UWA 1/92

The University of Western Australia

AGRICULTURAL ECONOMICS

SCHOOL OF AGRICULTURE

FUTURES MARKETS, STORAGE AND CONVENIENCE YIELD*

Donna Brennan and Brian Wright#

Agricultural Economics
Discussion Paper: 1/92

Nedlands, Western Australia 6009

FUTURES MARKETS, STORAGE AND CONVENIENCE YIELD*

Donna Brennan and Brian Wright#

Agricultural Economics
Discussion Paper: 1/92

The University of Western Australia
#University of California, Berkeley
*This research has been supported by a Charles & Annie Neumann Post Doctoral Fellowship in Agriculture

Futures Markets, Storage and Convenience Yield

by Donna Brennan* and Brian Wright#

Why are stocks of a commodity held when the spot price exceeds the price for future delivery? Working's (1948) supply of storage hypothesis attributes a non-market benefit to storage, based on Kaldor's (1939) discussion of convenience yield.

This paper examines stockholding behaviour at the micro-level and shows that stocks are never held at a loss when the appropriate local price spread, net of transport costs, is considered. In an empirical model of a spatial wheat market, aggregate stockholding in the presence of a negative price spread at the port is observed, but this is attributed totally to non-linearities in transport costs. Thus the convenience yield hypothesis, which attributes an intangible non-market benefit to stockholding, is not a micro-economic phenomenon, but the result of an aggregation problem.

The empirical model is also used to examine the nature of price spreads in a competitive equilibrium, with non-linear transport costs. Local price spreads differ enormously across locations and this has important implications in the regulation of futures markets. Some "unusual" local price spreads are observed, which resemble those that have been attributed to non-competitive behaviour in the past (Gray and Peck 1981). However, these price spreads are a result of normal competitive behaviour in the face of non-linear transport costs.

* University of Western Australia
# University of California, Berkeley

Contributed Paper presented at the 36th Annual Conference of the Australian Agricultural Economics Society, February 10-12, Canberra.

The hypothesis of Kaldor (1939) that holders of commodity stocks obtain a nonmarket "convenience yield" from their holdings, is central to the model of commodity prices of Working (1948,1949), Brennan (1958) and Telser (1958) currently dominant in the literature. (See, for example, Fama and French 1988 p 1075-6).

Convenience yield is empirically defined rather indirectly; its marginal value is the positive differential, (if any) needed to raise the return to a holder of commodity stocks to the holder's opportunity cost of capital. The current consensus is that convenience yield is an important feature of the returns to holders of commodity stocks[1] . This consensus is no doubt based on the general belief that positive stocks of commodities are held voluntarily even when their net direct monetary return is expected to be negative[2]. This belief in turn is based on the well established characteristics of supply of storage curves relating aggregate stocks to the expected change in price, pioneered by Working (1933,1948) in studies of the United States wheat market. (See Figure 1 for an example). From inspection of the supply of storage relation in Figure 1, the implication that stocks are voluntarily held at a loss when supplies are moderate or lower seems so obvious it is easy to see why it is widely accepted as a fact, though it has never been tested with micro-level data. In any case attempts to conduct a rigourous test would usually be thwarted by lack of appropriate micro-level data on costs, returns and expectations.

In this paper we follow Holbrook Working's footsteps in studying supply of storage in a spatially dispersed wheat market. But we, unlike Working, have access to detailed information on the distribution of harvest and costs of storage and transportation in the market we study, the Kwinana region in Western Australia. This information allows us to attack the question from a different perspective. We use the cost information to model the optimal allocative response to expected price changes, and obtain a supply of storage relation with those typical characteristics on which the convenience yield hypothesis has been based for half a century. But there is no convenience yield in our model; stocks are never held at a loss to the holder. Working's supply of storage relation does not necessarily imply any convenience yield to the holders of stocks.

---

[1] For example, "the notion of convenience yield, viewed as a net 'dividend' yield accruing to the owner of the physical commodity at the margin, has already proven to drive the relationship between futures and spot prices for many commodities" (Gibson and Schwartz, 1990).

[2] Telser (1958 p23b) is typical: "The reader may wonder why it is necessary to introduce the concept of convenience yield to explain the holding of stocks. Why is it not sufficient to consider only the marginal cost of storage? The answer lies in the fact that stocks are held even when prices are expected to fall. Hence those holding stocks may expect to suffer a "capital loss" and in addition incur storage charges".

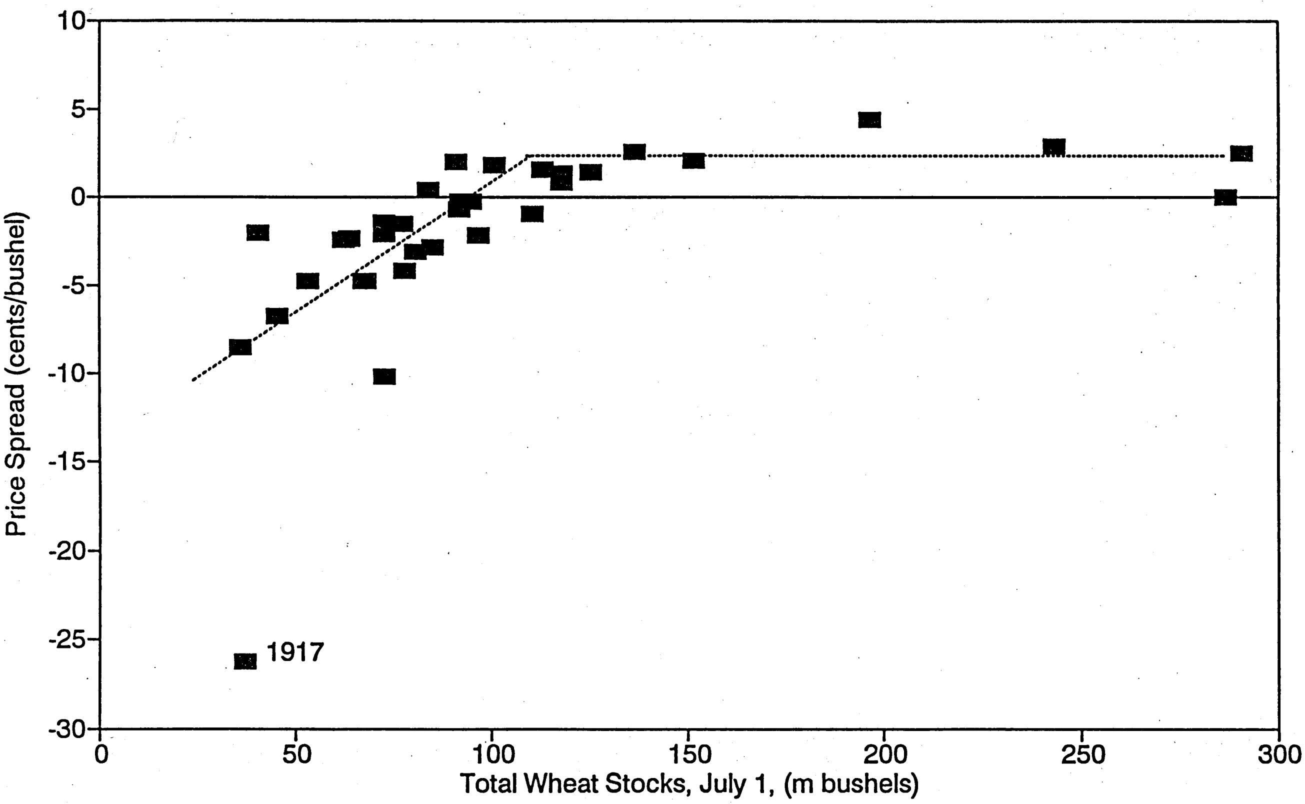

Figure 1: Relation between Chicago July-September Spread in June and Total United States Wheat Stocks, July 1, 1895-1930

(based on Working (1933))

Thus the estimation of a similar aggregate supply of storage relation in other markets, as in Brennan (1958), Telser (1958) and many subsequent studies, is not sufficient to establish the significance or eve the existence of convenience yield in those markets. Detailed microeconomic information of the type used here are necessary to distinguish between convenience yield and the competing hypothesis, first formulated in Wright and Williams (1989) and given empirical force here, namely that the supply of storage relation is explained by aggregated effects of nonlinearities in the costs of storage and transportation related to the importance of fixed capital in both activities.

Our results also shed new light on the type of evidence traditionally considered sufficient to establish market manipulation by "hoarders". In particular, a very wide variety of responses to stockholding at different locations when the spread at the terminal location places an increased premium on early delivery is consistent with socially optimal resource allocation in the market we have studied.

Background

The empirical evidence supporting the convenience yield hypothesis rests on the common qualitative features of the empirical "supply of storage" relationship between stocks and market price spreads observed for many commodities over many time periods. The data underlying an influential early example presented in Working "Supply of Storage" (1933), are reproduced in Figure 1. After rejecting some observations and other adjustments, he fitted a curve similar to that drawn in the figure. (See also Working 1953). The data, and the curve drawn to represent it, indicate that positive stockholding, above the minimum level in the sample, can occur when the price spread is negative. That standard explanation for this phenomenon is that holders of stock receive a "convenience yield" as a non-monetary compensation for the declining value of their stocks. (See, for example, Houthakker 1987 p448).

However, observation of a "supply of storage" relation of the usual shape does not constitute a rejection of the hypothesis that stockholders receive no convenience yield. First, there is a timing problem with the data. In Figure 1, the vertical axis measures an average expectation of a futures price differential between two futures contracts, maturing in July and September, over the month of June. Stocks are measured in July 1st. Hence the rate of change of price on the date of stocks measurement is not observed, but represented by a proxy, (an earlier expectation of a later price change), with attendant, unexamined, errors-in-variables problems.

Second, commodity stocks data, where available, are insufficiently accurate to test the hypothesis. They are usually aggregated across grades and locations, whereas the price data refer to one particular grade (whatever is currently the "cheapest to deliver", given the relation of current price differential to grade differential specified in the contract) in a specific market. The stockholder's own holdings, and the returns on them, are not directly observed.

For other commodities stocks data, if available at all, are less reliable than they are for grains. (For example, *Fruit and Tropical Products* ceased publication of United States coffee stocks in 1984, when the series turned negative!) The severity of this problem has led recent authors (Fama and French 1987,1988; see also Gibson and Schwartz 1989,1990) to try to test for convenience yield in an even more indirect fashion, using price data alone. As argued below, their results are consistent with the theory of profit-maximising storage where stocks are constrained to be non-negative, (Gustafson 1958, Samuelson 1971, Garner 1979) regardless of the existence of convenience yield.

The Transportation and Storage Model

The subject of this study is an empirically based mathematical programming model of the storage and transportation network for wheat marketed through the port of Kwinana in Western Australia[3]. The primacy of export demand, the separation of the marketing network from other production regions, the dominance of wheat in grain production and the insignificance of inter-year carryover make this market especially simple to analyze. But its particular attraction, for our purposes, lies in the availability of necessary microeconomic data. Detailed physical data on the entire system was available because of the recent Royal Commission into Grain Handling and Storage.

The storage and transportation network is shown in Figure 2. There are one hundred and three hinterland receival points with storage facilities of various types. These receival points are linked by rail lines of two different gauges and/or by road to the single terminal, the port city of Kwinana. Most of the gain is sold on the export market on which the world price is exogenous. The annual crop is harvested over a short two month period.

Because insect control problems make on-farm storage infeasible, all of the grain is delivered to receival points or subterminals during the harvest period. This results in an enormous demand for internal transport and storage facilities which are in limited supply during this time. This peak load problem means that there are two economically distinct periods in the year. Grain that is delivered to a colleciton points in the harvest period can either be transported directly to the port in the peak period, or it can be stored until the off-peak period and transported to the port at a lower cost. There is no peak load problem for port shipping services. All stocks that arrive at the port can be immediately exported.

The empirical model contains detailed information, provided from industry sources, about the costs and capacities of alternative methods of storing grain at each receival point. It also contains details on the costs of transporting from each site along the alternative routes to the port in

---

[3] The model was originally constructed to study a rationalisation of the rail transport and grain storage system (Brennan 1990). Here we assume the least efficient rail lines have been closed in line with that rationalisation, but the model has not otherwise altered in producing the results reported here.

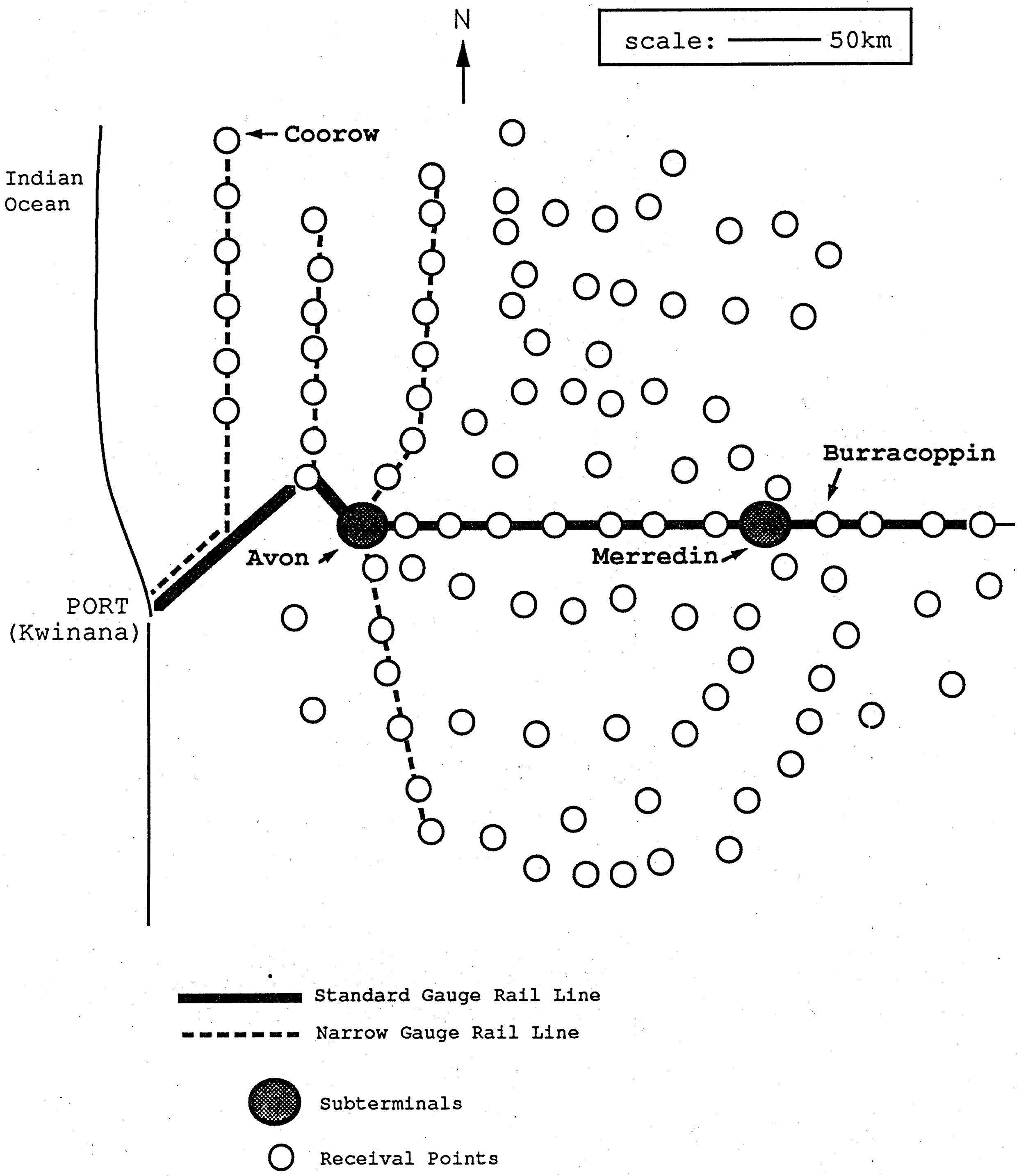

Figure 2: The Western Australian Wheat Transport System
(Kwinana Region)

each period. Short run costs are determined endogenously according to the scarcity of transport and storage capacity.

Transport

As shown in Figure 2, there are a number of alternative transport routes in the system. Some sites are located on rail lines, others are serviced only by road links. There are two rail networks, one narrow gauge and the other standard gauge. Trains on the standard gauge rail line have relatively low operating costs because they are larger and travel faster. There are two sub-terminal located on the standard gauge line, which can receive grain from road linked receival points, and from receival points linked by the narrow gauge line. These subterminals are large capacity, high throughput sites that can handle a substantial proportion of the total harvest, and can load large, economical unit trains. Grain may also be transported by road from any of the receival sites directly to the port.

The transport operating cost t_j, is determined largely by fuel use and labour costs (assumed constant across peak and off-peak periods) and upon the distance of the trip and the mode/s of transport, and whether inter-modal transfer is required at the sub-terminal. This cost is the same, for a given route, regardless of whether the grain transportation occurs in the peak or off-peak periods.

There is also an opportunity cost associated with using transport capacity in the peak period. For rail transport, the opportunity cost per tonne along a given route depends on the opportunity costs of wagon time and locomotive time (which are in limited supply for the system as a whole) and the amount of time used up in completing a round trip. Sites that are closer to the port or have facilities that can load trains at a faster rate have relatively lower opportunity costs per tonne of grain hauled to the port. We shall refer to four sites in particular for purposes of exposition. They are denoted "close subterminal" (Avon), "far subterminal" (Merredin), "far receival point" (Burracoppin) and "branch line receival point" (Coorow) respectively. These sites are located on rail lines and can transport wheat directly by rail to the port. Peak transport premiums are also incurred in the road, road/rail and rail/rail routes to the port. The mobility of road transport facilities both spatially and between industries means that they are not in limited supply in the peak period, but a peak transport cost is incurred due to idle time spent waiting in truck queues at receival points and/or subterminals.

The cost of railing grain from site j to the port at Kwinana is the operating cost t_j, incurred in either period, plus an opportunity cost if the grain is transported in the peak period. The opportunity cost of transporting a tonne of grain from site j on the standard gauge rail system is given by $\alpha_j\lambda$ where λ is the shadow cost of a peak train hour and α_j is the number of train hours needed to transport a tonne of grain on the rail system from site j to the port. The shadow cost of a peak train hour is the sum of hourly shadow costs of locomotive services and wagon services used in the train, determined by the total demand in the system for these services and capacity constraints. Demand for rail capacity in the peak period is determined by the price spread at the port. Following convention, the port price spread S_p is defined as the second (off-peak) period price P_2 minus the current price P_1. Subscripts 1 and 2 refer to current and off-peak periods respectively. For simplicity, all second period prices are expressed here as discounted period one values; and the discount rate is assumed constant over all cases.

(1) $$S_p = P_2 - P_1$$

A fall in the price spread implies that the returns from marketing in the current period rise, increasing the demand for rail capacity in the peak period. Thus the opportunity cost of a peak standard gauge train hour can be expressed as a function of the price spread:

(2) $$\lambda = \lambda[S_p]$$
where $\delta\lambda / \delta S_p < 0$ and $\delta^2\lambda / \delta S^2_p > 0$

The cost of transporting a tonne of wheat from site j in the peak period is the sum of the operating cost t_j and the opportunity cost:

(3) $$t_{j1} = t_j + \alpha_j\lambda[S_p]$$

Off peak transport costs (discounted to period 1, where r is the opportunity cost of capital) are:

(4) $$t_{j2} = t_j/(1+r)$$

The "peak premium" paid on grain that is transported to the port in the peak period is given by:

(5) $$t_{j1} - t_{j2} = rt_j/(1+r) + \alpha_j\lambda[S_p]$$

In the model, calculations of the type shown here for standard gauge rail services are made for each transport option relevant to a given site, including some or all of standard gauge rail, narrow gauge rail, road, road/standard gauge rail, road/narrow gauge rail, and narrow gauge/standard gauge rail. For each site the model select t_{j1} and t_{j2} as the cheapest transport modes in each period.

Storage

There are three types of possible storage technologies in place in the system each with distinct but constant short run marginal costs. These costs are principally the costs of input and retrieval of grain, and pest control, which are lowest for the most capital intensive technology (vertical elevators), highest for the least capital intensive alternative (horizontal open bunkers with plastic covers), with enclosed horizontal storage in an intermediate position. The marginal cost s_j curve at each receival point is an increasing stepped function determined by the amount of each technology at that site. The marginal cost of storing grain at site j depends on the amount stored X_j and is given by a step function with the location of each step determined by storage capacity of the three types.

(6) $$s_j = s_j[X_j] > 0, \quad X_j > 0$$

$$s_j[X_j'] \geq s_j[X_j], \quad X' > X > 0$$

In the model, the returns from marketing the grain over the two periods are maximised, given the price spread at the port and the fixed capital investment in transport and storage. The returns from marketing in the peak period depend on the current port price and the current cost of transporting grain to the port. The discounted returns from marketing in the off-peak period depend on the cost of storing grain until then, the discounted expected off-peak price and the discounted off-peak transport cost, all assumed known with certainty. As recognized in Equations 3-6, each grain collection point j has its own transport and storage costs. This means that the decision to market grain in the current period or store it until the off-peak period is specific to each site. The price spread S_j at each location is expressed net of transport costs:

(7) $$S_j = S_p + rt_j/(1+r) + \alpha_j\lambda[S_p]$$

The socially optimal decision to store grain at a site j depends on the local price spread S_j and the marginal cost of storage at the site. The local price spread is the return from marketing grain in the off-peak period, including the opportunity cost of not marketing it in the current period. If the local spread is less than the cost of storage, grain is stored at the site and vice versa. This can be written as the complementary inequalities:

(8) $$S_j = s_j[X_j], \quad X_j \geq 0$$

$$S_j < s_j[X_j], \quad X_j = 0$$

Thus storage behaviour at each location depends not only on the port price spread, but also on how well each site competes for scarce transport capacity in the peak period (determined by α_j) and the marginal cost of storage at the site. Deliveries to each receival point from the representative supply nodes were determined endogenously according to the cost of transport from farm to receival point (including the cost of congestion on heavily used sites) and the cost of marketing (storage and transport) through the alternative sites. (For details, see Brennan (1991)).

The model determines storage and transport behaviour simultaneously at each site, generating a general equilibrium social planner's solution for the market given the resource constraints and the exogenous port price spread. Aggregate behaviour is determined by summing over the micro decision made at each receival point. The model is used to derive optimal storage and transport allocations at each site under a number of alternative f.o.b. port price spreads, given the actual amount of grain harvested and its spatial distribution in the 1983/4 season.

The Supply of Storage Curve

Optimal storage and transport allocations were derived under a range of exogenous port spreads. A plot of the port spread against aggregate stocks shown in Figure 3 reveals an aggregate "supply of storage" relationship. Aggregate grain stocks decline as the port spread falls, but remain positive as the spread falls below zero [4]. But there is no "convenience yield" at any storage site to explain the slope of the "supply of storage curve".

To explore the micro-economic behaviour underlying the aggregate supply of storage relation, consider the three reference sites in Figure 2 located on the standard gauge rail line and therefore competing directly for the same scarce rail resources (standard gauge locomotive and wagon hours). Train turnaround times differ between the sites, because of distance from the port and site characteristics affecting grain loading rates. The close subterminal is the most efficient user of standard gauge trains in the system, because of its proximity to the port and the technology available for the rapid loading of unit trains. The far receival point is the least efficient of the three sites because of distance from the port and the relatively slow train loading technology used at the elevator. The slow train loading technology and the site limitations on train size have an important effect of train efficiency. For example, one locomotive hour can only haul half as much grain from the far receival point, compared to its neighbour, the distant subterminal. These differences in the transport efficiency account for large differences in the behaviour of price spreads and storage at the three case sites.

The peak transport premia for the close and distant subterminals and the distant elevator are plotted against the port price spread in the upper part of Figure 4. As the port spread falls, competition for rail capacity drives up the opportunity cost of a locomotive hour. The far receival point is inefficient in its use of rail capacity. The peak transport premium at the close subterminal is least affected by the rise in the cost of train hours because it is the most efficient (least intensive per unit of grain received) used of rail in the whole system.

As shown in Equation (7), the local spread at a particular site j is the sum of the port spread and the transport premium. The 45° line in the lower part of Figure 4 translates the port spread to the vertical axis, which allows it to be compared directly with the transport premium. The sum

---

[4] If the exercise is repeated using the harvest data for the five other years available to us, similar results are obtained. If results for all six harvests are plotted on the one graph, there is a wider scatter, but the "evidence" of convenience yield remains.

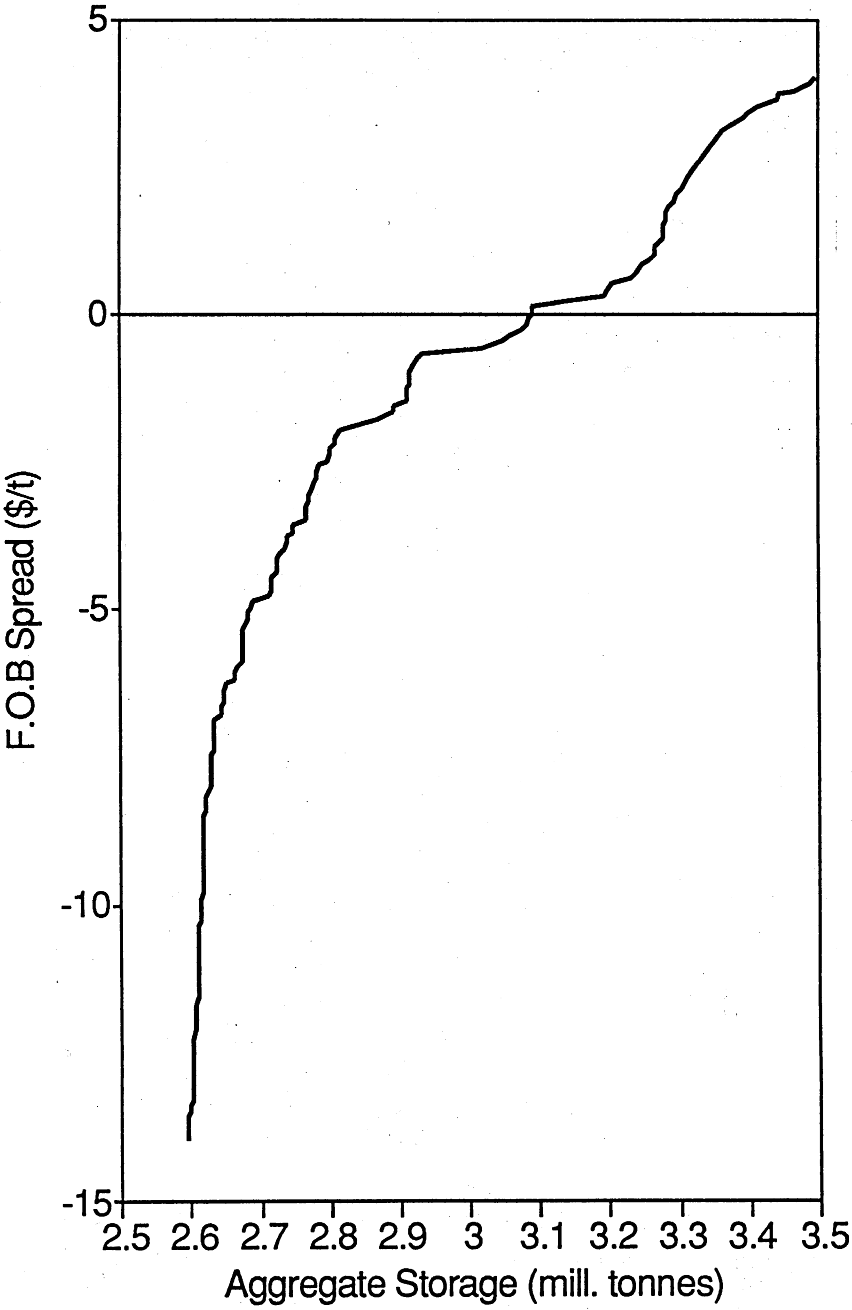

Figure 3: Aggregate "Supply of Storage"

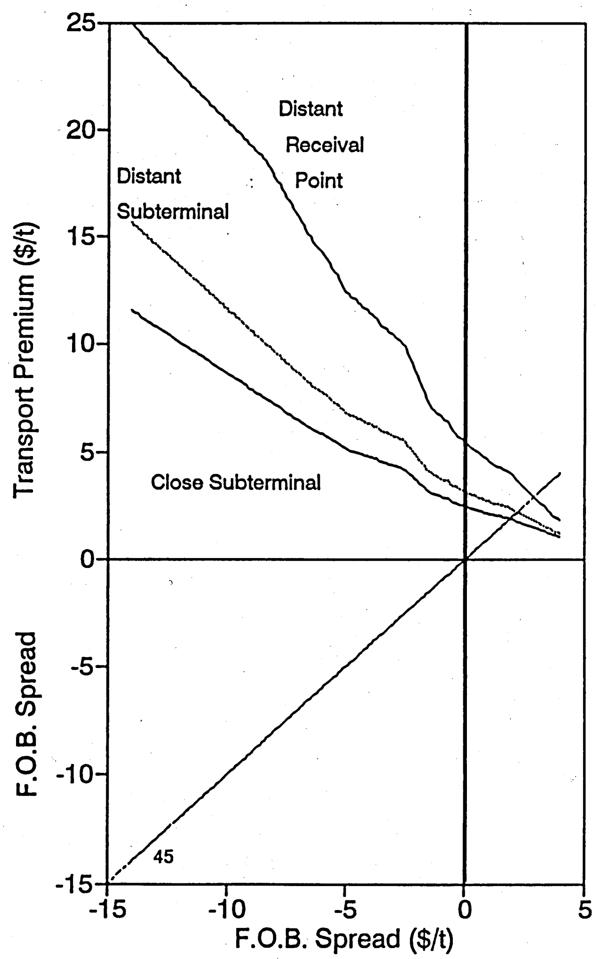

Figure 4: Transport Premiums

of the transport premium at the site and the port spread give the local spreads for each site, as plotted in Figure 5.

The local spread at the close subterminal moves in the same direction as the port spread. As revealed in Figure 4, as the port spread becomes increasingly negative, the costs of peak transport of grain to the port increase, but the fall in the port spread outweighs the rise in transport costs for the close subterminal, which uses the least transport services per tonne moved to the port. The local spread curve in Figure 5 for the far subterminal shown is flat on the left, then rises toward the right. At highly negative spreads, low-marginal-cost vertical storage at the far subterminal adjusts as a marginal source of supply to the port. When storage capacity of this type is not a binding constraint, the shadow price of transport and the amount transported adjust until the local spread equals the locally constant marginal storage cost. When the port spread is sufficiently higher, low cost storage capacity is filled, so the local spread is also higher to reflect the higher shadow price of local storage.

In contrast to our simple intuition about a spatial model, the local spread for the far receival point in Figure 5 moves in the opposite direction to the port price spread. This is explained by the underlying curves shown in Figure 4. As the port spread falls, the increase in the peak transport premium dominates the fall in the port spread because movement of wheat form the far receival point to the port is an intensive user of train time (locomotives and wagons). It becomes relatively more attractive to delay transportation from there, storing wheat until the off peak period, when the main line rail transport is relatively cheap. Transport services released by storing another tonne at the distant elevator can move several tonnes of grain from the close subterminal to the port in the peak period.

More complex relationships are evident for receival points that are not located on the standard gauge rail line. For these other sites, different transport resources are used (including combinations that change in response to spreads) so different opportunity costs are faced. For example, the transport premium for the branch line receival point (see Figure 2) is shown in Figure 6. The shadow price of narrow gauge train time is zero when the port spread is zero, whereas the elevator and the transport premia shown in Figure 4 for the standard gauge locations are still positive at zero port spread. Branch line sites use incompatible narrow gauge locomotives, and peak transport is relatively less elastic at the branch line receival points because of limits on loading rates that affect peak period train operating costs. Beyond a certain threshold of utilisation the transport premium rises sharply to the left. As a result the local price spread rises for the train-time-intensive locations as the port spread falls, as in the case of the far receival point on the standard gauge line.

The "Supply of Storage" and "Convenience Yield"

Suppose a competitive futures market existed with the close subterminal as delivery point. Then if supply were

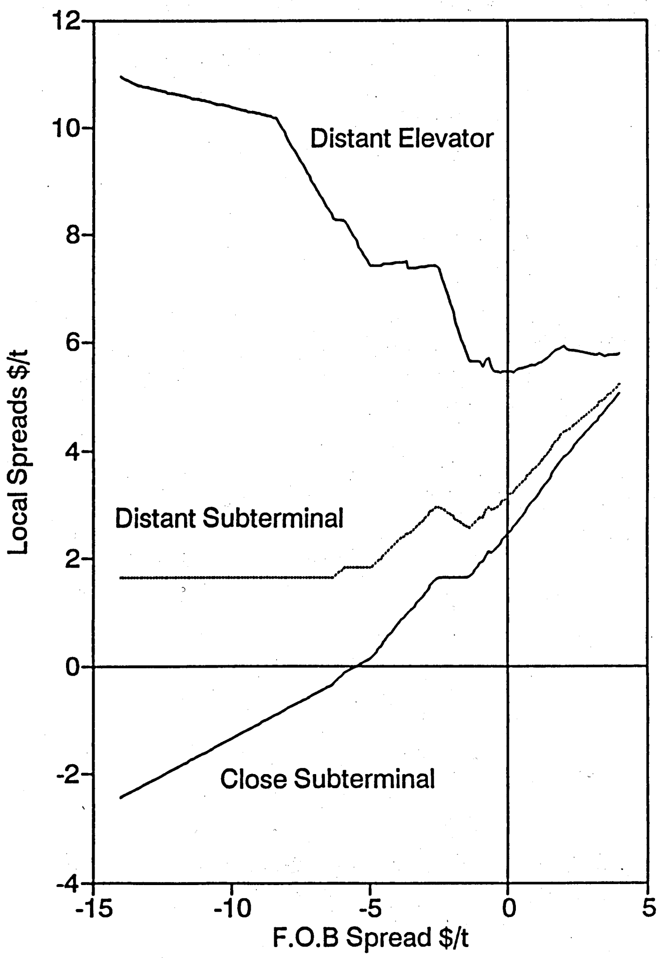

Figure 5: Local Spreads and Port Spread

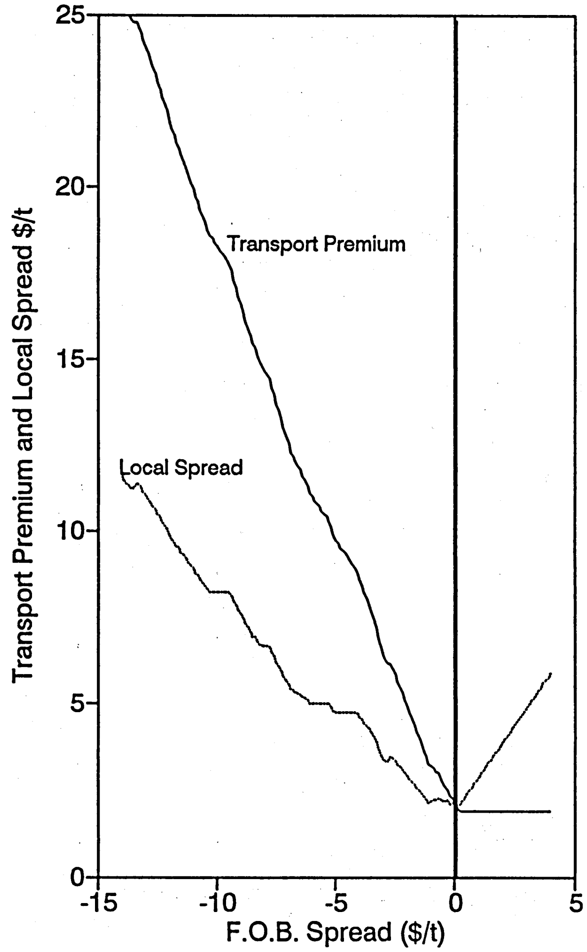

Figure 6: Premium and Spread at Branch Line R.P.

constant but demand fluctuated period to period, the relation between aggregate stocks (lower scale) and price spread traces out the "supply of storage curve" shown in Figure 7, with some above-minimum storage at negative spreads. Because of the strong monotonic relation between the close subterminal spread and the port spread, the curve is quite similar to that in Figure 3.

But no storage takes place at this location under local backwardation. This is obvious from inspection of the kinked, piecewise linear curve in Figure 7 that shows the response of *local* stocks to the *local* spread. In fact nowhere in the system are stocks ever held at negative local spreads. Any inference of "convenience yield" from "Supply of Storage" curves would have no microeconomic basis in this market.

Note however that the broader stock definition would show a higher correlation between stocks and spread, just as Working (1933) reported a stronger correlation using "national stocks" than "visible supply". As our model would predict, a plot of the latter more narrowly defined variable against the spreads in Figure 1 (available from the authors) shows less evidence of "convenience yield".

Fama and French's (1988) method of "proving" convenience yield was based on comparing the variability of price spreads at low and high levels of spread. According to Working's supply of storage relationship price spreads should be more variable when spreads are in backwardation, and more stable when spreads are at full carry. However, a model in which storage cost is constant for positive stocks, and storage is constrained to be nonnegative, has similar implications even though no storage is ever held during a backwardation. (See Williams and Wright (1991). While Fama and French's results reflect the fact that the correlation between spot and futures price depends on the current price level in a storage model, they do not prove that firms holding stocks ever receive non-market benefits.

The problems with using aggregate storage behaviour to formulate theories on microeconomic storage behaviour are further emphasized by storage behaviour at our chosen example of a branch line receival point (Coorow). The true storage behaviour at that location, storage against the local spread, is compared with storage behaviour against the port spread in Figure 8. The amount of grain stored at the site falls as the port spread falls. This seemingly perverse behaviour might well lead to inference of market manipulation. But it is actually explained by the fact that the local price spread moves in the opposite direction to the port spread, as a result of the sharply rising peak transport premium shown in Figure 6.

The conclusion of market manipulation arises from observation of the wrong variables. It can be seen in Figure 8 that storage behaviour at the branch line elevator is consistent with the local price spread, a competitive operator of the branch line receival point would offer a lower price for wheat (and store what he buys) as the current (peak) price

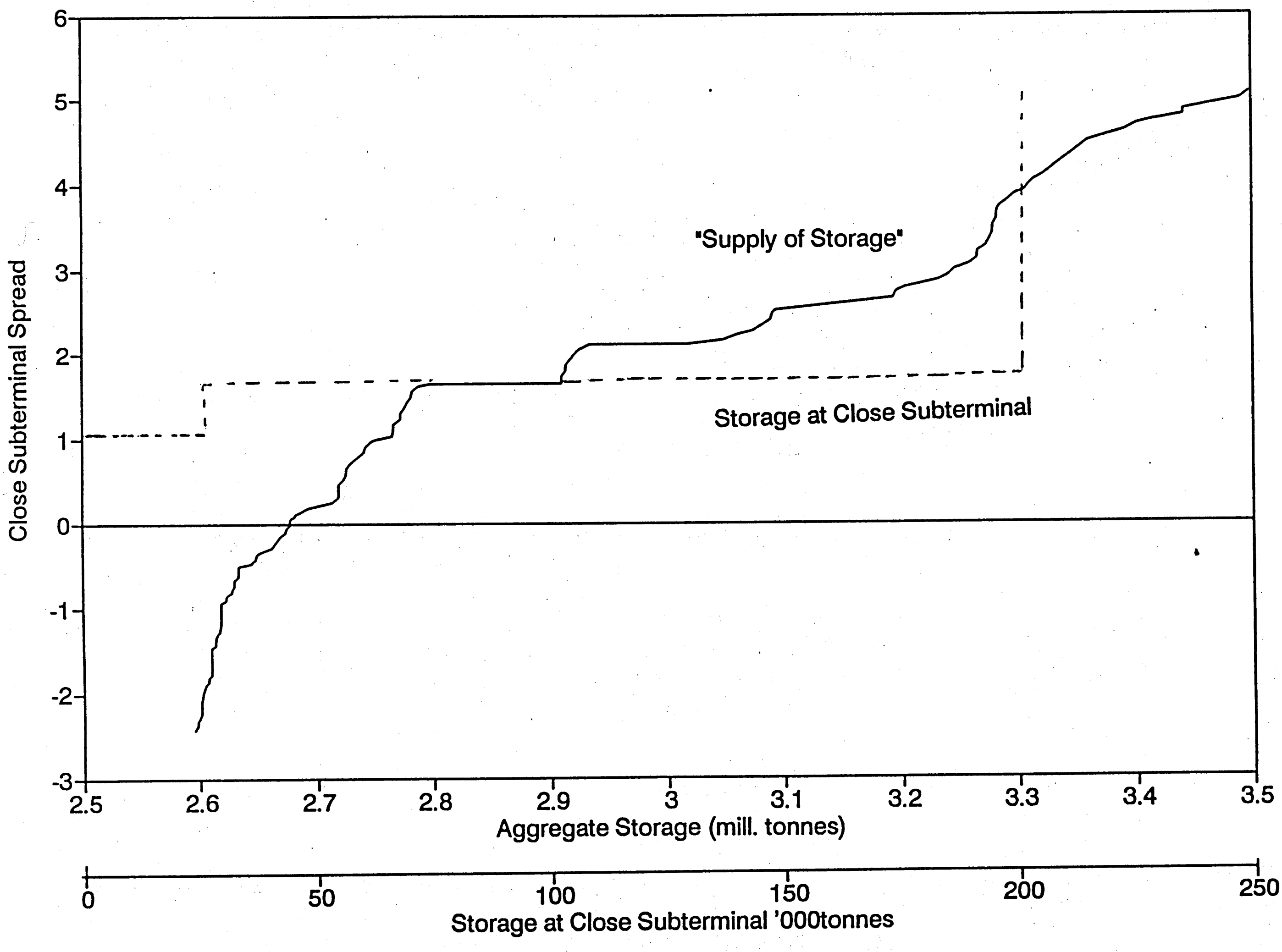

Figure 7: "Aggregate" and Micro Economic Storage Behavior

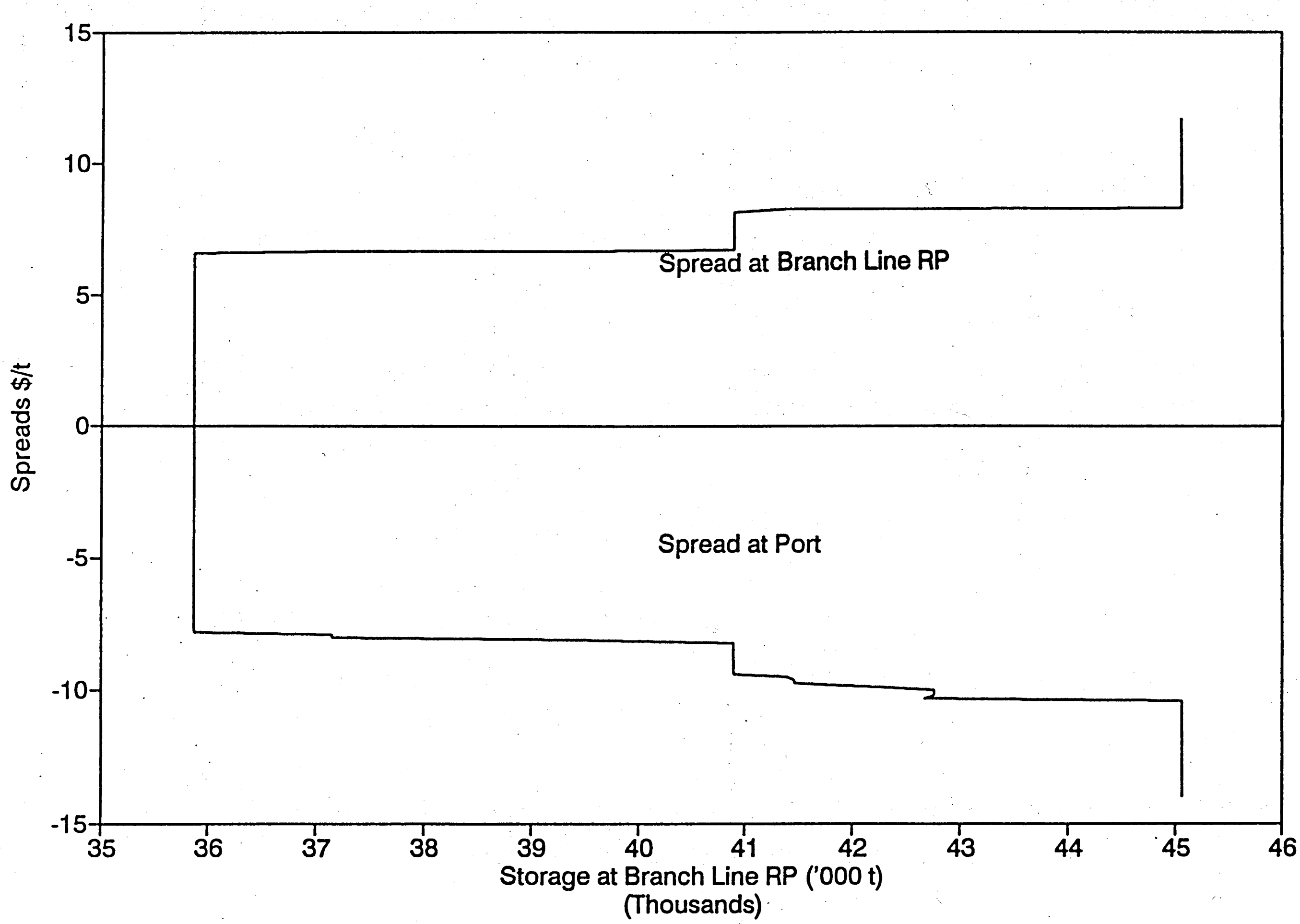

Figure 8: "Perverse" Storage Behaviour

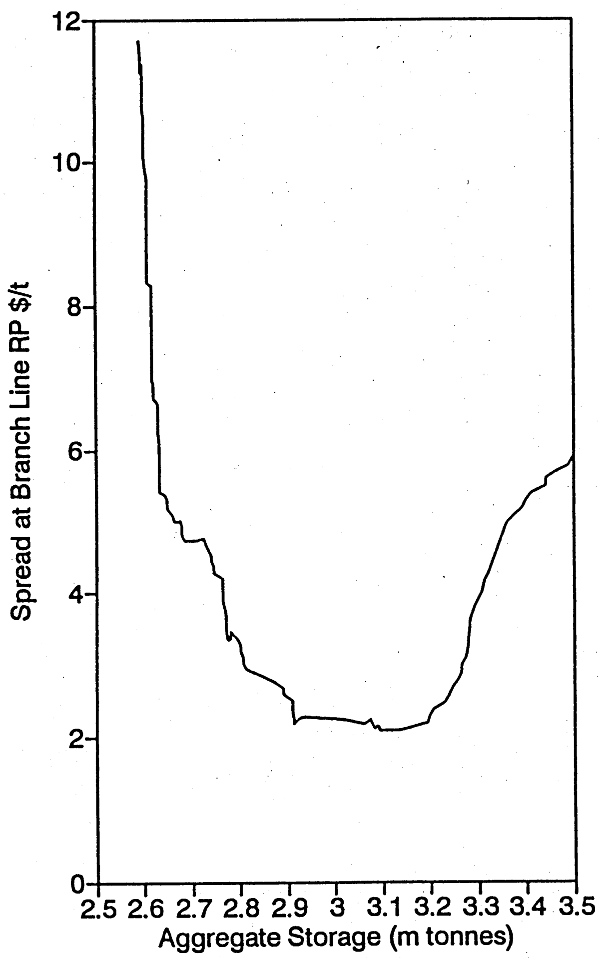

Figure 9: Another "Supply of Storage" Curve

at the port rises holding the off-peak price constant. A similar result was found for the far receival point on the standard gauge line, and for a large number of other sites that were relatively inefficient users of transport resources. The price relationship shown in Figure 8 is consistent with global cost minimizing behaviour in the presence of bottlenecks. In the absence of accurate peak load pricing in agricultural transport markets, such behaviour is likely to be misinterpreted as "non-competitive hoarding" in the face of strong pressure for current delivery.

In Figure 9, a "supply of storage" curve is plotted using the price spread at the branch line receival point. This U-shaped curve would result in inferences about the firm's storage incentives markedly different from the currently popular conception of convenience yield.

Conclusions and Caveats

In the Western Australian wheat market studied here, optimal peak-period storage responds to market price spreads in the manner familiar from supply of storage curves estimated for many other markets. Yet there is no convenience yield at any location in the market model we use. Stocks are held at a given location if and only if the net costs of marketing now (including transport costs) exceed the cost of storage and marketing in the off-peak period.

The widespread impression that adoption of the convenience yield hypothesis is necessary to explain the observed characteristics of supply of storage curves in commodity markets is incorrect.

In this unusually simple market the peak load transportation problem was very pronounced because the wheat crop is the main user of limited transportation resources in the region studied. The bulky nature of commodities, the geographical separation of production regions and markets, and the role of fixed capital in transport and storage imply that the effect of spatial and temporal interactions may have an important influence on commodity price spreads in other markets also.

Further investigation might show convenience yield to be a significant feature of returns to commodity storage in some wheat markets or markets for other commodities, but this must now be recognised as an open question rather than an established fact.

The spatial-temporal model presented here casts a new perspective on efficient allocative response in a commodity market. Even in the very simple market considered here, the range of efficient spatial responses to changes in the terminal spread is much more rich and diverse than in a static location model. Effects on storage and transportation may even differ in sign at different locations, calling into question intuitive notions of *per se* evidence on noncompetitive market behaviour during times of unusual pressure for early delivery.

REFERENCES

Brennan, D.C. (1991) *Peak Load Problems in the Australian Grain Distribution Industry: An Economic Analysis*. Ph. D. Thesis, University of Western Australia

Brennan, M.J. (1958) "The Supply of Storage" *American Economic Review* XLVIII p50-72

Chicago Board of Trade (1977) *Selected Writings of Holbrook Working* Chicago: Board of Trade of the City of Chicago.

Fama, E.F. and French, K.R. (1987) "Commodity Futures Prices: Some Evidence on Forecast Power, Premiums and the Theory of Storage" *Journal of Business* 60(1) p55-73.

Fama, E.F. and French, K.R. (1988) "Business Cycles and the Behaviour of Metals Prices" *Journal of Finance* XLIII 5 p1075-1093

Gardner, B.L. (1975) *Optimal Stockpiling of Grain*. Lexington, Mass.: Lexington Books.

Gibson, R. and Schwartz, E.S. (1990) "Stochastic Convenience Yield and the Pricing of Oil Contingent Claims" *Journal of Finance* XLV.3 p959-976

Gray, R.W. and Peck, A.E. (1981) "The Chicago Wheat Futures Market: Recent Problems in Historical Perspective" *Food Research Institute Studies*. Stanford, California; Stanford University 18:89-115.

Gustafson, R.L. (1958a) *Carryover Levels for Grains: A Method for Determining Amounts that are Optimal under Specified Conditions*. Technical Bulletin. 1178. Washington D.C. United States Department of Agriculture.

Gustafson, R.L. (1958b) "Implications of Recent Research on Optimal Storage Rules" *Journal of Farm Economics* 40 p290-300.

Houthakker, H.S. (1987) "Futures Trading" *The New Palgrave: A Dictionary of Economics*. Ed. J. Eatwell, Milgate, M. and Newman, P. London, England: McMillan, 447-49.

Samuelson, P.A. (1971) "Stochastic Speculative Price" *Proceedings of the National Academy of Sciences*. p335-7

Telser, L.G. (1958) "Futures Trading and the Storage of Cotton and Wheat" *Journal of Political Economy* LXVI.3 p105-113.

Williams, J.C. and Wright, B.D. *Storage and Commodity Markets*. Cambridge, England: Cambridge University Press.

Working, H. (1933) "Price Relations Between July and September Wheat Futures at Chicago since 1885" *Wheat Studies*. Stanford, California: Stanford University, Food Research Institute, IX,6:187-237

Working, H. (1948) "Theory of the Inverse Carrying Charge in Futures Markets". *Journal of Farm Economics* 30:1-28.

Working, H. (1949) "The Theory of the Price of Storage" *American Economic Review* December. 1254-1262

Wright, B.D. and Williams, J.C. (1989) "A Theory of Negative Prices for Storage." *Journal of Futures Markets* 9(1):1-13