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FACULTY OF AGRICULTURE

The Economics of Crop Hail Insurance

Alfred
James Vercammen and David J Pannell

Agricultural & Resource Economics
Discussion Paper: 2/96

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The Economics of Crop Hail Insurance

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Abstract

A stylized model of crop hail insurance is developed to examine the properties of the demand for such insurance. Only in the simplest case (i.e., a static model with no hail-free crop revenue uncertainty) do the results resemble those conforming to the standard theory of insurance. Allowing for crop revenue uncertainty and for dynamic updating both induce the farmer to underinsure despite an actuarially fair premium rate. The underinsurance result also emerges in a more general model where hail insurance is purchased along with an all-risk insurance contract. In this more general model, two alternative approaches currently in use in North America are examined and their efficiency properties compared.

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The Economics of Crop Hail Insurance

I. Introduction

Although all-risk/multiple peril crop insurance has been studied extensively in recent years, very little research has been devoted to the economics of crop hail insurance.¹ In particular, there has been no specific analysis of the farmer's decision problem regarding the optimal level of hail insurance coverage in different circumstances.² This neglect is despite the fact that hail is an important source of production risk throughout North America, Australia and other agricultural regions. For example, in western Canada total indemnities paid by the Saskatchewan Crop Insurance Corporation (SCIC) for hail damage averaged \$308 million per year between 1986 and 1991. This value works out to an average of approximately \$2200 per claim and makes up about 18 percent of total annual indemnities paid by the SCIC (SCIC Annual Report).

The purpose of this paper is to rigorously investigate the farmer's decision regarding the optimal level of hail insurance and the factors affecting this decision. The first part of the paper covers the case where a farmer insures against hail damage only and does not participate in an all-risk crop insurance program. This scenario is the norm in Australia, where all-risk crop insurance is not available and, depending on the crop in question, is often observed in North America as well.³ In the second part of the paper, we examine a situation where the farmer obtains hail insurance protection as part of an all-risk crop insurance contract and may purchase supplementary insurance from a private broker. This scenario has relevance for North America where typically the majority of farmers who produce grain purchase crop insurance (e.g., see note 3).

One possible reason why comparatively little research has been devoted to the economics of hail insurance is that the hail insurance market is generally viewed as functioning well. Problems of moral hazard and individual adverse selection are likely to be virtually non-existent in hail insurance markets because the frequency and severity of a hail storm is not affected by the actions taken by a farmer and the risk of loss facing farmers within a particular risk zone face is known by the supplier of insurance with considerable accuracy.⁴ Also, suppliers of hail insurance outside of the all-risk crop insurance program are generally comparatively large in number and hence are likely to behave reasonably competitively.⁵

Despite these features of the hail insurance market, a number of interesting and rather unexpected results have emerged from the analysis of a stylized model of hail insurance. For

¹ See Hueth and Furtan (1994) and Goodwin and Smith (1995) for a good review of the literature on all-risk crop insurance.

² McMaster and Blong (1992) examine the issue of setting premium rates for crop hail insurance but do not consider the demand for such insurance.

³ For example, in 1985-86, the percent of cropped acres for which Saskatchewan farmers purchased crop insurance was 62.5 (barley), 61.4 (canola), 52.2 (oats), 72.9 (wheat), 89.0 (mustard) and 61.6 (flax) (Sigurdson and Sin, 1994). Certainly a sizable fraction of the crop base not insured for all-risk would have been insured for hail damage only.

⁴ In this respect, hail insurance is similar to area yield crop insurance as discussed by Carriker (1991) and Miranda (1991).

⁵ For example, in Saskatchewan a farmer would typically have the choice of purchasing hail insurance coverage from at least six different brokers.

example, in the case of hail insurance only, the demand for insurance tends to be more elastic rather than less elastic the greater the probability of hail. As well, uncertainty in crop revenue (net of the effects of hail damage) typically induces the farmer to purchase less than full coverage, even when the insurance premium is actuarially fair. In a multi-period setting where farmers can purchase additional insurance after obtaining more precise information about the distribution of crop revenues, farmers tend to remain underinsured throughout the production period for reasons distinct from those relating explicitly to crop revenue uncertainty.

In the more general case where farmers purchase all-risk crop insurance coverage and possibly supplementary hail insurance, the underinsurance result remains. However, in this case the provision for loss from hail within the all-risk crop insurance contract turns out to have some interesting properties. Two general types of provisions are currently in use in North America.⁶ Firstly, hail damage could be assessed separately from the damage due to all other perils. That is, when hail damage occurs, the crop insurance agency pays compensation to the farmer at the time of the hail damage and then reduces the post-harvest yield guarantee by a corresponding amount. Consequently, the farmer could potentially collect an indemnity from hail damage despite that fact that his or her actual yield is above the guaranteed level. Secondly, hail could be treated as just another peril (i.e., not assessed separately) in which case an indemnity from hail damage would not be paid to a farmer if his or her actual yield after hail damage exceeds the yield guarantee. We show in this paper that the former scheme is generally less efficient than the latter one because the variability of the farmer's net profits is relatively higher with the former.

The results of this analysis are largely attributable to the structure of a typical hail insurance contract. Farmers purchase λ dollars per acre worth of coverage and if hail occurs and a proportion α of the crop is damaged, then the payment to the farmer is $\alpha\lambda$, regardless of the actual value of the crop. Moreover, the premium cost is unrelated to the expected or actual value of the crop. In standard all-risk crop insurance, the level of coverage for the farmer and hence the premium rate is directly proportional to the expected value of the crop and indemnities are triggered by a sufficiently low crop value rather than the mere existence of a peril such as hail which reduces the crop's yield.

In the next section of the paper the basic assumptions of the model are set out and the solution for the static hail insurance model is characterized. The model is initially solved assuming that hail is the only source of crop revenue uncertainty in order to construct a point of reference. In the third section, the dynamic version of the model is developed and contrasted to the static case. A combined model of all-risk crop insurance and hail insurance is constructed and analyzed in the fourth section. Concluding comments are provided in the last section of the paper.

II. Static Model of Hail Insurance

Consider a farmer who produces a crop that is subject to hail damage at some point in the growing season. The farmer is assumed to have a homogenous land base, to grow only one type of crop and to receive equal hail damage on all portions of the land. These assumptions imply that the entire farm can be treated as a single unit and yield can be defined as total production for the entire farm. Let Π denote the product of the crop's yield and market price. The farmer's net

⁶ For example, the province of Saskatchewan uses the first type of scheme described below whereas the state of Kansas uses the second type.

post-harvest revenue, W , can be expressed as $W = \Pi + I - P$ where I denotes total insurance indemnities from hail damage and P denotes the total cost of the insurance.

Farmers are assumed to be risk averse expected utility maximizers who do not face binding credit constraints at the time of purchasing the hail insurance.⁷ Specifically, a farmer chooses the level of hail insurance coverage that maximizes $E\{U(W)\}$ where E denotes the expectations operator and $U(\cdot)$ is the farmer's von Neumann-Morgenstern expected utility function with the standard properties (i.e., $U' > 0$ and $U'' < 0$).⁸ As is shown below, the sign of U''' is important in this analysis. The standard assumption is that $U''' > 0$ because this condition is necessary for constant or decreasing absolute risk aversion.

Non-Stochastic Yields

The simplest case to analyze is one in which crop revenues (excluding the effects of hail damage) are non-stochastic, hail damage occurs at most once in the growing season, the probability of a hail event during the growing season is θ and the proportion of the crop that is destroyed when hail damage does occur is a fixed parameter, α . That is, crop revenues, Π , equal π with probability $1-\theta$ and $(1-\alpha)\pi$ with probability θ . From this point on, π will be referred to as "hail-free crop revenue".

Let λ denote the dollar value of the hail insurance coverage purchased by the farmer. When hail occurs and a fraction α of yield is destroyed by hail, the indemnities paid to the farmer equal $\alpha\lambda$. The actuarially fair premium rate per dollar of coverage is $\theta\alpha$ (i.e., the probability of hail multiplied by the proportion of each dollar of coverage that is paid out as an indemnity when hail occurs). The actual premium rate is likely to be higher than the actuarially fair rate in most cases because of the fixed transaction and administration expense incurred by the companies who supply the hail insurance. The actual premium rate can be expressed as $\gamma\theta\alpha$ where $\gamma \geq 1$. The total premium expense for the farmer is therefore $\gamma\alpha\theta\lambda$.

The expected value of the farmer's utility can now be expressed as

$$(1) \quad E(U) = \theta U((1-\alpha)\pi + \alpha\lambda - \gamma\theta\alpha\lambda) + (1-\theta)U(\pi - \gamma\theta\alpha\lambda).$$

The problem facing the farmer is to choose the level of hail insurance coverage, λ , that maximizes expected utility. The first order condition for this problem reduces to

$$(2) \quad (1-\gamma\theta)U'(\pi - \gamma\theta\alpha\lambda + \alpha(\lambda - \pi)) = \gamma(1-\theta)U'(\pi - \gamma\theta\alpha\lambda).$$

Equation (2) can now be used to establish the following result:

Result 1: *For the case of non-stochastic hail-free crop revenues and non-stochastic hail damage, it is optimal for farmers to purchase full coverage (i.e., $\lambda^* = \pi$) when the insurance is*

⁷ The utility function is assumed to be defined over net revenues only rather than net revenues minus production costs plus other wealth. The qualitative nature of the results are not affected by this particular specification because production costs and other wealth are assumed non-stochastic.

⁸ A single, double and triple prime denotes a first, second, and third total derivative, respectively.

actuarially fair (i.e., $\gamma = 1$) and to purchase less than full coverage (i.e., $\lambda^* < \pi$) when the price of the insurance is greater than the actuarially fair value (i.e., $\gamma > 1$).

Proof: Notice that when $\gamma = 1$ then $\lambda = \pi$ is necessary for equation (2) to hold. When $\gamma > 1$ and $\lambda = \pi$, equation (2) is not satisfied; the left side is smaller in value than the right. Hence, $\lambda^* < \pi$ because the expression on the right hand side of equation (2) is increasing in λ (since $U'' < 0$) and, assuming $\gamma\theta < 1$, the left side is decreasing in λ . Q.E.D.

Result 1 is not surprising and is certainly consistent with the standard theory of insurance (e.g., Arrow, 1963.). The intuition is that in the case of $\gamma = 1$, expected net revenues do not depend on the amount of insurance purchased. Consequently, the farmer chooses $\lambda = \pi$ because this level of coverage minimizes the variance in net revenues (to zero in this particular case). When $\gamma > 1$, as would be expected for any commercially offered insurance contract, net expected revenues decrease as more insurance is purchased. Consequently, the farmer trades off at the margin a lower variance of net revenues with a lower expected net return and as a result purchases less than full coverage.

It is informative to examine the comparative static results with respect to the exogenous parameters π , α and θ for the case of $\gamma > 1$. These results are most easily established for the case of constant absolute risk aversion (CARA).⁹

Result 2: For the case of CARA, (a) $d\lambda^*/d\pi = 1$, (b) $\frac{d\lambda^*}{d\alpha}\big|_{\gamma>1} > 0$ and (c) $\frac{d\lambda^*}{d\theta}\big|_{\gamma>1} < 0$.

In understanding the reason why λ^* is positively related to π in result 2(a), it is helpful to think of insurance as an investment which is negatively correlated with the returns from the crop. When the value of π increases, so too does the variability of revenue, thus increasing the size of the negatively correlated investment in insurance. Result 2(a) is particularly interesting because it implies that as π changes from year to year, farmers with CARA should optimally change their coverage by the same dollar value. In other words, the gap between π and λ^* which occurs when $\gamma > 1$ is not dependent on the size of π . Of course a constant absolute gap implies that the percent gap between π and λ^* decreases as π increases. Although not formally established, it can be shown that the absolute gap between π and λ^* increases as π increases for farmers with decreasing absolute risk aversion (DARA).

Result 2(b) states that the larger the fraction of the crop lost when hail does occur, the smaller the gap between optimal and full coverage. On the other hand, result 2(c) states that the greater the probability of hail, the larger the gap between optimal and full coverage. In other words, the demand for hail insurance tends to be less elastic the greater the severity of hail damage and more elastic the greater the probability of hail. The explanation of results 2(b) and 2(c) is that when $\gamma > 1$, the excess of the actual premium over the actuarially fair premium, $(\gamma-1)\theta\alpha\lambda$ is rising with both α and θ .¹⁰ Thus, an increase in either α or θ will have opposing effects on λ^* : a positive

⁹ The proofs of the following three comparative static results are not complex but yet are quite long. To save on space, these proofs are not included in the paper but are available from the authors on request.

¹⁰ Of course, if the actual premium rate is not proportional to the actuarially fair premium rate then results 2(b) and 2(c) may not hold.

effect because of the added risk and a negative effect because of the added excessive premium cost. In the case of α , the positive effect outweighs the negative effect whereas in the case of θ the opposite is true. With reference to equation (2), because $1-\gamma\theta < \gamma(1-\theta)$ when $\gamma > 1$ and the difference between $1-\gamma\theta$ and $\gamma(1-\theta)$ does not depend on the value of either α or θ , it follows that marginal utility must change more in the hail state versus the no-hail state to maintain the first-order condition when either α or θ is increased. Given an increase in α , net revenues decline faster in the hail state than the no-hail state when $\lambda < \pi$, but the imbalance is too large so λ must be increased to reduce the differential in the rates of decline. Given an increase in θ , net revenues decline at the same rate across both states so λ must be decreased to ensure that net revenues in the hail state decline relatively faster.

Stochastic Crop Revenue

Now consider the more realistic case where both the occurrence of hail and hail-free crop revenue (π) are stochastic. The variance in π is due to both yield and price. For convenience of illustration we use here a two-level discrete distribution for π . A more realistic distribution is used in the analysis discussed below. Assume that hail-free crop revenues are given by π_L with probability ϕ and π_H with probability $1-\phi$. To simplify matters, attention is restricted to the case of $\gamma = 1$ (i.e., an actuarially fair premium rate). Expected utility in this case can be expressed as

$$(3) \quad E(U) = \theta\phi U((1-\alpha)\pi_L + \alpha\lambda - \theta\alpha\lambda) + \theta(1-\phi)U((1-\alpha)\pi_H + \alpha\lambda - \theta\alpha\lambda) \\ + (1-\theta)\phi U(\pi_L - \theta\alpha\lambda) + (1-\theta)(1-\phi)U(\pi_H - \theta\alpha\lambda).$$

The first-order condition for choosing λ to maximize equation (3) reduces to

$$(4) \quad \phi[U'(a) - U'(c)] = (1-\phi)[U'(d) - U'(b)]$$

where: $a = (1-\alpha)\pi_L + \alpha\lambda - \theta\alpha\lambda$, $b = (1-\alpha)\pi_H + \alpha\lambda - \theta\alpha\lambda$, $c = \pi_L - \theta\alpha\lambda$ and $d = \pi_H - \theta\alpha\lambda$.

The following result can now be established:

Result 3: *For an actuarially fair premium rate,*

$$\lambda^* = \phi\pi_L + (1-\phi)\pi_H \text{ (i.e., the farmer fully insures) if } U''' = 0$$

$$\lambda^* < \phi\pi_L + (1-\phi)\pi_H \text{ (i.e., the farmer underinsures) if } U''' > 0$$

$$\lambda^* > \phi\pi_L + (1-\phi)\pi_H \text{ (i.e., the farmer overinsures) if } U''' < 0$$

Proof: To save space, result 3 is formally established here only for the case of $\phi = 0.5$ (i.e., there is an equal probability of obtaining high and low yields). Given $\phi = 0.5$, if

$\lambda = \phi\pi_L + (1-\phi)\pi_H$ then manipulation of the above expressions for a , b , c and d reveals that $a - c = d - b = 0.5\alpha[\pi_H - \pi_L]$. Now in the case where $U''' = 0$ (i.e., U' is linear), $a - c = d - b$ implies that $U'(a) - U'(c) = U'(d) - U'(b)$. Hence, the first-order condition in (4) is satisfied and therefore $\lambda = \phi\pi_L + (1-\phi)\pi_H$ is indeed the solution. If $U''' > 0$ then the U' schedule is downward sloping and convex to the origin. For this particular case, because $d - a = b - c = (1-0.5\alpha)(\pi_H - \pi_L) > 0$, when $\lambda = \phi\pi_L + (1-\phi)\pi_H$ it follows that $U'(c) - U'(a) > U'(b) - U'(d)$. Hence, the first-order condition is violated and therefore $\lambda = \phi\pi_L + (1-\phi)\pi_H$ is not a solution. Decreasing λ below $\phi\pi_L + (1-\phi)\pi_H$

increases c and d (thus making $U'(c)$ and $U'(d)$ smaller in value) and decreases a and b (thus making $U'(a)$ and $U'(b)$ larger in value), eventually satisfying equation (4) for λ sufficiently less than $\phi\pi_L + (1-\phi)\pi_H$. Using a similar argument, it follows that if $U''' < 0$ then $\lambda^* > \phi\pi_L + (1-\phi)\pi_H$. Q.E.D.

Result 3 shows that when hail-free crop revenues are uncertain and $U''' > 0$ for the realistic cases of constant and decreasing absolute risk aversion, the farmer underinsures when the insurance premium is actuarially fair. In contrast, the farmer chooses to exactly insure in the previous case where hail-free revenues were certain and the insurance premium was actuarially fair. Thus, crop revenue uncertainty induces the farmer to underinsure when $U''' > 0$. Result 3 also emerges in a more general framework within which the distribution of yield is continuous and the level of hail damage, α , is also a continuous random variable (see the Appendix).

The explanation of result 3 is as follows. If the farmer fully insures and the premium is actuarially fair, then expected net revenues are the same in the hail state as in the no-hail state. However, from equation (3) it can be seen that the difference in net revenue between high and low yield states is greater in the no-hail state than in the hail state (i.e., $\pi_H - \pi_L > (1-\alpha)[\pi_H - \pi_L]$). Consequently, if the farmer fully insures and $U''' > 0$, the expected value of marginal utility is greater in the no-hail state than the hail state. This relationship is inconsistent with the first-order condition, which requires expected marginal utility to be equated with and without hail. Decreasing the amount of insurance purchased decreases expected net revenues and hence increases expected marginal utility in the hail state and increases expected net revenues and hence decreases expected marginal utility in the no-hail state, so λ^* is necessarily less than full insurance. The opposite result emerges with $U''' < 0$. With $U''' = 0$, expected marginal utility does not depend on the difference in marginal utility across the two crop revenue states and thus full insurance equalizes expected marginal utility across the hail and no-hail states.

A more general interpretation of this result is that farmers choose to underinsure because the variability of revenues is lower in the hail state than the no-hail state. It is important to ask whether this phenomena is likely to be observed in reality. Specifically, it is important to ask whether it is reasonable to assume that crop yield has a lower variance after hail damage than before. The current specification assumes that all potential yield outcomes are reduced by the same proportion when hail occurs (i.e., the random yield and the hail damage variables are multiplicative). Although the actual biological relationship between hail damage and yield outcome is likely to be much more complex than the one assumed here, it seems reasonable to assume that a crop with a lower expected yield because of hail damage will also have a lower yield variance. If situations exist where yield tends to have a higher variance after hail damage, then results opposite to those reported here will emerge (i.e., farmers will overinsure when $U''' > 0$ and underinsure when $U''' < 0$).

III. Dynamic Model of Hail Insurance

In the previous section it was assumed that farmers purchase insurance only at the beginning of the production period. In reality it is common for farmers to purchase additional insurance within the growing season. Presumably the reason for these mid-season adjustments is that farmers become better informed about the distributions of yield and price for their crop and/or their assessment about the probability or severity of hail damage changes. In this section the first of these two reasons is formally incorporated into the model. The particular problem at hand is

determining how a farmer's overall demand for coverage is affected by including the assumption that additional hail insurance can be purchased during the growing season after more accurate information about crop yield and price is obtained.

The problem considered has several dimensions. Firstly, farmers may wish to insure less than the expected value of their crop because additional insurance can be purchased but not sold at a later date (i.e., there is an option value from waiting to insure). However, deferring the purchase of insurance is also risky because hail may occur prior to the purchase of the additional insurance. The other issue is that hail insurance companies typically do not offer discounts of premium rates for mid-season purchases of hail insurance. Because an actuarially fair premium rate reflects the expected loss from hail damage throughout the entire growing season, the marginal price of the insurance purchased mid-season will generally exceed its expected marginal benefit.

Additional Assumptions

For illustrative purposes, we assume that there are two distinct periods within the growing season. At the beginning of the first period, the farmer's hail-free crop revenue assessment is given, as before, by π_L with probability ϕ and π_H with probability $1-\phi$. By the beginning of the second period, the farmer knows with certainty whether hail-free crop revenues will equal π_L or π_H . The farmer can purchase insurance at the same premium rate at the beginning of each of the two periods. Let θ_1 and θ_2 denote the probability of hail damage in periods 1 and 2, respectively, and α_1 and α_2 denote the corresponding fractions of damage. The assumption that the premium rate is actuarially fair is maintained.

Let P_1^f denote the actuarially fair premium rate for insurance purchased at the start of period one:

$$(5) \quad P_1^f = \theta_1(1 - \theta_2)\alpha_1 + (1 - \theta_1)\theta_2\alpha_2 + \theta_1\theta_2(\alpha_1 + \alpha_2(1 - \alpha_1)).$$

The first two terms in equation (5) are the probabilities of hail in period one only and period two only multiplied by the respective proportions of yield loss. The last term is the probability of receiving hail in both periods multiplied by the cumulative percent damage. Equation (5) can be simplified to $P_1^f = \theta_1\alpha_1 + \theta_2\alpha_2 - \theta_1\theta_2\alpha_1\alpha_2$. The premium rate implied by equation (5) is actuarially fair in the sense that a farmer who purchases a dollar of coverage at the beginning of period one expects to receive P_1^f dollars in indemnities over the course of both periods.

Let λ_1 denote the level of coverage purchased by the farmer in period one. Also, let λ_2^0 denote the coverage level in period two given that $\pi = \pi_H$ and hail never occurred in period one and let λ_2^1 denote the coverage level in period two given that $\pi = \pi_H$ and hail did occur in period one. It is assumed that the spread between π_L and π_H is sufficiently large such that no additional insurance is purchased in period two when $\pi = \pi_L$.¹¹ As before, a superscripted * on these variables denote the optimal values. The farmer's expected utility as a function of λ_1 and λ_2^0 and λ_2^1 can be written as

¹¹ The only time a farmer may want to purchase additional insurance in period two when $\pi = \pi_L$ is if first period purchases of insurance are sufficiently less than π_L . If the spread between π_L and π_H is sufficiently large then in the equilibrium described below the optimal level of insurance in period one will not be less than π_L .

$$(6) \quad E(U) = \theta_1(1-\theta_2)[\phi U(a_L) + (1-\phi)U(a_H)] + (1-\theta_1)\theta_2[\phi U(b_L) + (1-\phi)U(b_H)] \\ + \theta_1\theta_2[\phi U(c_L) + (1-\phi)U(c_H)] + (1-\theta_1)(1-\theta_2)[\phi U(d_L) + (1-\phi)U(d_H)]$$

where: $a_L = (1-\alpha_1)\pi_L + \alpha_1\lambda_1 - P_1^f\lambda_1$; $a_H = (1-\alpha_1)\pi_H + \alpha_1\lambda_1 - P_1^f(\lambda_1 + \lambda_2^1)$;
 $b_L = (1-\alpha_2)\pi_L + \alpha_2\lambda_1 - P_1^f\lambda_1$; $b_H = (1-\alpha_2)\pi_H + \alpha_2(\lambda_1 + \lambda_2) - P_1^f(\lambda_1 + \lambda_2^0)$;
 $c_L = (1-\alpha_1)(1-\alpha_2)\pi_L + \alpha_1\lambda_1 + \alpha_2(1-\alpha_1)\lambda_1 - P_1^f\lambda_1$;
 $c_H = (1-\alpha_1)(1-\alpha_2)\pi_H + \alpha_1\lambda_1 + \alpha_2[(1-\alpha_1)\lambda_1 + \lambda_2] - P_1^f(\lambda_1 + \lambda_2^1)$;
 $d_L = \pi_L - P_1^f\lambda_1$; $d_H = \pi_H - P_1^f(\lambda_1 + \lambda_2^0)$.

Solution for the Case of $U''' = 0$

To simplify the analysis, attention is restricted to the case of $U''' = 0$. Result 3 showed that in the case where the farmer was only able to purchase at the beginning of the growing period, then full insurance is optimal when $U''' = 0$. Hence, in the current analysis with the assumption of $U''' = 0$, any deviation from full insurance will necessarily be attributable to the potential to purchase additional insurance during the growing season.

First it is useful to examine the total level of insurance purchased across both periods. It is straight forward to establish that $\lambda_1^* + (\lambda_2^0)^* < \pi_H$ when hail did not occur in period one and $\lambda_1^* + (\lambda_2^1)^* < (1-\alpha_1)\pi_H$ when hail did occur in period one. That is, when $\pi = \pi_H$, total coverage after making additional purchases in period two is always less than the value of the crop. The reason for this is that the cost of purchasing insurance at the beginning of period two is $P_1^f = \theta_1\alpha_1 + \theta_2\alpha_2 - \theta_1\theta_2\alpha_1\alpha_2$ (i.e., the actuarially fair rate as of the beginning of period one) because the companies do not offer mid-season discounts. The true actuarially fair value of the insurance as of the beginning of period two, $P_2^f = \theta_2\alpha_2$, is less than P_1^f . Thus, from the perspective of the farmer purchasing hail insurance at the beginning of period two, the insurance is over-priced so consequently, as was established in result 2, the farmer will obtain less than full coverage.

By substituting P_1^f/P_2^f for γ in equation (2) with $\theta \equiv \theta_2$ and $\alpha \equiv \alpha_2$, an implicit expression for the total amount of insurance purchased is obtained. That is, with $\pi = \pi_H$ the solution value for λ in equation (2) after making the above substitutions equals $\lambda_1^* + (\lambda_2^0)^*$ by construction. If hail occurred during the first period then the corresponding solution equals $\lambda_1^* + (\lambda_2^1)^*$. Below it is shown that $\lambda_1^* \leq \phi\pi_L + (1-\phi)\pi_H$. Thus, unless

P_1^f/P_2^f is excessively large, additional insurance will be purchased at the beginning of period two when $\pi = \pi_H$, especially when there was no hail damage during period one. That is, despite the "excessively" high premium cost, the farmer will generally purchase some additional insurance at the beginning of period two if it is revealed at that point that hail-free revenues are above average. For this outcome to emerge for the case of no hail in period one, the parameters of the model are assumed to be such that P_1^f/P_2^f is sufficiently small to ensure that the solution value of λ in equation (2) with $\theta \equiv \theta_2$, $\alpha \equiv \alpha_2$, $\pi \equiv \pi_H$ exceeds $\phi\pi_L + (1-\phi)\pi_H$ and hence $(\lambda_2^0)^* > 0$.

The question of whether or not the farmer purchases full insurance in period one can now be addressed. In order to answer this question, the derivative of equation (6) with respect to λ_1 evaluated at the full insurance level, $\lambda_1 = \phi\pi_L + (1-\phi)\pi_H$, should be examined. If this derivative

vanishes, then full insurance in period one is optimal. If the derivative takes on a negative value, then $\lambda_1 = \phi\pi_L + (1-\phi)\pi_H$ exceeds the optimal value and hence $\lambda_1^* < \phi\pi_L + (1-\phi)\pi_H$ (i.e., the farmer underinsures in period one). To simplify the analysis, ϕ is set equal to 0.5 (i.e., π_L and π_H are equally likely). Also, to incorporate the restriction that $U''' = 0$, it is assumed that $U' = K - \beta W$ (i.e., utility is quadratic). After a great deal of manipulation, the derivative in question can be expressed as

$$(7) \quad \frac{\partial E(U)}{\partial \lambda_1} = -\beta \left\{ \begin{aligned} &2\theta_1\theta_2\alpha_2(\lambda_2^0 - \lambda_2^1)P_1^f + [(1-\theta_1)\lambda_2^0 + \theta_1(1-\theta_2\alpha_1\alpha_2)\lambda_2^1](P_1^f)^2 \\ &+ \theta_1\alpha_1[\theta_2\alpha_2(1+\alpha_2P_1^f) - \theta_1\alpha_1P_1^f]\lambda_2^1 + \theta_2\alpha_2^2(1-\theta_1)\lambda_2^0 \end{aligned} \right\}.$$

The following result can now be established:

Result 4: *When farmers are able to make mid-season purchases of insurance and the premium rate is actuarially fair, then $\lambda_1^* < \phi\pi_L + (1-\phi)\pi_H$ (i.e., the farmer underinsures) when $U''' = 0$ and $\phi = 0.5$. In contrast, when mid season purchases were not allowed as in the previous section, then the farmer choose to fully insure when $U''' = 0$ and $\phi = 0.5$.*

Proof: Earlier it was established that $(\lambda_2^0)^* > 0$ and $(\lambda_2^1)^* \geq 0$. These two restrictions ensure that equation (7) takes on a negative value¹² which in turn implies that expected utility is decreasing in λ_1 when evaluated at $\lambda_1 = \phi\pi_L + (1-\phi)\pi_H$. Hence, $\lambda_1^* < \phi\pi_L + (1-\phi)\pi_H$. Q.E.D.

Result 4 is important because it provides yet another reason why farmers may choose less than full hail coverage even though the premium rate is actuarially fair. That is, in the previous section it was shown that uncertainty in hail-free revenues resulted in exact coverage when $U''' = 0$ and less than full coverage when $U''' > 0$. Result 4 shows that the potential for mid-season insurance purchases induces the farmer to remain underinsured during the first period when $U''' = 0$. Although not formally analyzed, it would appear that in the case where mid-season purchases were allowed and $U''' > 0$, the level of insurance purchased in the first period would be less than either the static model with $U''' > 0$ or the dynamic model with $U''' = 0$.

The remaining question to be analyzed in this section is whether or not the farmer is under, over or exactly insured on average in the second period. That is, it was established above that when $\pi = \pi_H$ the farmer chooses to underinsure in period two and when $\pi = \pi_L$ the farmer is automatically overinsured in period two because $\lambda_1^* > \pi_L$. Therefore, it makes sense to ask whether the farmer will be underinsured or overinsured on average in period two.

The expected value of the crop at the beginning of period two, denoted $\bar{v}$, can be expressed as $\bar{v} = \phi[\theta_1(1-\alpha_1) + (1-\theta_1)]\pi_L + (1-\phi)[\theta_1(1-\alpha_1) + (1-\theta_1)]\pi_H$. This expression can be simplified to $\bar{v} = (1-\theta_1\alpha_1)[\phi\pi_L + (1-\phi)\pi_H]$. The average value of total insurance coverage chosen at the beginning of period two, denoted $\bar{z}$, can be expressed as $\bar{z} = \phi\lambda_1^* + (1-\phi)\{\theta_1[\lambda_1^* + (\lambda_2^0)^*] + (1-\theta_1)[\lambda_1^* + (\lambda_2^1)^*]\}$. The results derived earlier imply the following restrictions: $\lambda_1^* = \phi\pi_L + (1-\phi)\pi_H - c$; $\lambda_1^* + (\lambda_2^0)^* = \pi_H - c^0$; and $\lambda_1^* + (\lambda_2^1)^* = (1-\alpha_1)\pi_H - c^1$ where c , c^0 and c^1 are variables with

¹² Within equation (7), $(\lambda_2^0)^* > (\lambda_2^1)^*$ because the coverage level on the left (right) side of this inequality corresponds to the case where hail damage did not (did) occur in the first period and certainly the value of the former will exceed the value of the latter.

strictly positive values. Substituting these last three expressions along with the expression for $\bar{v}$ into the expression for $\bar{z}$ results in:

$$(8) \quad \begin{aligned} \bar{z} = & \bar{v} - \phi(1 - \phi - \theta_1\alpha_1)\pi_L - (1 - \phi)(1 - 2\theta_1)(\alpha_1 - \phi)\pi_H \\ & - \phi c - \theta_1(1 - \phi)c^0 - (1 - \theta_1)(1 - \phi)c^1 \end{aligned}$$

Equation (8) can be used to establish the following result:

Result 5: When $U''' = 0$, the average level of insurance purchased at the beginning of period two, $\bar{z}$, is generally less than the average value of the crop as of the beginning of period two. In other words, on average farmers are underinsured during the second period.

Proof: Equation (8) shows that for reasonable parameter values, $\bar{z} < \bar{v}$. Q.E.D.

Result 5 confirms that the overinsurance which occurs in period two when $\pi = \pi_L$ is relatively less severe than the underinsurance which occurs in period two when $\pi = \pi_H$ because, on average, the value of the insurance coverage held in period two is less than the value of the crop. Results 4 and 5 together imply that for the case of $U''' = 0$, the farmer tends to be underinsured throughout the entire production period. Although not rigorously established, the previous results also imply that a qualitatively similar result will emerge for the more realistic case of $U''' > 0$.

An explanation for these results can be summarized as follows. The farmer underinsures in period one in order to preserve the option value of not carrying excessive insurance if hail-free crop revenues end up being below average. But once the farmer arrives at the beginning of period two, there is an incentive to remain underinsure for above-average hail-free crop revenues because at that point in time the price of the insurance is relatively over priced. Regarding sensitivity of these results to the parameters, high values of θ_1 and α_1 relative to θ_2 and α_2 , respectively, will tend to be associated with relatively low levels of underinsurance in period one and relatively high levels of underinsurance in period two when $\pi = \pi_H$. The reason is that such a parameter configuration is associated with relatively high risk in period one and hence the implicit cost of remaining underinsured in period one is relatively higher. Moreover, in this situation $P_1^f/P_2^f = (\theta_1\alpha_1 + \theta_2\alpha_2 - \theta_1\theta_2\alpha_1\alpha_2)/(\theta_2\alpha_2)$ takes on a relatively large value implying that mid-season purchases of insurance are relatively unattractive.

IV. Combined Hail and All-Risk Crop Insurance

Now suppose that in addition to obtaining hail insurance, the farmer can also purchase all-risk crop insurance. Because farmers who purchase all-risk crop insurance operate with less than full coverage (i.e., their yield guarantee is generally less than their mean yield), additional hail insurance is typically purchased from a private broker. The typical all-risk contract is structured as follows. Suppose a farmer's expected yield is y and the market price of the crop being grown is non-stochastic and equal to 1. Also suppose that this farmer purchases ψ *100 percent coverage from the supplier of all-risk crop insurance where ψ typically ranges between 0.5 and 0.8. If hail does not occur in the growing season and the farmer's actual yield is $\tilde{y} < \psi y$, then this farmer

receives an indemnity equal to $\psi y - \tilde{y}$.¹³ If hail does occur and α percent of the yield is lost because of the hail, then the contract can be specified in one of two ways. One possibility (method A) is that an indemnity equal to $\alpha\psi y$ can be paid to the farmer when the damage occurs and the yield guarantee for other sources of risk can then be reduced to $(1-\alpha)\psi y$. The other possibility (method B) is that hail can be treated as just another risk and thus an indemnity is paid after harvest only if $(1-\alpha)\tilde{y} < \psi y$. Of interest in this section is comparing the efficiency properties of methods A and B.

Method A

To capture the essence of the problem, it is necessary to specify a model with a continuum of yield outcomes. To preserve simplicity, the market price of the commodity is assumed constant and equal to 1. Suppose actual yield, $\tilde{y}$, is equal to $y\varepsilon$ where ε is a random variable with mean 1, probability density function, $f(\varepsilon)$, and cumulative density function, $F(\varepsilon)$. The yield guarantee for the farmer is ψy and the actuarially fair insurance cost of the all-risk insurance is denoted P_A . Given the above description of how spot-loss hail claims are currently handled within an all-risk crop insurance system, it follows that in years without a hail claim, indemnities equal 0 if $\varepsilon \geq \psi$ and equal $(\psi - \varepsilon)y$ if $\varepsilon < \psi$. In years with hail damage, indemnities for hail damage only from the all-risk crop insurance agency equal $\alpha\psi y$ and indemnities from the private supplier of hail insurance equal $\alpha\lambda$ (as before, λ denotes the level of private hail insurance coverage). Both of these payments are made at the time of the hail damage. Additional indemnities equal to $(1-\alpha)(\psi - \varepsilon)y$ are paid by the all-risk agency post harvest if $\varepsilon < \psi$ and no additional indemnities are paid when $\varepsilon \geq \psi$.

The expected utility function for the farmer in model A can now be written as

$$(9) E\{U\} = \theta \left[\int_0^{\psi} U(X_1^A) dF(\varepsilon) + \int_{\psi}^{\infty} U(X_2^A) dF(\varepsilon) \right] + (1-\theta) \left[\int_0^{\psi} U(X_3^A) dF(\varepsilon) + \int_{\psi}^{\infty} U(X_4^A) dF(\varepsilon) \right]$$

Where: $X_1^A = \psi y - P_A + (1-\theta)\alpha\lambda_A$; $X_2^A = (1-\alpha)\varepsilon y + \alpha\psi y - P_A + (1-\theta)\alpha\lambda_A$;

$X_3^A = \psi y - P_A - \theta\alpha\lambda_A$; and $X_4^A = \varepsilon y - P_A - \theta\alpha\lambda_A$.

The first set of expressions in large square brackets in equation (9) is the expected utility for the farmer in hail states and the second set is expected utility in non-hail states. The subscript A on λ is used to distinguish the equilibrium value of this variable from that in an alternative specification of the problem presented below. Notice that the cost of the private hail insurance, $\theta\alpha\lambda_A$, has been subtracted from net revenues within each of the four conditional utility functions. The percent coverage variable, ψ , is exogenous to the problem¹⁴ so the problem reduces to selecting the value of private hail insurance, λ_A , that maximizes equation (9).

¹³ For simplicity, assume that the probable yield assigned by the supplier of all-risk crop insurance exactly equals the farmer's actual expected yield.

¹⁴ Because the insurance premium is actuarially fair and $\psi < 1$, the farmer will optimally choose the maximum allowable value for ψ (normally 80 percent or less).

Method B

With method B, the yield which triggers an indemnity and the size of the indemnities are the same as in method A in non-hail years. However, when hail does occur, no indemnities are paid at the time of the hail damage but post-harvest indemnities equal to $\psi y - (1-\alpha)\epsilon y$ are paid if $(1-\alpha)\epsilon y < \psi y$ or $\epsilon < \psi/(1-\alpha)$ and zero otherwise. Letting P_B and λ_B denote the equilibrium values for the actuarially fair premium rate for all-risk insurance and the level of private insurance purchased, respectively, it follows that the farmer's expected utility for this particular specification of the problem can be expressed as follows:

$$(10) \quad E\{U\} = \theta \left[\int_0^{\psi^*} U(X_1^B) dF(\epsilon) + \int_{\psi^*}^{\infty} U(X_2^B) dF(\epsilon) \right] + (1-\theta) \left[\int_0^{\psi} U(X_3^B) dF(\epsilon) + \int_{\psi}^{\infty} U(X_4^B) dF(\epsilon) \right]$$

$$\begin{aligned} \text{Where: } X_1^B &= \psi y - P_B + (1-\theta)\alpha\lambda_B; & X_2^B &= (1-\alpha)\epsilon y - P_B + (1-\theta)\alpha\lambda_B; \\ X_3^B &= \psi y - P_B - \theta\alpha\lambda_B; & X_4^B &= \epsilon y - P_B - \theta\alpha\lambda_B; \\ \psi^* &= \psi/(1-\alpha) \end{aligned}$$

Method A versus Method B

When comparing the expected utility of the farmer in method A and B, it is important to note that expected net revenues are equal under both schemes because the insurance is actuarially fair by construction. Therefore, for at least a restricted class of utility functions, the model that generates the highest expected utility for the farmer is the one with the lowest variability in net returns.¹⁵

When comparing the variance of net returns for the two models, notice that attention can be restricted to the states within which hail occurs because, except for possible differences in values of the variables which do not affect the variance (P_A , P_B , λ_A and λ_B), the return in the no-hail states are identical for the two models. Therefore expected utility for the farmer will be lower in method A than in method B if the variance of net returns in the hail state are higher with the former than the latter.

To calculate an expression for variance of net returns in hail states, an expression for expected net returns in hail states is required. This expression is constructed by noting that

$$(11) \quad E\{W_{\text{Hail}}\} = E\{W_{\text{Hail}} | \epsilon \leq \psi\} \text{PR}(\epsilon \leq \psi) + E\{W_{\text{Hail}} | \epsilon > \psi\} \text{PR}(\epsilon > \psi)$$

where PR denotes probability. After substituting the appropriate equations into equation (11), expressions for expected net returns in the hail state for method A, $E\{W_{\text{Hail}}\}^A$, and method B, $E\{W_{\text{Hail}}\}^B$, are given by

$$(12) \quad E\{W_{\text{Hail}}\}^A = y \left[\psi F(\psi) + (\alpha\psi + [1-\alpha]\bar{\epsilon}_+) (1 - F(\psi)) \right] - P_A + (1-\theta)\alpha\lambda_A$$

and

$$(13) \quad E\{W_{\text{Hail}}\}^B = y \left[\psi F(\psi^*) + [1-\alpha]\bar{\epsilon}_+^* (1 - F(\psi^*)) \right] - P_B + (1-\theta)\alpha\lambda_B$$

where $\bar{\epsilon}_+ = E\{\epsilon | \epsilon > \psi\}$ and $\bar{\epsilon}_+^* = E\{\epsilon | \epsilon > \psi^*\}$.

¹⁵ That is, the following analysis implicitly assumes a mean-variance utility function.

An expression for the variance of net revenues during hail states for method A, $VAR(W_{Hail})^A$, can now be written as

$$(14) \quad VAR(W_{Hail})^A = \int_0^{\psi} [X_1^A - E\{W_{Hail}\}^A]^2 dF(\varepsilon) + \int_{\psi}^{\infty} [X_2^A - E\{W_{Hail}\}^A]^2 dF(\varepsilon)$$

The expression for $VAR(W_{Hail})^B$ is analogously defined. After substituting the appropriate expressions and considerable manipulation, equation (14) can be rewritten as

$$(15) \quad VAR(W_{Hail})^A = y^2(1-\alpha)^2 \left[(\bar{\varepsilon}_+ - \psi)^2 F(\psi)[1 - F(\psi)] + \int_{\psi}^{\infty} (\varepsilon - \bar{\varepsilon}_+)^2 dF(\varepsilon) \right].$$

The corresponding expression for method B is identical to equation (15) except substitute ψ^* for ψ and $\bar{\varepsilon}_+^*$ for $\bar{\varepsilon}_+$.

Now differentiate equation (15) with respect to ψ . After considerable simplification, the resulting expression can be written as

$$\frac{\partial VAR(W_{Hail})^A}{\partial \psi} = -2y^2(1-\alpha)f(\psi)F(\psi)(\bar{\varepsilon}_+ - \psi) \left[\frac{1 - F(\psi)}{f(\psi)} + \frac{\bar{\varepsilon}_+}{1 - F(\psi)} \right].$$

The following result can now be established.

Result 6: Expected net revenues for the farmer are the same with methods A and B but the variability of profits is lower with method B. Hence, for at least a certain class of utility functions, expected utility for the farmer is higher with method B than with model A.

Proof: The expression for $VAR(W_{Hail})^A$ and $VAR(W_{Hail})^B$ are identical except that the former contains the parameter ψ and the latter the parameter $\psi^* = \psi/(1-\alpha)$. Because $\psi^* > \psi$, it follows that if equation (16) takes on a negative value then $VAR(W_{Hail})^A < VAR(W_{Hail})^B$. The sign of equation (16) is obviously negative. Q.E.D.

The explanation for result 6 is as follows. With method A, indemnities for hail damage are paid to the farmer even if the crop's yield after the hail damage is above the average crop yield (without hail damage). The impact of the hail payouts in these years is to actually increase the overall variance of net revenues in method A. In contrast, with method B indemnities for hail damage are only paid if yield after hail damage falls below the guarantee so their impact can only ever be to decrease the variance of net revenues. The combined effect of these differences is that net revenues tend to be more variable with method A than with method B.

Finally, without formal analysis, it follows from the previous results and is illustrated in the simulation results presented below that the level of underinsurance will be more severe with method A than with method B. The reason is that variability in net crop revenues induces farmers to underinsure (see result 3). Consequently, the level of underinsurance is expected to be higher with method A than with method B because, as demonstrated above, the variability in net revenues is higher with the former.

Simulation Results

Table 1 shows numerical results from methods A and B above for the optimal level of total hail coverage and the difference in certainty equivalent value of the two schemes. The simulations are based on a CARA utility function with absolute risk aversion parameter (R) set equal to 2.5 and a normally distributed yield error term (ϵ) with mean 1 and standard deviation σ . Sensitivity analysis is conducted for four parameters: probability of hail (θ), standard deviation of yield (σ), guaranteed proportion of average crop yield in the all-risk insurance (ψ), and the proportion of yield lost when hail occurs.

The results illustrate the previously discussed findings: whenever yield is uncertain, it is optimal to underinsure against hail; the extent of underinsurance is lower for method B than for method A; and scheme B dominates scheme A in terms of its certainty equivalent value for farmers. We are confident that these findings are robust. In further wide-ranging sensitivity analyses (not presented here) the only violation of these conclusions occurred in a formulation where price and yield were both uncertain and strongly negatively correlated. Of the parameters examined, that with the greatest impact on the results was the standard deviation of yield. Apart from this effect, plausible variations in parameters had minor impacts.

VI. Conclusions

Results with the simplest version of the model indicated that it was optimal for farmers to fully insure against hail if the premium rate was actuarially fair. Subsequently, however, a number of realistic complications included in the model each provided an impetus for insuring less than the full value of the crop. In particular, this was true of (a) higher premium rates (e.g., to cover transaction costs, administrative costs and normal profits), (b) stochastic hail-free crop revenue (e.g., due to stochastic yield and/or price), (c) dynamic refinement of the level of hail insurance coverage within the growing season, and (d) integration of hail insurance coverage with all-risk crop insurance. In combination, these analyses would appear to suggest that farmers are likely to be underinsured against hail damage. It would certainly be of interest to test this hypothesis with empirical data.

The study also examined the efficiency properties of two alternative ways through which hail loss is handled within the context of an all-risk crop insurance program. Method A (currently in use in the province of Saskatchewan) entails making a separate adjustment for hail and method B (currently in use in the state of Kansas) entails treating hail as just another peril. The latter method was shown to be more efficient than the former because the variability of profits is relatively lower.

An industry expert within Saskatchewan suggested that a possible reason why the scheme in question is in use is that it could potentially be responsible for higher participation rates in the all-risk crop insurance program. Specifically, given the subjective nature of measuring hail damage, mid-season crop insurance adjusters have the flexibility to bias hail adjustments in favour of the farmer as a mechanism for enticing the farmer to participate in the crop insurance program. Another reason is that farmers may prefer protection against hail damage on a field by field basis. That is, if the farmer received hail damage in one field but the average yield from all fields did not fall below the yield guarantee an indemnity would not be triggered with method B whereas it would be triggered with method A.

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Appendix

The purpose of this appendix is to examine the optimal level of hail coverage when: (i) the farmer does not purchase all-risk crop insurance; (ii) hail insurance can only be purchased at the beginning of the production period; and (iii) both hail-free revenues and the proportion of hail damage are continuous random variables. Let $\pi\varepsilon$ denote hail-free revenues where π is a fixed constant and ε is a continuous random variable defined over $[0, \infty]$ with mean 1 and cumulative distribution function $F(\varepsilon)$. Also, let $\tilde{\alpha}$ denote the proportion of the crop lost when hail occurs. This variable, now assumed to random, is defined over $[0, 1]$ with mean $\bar{\alpha}$ and cumulative distribution function $G(\tilde{\alpha})$. Assuming an actuarially fair premium rate, an expression for expected utility can be written as

$$(A.1) \quad E(U) = \theta \int_0^1 \int_0^\infty U([1 - \tilde{\alpha}]\pi\varepsilon + \tilde{\alpha}\lambda - \theta\bar{\alpha}\lambda) dF(\varepsilon) dG(\tilde{\alpha}) \\ + (1 - \theta) \int_0^1 \int_0^\infty U(\pi\varepsilon - \theta\bar{\alpha}\lambda) dF(\varepsilon) dG(\tilde{\alpha}).$$

The first-order condition with respect to λ evaluated at $\lambda = \pi$ can be written as

$$(A.2) \quad \int_0^1 \int_0^\infty U'(A) \left[\frac{\tilde{\alpha}}{\bar{\alpha}} - \theta \right] dF(\varepsilon) dG(\tilde{\alpha}) = \int_0^1 \int_0^\infty U'(B) dF(\varepsilon) dG(\tilde{\alpha})$$

where $A = \pi[\varepsilon + \tilde{\alpha}(1 - \varepsilon) - \theta\bar{\alpha}]$ and $B = \pi[\varepsilon - \theta\bar{\alpha}]$.

The intent of this analysis is to show that with $\lambda = \pi$, the left-hand side of equation (A.2) takes on a smaller value than the right-hand side when $U''' > 0$. Reducing λ below π will reduce and eventually eliminate this imbalance thus ensuring that the optimal strategy is to underinsure. To show that the left-hand side of equation (A.2) takes on a smaller value than the right-hand side when $\lambda = \pi$, assume initially that the variance of A and B are the same. With this assumption, because $E(A) = E(B)$, the two sides of equation (A.2) will differ in value only because A is correlated with α on the left-hand side of equation (A.2). When $\varepsilon < 1$, A and α are positively correlated thus reducing the left-hand side of equation (A.2) relative to the right-hand side because $U'(A)$ is a decreasing function. when $\varepsilon > 1$, A and α are negatively correlated thus increasing the left-hand side of equation (A.2) relative to the right-hand side. However, A is increasing in ε and $U'(A)$ is decreasing in A so the value-increasing effects from the negative correlation between A and α when $\varepsilon > 1$ will be less than the value-decreasing effects from the positive correlation between A and α when $\varepsilon < 1$. Hence, if A and B had equal variances, the left-hand side of equation (A.2) would take on a smaller value than the right-hand side.

In fact, the variance of A is less than the variance of B when $\lambda = \pi$ implying via Jensen's inequality that the expected value of $U'(A)$ is less than the expected value of $U'(B)$ for $U''' > 0$. This result further strengthens the case that the left-hand side of equation (A.2) takes on a smaller value than the right hand side. Comparing the variances of A and B is straight forward:

$$VAR(A) = \pi^2 \int_0^1 (1 - \alpha)^2 dG(\alpha) \int_0^\infty (1 - \varepsilon)^2 dF(\varepsilon) < VAR(B) = \pi^2 \int_0^\infty (1 - \varepsilon)^2 dF(\varepsilon)$$

Decreasing λ below π increases the left-hand side value of equation (A.2) and decreases the right-hand side value, thus ensuring that $\lambda^* < \pi$. The left-hand side value increases because net revenues in the no-hail state is increasing in λ on average and net revenues in the hail state is decreasing in λ .

Table 1: Simulation Results for Combined Model with Normally Distributed Yield and CARA Utility

σ	ψ	α	$\theta=0.1$			$\theta=0.2$		
			cover _A *	cover _B *	CE _B -CE \$ $\times 10^{-3}$	cover _A *	cover _B *	CE _B -CE \$ $\times 10^{-3}$
0	0.7	0.5	1.000	1.000	0	1.000	1.000	0
0.2	0.7	0.5	0.942	0.948	0.761	0.937	0.939	1.526
0.2	0.7	0.7	0.951	0.957	0.088	0.945	0.951	0.078
0.2	0.8	0.5	0.959	0.969	0.732	0.951	0.959	1.506
0.2	0.8	0.7	0.964	0.970	0.141	0.955	0.960	0.223
0.4	0.7	0.5	0.895	0.914	2.295	0.878	0.892	4.659
0.4	0.7	0.7	0.911	0.932	0.636	0.888	0.910	1.357
0.4	0.8	0.5	0.947	0.968	2.095	0.925	0.945	4.395
0.4	0.8	0.7	0.956	0.973	0.561	0.926	0.945	1.219

Cover represents combined hail coverage from both all-risk crop insurance and privately-supplied hail-only insurance. CE denotes the certainty equivalent of net revenues associated with expected utility in the optimized model. For these simulations, $y = 1$ and the coefficient of absolute risk aversion, R , was set equal to 2.5.

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