depending on z_1. The subscripts 0 and 1 denote the practices adopted at times t_0 and t_1. The two practices are linked because the final stock of the resource from the first practice is the initial stock for the second.

In general, there could be n different farming practices, each with a different profit function and rate of change. There would be n successive control problems, J_0, J_1, through J_{n-1}, linked together. Substitute these n control problems into Problem (1).

3) $$J_0(X_{t_0}) = \sum_i \text{Max} \int_{t_i}^{t_{i+1}} e^{-\delta(t-t_0)}\ \Pi_i(X_t,z_i)dt + e^{-\delta(t_n-t_0)}J_n(X_{t_n})$$

subject to:

$$\dot{X}_t = g_i(X_t,z_i);\ i=0,\dots,n-1;\quad t_i \le t \le t_{i+1};\ \text{and}$$

X_{t_0} given.

Problem (3) is the discrete choice model in real time. Practices are listed in the order in which they will be adopted. Real time starts and stops for the first practice at t_0 and t_1, for the second practice at t_1 and t_2, and so on until real time starts and stops for the n^{th} practice at t_{n-1} and t_n. However, the order of the practices may not be known *a priori* and the sudden starting and stopping makes analysis intractible.

These difficulties can be overcome by transforming the problem into artificial time which runs continuously and doesn't require ordering of the practices. Robson (1981) and Seierstad (1984) introduce artifical time to derive sufficient conditions for a free-time control problem. Kamien and Schwartz (1981, p 226) discuss artificial time for discrete jumps in the state variable. Problem (3) is more elaborate than these models but still can be transformed. Let s denote artificial time running from 0 to T. As artificial time runs, the farmer chooses whether or not real time also runs.

$$dt/ds = \sum_i \phi_i.$$

The ϕ variables control real time. Choosing a ϕ_i to be positive means the i^{th} practice has been adopted and the real-time clock is running. The ϕ's cannot be negative and make the clock run backwards nor can they collectively exceed unity and make the clock run faster than artificial time. The clock will run at the same rate as artificial time until none of the farming practices are as profitable as selling out. Then the clock will stop.

Only when the clock is running can profits be earned and the land resource changed. In artificial time, profits and the rate of change for the i^{th} practice become $\phi_i\Pi_i$ and $\phi_i g_i$. Transform Problem (3) into artificial time.

4) $$J_0(X_0,t_0) = \text{Max}\int_0^T e^{-\delta(t_s-t_0)}\sum_i \phi_i\Pi_i(X_s,z_i)\, ds + e^{-\delta(t_n-t_0)}J_n(X_T)$$

subject to:

$$dX/ds = \sum_i \phi_i g_i(X_s,z_i);$$

$$dt/ds = \sum_i \phi_i;$$

$$0 \leq \phi_i;\ i=0,\ldots,n-1;$$

$$\sum_i \phi_i \leq 1; \text{ and}$$

X_0 and t_0 given.

In Problem (3) there were n independent practices, each with a distinct maximization in the objective and a distinct equation of motion for changing the land resource. Because the clock could run for only one practice at a time, there was no way of evaluating which of the practices might be most profitable. The critical distinction of Problem (4) is a single maximization subject to only one equation of motion for the land resource. Artificial time runs the same for all practices which are evaluated simultaneously. The real clock runs only for that practice with a ϕ of one.

Miranowski (1984) and Lazarus and Swanson (1983) developed models with fixed time horizons and with control variables, not for adoption times, but for the acreages devoted to each practice. A ϕ also could be interpreted as an acre of land if the Π's were profits per acre and, collectively, the ϕ's always summed to unity. Then the clock couldn't stop. Elapsed time, t_s-t_0, would always equal artificial time, s, and the equation of motion relating real to artificial time would be unnecessary. Otherwise, a ϕ must be interpreted as a time variable. Depending on the choice of ϕ, elapsed time may be less than artificial time and discounting of the future may be effected.

Optimal Adoption

Maximizing the value of the farm in Problem (4) is similar to solving an ordinary control problem. Instead of maximizing the value directly, a Hamiltonian can be maximized for each time period. A Hamiltonian is a dynamic profit function. It is formed by subtracting total user-costs from annual profit. Total user-cost of the land resource equals the implicit price of the resource multiplied by the quantity used. The implicit price is a costate variable and the quantity used is the right-hand side of the equation of motion. Total user-cost of real time is analagous. Finally, the Hamiltonian must be augmented for contraints on the times of adoption.

$$H_S = e^{-\delta(t_s-t_0)}\sum_i \phi_i \Pi_i + \lambda_s \sum_i \phi_i g_i + \psi_s \sum_i \phi_i + \sum_i \mu_i \phi_i + \nu_s[1 - \sum_i \phi_i].$$

λ and ψ are costate variables for the implicit prices of the land resource and of time. μ and ν are Lagrange multipliers for the inequality constraints. Because annual profits are discounted, the Hamiltonian, costate variables and multipliers are denominated in dollars at time t_0.

In addition, each practice has its own dynamic profit.

$$5) \qquad H_i = e^{-\delta(t_s-t_0)}\Pi_i + \lambda_s g_i; \quad i=0,...,n-1.$$

Thus the overall Hamiltonian is a combination of dynamic profits from the individual practices, augmented for time constraints.

6) $H_s = \sum_i \phi_i H_i + \sum_i \phi_i[\psi_s + \mu_i - \nu_s] + \nu_s$

The Hamiltonian is to be maximized for intensities, z, and adoption times, ϕ. It is not possible, however, to maximize for intensities by simply differentiating equation (6) with respect to the z's and setting the derivatives to zero. The derivatives may already be zero because the ϕ's may be zero. It would then be impossible to know which practice was the most profitable and should be adopted. Instead, the optimality conditions for the z's must maximize dynamic profit for each practice in equation (5). The optimality conditions for the ϕ's and for the state variables X and t, on the other hand, are derived by differentiating equation (6).

7a) $\partial H_i/\partial z_i = 0 = e^{-\delta(t_s-t_0)}\partial\Pi_i/\partial z_i + \lambda_s \partial g_i/\partial z_i$; $i=0,\ldots,n-1$;

7b) $\partial H_s/\partial\phi_i = 0 = H_i + \psi_s + \mu_i - \nu_s$; $i=0,\ldots,n-1$;

7c) $-\partial H_s/\partial X_s = -\sum_i \phi_i \partial H_i/\partial X_s = d\lambda/ds = -\sum_i \phi_i[e^{-\delta(t_s-t_0)}\partial\Pi_i/\partial X_s + \lambda_s \partial g_i/\partial X_s]$;

7d) $-\partial H_s/\partial t_s = -\sum_i \phi_i \partial H_i/\partial t_s = d\psi/ds = \delta \sum_i \phi_i e^{-\delta(t_s-t_0)}\Pi_i$;

7e) $\lambda_T = e^{-\delta(t_n-t_0)}\partial J_n/\partial X_T$;

7f) $\psi_T = -\delta e^{-\delta(t_n-t_0)}J_n$.

Optimality conditions also include the equations of motion and initial conditions in Problem (4) plus complementarity slackness conditions for the time constraints.

(7g) $0 \leq \mu_i$; $\mu_i\phi_i = 0$; $i=0,\ldots,n-1$;

(7h) $0 \leq \nu_s$; $\nu_s[1 - \sum_i \phi_i] = 0$.

How profitable is each practice? Condition (7a) maximizes each dynamic profit with respect to its intensity, z, by equating annual marginal profit to marginal user-cost. Deriving this condition from equation (5) and not (6) has a practical implication. Mathematical programming cannot simultaneously maximize the profits from each discrete practice and choose among those practices without special gradient calculations.

Should the land be farmed or sold? Condition (7b) compares each dynamic profit, maximized for intensity, to the costate variable for time. The costate is interpreted as the change in the farm's value due to the passage of time and, after integrating conditions (7d) and (7f), equals the negative of the interest rate multiplied by the value of the farm.

$$8) \qquad \psi_s = -\delta\left[\int_s^T e^{-\delta(t_\tau - t_0)} \sum_i \phi_i \Pi_i \, d\tau + e^{-\delta(t_n - t_0)} J_n\right].$$

Thus, condition (7b), compares dynamic profits to the opportunity cost of interest foregone by investing on the farm rather than off. If none of the practices meet the opportunity cost, the μ multipliers must be positive and, by complementary slackness in condition (7g), the ϕ's must equal zero. None of the practices should be adopted and the farm should be sold. If any practice is to be adopted, its multiplier, μ, must be zero and its dynamic profit from production must meet or exceed the opportunity cost of investment.

Which practice should be adopted? Any two of the n practices can be compared by combining equations from (7b).

$$9) \qquad 0 = H_j - H_k + \mu_j - \mu_k; \quad j=1,\ldots,n-1; \; k=0,\ldots,j-1.$$

If practice k is not as profitable a practice j, its multiplier, μ, must be positive and, by complementary slackness, it cannot be adopted. If practice j is to be adopted it must be at least as profitable as every other practice.

A practice may not be the most profitable now, but will it be in the future? Dynamic profitability can change over time for two reasons. First, degradation or renewal changes the land resource. Second, discounting decreases the value of future profits denominated in dollars at time t_0. Discounting applies equally to all practices and can be eliminated by denominating in dollars at time t_s. Then dynamic profitability changes only because of degradation or renewal.

$$d(e^{\delta(t_s - t_0)} H_j)/ds = e^{\delta(t_s - t_0)}[\delta \lambda_s g_j \sum_i \phi_i \; + \; (\partial H_j/\partial X_s)\sum_i \phi_i g_i \; - \; (\sum_i \phi_i \partial H_i/\partial X_s) g_j].$$

If the farm is to be sold and none of the practices adopted, dynamic profitability can no longer change and the right-hand side of this equation is zero. If practice j is currently being used, dynamic profitability must change to match the interest expense on the total user-cost of the land resource.

10) $$d(e^{\delta(t_s-t_0)}H_j)/ds = e^{\delta(t_s-t_0)}\delta\lambda_s g_j.$$

The costate on the right-hand side is the change in the value of the farm per unit change in the land resource. Total user-cost, the costate times the amount used of the resource, is the total change in the value of the farm from using rather than selling it. Multiplying by the interest rate gives the change in the opportunity cost of investment. Therefore in equation (10), dynamic profit must change to match the changing opportunity cost. If the land is degraded with g_j negative, dynamic profit of practice j will decline in Figure 1 along the path labeled H_j. Usually, profit will fall more rapidly over time as the increasingly scarce land resource becomes more costly to use.

Because practice k is not currently used, its dynamic profit will be shifted as practice j degrades or renews the land.

11) $$d(e^{\delta(t_s-t_0)}H_k)/ds = e^{\delta(t_s-t_0)}[\delta\lambda_s g_k + (\partial H_k/\partial X_s)g_j - (\partial H_j/\partial X_s)g_k].$$

The first term in square brackets is the change that would occur if practice k was adopted. The second two terms shift the profit because practice j is used instead. The more the dynamic profitability of practice k is affected by a change in the land resource, the greater the damage and the greater the magnitude of the shift.

Practice k may be less degrading but initially less profitable. As the land resource becomes more costly to use, practice k might become relatively more profitable along path H_k in Figure 1 and be adopted at time t_k. Profit of practice k would decline even more slowly if it were not being shifted down by practice j. An example of a practice evolving along path H_k is conservation tillage following conventional tillage.

The dynamic profit of a third practice, practice ℓ, might follow path H_ℓ. Initially, practice ℓ is not only less profitable but more degrading. In an equation analogous to (11), however, that profitability may decline slowly if it is not damaged by a change in the land resource. Eventually, practice ℓ could be adopted. This result is more than a curiosity. One example is a drought-resistant crop such as beans, peas or safflower, which would leave little residue on the surface for erosion control but will withstand the reduced water-holding capacity of soil after years of erosion. Another example is a salt-tolerant crop such as barley or salt bush, which would transpire less moisture than other crops and contribute to a higher water table if it were used early on but will withstand the salts after years of salinization by other practices.

Of course, a practice that is easily damaged by degradation, like practice m along path H_m in Figure 1, may never be adopted. A practice will converge toward adoption if its profit declines more slowly than that of the currently used practice--if equation (11) subtracted from equation (10) is less than zero.

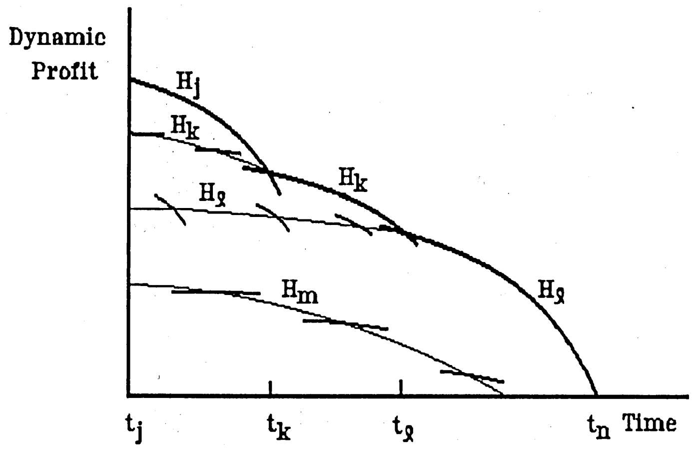

Figure 1. Degradation and Optimal Adoption.

$$12)\quad d(e^{\delta(t_s-t_0)}(H_j - H_k))/ds = e^{\delta(t_s-t_0)}[\delta\lambda_s(g_j - g_k) - (\partial H_k/\partial X_s)g_j + (\partial H_j/\partial X_s)g_k].$$

In Figure 1, equation (12) is less than zero as practice k converges toward adoption After the switch at time t_k, a similar equation could be derived to compare the newly adopted practice k with the old practice j. This equation, however, would be the negative of equation (12). Because equation (12) remains less than zero after the switch, practice j becomes increasingly unprofitable and, probably, will not be used again. Subsequently, practice k is replaced by practice ℓ and eventually the farm is either sold or abandoned.

Smooth convergence toward adoption depends upon the good behavior of the costate. The costate is interpreted as the implicit price of the land resource or the change in the farm's value due to a change in the resource. After integrating conditions (7c) and (7e), it equals the net present value of damage to the future caused by a current change.

$$13)\quad \lambda_s = e^{-\delta(t_s-t_0)}\left[\int_s^T e^{-\int_s^\tau \delta-\Sigma\phi_i\partial g_i/\partial X\,dT}\sum_i\phi_i\partial\Pi_i/\partial X_\tau d\tau + e^{-\int_s^T \delta-\Sigma\phi_i\partial g_i/\partial X\,dT}\partial J_n/\partial X_T\right].$$

Damage to the future takes two forms: lower annual profits and faster rates of degradation. The smaller the land resource, the smaller the annual profits and, perhaps, the faster the rate of degradation. The effect of a faster rate of degradation is similar to discounting the future more heavily.

Like dynamic profit, the costate can change because of degradation and discounting. But denominated in dollars at time t, the costate changes only because of degradation.

14) $$d(e^{\delta(t_s-t_0)}\lambda_s)/ds = [\delta - \sum_i \phi_i \partial g_i/\partial X_s]e^{\delta(t_s-t_0)}\lambda_s - \sum_i \phi_i \partial \Pi_i/\partial X_s.$$

If both annual profits and the rate of degradation were linear in the land resource and independent of which practice might be adopted, and if the time horizon were infinite, then the integration of equation (13) would simplify in the limit to the marginal annual profit divided by a discount factor equal to the rate of interest minus the marginal rate of degradation. Substituting into equation (12), the change in the costate would be zero and the value of the land resource would be constant over time. But if annual profits are concave and increasing in the land resource or the rate of degradation is convex and decreasing, damage intensifies as degradation proceeds. The costate will be large in equation (13) and increase over time in equation (14). As the land resource becomes scarce and its value increases, alternative practices will converge toward adoption in Figure 1. A finite time-horizon would have a small countervailing effect. Near the time a farm is to be abandoned, however, the value of the land must fall to zero. It might become optimal to switch back to a previous practice.

Renewal of the land is discussed only briefly in the literature but examples abound. Cover crops, fallowing and deep ripping can renew organic matter and soil tilth. Legumes replace lost nitrogen. Small grains can allow better weed and pest control. Periodic liming ameliorates acidity. Trees and drainage can restore salt land.

If practice j renews instead of degrades the land, with g_j positive, it will receive total user-benefits from renewal rather than pay total user-costs. Dynamic profit will rise in Figure 2 along path H_j, but rise more slowly over time as it becomes less beneficial to renew an increasingly abundant resource. Initially, practice k may be less renewing and also less profitable because it receives fewer total user-benefits. As the benefits of renewal decline, practice k could become relatively profitable along path H_k and be adopted at time t_k.

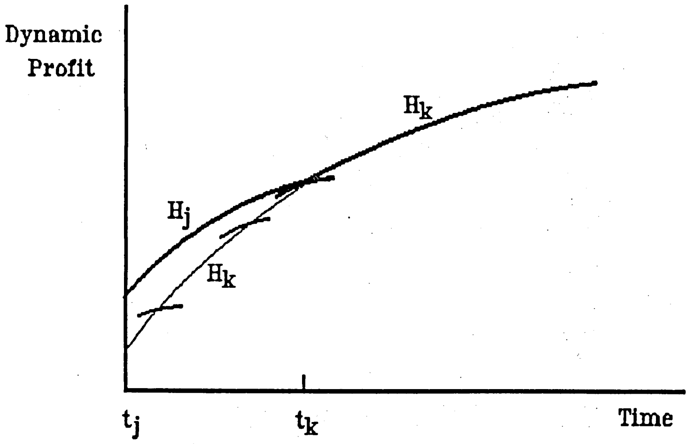

Figure 2. Renewal and Optimal Adoption.

In the literature, crop rotations are invariably at a steady-state. Soil erosion, acidification and salinization will have a lengthy phase of either degradation or renewal. But this phase may eventually be followed by a steady-state rotation if both degrading and renewing practices are available.

In Figure 3, practice j degrades and practice k renews the land. Practice j, with g_j negative, pays total-user costs for its degradation. Practice k, with g_k positive, receives total user-benefits. As the land resource becomes scarce and its value increases in equation (14), the costs to practice j and the benefits to practice k increase. Practice k becomes relatively more profitable in equation (12) and is adopted at time t_k. But in this instance, the dynamic profit of practice j doesn't continue to decline. As the land is renewed by practice k its value falls. The total user-costs to practice j and the total user-benefits to practice k fall. Practice j becomes relatively more profitable and will be adopted again. Then practice j will degrade the land making practice k more profitable in rotation.

The proportion of the time each practice will be used in rotation can be solved from the equation of motion for the land resource in the steady-state.

$$\phi_j/\phi_k = g_k/g_j.$$

If degradation is rapid and renewal is slow, the degrading practice, practice j, will be used very little of the time. Only two practices will be rotated if the land resource is described by a single equation of motion. More complex rotations

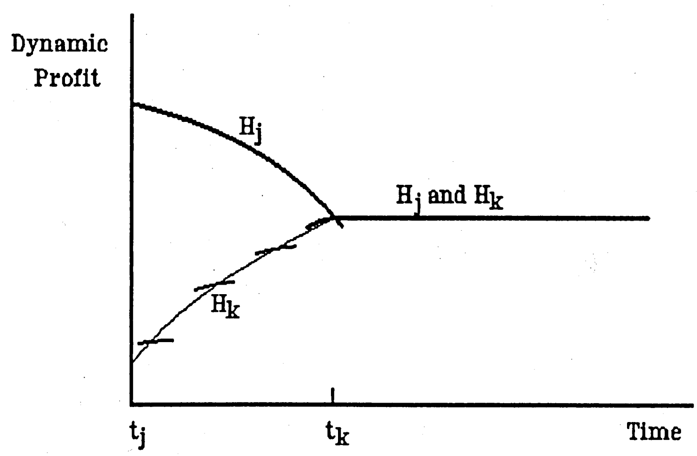

Figure 3. Degradation, Renewal and Optimal Rotation.

require two or more equations of motion describing, for example, soil depth, acidity, salinity, nutrients, pests or diseases.

Finally, how well can either a static but discrete or a dynamic but continuous model approximate the adoption of discrete practices? More technically, in Problem (4) can the equation of motion be eliminated to make the model static or can one function interpolate between the many annual profit functions and another function interpolate between the rates of degradation to make the model continuous? Figure 4 shows the annual profits at time t_0 of four practices sorted by increasing rates of degradation. As an approximation, a smooth frontier is fit and passes through the annual profits for practices j and m. Practice ℓ seems most unpromising and is eliminated from the approximation. But practice k, with the second highest annual profit and next to lowest rate of degradation, must also be eliminated to maintain concavity of the frontier function.

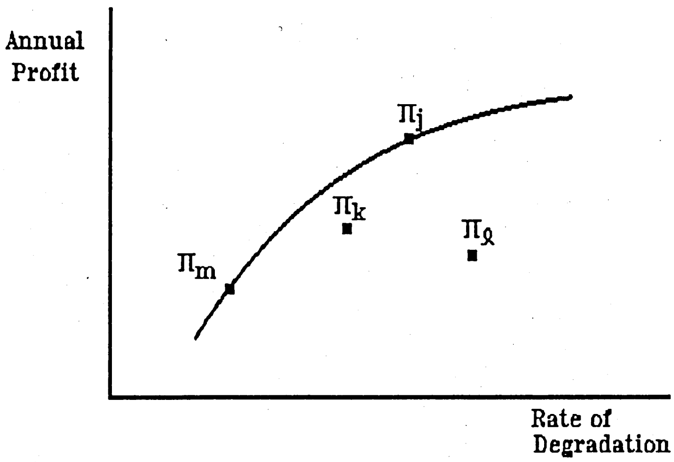

Figure 4. Annual Profits Sorted by Increasing Rates of Degradation.

The static model will choose practice j. This may or may not be correct. There is no guarantee that a less degrading practice paying lower total user-costs won't have a higher dynamic profit. The continuous control model will choose a combination of practices j and m and decrease the rate of degradation over time by using more of practice m. Unfortunately, the optimal order of adoption could be the same as in Figure 1: practice j followed by practice k and finally practice ℓ. Practice m might never be used. Or it might be used exclusively. Or any of 13 other combinations might occur. The frontier function has no meaning. Over time, annual profits can shift up, down, left or right relative to each other. Even if the practices are damaged equally and shifts don't occur, practice k will often be optimal. To answer the question, either a static model or a continuous control model may happen on the correct practice by serendipity but there are no guarantees.

Conclusions

Practices to manage the land are discrete and linked over time by the degradation or renewal they cause. The difficulties of modelling both dynamic and discrete practices have forced most authors to assume away either the dynamics or the discreteness. A static model or a dynamic but continuous model may select the optimal practice by happenstance. Neither can guarantee optimality.

In this study, a model of the dynamically optimal adoption and rotation of discrete practices is constructed and applied. An initial phase of degradation or renewal should often be followed by a steady-state rotation of degrading and renewing practices. Soil erosion seldom may be renewable but acidity and salinity usually will be. The possibility of a steady-state has received little attention in the literature but has profound implications for public policy. Policy would no longer be concerned with conserving an exhaustible land resource to forestall eventual starvation. Instead, policy would be concerned with the much less urgent task of achieving the optimal steady-state for a sustainable agriculture.

Finally, farm decisions other than land management require a dynamic discrete-choice model. An optimal decision must not only equate the marginal conditions for a given production function but choose among production functions and the choice may affect the future. Models of machinery and livestock investment will differ in their equations of motion, but the method of analysis developed here, linking free-time control problems and translating to artificial time, should apply equally well.

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