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HEDGING EFFECTIVENESS OF U.S. WHEAT FUTURES MARKETS

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FOREWARD

Futures markets as a tool for risk management have become increasingly important in recent years. The effectiveness of hedging in the U.S. wheat futures market is evaluated in this study. Helpful suggestions from W. Koo and C. Carter of North Dakota State University and University of Manitoba, respectively, are greatly appreciated. This research was conducted under Regional Project NC-160, Performance of the U.S. Grain Marketing System in a Changing Policy and Economic Environment.

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Highlights

Five classes of wheat, each with different quality characteristics are produced in the United States and hedging possibilities exist for each at the three futures exchanges which trade wheat contracts. Even though a particular class of wheat may not be deliverable against a futures contract, cross hedging is possible. In addition, it is possible to spread hedges across more than one futures contract. The purpose of this study is to evaluate the effectiveness of hedging wheat at major U.S. cash markets against the three U.S. wheat futures markets. Portfolio analysis is used and optimal hedge ratios and measures of hedging effectiveness are calculated for different qualities of cash wheat. The effect of spreading wheat contracts across more than one futures contract also is analyzed.

The results of the analysis for hedges placed in a single futures contract indicate that hedging in the inherent wheat futures contract generally offers greater risk protection than cross-hedging. The latter, however, becomes relatively more viable in longer term hedges. The optimal hedge ratios were generally less than one, indicating that hedged positions which minimize risk should be less than 100 percent of the cash position. In other words, as opposed to having equal and opposite positions as in traditional hedging, cash positions should be only partially hedged if the objective is to minimize risk.

Portfolio analysis also was used to determine the optimum hedge ratios for hedges spread across two or three wheat futures markets. The largest hedge ratio for each class of cash wheat was generally in its inherent futures market with smaller positions cross hedged in one or both of the other markets. In most cases, the additional risk reduction associated with spreading was 1 to 2 percent.

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Introduction

It is commonly recognized that two important roles of futures markets are to establish forward prices and to guide the temporal allocation of inventories. Much of the recent research in futures has evaluated their "efficiency" in establishing forward prices. Studies by Tomek and Gray, Kofi, Leuthold, Leuthold and Hartman, Martin and Garcia, Just and Rausser, Carter (1980), and others have evaluated the market performance of futures. Market performance as addressed in those studies implies forecasting ability. More recently Peck (1981) has questioned the appropriateness of these tests as measures of the performance of futures markets based on the logical interrelationships between cash and futures prices and the supply and demand for storage.

Futures markets also facilitate risk shifting via the hedging mechanism. Indeed, the hedging effectiveness of various futures markets may be a more appropriate question from a pragmatic commercial perspective. Hedgers are not concerned about market performance in the "forecasting ability of futures" sense, but rather in the effectiveness of using futures to manage risk. Total price risk is reduced by hedging. In particular, flat price risk is eliminated and basis risk is incurred. With increased commodity price volatility in the past decade, greater interest is placed on the futures markets as a tool in risk management. Particularly important is the hedging performance, or the effectiveness of particular hedges to reduce the variance in prices.

Hedging performance and associated questions on optimal hedging are especially important in the case of wheat. Five classes of wheat, each with different quality characteristics, are produced in the United States. These can be hedged on three futures exchanges which trade wheat contracts. In addition, different types of wheat grown around the world which have the potential to use U.S. futures markets as mechanisms for reducing risk.[1] Even though a particular type of wheat may not be a deliverable grade against a futures contract, cross-hedging is possible. Examples include different qualities of cash wheat hedged against the various futures contracts or hedging of qualities of wheat which are not deliverable at any of the futures markets. Durum and white wheat are examples of the latter.[2]

In light of the multitude of hedging alternatives, marketing firms, producers, professors, and extension workers are constantly faced with many questions regarding hedging wheat. Particularly important are which market to hedge in and how much should be hedged. The answers normally given to these questions are that hedges should be placed in the futures market which has delivery for the type of wheat being hedged, and the futures position should be equal and opposite to the cash position. A related question particularly important in the case of wheat is the economics of spreading hedges across more than one wheat futures contract.[3]

Many of these questions can be answered empirically. Portfolio analysis provides the theoretical and empirical base for evaluating the hedging effectiveness of futures. The concept of risk was integrated into traditional hedging theory by Johnson and Stein to explain observed behavior of those holding both hedged and unhedged positions. Ederington later-evaluated the hedging performance of GNMA and T-bill futures. Two and four-week hedges were evaluated in various contract months and several hypotheses were tested. Hill and Schneeweis (1982) recently evaluated the hedging potential of foreign currency futures markets. Comparisons were also made between hedging in futures versus forward markets. In each case, minimum risk hedge ratios and measures of hedging effectiveness were compared. The above studies assumed price uncertainty with quantity fixed. Rolfo went a step further and evaluated optimal hedging under price and quantity uncertainty for cocoa. A similar study was conducted by Carter (1981) for Canadian wheat.

The purpose of this study is to evaluate the effectiveness of hedging wheat at major U.S. cash markets against the three U.S. wheat futures markets. Specifically, optimal hedge ratios and measures of hedging effectiveness are calculated for different qualities of cash wheat at the three futures markets. The effectiveness of spreading hedges across the futures markets is also evaluated.

Theoretical Model

To put this study into perspective, three theories of hedging can be distinquished (Ederington). The first theory is that of traditional hedging, in which case cash positions are hedged by taking equal and opposite positions in the futures market. If the change in basis is zero, the hedge is perfect. It is normally assumed that the variance, a measure of risk, for basis changes is less than the variance of flat price changes. Consequently, risk is reduced by hedging versus not hedging. The major theme of traditional hedging theory is that the hedge ratio, the proportion of the cash position hedged, is one. Risk can be analyzed by comparing the variance of changes in the basis against the variance of changes in cash prices. The second theory of hedging, referred to as Working's Hypothesis, questions the motivation of pure risk minimization of hedgers. Working suggested that hedgers function much like speculators, but their concern is primarily in relative prices rather than absolute prices. In other words, hedgers speculate in changes in basis rather than changes in price levels.

The third theory of hedging is due primarily to Johnson and Stein in two independent papers. They explain theoretically why some hedgers may not be fully hedged. Risk of price changes is introduced into the hedging model in a variance function and a frontier is traced showing a relationship between variance (risk) and expected returns. Hedgers select the proportion of their cash position which is hedged according to their indifference curve between risk and expected returns. It is theoretically possible for the optimal hedge ratio to be equal to, less than, or greater than one. Spot and futures prices are treated as separate assets in a portfolio. The size of the position in the spot market can be viewed as fixed and the hedger's decision is what proportion of the spot position should be hedged.

Two theoretical models are developed below. The first is the two market, or two asset model, in which there is one spot and one futures market in the portfolio. The second includes one spot market and more than one futures market. The latter is applicable to spreading spot positions across two or three wheat futures markets.

Two Market Case

Two important equations are presented throughout the theoretical model developed below. One is an equation of expected revenue from holding a commodity which in the simple cash market case is:

$$E(R) = X_i E(P_{i_{t+n}} - P_{i_t}) \qquad (1)$$

where E(R) is expected revenue, X_i is quantity of the commodity held in the cash position, and $P_{i_{t+n}}$ and P_{i_t} are expected cash prices for commodity i in periods t+n and t respectively. The equation essentially says the returns from storing the cash commodity from time period t to t+n is equal to the expected change in prices between the two periods. In this case, X_i is positive indicating a long cash position.

The other equation indicates the variance of return which can be used as an indicator of price risk. More specifically, the variance of price risk is the variance of a subjective probability distribution of price changes from t to t+n which is held by the trader at t when the actual price change from t to t+n is random. Variance of a price change in the ith market, σ_i^2, indicates the risk of holding one unit in that market.[4] The price risk of holding X_i units in the ith market is:

$$V(R) = X_i^2 \sigma_i^2 \qquad (2)$$

The two market case expands on the equation above including separate variables for the cash market and futures market. The price risk of holding a one unit position in market j (i.e., futures market) is σ_j^2 and the variance is:

$$V(R)_j = X_j^2 \sigma_j^2 \qquad (3)$$

where X_j indicates the quantity or size of the position in the jth market. The covariance of price changes between market i and market j, σ_{ij}, indicates the extent price changes vary together. Then the E(R) and V(R) for a combination of positions held in the two markets are:[5]

$$E(R) = X_i\ E(P_{i_{t+n}} - P_{i_t}) + X_j\ E(P_{j_{t+n}} - P_{j_t}) \qquad (4)$$

$$V(R) = X_i^2 \sigma_i^2 + X_j^2 \sigma_j^2 + 2X_iX_j\ \sigma_{ij} \qquad (5)$$

Recalling that X_i and X_j indicate the size of the positions held in market i and market j respectively, (4) and (5) represent a typical hedging situation. In a long cash/short futures position, $X_i > 0$ and $X_j < 0$. Thus, (4) indicates the expected return from the short hedge and (5) indicates the variance in the return. If $X_i = 1$, then X_j can be interpreted as the proportion of the cash position which is hedged in the jth market, i.e., futures market, and would normally be negative indicating a short futures position.

In "traditional theory" and as conventionally taught, hedges should be equal and opposite to the cash position. Thus $X_i = -X_j$, or if $X_i = 1$, $X_j = -1$.

However, as developed below an optimal hedge may be either completely or partially hedged. To find the optimal hedge ratio, (5) is minimized with respect to X_j. The derivation is as follows:[6]

$$\frac{\partial V(R)}{\partial X_j} = 2\, x_j\, \sigma_j^2 + 2\, X_i\, \sigma_{ij} \qquad (6)$$

which when set equal to zero and solved for X_j yields:

$$X_j^* = \frac{-X_i\, \sigma_{ij}}{\sigma_j^2} \qquad (7)$$

If $X_i = 1$, then

$$X_j^* = -\frac{\sigma_{ij}}{\sigma_j^2} \qquad (8)$$

which is the optimal hedge ratio. The second order conditions are satisfied so long as $2\, \sigma_i^2 > 0$. X_j^* as calculated in (8) indicates the proportion of the cash position which should be hedged in order to minimize risk. Thus, it may be termed the minimum risk hedge ratio and may or may not equal 1. If $\sigma_{ij} < \sigma_j^2$ then $X_j^* \leq 1$ indicating the optimal hedged position should be less than the cash position. So long as $\sigma_{ij} > 0$, X_j^* will be opposite in sign from X_i.

The optimally hedged position in (8) is the hedged proportion of the cash position which yields the least risk. This may be equal to, less than, or greater than the absolute value of one. Actual positions of traders differ from this minimum risk hedge ratio to the extent they may be willing to accept some risk for the possibility of increased revenue. The actual hedge ratio chosen depends on the traders' indifference between risk and return.

The variance of return in the optimal portfolio can be derived by substituting (8) into (5) as follows:[7]

$$V(R)^* = \sigma_i^2 - \frac{\sigma_{ij}^2}{\sigma_j^2}$$

and since $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$, the variance for the optimal hedge is:

$$V(R)^* = \sigma_i^2 \, (1 - \rho_{ij}^2) \qquad (9)$$

where ρ_{ij} is the correlation coefficient of price changes in the two markets. $V(R)^*$ is the variance of return from a long cash position ($X_i = 1$) when the hedged proportion is given by (8). Several cases can be illustrated. In an unhedged position $X_j = 0$, then $V_1(R) = \sigma_i^2$, which is greater than $V(R)^*$ so long as $\rho_{ij} = 0$. In the traditional hedge, cash and futures positions are equal and opposite i.e., $X_i = 1$ and $X_j = -1$, and the variance is:

$$V_2(R) = \sigma_i^2 + \sigma_j^2 - 2\,\sigma_{ij} \qquad (10)$$

which may or may not be the same as $V(R)^*$, the variance of the optimal hedged position. The level of the variances of these alternative hedged positions is an empirical question largely depending on ρ_{ij}. If $\rho_{ij} = \pm 1$, the $V(R)^*$ $= 0$ and if $\rho_{ij} = 0$, then $V(R)^*$ equals that of the unhedged position.

A measure of the effectiveness of hedging can be derived from the relationships developed above. The variance of return in an unhedged position is given by $V_1(R) = \sigma_i^2$ and that in an optimal hedged position is $V(R)^* = \sigma_i^2$ $(1 - \rho_{ij}^2)$. The effectiveness of the hedge is the extent to which risk is reduced in the optimal hedge case, relative to an unhedged position. An empirical measure of the effectiveness of a hedge is as follows:

$$E = 1 - \frac{V(R)^*}{V_1(R)} \qquad (11)$$

which in the two market case can be reduced to:

$$E = \rho_{ij}^2$$

where ρ_{ij} is the correlation coefficient between price changes in the two markets. E is a measure of hedging effectiveness and can be interpreted as the average proportional decrease in spot price risk that could be realized by hedging at X_j^*. A large value of E indicates a more effective hedge in terms of risk reduction. As E approaches 0, less risk reduction is obtained from the hedge. Note that E as defined in (11) is for the optimally hedged position, X_j*, and differs if the hedged position is different than X_j*.

A further elaboration on the above concepts is to incorporate the expected change in basis, E(ΔB), in the expected return function. Frequently, in hedging analysis the expected gain or loss is equal to the expected change in basis. This does not affect the variance equation but it does affect the expected return function. Let the expected change in basis E(B) be defined as:

$$E(\Delta B) = E[(P_{j_{t+n}} - P_{i_{t+n}}) - (P_{j_t} - P_{i_t})] \qquad (13)$$

where the j and i subscripts indicate the futures and spot market respectively and prices in the two markets are expected in time period t+n and t. The expected revenue function in (4) can be rewritten as:

$$E(R) = X_i\ [(1 - \delta)\ E(P_{i_{t+n}} - P_{i_t}) + \delta\ E(P_{i_{t+n}} - P_{i_t}) - \delta\ E(P_{j_{t+n}} - P_{j_t})]$$

where δ is the proportion of the cash position hedged. Letting E(S) = $E(P_{i_{t+n}} - P_{i_t})$ which is the expected change in the spot price, E(R) can be rewritten as:

$$E(R) = X_i\ [(1 - \delta)\ E(S) - \delta\ E(\Delta B)] \qquad (14)$$

The expected return is the difference between the unhedged portion times the expected change in the spot price, (1 - δ) E(S), and the hedged portion times the expected changes in basis, δ E(ΔB). If δ = 1 then the expected return reduces to -E(ΔB).

Multi-Market Case

In most commodities, there is one market for hedging and possibilities may or may not exist for cross-hedging. In wheat, there are three futures in which hedges can be placed. A particular quality of wheat can be hedged in any of the three individually or spread with part of the hedge placed in each. In the former case the effectiveness of the hedge can be evaluated using the framework outlined above. To evaluate the effectiveness of spreading hedges across more than one market the model is expanded. In particular, four markets are used in the portfolio. The expected return and variance equations are as follows:[8]

$$E(R) = X_1\, E(P_{1_{t+n}} - P_{1_t}) + X_2\, E(P_{2_{t+n}} - P_{2_t}) + X_3\, E(P_{3_{t+n}} - P_{3_t}) + X_4\, E(P_{4_{t+n}} - P_{4_t}) \qquad (15)$$

$$V(R) = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + X_3^2 \sigma_3^2 + X_4^2 \sigma_4^2 + 2X_1 X_2 \text{Cov}_{12} + 2X_1X_3 \text{Cov}_{13} + 2X_1X_4 \text{Cov}_{14} + 2X_2X_3 \text{Cov}_{23} + 2X_2X_4 \text{Cov}_{24} + 2X_3X_4 \text{Cov}_{34} \qquad (16)$$

where the X_i's represent the quantity or size of the position held in the cash market, X_1, and each of the futures markets X_2, X_3, and X_4. X_1 is the size of the cash position and can for simplicity be set equal to one. Then, X_2, X_3, and X_4 indicate the size of the position in each of the futures markets. In the case of analyzing other applications of portfolio analysis (i.e., stocks) the sum of positions in the other assets are constrained to equal one. However, this constraint is unnecessary in the case of grain merchandising.

To determine the optimal hedge in each of the futures markets, (16) is minimized and solved for the position in each of the futures markets. The partial derivatives are:

$$\frac{\partial V(R)}{\partial X_2} = 2\, X_2\, \sigma_2^2 + 2\, X_1\, \text{Cov}_{12} + 2\, X_1\, X_3\, \text{Cov}_{23} + 2\, X_4\, \text{Cov}_{24} = 0$$

$$\frac{\partial V(R)}{\partial X_3} = 2\, X_3\, \sigma_3^2 + 2\, X_1\, \text{Cov}_{13} + 2\, X_2\, \text{Cov}_{23} + 2\, X_4\, \text{Cov}_{34} = 0$$

$$\frac{\partial V(R)}{\partial X_4} = 2\, X_4\, \sigma_4^2 + 2\, X_1\, \text{Cov}_{14} + 2\, X_2\, \text{Cov}_{24} + 2\, X_3\, \text{Cov}_{34} = 0$$

Setting $X_1 = 1$ and solving for each futures position gives a system of three equations in three unknowns and can be stated in matrix form as follows:

$$\begin{bmatrix} \sigma_2^2 & Cov_{23} & Cov_{24} \\ Cov_{23} & \sigma_3^2 & Cov_{34} \\ Cov_{24} & Cov_{34} & \sigma_4^2 \end{bmatrix} \begin{bmatrix} X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -Cov_{12} \\ -Cov_{13} \\ -Cov_{14} \end{bmatrix} \quad (17)$$

and solved for X_2, X_3, and X_4. Analysis of wheat spreads across two futures markets is similar except there are two equations and two unknowns. The second order conditions are satisfied as long as $\sigma_i \geq 0$.

The solution to the equation system yields values for X_2^*, X_3^*, and X_4^*. These can be interpreted as the size of the positions held in each of the respective futures markets which minimizes the variance, or risk. If $Cov_{14} = Cov_{24} = Cov_{34} = 0$, the problem reduces to the three market problem--one cash and two futures. If, in addition, $Cov_{13} = Cov_{23} = 0$, the results reduce to the two market problem as developed in the previous section. Thus, the size of the position held in each of the three markets in a spreading situation can be answered empirically.

The variance of returns when the optimal futures positions are taken can be calculated by substituting X_2*, X_3*, and X_4* from (17) into (16) to derive V(R)*. The effectiveness of the hedge then is defined as:

$$E = 1 - \frac{V(R)^*}{V(R)}$$

where V(R)* is the minimum variance and V(R) is the variance in an unhedged position. E retains the same interpretation as in the two market case.

Empirical Procedures

Dimensions of Analysis and Data Sources

The three wheat futures markets in the United States each have different deliverable grades. The grades which are deliverable against futures at the Chicago Board of Trade include No. 2 Soft Red Winter, No. 2 Hard Red Winter, No. 2 Dark Northern Spring, and No. 1 Northern Spring. At the Kansas City

Board of Trade the deliverable grade is No. 2 Hard Red Winter, and that at the Minneapolis Grain Exchange is No. 2 Northern Spring with 13½% protein or higher. In addition, each exchange has established premiums and discounts for grades which deviate from that which is deliverable. So long as hedgers do not have intentions of making or accepting delivery of a contract, any type of cash wheat can be hedged at any of the three futures markets. In this analysis the effectiveness of the three futures markets is analyzed for hedging three classes of wheat with different protein levels. These are listed in Table 1.

TABLE 1. QUALITIES OF CASH WHEAT AND MARKETS USED IN ANALYSIS[a]

Delivery Market	Minneapolis	Kansas City	Chicago
Class of Wheat and Protein Level	HRS 13% HRS 14% HRS 15% HRS 17%	HRW Ordinary HRW 12% HRW 13%	SRW

[a]HRS, HRW, and SRW stand for Hard Red Spring, Hard Red Winter, and Soft Red Winter respectively.

The analysis included four qualities of wheat at Minneapolis, three at Kansas City and one at Chicago. The qualities of wheat which predominate the trade at each of the futures markets were chosen. Each quality of wheat in Table 1 can be hedged against wheat futures at any of the three exchanges. For example, even though HRS 13% and HRS 14% are the inherent qualities of wheat traded in the spot market at Minneapolis, it is possible to hedge these positions against futures at the Minneapolis Grain Exchange (MGE), Kansas City Board of Trade (KCBT), or the Chicago Board of Trade (CBT). The empirical analysis indicates which futures market gives the most protection.

It is frequently stated that hedging of higher protein spring wheats is problematic because they do not correspond with the deliverable grade.[9]

The analysis tests the extent that HRS 17% can be hedged at each of the markets. In addition, the analysis examines the applicability of spreading wheat hedges across the three futures. For example, a long position in HRS 15% at Minneapolis could be offset by partial futures positions at the MGE, KCBT, and CBT.

Optimal hedge ratios and measures of hedging effectiveness were derived for hedges of different length and for hedges placed in different futures contracts. Hedges of three different lengths were analyzed. These included 4-week, 13-week, and 26-week hedges. A 4-week hedge, for example, is one placed and lifted four weeks later. Hedges can be placed in near or distant contracts. To allow comparison, hedges in nearby contracts, in contracts two to six months forward, and in contracts six to ten months forward were analyzed separately. For example, a 4-week hedge in a nearby contract means calculation of the variance/covariance matrix of price changes over a 4-week period for both cash and nearby futures prices. Cash and futures prices in the delivery option month were not included because of the abnormal hedging risk.

Several hypotheses about the behavior of E, the measure of hedging effectiveness, were posed. Nearby futures normally are affected more by unexpected changes in cash prices than are the more distant futures. The first hypothesis is that E declines for hedges placed in more distant futures. The second hypothesis, following Ederington, is that E will be greater for longer term hedges than shorter term hedges. The logic of the second hypothesis is that futures prices in longer term hedges would have more time to respond to potentially greater changes in cash prices.

Weekly data were used in the analysis covering the period from the first week of 1977 to the last week of 1981. The prices on Wednesday of each week were collected for each respective spot commodity and futures contract. All of the data were from annual reports of the exchanges.

Calculation of Hedging Parameters

The analysis is ex post since historical data were used to calculate the hedging parameters. Weekly data lags of 4, 13, and 26 weeks were created for each of the eight spot prices and prices for the three futures contracts. For each spot price a variance/covariance matrix between itself and each of the future prices was derived. This was done for each duration of hedge and contract used (e.g., nearby). The results were used to calculate optimal hedge ratios (X_i*), the variance associated with the optimal hedge (V(R)*) and the measure of hedging effectiveness (E) using the formulas developed in the previous section.

Hill and Schneeweis (1982) and Ederington, as well as others, have shown that the parameters estimated from ordinary least squares regression of spot price changes on futures price changes are equivalent to the results from the minimization problem developed above. In particular, the slope coefficient and the coefficient of determinization are equivalent to the optimal hedge ratio and measure of hedging effectiveness, respectively. Results from these procedures were used to statistically test various hypotheses about the calculated hedge ratios and measures of hedging effectiveness.

Empirical Results

Hedging effectiveness for various spot commodities is essentially determined by the extent to which the spot price and the price of the different futures contracts move together. Prior to presenting the empirical results, a brief description of the historical data used in this study is provided. The relationship between cash and futures prices is referred to as the basis and is defined, for purposes here, as the spot price minus the futures price.

Simple means and standard deviations for each possible basis were calculated over the sample period and are presented in Table 2. Examination indicates that for each quality of wheat the standard deviation is smallest for the basis calculated relative to its inherent futures market.

TABLE 2. MEANS AND STANDARD DEVIATION OF EACH BASIS RELATIVE TO THE NEARBY WHEAT CONTRACTS AT THE THREE FUTURES MARKETS, 1977-1981 (N=255)[a]

	Futures Market					
	MGE		KCBT		CBT	
Spot Market	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
	¢/bushel					
Minneapolis						
HRS 13%	8	12	9	19	2	24
HRS 14%	12	12	14	21	6	26
HRS 15%	27	15	28	22	21	28
HRS 17%	44	19	45	25	38	30
Kansas City						
HRW Ordinary	6	20	8	14	0.6	20
HRW 12%	4	21	5	14	-2	20
HRW 13%	10	20	11	15	4	21
Chicago						
SRW	-7	34	-6	31	-13	21

[a]Basis is defined as spot price minus the relevant nearby futures.

Thus, there is less basis risk if hedged at the inherent market than at one of the other markets. One further observation in the case of the Minneapolis spot market is that the basis for HRS 17% is larger, on average, and also has a greater standard deviation. The coefficient of variation is a measure of relative variability and is 1.5, 1.0, .55, and .43 for HRS 13%, 14%, 15%, and 17% basis relative to the MGE futures. The basis for the lower protein wheats, therefore, have greater variability relative to their means compared to that of the higher protein wheats. Variances and covariances were calculated for all combinations of price changes. These were used to derive values for the optimal hedge ratios and measures of hedging effectiveness and are shown in the Appendix Tables A1-A4.

Hedging in the Two Market Case

The results presented in this section assume hedges for the various qualities of wheat are placed in one of the three wheat futures markets. For each cash wheat, optimal hedge ratios and measures of hedging effectiveness were calculated at the three wheat futures markets. Variances and covariances of price changes were derived for hedges held for 4 weeks, 13 weeks, and 26 weeks. In addition, hedges were evaluated when placed in nearby contracts, in contracts two to six months forward and in contracts six to ten months forward. The results are reported in Tables 3, 4, and 5.[10]

In each case the optimal hedge ratios and the measures of hedging effectiveness were calculated using equations (8) and (11) and the variances and covariances in the Appendix. The optimal hedge ratios, X_2^*, X_3^*, and X_4^*, represent those calculated for a particular type cash wheat which is hedged against the wheat futures contracts at Minneapolis (MGE), Kansas City (KCBT), and at Chicago (CBT), respectively. They indicate the proportion of a cash position which should be hedged in order to minimize risk. In deriviation of the formulas, a long cash position equal to one was assumed. Thus, a negative hedge ratio indicates a short futures position should be used to offset the long cash position. A -.94, for example, indicates that 94 percent of a long spot position should be hedged with short futures in order to minimize risk.[11] E is the measure of hedging effectiveness and can be interpreted as the percentage decrease in spot price variability because of the hedged position, relative to an unhedged position. Greater values of E indicate greater risk protection from a particular hedge relative to another with a lower value. E was calculated assuming the value of the optimal hedge ratio.[12]

In Table 3 the effectiveness of hedges held for four weeks placed in nearby contracts, contracts two to six months forward, and contracts six

TABLE 3. FOUR-WEEK HEDGES IN THREE WHEAT FUTURES MARKETS INDIVIDUALLY

Cash Market and Type of Wheat	Hedging Market: Minneapolis X_2*	E	Kansas City X_3*	E	Chicago X_4*	E
Minneapolis						
HRS 13%						
Nearby Contract	-.94	.74	-.62	.44	-.64	.52
2-6 Month	-.84	.71	-.76	.59	-.65	.61
6-10 Month	-.79	.59	-.69	.46	-.62	.48
HRS 14%						
Nearby	-.93	.73	-.61	.44	-.64	.52
2-6 Month	-.83	.70	-.75	.58	-.64	.61
6-10 Month	-.79	.60	-.70	.48	-.63	.51
HRS 15%						
Nearby	-.96	.59	-.64	.36	-.68	.44
2-6 Month	-.84	.54	-.76	.46	-.66	.48
6-10 Month	-.83	.51	-.75	.42	-.69	.40
HRS 17%						
Nearby	-.98	.65	-.66	.41	-.67	.49
2-6 Month	-.83	.55	-.75	.46	-.65	.49
6-10 Month	-.81	.52	-.71	.39	-.65	.43
Kansas City						
HRW Ordinary						
Nearby	-.96[a]	.58	-.93	.74	-.75	.54
2-6 Month	-.87	.56	-.87	.58	-.71	.54
6-10 Month	-.85	.52	-.82	.50	-.70	.46
HRW 12%						
Nearby	-.88	.58	-.87	.79	-.70	.57
2-6 Month	-.81	.60	-.82	.63	-.66	.58
6-10 Month	-.81	.57	-.79	.56	-.67	.52
HRW 13%						
Nearby	-.87	.58	-.87	.78	-.70	.56
2-6 Month	-.81	.60	-.81	.61	-.66	.57
6-10 Month	-.81	.57	-.79	.56	-.68	.52
Chicago						
SRW						
Nearby	-.97[a]	.51	-.78	.47	-.89	.66
2-6 Month	-.92[a]	.56	-.92[a]	.57	-.78	.59
6-10 Month	-.92[a]	.54	-.94[a]	.57	-.86	.61

[a] Indicates not significantly different than one at the 5 percent level.

TABLE 4. THIRTEEN-WEEK HEDGES IN THREE WHEAT FUTURES MARKETS INDIVIDUALLY

Cash Market and Type of Wheat	Hedging Market: Minneapolis X_2^*	Minneapolis E	Kansas City X_3^*	Kansas City E	Chicago X_4^*	Chicago E
Minneapolis						
HRS 13%						
2-6 Month	-.80	.69	-.70	.56	-.56	.55
6-10 Month	-.77	.64	-.71	.52	-.65	.55
HRS 14%						
2-6 Month	-.81	.71	-.69	.54	-.55	.52
6-10 Month	-.79	.66	-.77	.52	-.66	.56
HRS 15%						
2-6 Month	-.84	.60	-.72	.46	-.57	.44
6-10 Month	-.85	.62	-.81	.49	-.70	.51
HRS 17%						
2-6 Month	-.88	.57	-.78	.47	-.61	.44
6-10 Month	-.87	.55	-.81	.47	-.72	.46
Kansas City						
HRW Ordinary						
2-6 Month	-.83	.65	-.85	.71	-.67	.68
6-10 Month	-.79	.58	-.74	.59	-.72	.60
HRW 12%						
2-6 Month	-.76	.62	-.78	.68	-.62	.65
6-10 Month	-.72	.56	-.77	.57	-.66	.57
HRW 13%						
2-6 Month	-.79	.65	-.80	.69	-.63	.65
6-10 Month	-.76	.59	-.78	.60	-.69	.60
Chicago						
SRW						
2-6 Month	-.78	.53	-.81	.59	-.68	.63
6-10 Month	-.74	.47	-.65	.50	-.74	.57

TABLE 5. TWENTY-SIX-WEEK HEDGES IN THREE WHEAT FUTURES MARKETS INDIVIDUALLY

Cash Market and Type of Wheat	Hedging Market: Minneapolis X_2^*	Minneapolis E	Kansas City X_3^*	Kansas City E	Chicago X_4^*	Chicago E
Minneapolis						
HRS 13% 6-10 Month	-.80	.75	-.79	.68	-.70	.73
HRS 14% 6-10 Month	-.83	.75	-.80	.66	-.71	.70
HRS 15% 6-10 Month	-.92[a]	.75	-.90	.66	-.79	.68
HRS 17% 6-10 Month	-1.04[a]	.73	-1.03[a]	.67	-.89	.67
Kansas City						
HRW Ordinary 6-10 Month	-.79	.61	-.86	.68	-.75	.70
HRW 12% 6-10 Month	-.75	.60	-.81	.66	-.71	.68
HRW 13% 6-10 Month	-.80	.63	-.87	.70	-.76	.72
Chicago						
SRW 6-10 Month	-.69	.43	-.73	.46	-.67	.52

[a]Indicates not significantly different than one at the 5 percent level.

to ten months forward are compared. Several observations are apparent. First, the measure of hedging effectiveness, E, is generally greater for hedges placed in the inherent futures market--i.e., in the futures market with delivery specifications compatible with the cash market--as opposed to cross-hedging. For example, hedging of Minneapolis HRS 14% in contracts two to six months forward against MGE wheat futures is more effective (E = .70) than if the same cash wheat were hedged against either KCBT wheat futures contracts (E = .58) or CBT wheat futures contracts (E = .61). This is generally true for each of the eight cash wheats, but in some cases the differences are not very great. The most effective hedges, as measured by E, are for Kansas City HRW 12% and 13% when hedged against nearby KCBT wheat futures contracts.

A second observation is the measures of hedging effectiveness for wheat of different protein levels. It is frequently stated that hard red spring wheat of higher protein (i.e., 17%) cannot be hedged. The results indicate that hedges placed against HRS 17% do reduce the risk of price changes (i.e., they can be hedged). However, hedges for HRS 17% are somewhat less effective than hedges for HRS 13% and HRS 14%, but this is not true compared to HRS 15%.

In the previous section a hypothesis was stated that the hedging effectiveness decreases for hedges placed in more distant contracts. Greater risk protection is expected if hedged in nearby than distant contracts. Comparisons of E for hedges placed in the inherent futures indicate that in all cases this is true. In other words, the effectiveness of hedging does decrease for hedges placed in more distant contracts. This is not always the case, however, for cross-hedges.[13]

In all cases the hedge ratios shown in Table 3 are less than 1.00. Statistical tests were used to determine in which cases the optimal hedge ratios were significantly different than one and in most cases they were.

Recall that hedges, as traditionally presented, should be equal and opposite the cash position (i.e., hedge ratios should equal 1). The results here indicate that hedge ratios which minimize risk are usually less than 1, or that the cash position should only be partially hedged. The hedge ratios were generally greater for the inherent hedges compared to the cross-hedges.

The results of the 13-week hedges placed in contracts two to six months forward and contracts six to ten months forward are presented in Table 4. The general conclusions are similar to the results of the 4-week hedges. The hedging effectiveness of 4-week and 13-week hedges are about the same. The hedging effectiveness does decrease however, for hedges placed in more distant contracts except in the case of HRS 15% at Minneapolis. Also, as in the 4-week hedges, the effectiveness of hedging each cash wheat is greater for hedges placed in the inherent futures contract, than for cross-hedges. The optimal hedge ratios are generally the same as in the 4-week hedges. All are significantly less than one indicating that futures positions equal and opposite the cash positions do not result in the least risk.

The results for 26-week hedges placed in contracts six to ten months forward are presented in Table 5. Again, the general results are similar to those for the 4- and 13-week hedges. In each case except SRW at Chicago, the effectiveness of hedging for the longer term is greater than for shorter hedges. This merely indicates that absolute changes in cash prices are greater and the futures have a longer time to respond. Thus, hedges placed in contracts six to ten months forward over the longer term (e.g., 26 weeks) are relatively more effective than shorter term hedges (e.g., 4 or 13 weeks). Again, for each class of cash wheat, hedges placed in the inherent futures market are more effective than cross-hedging. However, these differences are less for 26-week hedges than for shorter term hedges, indicating that cross-hedging becomes relatively more viable in the longer term. In most cases the hedge ratios for the 26-week hedges are less than one. However, the hedge ratios are generally greater for the 26-week hedges than for the shorter term hedges.

The measure of hedging effectiveness used in the above comparisons is dependent on the optimal hedge ratio. In most cases however, the optimal hedge ratio varies from hedge to hedge and consequently, affects the calculation of the measure of hedging effectiveness. In commercial transactions, hedging ratios are implicitly determined by traders according to their indifference curve between expected return and risk. The sensitivity of the measure of hedging effectiveness to the assumed hedge ratios is evaluated below. The variance of a portfolio for each type of cash wheat is calculated in three situations. In the first case the hedge ratio is equal to 0. This is the same as an unhedged cash position and reduces to the variance of the spot price change. In the second case the hedge ratio is assumed equal to one indicating a fully hedged position. In the third case the hedge ratio equals the optimal hedge ratio (i.e., $X_2 = X_2{}^*$) as presented in Tables 3-5. In other words, the comparisons made below are between two extreme situations and one which is optimal. The first is an unhedged situation and is perhaps closest to the position that many producers hold. The second assumes a fully hedged situation as traditionally taught, and as practiced by many merchandisers. Finally, comparisons are made of the risk assuming an optimally hedged position. In the latter case risk is minimized. The variance of a portfolio assuming the three situations above was calculated for 4-, 13-, and 26-week hedges at each of the futures markets.

The results in Table 6 are for 4-week hedges placed in nearby contracts, contracts two to six months forward, and contracts six to ten months forward. The variance of a change in spot price is the inherent risk of an unhedged position (i.e, a hedge ratio equal to zero). At the Minneapolis cash market the

TABLE 6. VARIANCES IN UNHEDGED SPOT MARKET POSITIONS AND FULL AND PARTIALLY HEDGED POSITIONS, 4-WEEK DURATION (1977-1981)

Cash Market and Type of Wheat	Spot Price Change	Hedges Placed in Nearby Months						Hedges Placed in Contracts 2-6 Months Forward						Hedges Placed in Contracts 6-10 Months Forward					
		MGE		KCBT		CBT		MGE		KCBT		CBT		MGE		KCBT		CBT	
		$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$	$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$	$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$
		¢/bushel																	
Minneapolis																			
HRS 13%	430	113.5	112.3	313.8	242.4	274.1	205.0	136.7	125.8	199.5	174.7	241.4	166.2	193.3	174.4	271.2	230.2	302.0	223.8
HRS 14%	425	114.8	113.3	317.7	240.5	273.0	202.6	138.6	126.7	207.3	179.5	246.1	167.5	188.0	169.1	259.7	220.7	281.8	209.3
HRS 15%	563	231.3	230.4	425.7	362.1	369.5	313.2	267.8	257.3	328.5	304.7	361.6	290.7	288.2	276.5	350.6	324.7	358.2	306.4
HRS 17%	533	188.1	188.0	368.2	314.3	324.2	272.8	251.0	238.3	313.2	286.0	346.8	270.9	272.0	257.9	356.4	320.8	367.9	303.3
Kansas City																			
HRW Ordinary	575	243.5	243.0	152.7	150.3	297.7	265.1	256.5	249.3	248.1	240.9	317.3	264.2	282.2	273.3	301.8	288.7	354.7	307.7
HRW 12%	472	201.8	196.6	105.1	97.3	249.2	202.0	205.2	189.8	190.8	176.7	269.6	199.2	218.4	202.9	228.2	209.5	286.3	228.6
HRW 13%	476	207.4	201.1	115.0	106.5	261.7	212.0	207.8	192.7	200.1	185.0	279.0	206.8	216.9	202.5	227.6	211.4	282.6	227.4
Chicago																			
SRW	651	318.8	242.4	369.6	346.7	225.6	219.8	290.8	288.0	282.2	279.4	299.2	270.0	299.0	296.7	279.9	278.4	261.4	251.0

variance for HRS 14% is the smallest and that for HRS 15% is the largest. The variance for HRW Ordinary at Kansas City is greater than that for HRW 12% or HRW 13%. The variance for Chicago SRW is the largest compared to all the cash wheats. The remainder of the table shows the variance of the portfolio for fully and partially hedged positions for each type of cash wheat hedged at the three futures markets.

In each case the variance of the hedged portfolio is less than that of the unhedged portfolio illustrating the risk reduction capability of hedging. In addition, the variance of the optimally hedged portfolio ($X_i = X_i*$) is less than that of the fully hedged portfolio ($X_i = 1$). The difference between these indicates the extent risk is reduced further by using the optimal hedge ratio. However, this difference is relatively slight in the 4-week hedges presented in Table 6. In all cases the variance of the portfolio for a particular type of cash wheat is less when hedged at its inherent futures market as opposed to being cross-hedged. For example, the variance of a fully hedged position of Minneapolis HRS 14% is 114.8, 317.7, and 273.0 when hedged against the MGE, KCBT, and CBT nearby wheat futures respectively. This reaffirms the same conclusions as the comparisons of the measures of hedging effectiveness.

The variance of the portfolios referenced in Table 6 is a statistical measure of the risk inherent in a particular position. It also can be used to illustrate the interpretation of risk. The standard deviation is equal to the square root of the variance and can be used to calculate confidence intervals. The 95 percent confidence intervals of price changes are:

$$\overline{\Delta P} \pm 1.96 \text{ (St. Dev.)}$$

where $\overline{\Delta P}$ is the average price change over a given (i.e., 4-week) period. For example, the 95 percent confidence intervals for Minneapolis HRS 13% ($\overline{\Delta P}$ = 1.95 cents, see Table A1) are -38.7 cents to +42.6 cents in the hedged position; -18.9 cents to +22.8 cents in a fully hedged position against MGE wheat futures; -32.7 cents to +36.6 cents in a fully hedged position against KCBT wheat futures; and -30.5 cents to +34.4 cents in a fully hedged position against CBT wheat futures. Interpretation of these 95 percent confidence intervals is that there is only one chance in 20 an actual price change would fall outside the values.

Variance of portfolios for 13-week hedges are shown in Table 7 in a similar format to the above. The results for 26-week hedges are shown in Table 8. The general conclusions and observations are the same. However, several points should be noted. First, the variances for both the spot price changes (i.e., unhedged position) and the fully and partially hedged portfolios are greater for longer term hedges than 4-week hedges. This indicates the greater inherent risk associated with positions held for a longer term. Second, the difference in the variance between a fully hedged and an optimally hedged position is greater for the longer term hedges than the 4-week hedge. This reaffirms the conclusions drawn with respect to the hedging effectiveness of longer term hedges. Finally, a comparison of the fully hedged SRW wheat against the KCBT and CBT wheat futures indicates that less risk is incurred by hedging in the former for; 1) 13-week hedges in futures contracts two to six months forward and 2) 26-week hedges.

Hedging in the Multi-Market Case

Traditionally hedgers take futures positions equal and opposite their cash position in the futures market perceived to have the highest correlation with the cash price. The analysis above illustrated that optimal positions normally should be less than fully hedged and that cross-hedging of different types of wheat in the three futures markets was feasible. However, as a general rule the least risk was attained by hedging in the inherent futures market. It is also possible that traders may spread their hedges of cash

TABLE 7. VARIANCE IN UNHEDGED SPOT POSITION AND FULL AND PARTIALLY HEDGED POSITION, 13-WEEK DURATION (1977-81)

Cash Market and Type of Wheat	Spot Price Change	Hedges Placed in Contracts 2-6 Month Forward						Hedges Placed in Contracts 6-10 Month Forward					
		MGE		KCBT		CBT		MGE		KCBT		CBT	
		$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$	$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$
Minneapolis													
HRS 13%	1,098	381	333	592	483	848	496	453	393	630	529	672	493
HRS 14%	1,102	369	324	630	510	896	524	433	378	622	524	652	481
HRS 15%	1,385	579	547	846	746	1,124	777	559	533	760	701	812	681
HRS 17%	1,672	719	701	922	862	1,182	907	747	728	902	862	976	866
Kansas City													
HRW Ordinary	1,257	470	435	397	367	607	407	578	525	555	512	613	503
HRW 12%	1,097	482	412	407	345	665	387	576	425	555	475	639	471
HRW 13%	1,148	453	401	406	355	664	406	535	468	524	464	590	454
Chicago													
SRW	1,382	706	650	613	568	709	517	810	733	741	685	691	593

TABLE 8. VARIANCE IN UNHEDGED SPOT POSITIONS AND FULL AND PARTIALLY HEDGED POSITIONS, 26-WEEK HEDGES (1977-81)

Cash Market and Type of Wheat	Spot Price Change	Hedges Placed in Contracts 6-10 Months Forward					
		MGE		KCBT		CBT	
		$X_2=1$	$X_2=X_2^*$	$X_3=1$	$X_3=X_3^*$	$X_4=1$	$X_4=X_4^*$
Minneapolis							
HRS 13%	1,882	561	475	700	606	755	512
HRS 14%	2,006	569	504	774	691	829	601
HRS 15%	2,500	635	623	886	843	915	789
HRS 17%	3,263	894	890	1,081	1,079	1,126	1,090
Kansas City							
HRW Ordinary	2,251	962	869	771	730	846	679
HRW 12%	2,046	963	823	772	697	881	649
HRW 13%	2,228	898	814	701	666	774	619
Chicago							
SRW	2,411	1,586	1,374	1,451	1,304	1,460	1,162

positions across two or three wheat futures markets. For example, a long cash position could be offset with short position at two or three exchanges. The sum of the short futures positions would approximately equal the long cash position.

Portfolio analysis was used to determine the optimal hedge ratios for hedges spread across two wheat futures markets, and hedges spread across three wheat futures markets. In addition, the measure of hedging effectiveness was calculated for comparison across markets and to single market hedges. For each cash wheat the best hedge in two futures markets--defined as that giving the greatest hedging effectiveness--is presented, as well as the results of the hedge across three futures markets. In each case two statistical tests were conducted. One is whether the summation of the hedge ratios were significantly different than one. These results indicate whether the sum of the optimal hedge ratios would be equal to the cash position. The second test is to determine whether the hedging effectiveness of a multi-futures market hedge is significantly greater than the single futures market hedge.

The results of the 4-week hedges placed in nearby contracts, contracts two to six months forward, and contracts six to ten months forward are presented in Table 9. Results for the 13- and 26-week hedges are presented in Tables 10 and 11, respectively. The minimization problem assumes a long spot position equal to one. Thus, a negative (positive) hedge ratio indicates the proportion which should be short (long) hedged in that particular futures market.[14] For example, optimal 4-week hedge ratios for Minneapolis HRS 13% in nearby contracts are -1.14 and .19 in the two futures markets cases. This indicates that risk minimization requires 114 percent of a long cash position to be short hedged against MGE wheat futures and 19 percent to be long hedged against KCBT futures. The sum of these is the proportion of the cash position which is hedged against the two futures markets. In this case, 95 percent

TABLE 9. FOUR-WEEK HEDGES SPREAD ACROSS MORE THAN ONE WHEAT FUTURES MARKET

Cash Market and Type of Wheat	Two Futures Markets				Three Futures Markets			
	X_2*	X_3*	X_4*	E	X_2*	X_3*	X_4*	E
Minneapolis								
HRS 13%								
Nearby	-1.14	.19		.75[b]	-1.18[a]	.17	.06	.75
2-6 Month	-1.16	.34		.72[b]	-1.10	.56	-.24	.72
6-10 Month	-1.25	.49		.62[b]	-1.24	.57	-.09	.62
HRS 14%								
Nearby	-1.13	.19		.75[b]	-1.16[a]	.17	.05	.75
2-6 Month	-1.23	.42		.72[b]	-1.16	.69	-.29	.72
6-10 Month	-1.19	.42		.62[b]	-1.13	.61	-.22	.62
HRS 15%								
Nearby	-1.14[a]	.18		.60[b]	-1.12[a]	.19	-.03	.60
2-6 Month	-1.14	.31		.55	-1.05	.65	-.36	.55
6-10 Month	-1.11	.29		.51	-1.01	.62	-.39	.52
HRS 17%								
Nearby	-1.11[a]	.13		.65	-1.11[a]	.13	-.005	.65
2-6 Month	-1.16	.34		.56[b]	-1.07	.69	.39	.57[c]
6-10 Month	-1.35	.56		.54[b]	-1.27	.82	-.31	.55
Kansas City								
HRW Ordinary								
Nearby	- .16[a]	-.82		.75[b]	- .24[a]	-.87	.12	.75
2-6 Month	- .32	-.57		.59[b]	- .35[a]	-.69	.13	.59
6-10 Month	- .64	-.22		.53	- .69	-.36	.16	.53
HRW 12%								
Nearby	- .05	-.84		.80	- .10	-.86	.06	.80
2-6 Month	- .22	-.62		.63	- .25	-.73	.13	.63
6-10 Month	- .50	-.32		.58[b]	- .54	-.45	.16	.58
HRW 13%								
Nearby	- .07	-.82		.78	- .13	-.86	.09	.78
2-6 Month	- .29	-.54		.62[b]	- .33	-.68	.15	.62
6-10 Month	- .54	-.28		.58[b]	- .56	-.37	.10	.58
Chicago								
SRW								
Nearby		.05	-.94	.66	.009	.05	-.94	.66
2-6 Month	- .27		-.58	.60	- .24	-.12	-.49	.60
6-10 Month		-.08	-.77	.60	- .03	-.01	-.83	.61

[a] Indicates the sum of the hedges are not significantly different than -1.0 at the 5 percent level of significance.
[b] Indicates increase in hedging effectiveness relative to the one futures market case is significant at the 5 percent level of significance.
[c] Indicates increase in hedging effectiveness relative to the two futures market case is significant at the 5 percent level of significance.

TABLE 10. THIRTEEN-WEEK HEDGES SPREAD ACROSS MORE THAN ONE WHEAT FUTURES MARKET

Cash Market and Type of Wheat	Two Futures Markets				Three Futures Markets			
	X_2*	X_3*	X_4*	E	X_2*	X_3*	X_4*	E
Minneapolis								
HRS 13%								
2-6 Month	-1.16	.37		.71[b]	-1.16	.36	.01	.71
6-10 Month	-1.25	.50		.66[b]	-1.26	.53	-.02	.67
HRS 14%								
2-6 Month	-1.35	.56		.74[b]	-1.36	.49	.06	.74
6-10 Month	-1.32	.57		.68[b]	-1.30	.60	-.05	.67
HRS 15%								
2-6 Month	-1.40	-.68		.64[b]	-1.42	.40	.17	.64
6-10 Month	-1.41	.59		.64[b]	-1.50	.48	.19	.64
HRS 17%								
2-6 Month	-1.19		.27	.58[b]	-1.21[a]	.09	.21	.58
6-10 Month	-1.25		.36	.56[b]	-1.32	.16	.27	.56
Kansas City								
HRW Ordinary								
2-6 Month	-.12	-.73		.71	-.11	-.64	-.09	.71
6-10 Month		-.33	-.43	.61[b]	-.15	-.27	-.36	.61
HRW 12%								
2-6 Month	-.06	-.72		.69	-.06	-.72	.001	.69
6-10 Month		-.34	-.36	.58[b]	-.18	-.26	-.28	.58
HRW 13%								
2-6 Month	-.19	-.62		.69	-.20	-.70	.07	.69
6-10 Month	-.30		-.43	.61[b]	-.22	-.21	-.32	.61
Chicago								
SRW								
2-6 Month	.17		-.83	.65	.09	-.05	-.70	.65
6-10 Month	.47		-1.14	.59[b]	.47	.05	-1.19	.59

[a]Indicates the sum of the hedges are not significantly different than -1.0 at the 5 percent level of significance.
[b]Indicates increase in hedging effectiveness relative to the one futures market case is significant at the 5 percent level of significance.

TABLE 11. TWENTY-SIX-WEEK HEDGES SPREAD ACROSS MORE THAN ONE WHEAT FUTURES MARKET

Cash Market and Type of Wheat	Two Futures Markets X_2	X_3	X_4	E	Three Futures Markets X_2	X_3	X_4	E
Minneapolis								
HRS 13% 6-10 Month	-.57		-.20	.75	-.72	.42	-.44	.76[c]
HRS 14% 6-10 Month	-1.16	.36		.76[b]	-1.03	.52	-.26	.76
HRS 15% 6-10 Month	-1.26	.36		.76[b]	-1.28	.34	.03	.76
HRS 17% 6-10 Month	-1.27[a]		.21	.73	-1.25[a]	-.05	.24	.73
Kansas City								
HRW Ordinary 6-10 Month	.49		-1.17	.71[b]	.63	-.41	-.95	.72[c]
HRW 12% 6-10 Month	.48		-1.13	.70[b]	.62	-.38	-.92	.71[c]
HRW 13% 6-10 Month	.49		-1.19	.74[b]	.65	-.45	-.95	.75[c]
Chicago								
SRW 6-10 Month	.69		-1.28	.55[b]	.60	.25	-1.42	.55

[a] Indicates the sum of the hedges are not significantly different than -1.0 at the 5 percent level of significance.
[b] Indicates increase in hedging effectiveness relative in the one futures market case is significant at the 5 percent level of significance.
[c] Indicates increase in hedging effectiveness relative to the two futures market case is significant at the 5 percent level of significance.

of the cash position is hedged. In the three market hedge, the hedge ratios are -1.18, .17, and .06, indicating that 118 percent of a long cash position should be short hedged in the MGE futures, and 17 percent and 6 percent of the cash position should be long hedged in KCBT and CBT wheat futures, respectively. In this case 95 percent of the cash position is hedged.

Comparison of the hedge ratios across the different qualities of cash wheat indicates that the largest hedge ratio is held in the futures market inherent to that cash market. Smaller positions are cross-hedged in one of the other two wheat futures markets. An exception to this is the HRW cash wheats for 26-week hedges. The optimal hedge ratios call for a large position in CBT wheat and a smaller and opposite position in MGE wheat. This result corresponds with the single market results in HRW wheats for longer term hedges receive greater risk protection when hedged against CBT wheat futures.

Generally, for hedges placed in two wheat futures markets, Minneapolis wheat is spread between MGE and KCBT wheat futures; Kansas City wheat is spread between MGE and KCBT wheat futures except in the 26-week hedges as discussed above; and Chicago wheat is spread either between CBT and MGE or between CBT and KCBT. Statistical tests were used to determine if the summation of the position across the futures markets was significantly different than one. In most cases they were, which is similar to the results of the single futures market hedge.

The measure of hedging effectiveness, E, retains the same interpretation as in the previous section. It is the percentage decrease in risk as a result of hedging using the optimal hedge ratios, relative to an unhedged position. In all cases, the effectiveness of multi-futures market hedging is equal to or greater than that of single futures market hedging. Statistical tests were used to determine if the increase in E was significant. In most cases it was for two futures market case, but not for hedges in three futures markets. In other words, risk reduction is enhanced in many cases by spreading hedges across two futures. However, in most cases little additional risk reduction stems from hedging in a third futures market. In most cases the additional reduction in risk is 1 to 2 percent greater in the two futures market case than with hedges in one futures market. There appears to be slightly more risk reduction potential by spreading in two markets for 13- and 26-week hedges. This is particularly true for hedging of Minneapolis wheats for 13-week hedges and Kansas City and Chicago wheats for 26-week hedges. For example, the hedging effectiveness of 13-week hedges in contracts two to six months forward for Minneapolis HRS 14% increases from .71 for hedges in MGE futures only, to .74 for hedges in MGE and KCBT wheat futures. In the former case .81 percent of the cash position is hedged in MGE futures. In the latter case 135 percent of a long cash position is short hedged against MGE futures and 56 percent is long hedged against KCBT futures--a net position which is 79 percent hedged. Only in one case in the 4-week hedge, and four cases in the 26-week hedge, did the hedging effectiveness for the three market hedge increase significantly more than in the two futures market hedge.

Summary and Conclusions

One of the functions of futures markets is to facilitate risk shifting An important empirical question is the hedging effectiveness of various futures markets. Hedging performance of the futures markets is particularly important in the case of wheat. Five classes of cash wheat exist, each with many different quality characteristics. These can be hedged on any of three future markets, each with different deliverable grades but affected by the same fundamental factors. In addition, it is possible for traders to spread hedges across more than one futures market. This study compares the *ex post* hedging effectiveness of the three wheat futures markets in reducing risk of three classes of cash wheat with different protein levels.

Traditional hedging theory states that futures positions should be equal and opposite the cash position, implying that the proportion of the cash position hedged is 100 percent. In addition, it is normally taught that hedges should be placed in the futures market which has deliverable grades corresponding to the type of wheat being hedged. Portfolio theory incorporates risk into hedging decisions and can be used to determine the optimal proportion of the cash position hedged. This may be equal to or different from 100 percent. It is optimal in the sense that it assumes the objective is to minimize risk. Optimal hedge ratios and measures of the effectiveness of hedging were calculated _ex post_ for each of the eight qualities of cash wheat hedged at each of the three futures markets. Weekly data were used from the period 1977-1981.

The results of the analysis for hedges placed in a single futures market indicate that cross-hedging is possible, but hedging in the inherent futures contract generally offered greater risk protection. Cross-hedging becomes relatively more viable in longer term hedges. The results also indicated that greater risk protection was attained if the hedges are placed in nearby contracts rather than in distant contracts. In other words, the hedging effectiveness decreases for hedges placed in more distant futures contracts. The optimal hedge ratios were generally less than one and in most cases were significantly less than one. Cash positions should be only partially hedged if the objective is to minimize risk, as opposed to having equal and opposite positions as in traditional hedging.

In the case of wheat it is possible for traders to spread hedges of cash wheat across two or three wheat futures markets. For example, a short cash position may be offset by long positions in more than one wheat futures contract. Portfolio analysis was used to determine the optimum hedge ratios for hedges spread across two wheat futures markets and for hedges spread across three wheat futures markets. The largest hedge ratio for each type of cash wheat was generally in its inherent futures market with a smaller position cross hedged in one or both of the other futures. In all cases, the hedging effectiveness was greater in multi-futures market hedges than in hedges placed in a single futures market. In most cases, the risk reduction attained by using two wheat futures markets was statistically significant, but this was rarely true when using three wheat futures markets. Risk reduction is enhanced in many cases by spreading hedges across two futures markets. In most cases, the additional risk reduction was 1 to 2 percent. However, little additional risk reduction stems from hedging in a third futures market.

The methodology presented in this report is applicable to other wheats and to other commodities in general. It could be used to evaluate the hedging effectiveness of durum and white wheats--neither of which has an actively traded futures market. Additionally, it could be used to evaluate hedging at local as well as other terminal markets. It could also be used by exporting agencies in other countries who are evaluating use of U.S. futures markets for hedging their exportable grains. The results reported here are *ex post* in the sense that historical data were used. It could be refined by developing forecasting models of basis relationships and integrating the expected values and standard errors in a similar decision-making framework.

Footnotes

[1]Hedging is only necessary when prices are variable. In many of the other exporting countries prices to producers are fixed. However, sales made in the international market entail price risk. Consequently, wheat exporting agencies may consider hedging as an alternative. The Australian Wheat Board has recently announced a change in policy allowing it to hedge export sales. The South African Maize Board has also expressed interest in hedging corn on U.S. exchanges.

[2]Dewbre and Blakeslee developed a model for forecasting white wheat basis relative to the wheat futures contracts and integrated to results to simulate alternative hedging strategies. In addition, a recent publication of the Minneapolis Grain Exchange promoted spreading as a potentially profitable speculative option.

[3]Gray indicates that one of the primary functions served by the Minneapolis and Kansas City wheat futures is to provide spreading opportunities for traders.

[4]As indicated recently by Hill and Schneeweis,(1981), it is theoretically and empirically incorrect to use price levels in the analysis of variance. It is important that price changes be used throughout in the calculation of variances and covariances.

[5]Brokerage and interest costs are not included in (4) for simplicity.

[6]This is similar to Markowitz's portfolio selection technique except that 1) the size of the long cash position, x_i is fixed at 1 and 2) the model is not constrained to various values of equation (4).

[7]For simplicity X_i is assumed equal to one throughout.

[8]In the general case of n position or assets in a portfolio the variance of return is:

$$V(R) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_i \sigma_{ij}^2$$

[9]The analysis could be expanded to any type of wheat, or other grains, at any of the terminal markets, or at local markets. Data are typically available at the terminal markets and analysis should be applied to that which is the location of the appropriate spot transaction.

[10]Hedges compared in Table 4 and 5 did not include the nearby contract or in the latter case did not include contracts two to six months forward, because some rule for rolling over the hedges would be necessary.

[11]The negative sign for the hedge ratios in Tables 3-5 indicate that opposite positions should be taken in the spot and futures markets. The numerical values are the same regardless whether the spot position is long or short. Thus, -.94 indicates 94 percent of the long (short) spot position should be hedged with a short (long) futures market.

[12]Empirical results of the sensitivity of E with respect to the hedge ratios are discussed later.

[13]In a previous study, Ederington measured the hedging effectiveness of the KCBT wheat future for 4-week hedges. His measure of hedging effectiveness was about .90 and was calculated for the two-year period ending December 1977. The results of this study indicate the hedging effectiveness of the KCBT wheat futures is less than that calculated by Ederington. This is presumably due to the more recent data used in the present study.

[14]The hedge ratios should be interpreted just the opposite if a short cash position were assumed.

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APPENDIX A TABLES

TABLE A1. MEANS AND VARIANCES OF CASH PRICE CHANGES, 1977-1981

	Four-Week (N=255)		Thirteen-Week (N=246)		Twenty-Six-Week (N=233)	
	Mean	Variance	Mean	Variance	Mean	Variance
Minneapolis						
HRS 13%	1.95	430	6.75	1,098	15.62	1,882
HRS 14%	1.73	425	6.43	1,102	15.50	2,006
HRS 15%	1.83	563	6.31	1,385	15.50	2,500
HRS 17%	1.53	533	5.77	1,622	14.60	3,263
Kansas City						
HRW Ordinary	2.32	575	7.94	1,257	18.31	2,251
HRW 12%	2.32	472	8.33	1,097	18.52	2,046
HRW 13%	1.93	476	7.23	1,148	16.63	2,228
Chicago						
SRW	0.93	651	6.27	1,382	15.00	2,411

TABLE A2. COVARIANCES BETWEEN CASH AND FUTURES FOUR-WEEK PRICE CHANGES, 1977-1981 (N=255)

	Futures Contracts								
	Nearby Months			2-6 Month Forward			6-10 Month Forward		
Cash Markets	MGE	KCBT	CBT	MGE	KCBT	CBT	MGE	KCBT	CBT
Minneapolis									
HRS 13%	336	303	349	361	334	405	325	290	333
HRS 14%	334	301	348	358	328	400	325	293	341
HRS 15%	344	314	368	362	336	411	344	317	372
HRS 17%	351	328	376	356	329	404	337	299	352
Kansas City									
HRW Ordinary	344	456	410	374	383	439	353	347	379
HRW 12%	313	429	383	348	360	412	334	333	362
HRW 13%	313	426	379	349	357	409	336	334	366
Chicago									
SRW	345	386	484	395	403	486	383	396	464
Futures Market									
MGE Nearby	356	352	389						
KCBT Nearby	352	490	441						
CBT Nearby	389	442	543						
MGE 2-6 Month				429	414	486			
KCBT 2-6 Month				414	439	506			
CBT 2-6 Month				486	506	621			
MGE 6-10 Month							414	396	442
KCBT 6-10 Month							396	422	459
CBT 6-10 Month							442	459	539

TABLE A3. COVARIANCES BETWEEN CASH AND FUTURES THIRTEEN-WEEK PRICE CHANGES, 1977-1981 (N=246)

Cash Market	Futures Contracts: 2-6 Months Forward MGE	KCBT	CBT	6-10 Months Forward MGE	KCBT	CBT
Minneapolis						
HRS 13%	955	873	1,062	909	808	933
HRS 14%	963	857	1,039	921	814	945
HRS 15%	1,000	891	1,068	1,000	886	1,008
HRS 17%	1,048	970	1,157	1,024	933	1,043
Kansas City						
HRW Ordinary	990	1,051	1,261	925	924	1,043
HRW 12%	904	966	1,152	846	845	949
HRW 13%	943	991	1,179	892	886	1,000
Chicago						
SRW	934	1,005	1,274	873	895	1,067
Futures Market						
MGE 2-6 Months	1,192	1,152	1,392			
KCBT 2-6 Month	1,152	1,242	1,481			
CBT 2-6 Month	1,392	1,481	1,873			
MGE 6-10 Month				1,173	1,108	1,245
KCBT 6-10 Month				1,108	1,148	1,240
CBT 6-10 Month				1,245	1,240	1,442

TABLE A4. COVARIANCES BETWEEN CASH AND FUTURES TWENTY-SIX-WEEK PRICE CHANGES, 1977-1981 (N=246)

Cash Market	Futures Contracts 6-10 Month Forward		
	MGE	KCBT	CBT
Minneapolis			
HRS 13%	1,754	1,621	1,945
HRS 14%	1,812	1,646	1,970
HRS 15%	2,026	1,847	2,174
HRS 17%	2,278	2,121	2,450
Kansas City			
HRW Ordinary	1,738	1,770	2,084
HRW 12%	1,635	1,667	1,964
HRW 13%	1,759	1,794	2,109
Chicago			
SRW	1,506	1,510	1,856
Futures Market			
MGE 6-10 Month	2,186	2,048	2,384
KCBT 6-10 Month	2,048	2,060	2,321
CBT 6-10 Month	2,384	2,322	2,762

List of Tables