RURAL ECONOMY

PROJECT REPORT

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Economic Replacement Strategies For Dairy Cattle[1]

Farming for the Future Report No. 87-1035

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Project Report No. 89-02

1 This research report draws heavily from the original master thesis work of W. Lohr. For the complete work see Lohr, Wayne. 1989. "Longevity and Genetic Improvement Issues in Economic Replacement Strategies For Dairy Cows Under Alberta Conditions". Master of Science Thesis, Department of Rural Economy, University of Alberta, Edmonton, Alberta.

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Acknowledgements

The authors are grateful for the technical advice provided to the project team by our University of Alberta colleagues Drs. Wictor Adamowicz, Mel Lerohl and John Kennelly as well as Dr. Alan McAllister of Agriculture Canada and Dr. Morris Freeman of Semex Canada. Mr. Ed Bristow of Alberta Agriculture was helpful in pointing the way to public and private sources of data. Mr. Mike Slompe of Alberta Dairy Herd Improvement Services provided access to industry data and four commercial dairy operators freely contributed their private herd data. The project would not have been possible without the contributions of these individuals and organizations and especially the financial support provided by Alberta Agriculture through its Farming for the Future program.

ABSTRACT

Dairy extension officers and the Alberta Dairy Seminar Committee have identified the economic life of a dairy cow as an important area of concern to the dairy industry. The purpose of this research was to develop an explanatory economic model to shed light on this question especially as it relates to the situation where cows are replaced by genetically superior ones.

Production data were collected from four co-operating dairy producers. A milk yield curve across lactations was developed from these data. This lactation curve, along with other data was used to specify the present value of net annual residual cashflows from the dairy cow. Included in the cashflows were the original value of the cow, her salvage value at the end of each lactation, the value of the calf produced plus the annual net operating revenue from milk production. The net present values for each feasible replacement pattern were evaluated using a finite Markov process to account for involuntary replacement of cows. This resulted in specification of a steady state herd composition in which cows would be replaced in such a manner as to maximize the net present value of the herd through time.

The model was tested for sensitivity to variations in basic parameters and although there was some sensitivity to variations in parameters, especially when varied jointly, the optimum replacement time remained at stable at lactation number three. Not only was the optimum replacement time stable, penalties for deviation from the optimum were economically insignificant.

The calculated economic replacement strategy was compared to the current Alberta situation as revealed in Alberta Dairy Herd Services data. No major differences were found between the replacement age predicted and that which is occurring in the industry. Furthermore, the average age of the herd calculated from the optimal strategy is almost identical to that observed in the Alberta dairy industry. It was concluded that there are no major differences between the calculated economic optimum replacement strategy and the current replacement practices being practiced by Alberta dairymen.

Certain restrictive assumptions were made in this research. Because of data limitations knowledge about likely performance in subsequent lactations gained during the current lactation was not incorporated into the model. Restrictions exist also because reliable health care costs, particularly as the cow ages, and data on involuntary culling rates are lacking for Alberta conditions. As a result these data were obtainable only from published secondary literature sources. Further investigation into veterinary costs relative to age and lactation number as well as production level is recommended. The rate of change in veterinary costs as the cow ages can be of importance in the replacement decision process.

While herd demography should be studied more deeply, there appears to be a lack of adequate data to facilitate such studies. Effort could profitably be directed towards developing a more adequate data base for farm level dairy research.

Knowledge about the lifetime lactation curve of dairy cows is a deficient area in the animal science literature and yet is the single most important determinant of replacement strategies. The data available are confounded because dairy farmers are operating in an economic environment making voluntary culling decisions while involuntary culling is present at the same time. Further controlled experimentation is suggested to resolve this issue.

Table of Contents

List of Figures

List of Tables

I. Introduction

The dairy industry plays a significant role in the agricultural economy of Alberta. In 1985, milk and cream sales amounted to over 246 million dollars or 6.2% of total farm sales. In 1986, milk and cream sales amounted to over 248 million dollars or 6.1% of total farm sales.[3]

Dairy extension specialists have expressed the concern that the rate of turnover of dairy cows in the Alberta dairy herd may be too rapid. The dairy extension specialists have hypothesized that there may be economic benefits which could be captured by individual dairy producers if they improved their replacement strategies. This concern may be accentuated when the issue of genetic improvement is introduced into the decision-making framework. The question of the optimum economic life of a dairy cow has also been identified by the Alberta Dairy Seminar Planning Committee as an important issue to the dairy industry of Alberta.

Statistics obtained from the Alberta Dairy Herd Improvement Services indicate that the average age of a dairy cow in Alberta is 4.78 years, with the average age of first calving at 28 months. The average calving interval is 13.5 months. It is suggested by these same statistics that the average time of disposal of dairy cows in Alberta is 5.3 years of age; sometime during the cow's third lactation.

The production cycle of the dairy cow is such that the output of milk from a particular cow increases at a decreasing rate from the time of first calving to maturity, then remains relatively constant over some time period, and subsequently decreases at an increasing rate with senescence. (Smith 1968; Lush and Shrode 1950; and Giaver 1966)

Dairy science literature suggests that a dairy cow reaches maximum production at around seven years of age (Mao, Burnside, Wilton and Freeman 1974; Lush and Shrode 1950) which places the point of attaining maximum production sometime during the fourth or fifth lactation. The production cycle, limited barn space and physical resources of dairymen, quota restrictions and other production constraints create an environment in which the net revenue generated by a mature or aging cow may no longer be sufficient to offset the net revenue that would be generated by a replacement cow, particularly if the replacement cow is genetically superior.

Several studies have been conducted on optimal replacement policies for dairy cows (Gardner, 1980; Giaver, 1966; Jenkins and Halter, 1964; Redman and Kuo, 1969; Smith, 1968; Stewart, 1975; and Van Arendonk, 1984). With the exception of Stewart and Van Arendonk, these studies have not included genetic improvement in the analysis. There appears to be a lack of research dealing with dairy cow economic replacement problems which take genetic improvement and questions of longevity of production into account, particularly as these concerns relate to Alberta producers and conditions (i.e., quota restrictions, climatic conditions, etc.).

The purpose of this research was to examine the behavior of Alberta dairy farmers with regard to their cow replacement strategies. The comparison of actual behavior to predicted behavior required the development of a capital asset replacement model for dairy cows.

3 Alberta Agriculture Statistics Handbook, 1985 and 1986.

II. The Analytical Model

A. Introduction

The analytical model is rooted in capital asset replacement theory. The underlying concepts have been well developed by a number of disciplines including agriculture, business, forestry, engineering and economics. Lohr (1989) provides a brief historical development including applications and extensions to dairy cow replacement issues.

A central issue of the theory was stated by Perrin (1972) who suggested that an asset (the dairy cow is a special class of capital asset) should be kept

> *"until the gain from keeping the current asset for another time interval no longer exceed the opportunity gains which could be realized from the replacement asset during the same period."*

This study considers the replacement problem within the context of profit-maximizing behavior. The objective is to find a net present value maximizing replacement strategy.

In the following discussions, it will be assumed that:

a) replacement will be defined as the replacement of an existing cow with a genetically superior cow.

b) all cashflows are received at the end of the lactation under consideration.

c) all replacement cows are purchased under the assumption that the rearing of ones own replacements is a separate economic activity.

d) the replacement cow is purchased at the beginning of the lactation.

e) the replacement cow has a calf and commences milk production on the day of purchase.

f) only integer values of the lactations are available for consideration.

g) the maximum productive life of the dairy cow is ten lactations.

The economic question to be answered can be expressed as: "Does one more year in a repeating cycle cause the annuity value to rise, fall, or remain the same."

B. Net Present Value Components

The replacement issue has a number of components, including the milk production component, the calf born at the onset of each lactation cycle, and the initial purchase of the replacement heifer and the subsequent sale or salvage of the cow at the end of the cycle.

1. The Milk Component and Genetic Growth

The first step in the development of the model is to determine the present value of the net cash flows associated with the milk production of the first cow in the replacement cycle.

Where:

V_t = the annual net cash flow from milk production associated with a specific cow in lactation "t".

i = the discount rate.

n = the number of lactations for which the cow is kept.

The present value of the milk component of the first cow can be expressed as:

$$\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \qquad (1)$$

This cow will be replaced at the end of "n" periods by a cow that is genetically superior by a factor of $(1+g)^n$. This is based on the notion that there is "g%" growth in the genetic pool per period and "n" periods have elapsed since the start of the first cow.

The present value of the second cycle will then be:

$$\left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t}\right]\left[\frac{(1+g)^n}{(1+i)^n}\right] \tag{2}$$

Where:

g = the rate of genetic improvement occurring per year expressed as a percentage increase in milk yield per year per cow.

The first cycle has grown by the factor "g" per lactation over the "n" periods of the first cycle but it starts "n" periods hence and so must be brought back to the present. By similar reasoning, the present value of cycle "k" is:

$$\left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t}\right]\left[\frac{(1+g)^{n(k-1)}}{(1+i)^{n(k-1)}}\right] \tag{3}$$

The sum of the present values of all future cycles can then be expressed as:

$$NPV_{milk} = \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t}\right]\left[1+\frac{(1+g)^n}{(1+i)^n}+\frac{(1+g)^{2n}}{(1+i)^{2n}}+\frac{(1+g)^{3n}}{(1+i)^{3n}}+\ldots+\frac{(1+g)^{(k-1)n}}{(1+i)^{(k-1)n}}\right]. \tag{4}$$

If this process is repeated for an infinite number of cycles (i.e. as "k" approaches infinity), then the net present value is:

$$NPV_{milk} = \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t}\right]\left[\frac{(1+i)^n}{(1+i)^n-(1+g)^n}\right] \tag{5}$$

2. The Calf and Capital Component

The capital component consists of the initial cost of the cow and her subsequent salvage value. For convenience, the sale of the calf will be included with the capital component of the model. It is assumed that there is no growth to either component. This assumption rests on the premise of an efficient market.

The present value of the first cycle is:

$$NPV_{capital} = \left[-C+\frac{SV_n}{(1+i)^n}+\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}\right] \tag{6}$$

Where:

C = the cost of the replacement heifer,

SV_n = the salvage value of the existing cow in period "n", and

B_t = the value of the calf in period "t".

It follows that the present value of the second cycle is:

$$NPV_{capital} = \left[-C+\frac{SV_n}{(1+i)^n}+\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}\right]\left[\frac{1}{(1+i)^n}\right] \tag{7}$$

and that the present value for cycle "k" is:

$$NPV_{capital} = \left[-C+\frac{SV_n}{(1+i)^n}+\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}\right]\left[\frac{1}{(1+i)^{(k-1)n}}\right] \tag{8}$$

The sum of all future cycles can then be expressed as:

$$NPV_{capital} = \left[-C+\frac{SV_n}{(1+i)^n}+\sum_{t=1}^{n} \frac{B_t}{(1+i)^t}\right]\left[1+\frac{1}{(1+i)^n}+\frac{1}{(1+i)^{2n}}+\frac{1}{(1+i)^{3n}}+\ldots+\frac{1}{(1+i)^{(k-1)n}}\right]. \tag{9}$$

Which in the limit reduces to:

$$NPV_{capital} = \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] \quad (10)$$

3. Total Net Present Value

Combining the two components into one equation results in:[4]

$$NPV_n = \left[\left\{ \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right\} \right] \quad (11)$$

If there was no involuntary culling of cows, (i.e. all cows would continue through to period "n"), then one would be justified in directly invoking the decision rule coming from

$$NPV^* = \underset{n=1}{\overset{N}{\mathrm{MAX}}} \; [NPV_n]. \quad (12)$$

$$NPV^* = \underset{n=1}{\overset{N}{\mathrm{Max}}} \left[\left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right] \quad (13)$$

If NPV is maximum, then keeping the cow for one more period will cause a decrease in NPV. That is to say that:

$$NPV_n > NPV_{n+1}. \quad (14)$$

C. Involuntary Disposals

To this point cows have been assumed to remain in the herd until voluntarily removed by the owner. Because of disease and other such factors, cows may be removed involuntarily at the end of a particular lactation, or even during a lactation.

For this reason, equation (13) must be modified to reflect the fact that involuntary replacement might occur. We assume that such replacement occurs after a lactation is completed. Equation (15), is the mathematical representation of the net present value incorporating both operating and capital flows for all cycles from one through "N" periods of "n" length:

$$NPV_n = \left[\left\{ \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{A_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right\} \right] \quad (15)$$

This results in a vector of all possible net present values:

$$[NPV] = \begin{bmatrix} NPV_1 \\ NPV_2 \\ \cdots \\ \cdots \\ NPV_N \end{bmatrix} \quad (16)$$

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4 Equation 11 can be expressed on an annual basis as:

$$ANPV_n = \left[\left\{ \left[-C + \frac{SV_n}{(1+i)^n} + \sum_{t=1}^{n} \frac{B_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - 1} \right] + \left[\sum_{t=1}^{n} \frac{V_t}{(1+i)^t} \right] \left[\frac{(1+i)^n}{(1+i)^n - (1+g)^n} \right] \right\} (i) \right]$$

1. Probability of Involuntary Disposal

The involuntary disposal situation can be described as a discrete - finite Markov process (Hillier and Lieberman, 1986; Agrawal and Heady, 1972). It is finite because there is a finite number of states (lactations) possible and it is discrete because only integer states (lactations) are to be considered. With each lactation there is a probability that the lactation will be completed and that the cow will proceed to the next lactation. The compliment of this probability is the probability that the cow will fail to advance to the next lactation. Thus, the progression of a dairy cow from one lactation to the next can be represented in the form of a transition probability matrix T with elements T_{ij}, $i = 1,2,...n;\ j = 1,2,...n$.
Thus, the appropriate Markov process can be represented by:

$$\begin{bmatrix} d_1 & d_2 & d_3 & \cdots & d_{n-1} & 1 \\ 1-d_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1-d_2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1-d_3 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1-d_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{k,1} \\ C_{k,2} \\ C_{k,3} \\ \cdots \\ C_{k,n-1} \\ C_{k,n} \end{bmatrix} = \begin{bmatrix} C_{k+1,1} \\ C_{k+1,2} \\ C_{k+1,3} \\ \cdots \\ C_{k+1,n-1} \\ C_{k+1,n} \end{bmatrix} \qquad (17)$$

With $[C_{k,1} + C_{k,2} + C_{k,3} + \dots + C_{k,n} = 1]$

where matrix T is post multiplied by vector $C_{k,n}$ containing the herd composition in the current period to obtain the herd composition in the subsequent time period $C_{k+1,n}$.

Of necessity, the herd composition vector must be exhaustive (i.e. account for 100% of the cows - sum to 1)

The cells in the first row of the matrix T represent the transition probabilities of a cow failing involuntarily in the specific lactation and being replaced by another cow. The cell $T_{2,1}$ in the succeeding row of the matrix T represents the probability $(1-d_1)$ that the cow will in be state 2 in the next period (i.e., that the cow will not be culled involuntarily). For the cow in lactation 2 there is a probability $(1-d_2)$ that she will advance to lactation 3 in the next period and a probability d_2 that she will be involuntarily culled and replaced with a cow that is in lactation 1. If a cow is in lactation $n-1$ then there is a probability $(1-d_{n-1})$ that she will advance to state "n" and a probability d_{n-1} that she will be culled. A cow in lactation "n" will be culled with certainty. C_j represents the proportion of cows in lactation "j"

Solution of the system in steady state results in:

$$\begin{bmatrix} C_{k,1} \\ C_{k,2} \\ C_{k,3} \\ \cdots \\ C_{k,n-1} \\ C_{k,n} \end{bmatrix} = \begin{bmatrix} C_{k+1,1} \\ C_{k+1,2} \\ C_{k+1,3} \\ \cdots \\ C_{k+1,n-1} \\ C_{k+1,n} \end{bmatrix} \qquad (18)$$

or

$$TC_s = C_s. \qquad (19)$$

Where C_j is the steady state vector representing herd composition.

Conditions for Steady State to Occur

$$1)\ [T][C_s] = [C_s] \qquad (20)$$

$$2)\ \sum_{j=1}^{N} C_j = 1$$

For example, in a two lactation pattern:

$$\begin{bmatrix} d_1 & 1 \\ 1-d_1 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \qquad (21)$$

$$C_1 + C_2 = 1$$

Multiplication results in three equations and two unknowns:

$$C_1 d_1 + C_2 = C_1$$

$$(1-d_1)C_1 + 0 = C_2 \qquad (22)$$

$$C_1 + C_2 = 1$$

This system of equations can be solved using the following two equations; the number of cows entering the second lactation and the total number of cows in lactation one and two.

$$C_1 + C_2 = 1 \qquad (23)$$

$$(1-d_1)C_1 - C_2 = 0$$

This can be expressed as:

$$\begin{bmatrix} 1 & 1 \\ (1-d_1) & -1 \end{bmatrix} \begin{bmatrix} C_{j,1} \\ C_{j,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad (24)$$

And solved by inversion as follows:

$$\begin{bmatrix} 1 & 1 \\ (1-d_1) & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{j,1} \\ C_{j,2} \end{bmatrix} \qquad (25)$$

In general, one can create the adjusted matrix T_a from the matrix T and solve the system:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & -1 \\ 1-d_1 & -1 & 0 & 0 & 0 \\ 0 & 1-d_2 & -1 & 0 & 0 \\ 0 & 0 & 1-d_3 & -1 & 0 \\ 0 & 0 & 0 & 1-d_{n-1} & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{s,1} \\ C_{s,2} \\ C_{s,3} \\ C_{s,n-1} \\ C_{s,n} \end{bmatrix} \qquad (26)$$

2. Cow Replacement Patterns

By examining the within the herd composition, it can be concluded that those cows in the herd at their "n"th lactation are part of a recycling pattern "n" lactations in length. This means that there must be C_n cows in this recycling pattern in each lactation up to and including the "n"th lactation. For example, if there are ten cows in the herd in their fifth lactation and there is a replacement strategy that dictates all cows are replaced after five lactations, it must be so that there are fifty cows in the herd predestined to be replaced after their fifth lactation. Of the cows (suppose for example there are sixteen of them) in their fourth lactation, there will be included those cows predestined to go on to their fifth lactation, (i.e. ten will go on to their fifth lactation, while six will be involuntarily culled after the fourth lactation). Thus, there must be 24 cows (4 x 6) to support this pattern. The cows in the remaining patterns can be calculated in similar fashion.

In general, the solution to the Markov system can be portrayed in matrix form as $C_1 > C_2 > C_3 > \ldots > C_{n-1} > C_n$.[5] In general if there are C_n cows in a pattern that recycles in "n" periods, then there must be C_n in each lactation stage to feed this cycle. There must, as a result be nC_n cows in the total system. If there are C_n cows in state n there must be C_{n-1} in state n-1 but C_n of these cows are already part of the "n" cycle pattern. Therefore, there are $C_{n-1} - C_n$ cows in the n-1 cycle at each stage in the system. The total number of such cows, predestined to be replaced in n-1 years is then $(n-1)(C_{n-1} - C_n)$. This pattern will continue until the last state where $1(C_2 - C_1)$. The cows in each pattern will then be:

$$\begin{bmatrix} 1(C_1 - C_2) \\ 2(C_2 - C_3) \\ \ldots \\ (n-1)(C_{n-1} - C_n) \\ n(C_n) \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ \ldots \\ Q_{n-1} \\ Q_n \end{bmatrix} \qquad (27)$$

where vector Q represents the number of cows in each pattern.

Having determined the proportion of the herd in various patterns of replacement, it is possible to determine the average net present value of the herd. This can be accomplished by weighting the net present values computed as the vector NPV_n in equation (19).

D. Optimum Replacement Age

The optimum replacement age is that age which will yield the maximum weighted annual net present value.

In terms of the vectors from Equation 16 and Equation 27 this age is:

$$[Q_1 \quad Q_2 \quad Q_3 \quad Q_{n-1} \quad Q_n] \begin{bmatrix} NPV_1 \\ NPV_2 \\ NPV_3 \\ NPV_{n-1} \\ NPV_n \end{bmatrix} = NPV_n^* \qquad (28)$$

This procedure will allow the maximization of the WNPV function such that the maximum value of the weighted average NPV of the stall can be determined.[6]

The optimum replacement time is determined by finding the maximum weighted perpetual net present value:

$$WNPV_{n^*} = \underset{n=1}{\overset{N}{\mathrm{MAX}}}\{Q'NPV\} \qquad (29)$$

where

Q' = the transposed vector of weighted proportions in the herd at each lactation,

NPV = the vector of perpetual net present values for cows kept for cycle "n", and

"n" = the optimum economic replacement cycle.

5 To reduce notational complexity, the subscript "s" will be dropped on the understanding that from here on the vector C is in steady state.

6 The multiplication of the WNPV by the discount rate (either on a per annum format or on a per lactation format) will give the annual or per lactation value of the associated perpetual cashflows.

III. Results and Analysis

The data used in the analytical model is described in detail in Appendix II. The lifetime lactation curve used in the model is derived in Section A. The relevant cashflows are determined in Section B. The veterinary costs associated with each lactation are set forth and discussed. Also a full discussion of the physical data is set forth. The appropriate discount is derived in Section C. Section D discusses the involuntary disposal of cows and the impact on the cashflows and the resultant weighted net present values.

A. The Base Case

Using the vector NPV obtained in Chapter 2, and multiplying by the vector Q from Appendix II, the following weighted net present values are obtained:

Lact.	1	2	3	4	5	6	7	8	9	10
NPV	11351	14000	14157	13261	12360	11354	10374	9563	8789	8021
% Max	.80	.99	1.00	.94	.87	.80	.73	.68	.62	.56
WNPV	11351	13825	14011	13331	12674	11998	11387	10915	10505	10145
% Max	.81	.91	1.00	.95	.90	.86	.81	.78	.75	.72

Maximization of the vector WNPV results in the determination of the optimal replacement time, which in our case occurs at the end of lactation three where the WNPV reaches a maximum value of $14011.

For ease of understanding the results are expressed on a per annum basis. Since the net present values are perpetual net present values, this transformation is accomplished simply through multiplication by the per annum discount rate.

The ANPV function for the base case is relatively insensitive to the changing values of n (lactation number). The value of the ANPV is 90% or greater of the maximum ANPV value achieved in lactation three (see Figure 5.1) for lactations two, three, four and five.

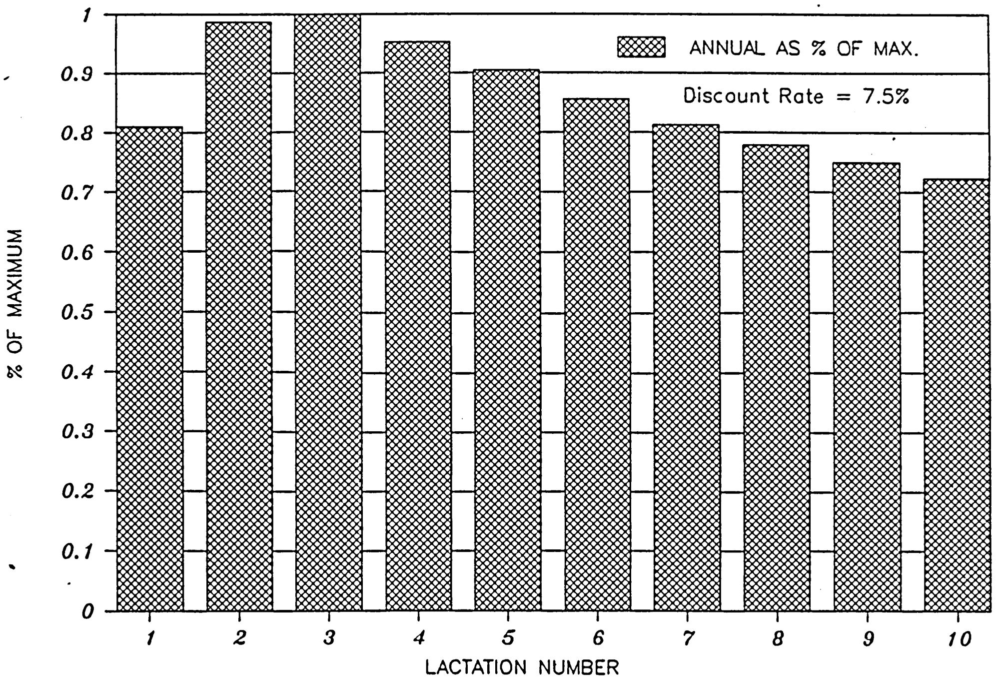

Figure 3.1 ANPV As A Per Cent Of Maximum For Discount Rate of 7.5%

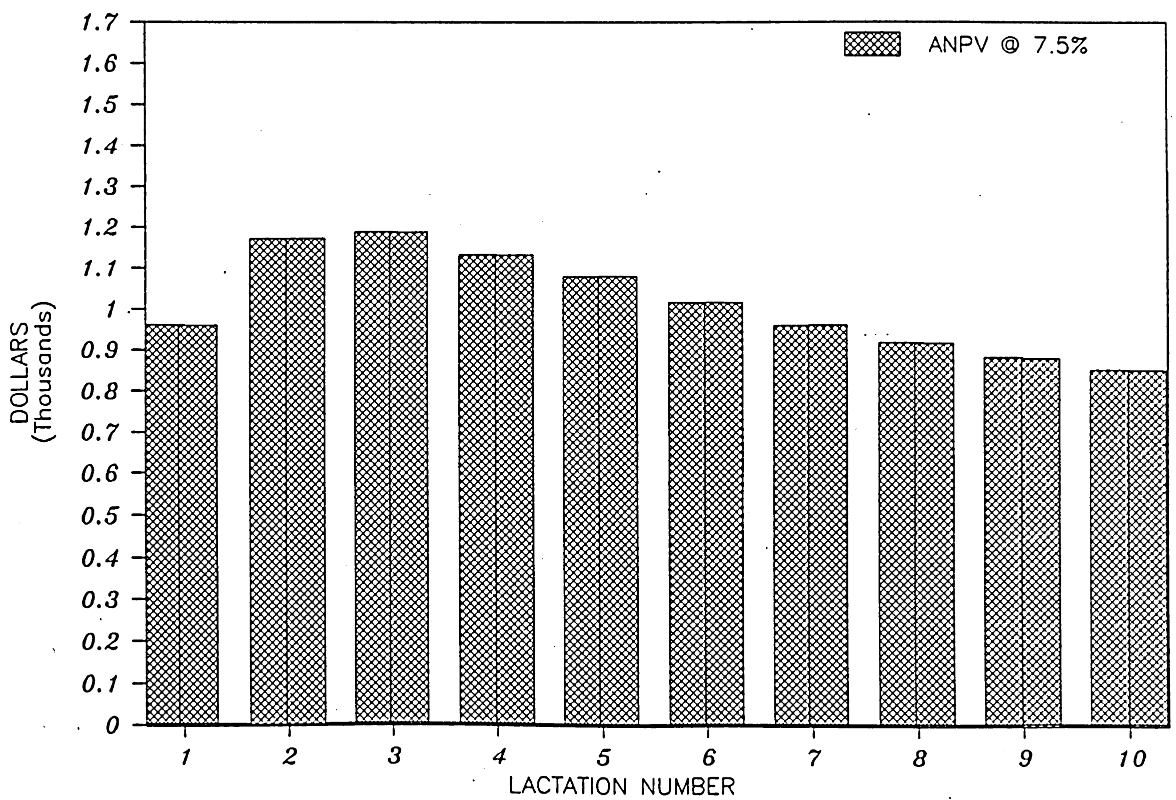

Figure 3.1a Annualized NPV Over Time.

The model predicts that with voluntary replacement occurring at the end of lactation three that the average age of cows in the herd would be 4.715 years which compares to the actual average age of cows in the provincial dairy herd, as obtained from ADHIS, of 4.78 years.

The model predicts replacement at the end of the third lactation. Data obtained from ADHIS indicates that the average age of disposal of cows from the dairy population in Alberta is 5.3 years of age which is during the cows third lactation. Given the age of disposal, the point of disposal in terms of lactations would be 2.64 lactations.

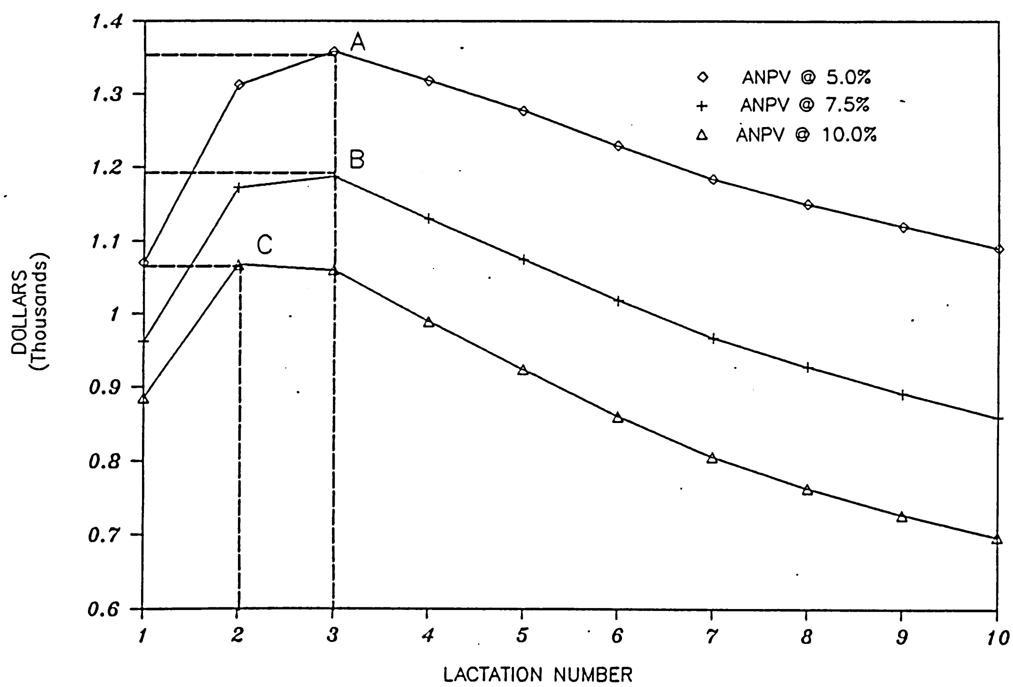

Figure 3.2. Changes in Replacement Time.

B. Discount Rates and Rates of Genetic Improvement.

An increase in the discount rate results in a decrease in the value of the ANPV function. As can be seen in Figure 3.2, an decrease in the ANPV function moves the function downwards.

While the ANPV function decreases due to the higher discount rate, it also reaches its maximum value sooner, resulting in the replacement point occuring earlier (point C in Figure 3.2) than at the lower discount rates.

When the ANPV function increases as the result of a lower discount rate, it moves upwards and shifts the point of maximization to the right (point A in Figure 3.2), resulting in an increase in the replacement time.

As can be seen in Figures 3.3 and 3.4, as the discount rate changes, so does the nature of the ANPV function. Figure 3.4 depicts a discount rate of 5.0%. Here the ANPV is at 90% or greater of the maximum ANPV from lactation two thru lactation six. This is a wider range of lactations than for the base case discount rate of 7.5%. When the discount rate increases to 15.0% (Figure 3.3) the ANPV is at 90% or greater of the maximum ANPV only for lactations two thru four. Thus, as the discount rate increases the ANPV function maximizes earlier but deteriorates quicker than at lesser discount rates.

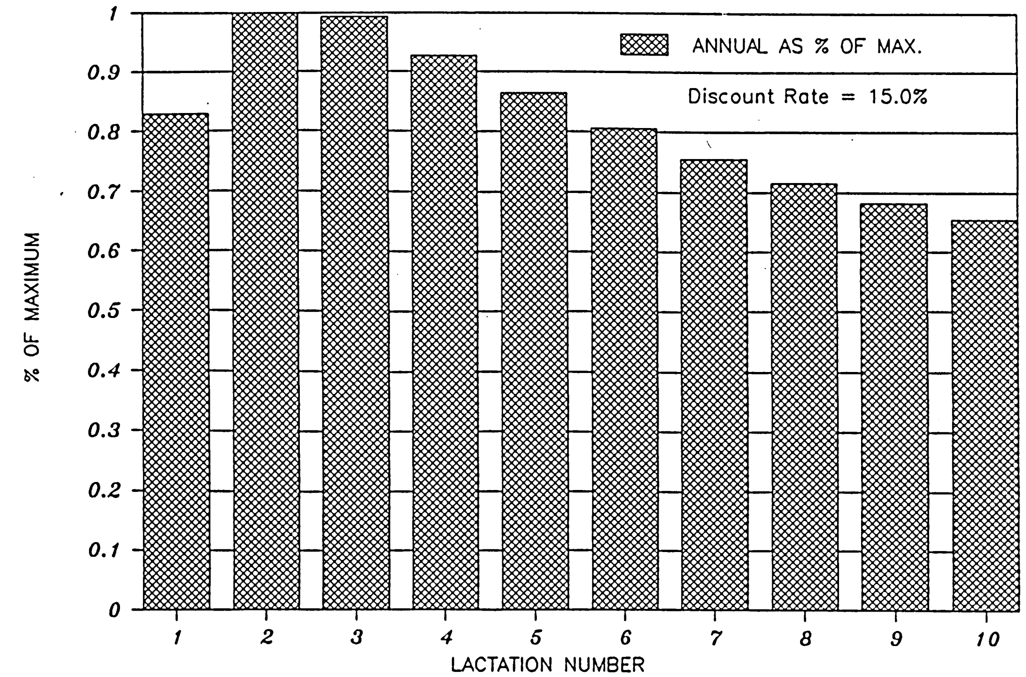

Figure 3.3 ANPV As A Per Cent Of Maximum For Discount Rate of 15.0%

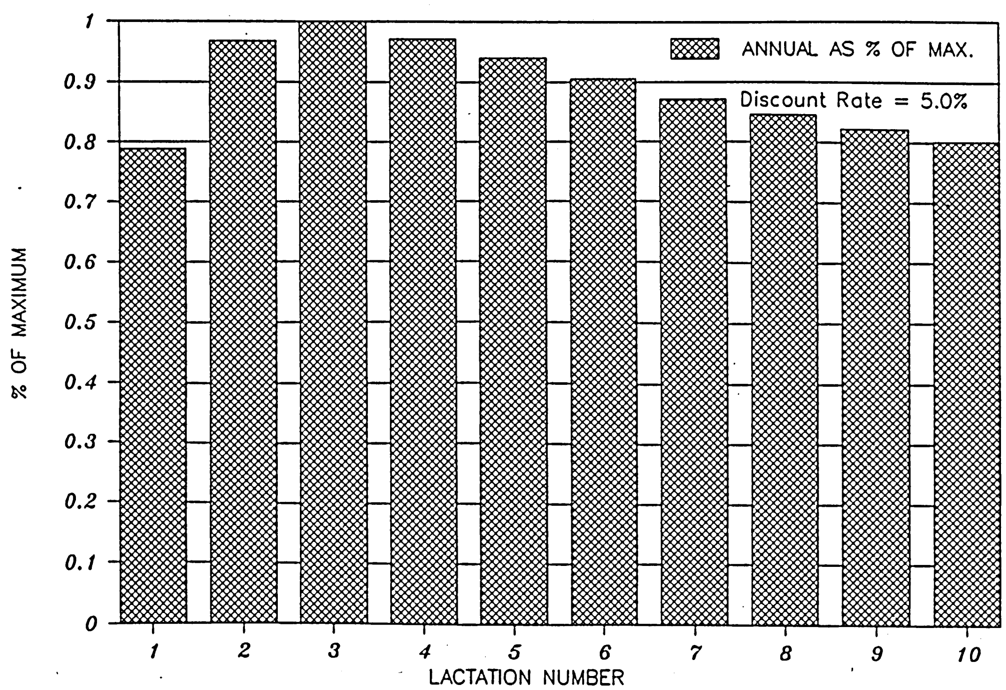

Figure 3.4 ANPV As A Per Cent Of Maximum For Discount Rate of 5.0%

Table 3.1 Discount Rate and Rate of Genetic Improvement			
Discount Rate	Genetic Improve.	Replacement Time	ANPV
5.0%	0.0%	3	$1219
	0.5%	**3**	**$1359**
	1.0%	3	$1533
	1.5%	3	$1758
	2.0%	3	$2057
7.5%	0.0%	3	$1107
	0.5%	**3**	**$1188**
	1.0%	3	$1281
	1.5%	2	$1391
	2.0%	2	$1530
10.0%	0.0%	2	$1007
	0.5%	**2**	**$1067**
	1.0%	2	$1134
	1.5%	2	$1210
	2.0%	2	$1294
15.0%	0.0%	2	$869
	0.5%	**2**	**$904**
	1.0%	2	$941
	1.5%	2	$982
	2.0%	2	$1025

1. Sensitivity To Discount Rates.

As indicated in Chapter II. Section B.3. "Determining The Cash Flows", the dairy cow replacement question is comprised of two parts. First is the capital component embodied in the initial cost and terminal value of the cow and value of calf produced and secondly, the annual operating component comprised of milk production

The discount rate affects both parts of this equation. As the discount rate increases, the present value of the stream of operating receipts decreases. But, if the replacement question is viewed as a question of capital intensity, then the higher the discount rate becomes, the greater the value of the capital (the cost of the cow). Thus, as the discount rate increases, the operator would rationally postpone the replacement process by keeping the existing cow longer than he would given a lower discount rate. This characteristic is then offset by the increased value (due to the higher discount rate) of the annuity value of the operating receipts taken in perpetuity. This perpetuity is also growing in value at each stage because of the impact of genetic improvement on the annuities.

The model is sensitive to the discount rate. At a discount rate of 5%, the optimal point of replacement is at the end of lactation three with a ANPV of $1359.

A rise in the discount rate to 7.5% (the base case) leaves the point of replacement unchanged and a decreases the ANPV to $1188.

Increasing the discount rate to 10% decreases the point of replacement to the end of the second lactation and decreases the ANPV to $1067.

Changing the discount rate to 15%, leaves the economical replacement point to the end of lactation two and further decreases the ANPV of the residual net income to a low of $904.

Thus, a change of 10% in the discount rate decreases the ANPV from $1359 to $904 or $455 in total while increasing the point of replacement to the end of the second lactation from the end of the third lactation.

2. Sensitivity To Genetic Improvement Rates.

When no genetic improvement is present the economical replacement point is at the end of lactation three with a ANPV of $1107. By increasing genetic improvement to a rate of 2% per year, the economical replacement point changes to the end of lactation two with a ANPV of $1530.

Again, a large change in the range of genetic improvement alters the ANPV of the residual net cashflow by $423 and shortens replacement cycle by one lactation.

If the discount rate used was very low (i.e. 2.5% or 3%) and the rate of genetic improvement per year was at the high level (2.0%), the replacement cycles would become shorter. The model would explode when the discount rate equaled the rate of genetic improvement. This would result in a 0 value in the denominator of the calculation of the perpetuity value. Effectively, the inclusion of a factor for genetic improvement lowers the discount rate used.

C. Using Varying Levels of Feed and Milk Prices

Table 3.2. Milk Price versus Feed Prices

Milk Price	Hay	Grain	Replacement Time	ANPV
$39.11	$0.08	$0.20	3	$ 946
	$0.11	**$0.25**	**3**	**$ 701**
	$0.17	$0.30	3	$ 311
$49.11	$0.08	$0.20	3	$1433
	$0.11	**$0.25**	**3**	**$1188**
	$0.17	$0.30	3	$ 798
$59.11	$0.08	$0.20	2	$1926
	$0.11	**$0.25**	**2**	**$1675**
	$0.17	$0.30	3	$1285

1. Sensitivity To Input and Output Prices.

Changes in input and output prices produce replacement timing changes. When milk prices were increased, the replacement timing decreased to the end of the second lactation from the base scenario replacement point of at the end of the third lactation.

Increasing or decreasing feed prices resulted in no change in the point of replacement but rather resulted only in increases or decreases in the ANPV.

D. Varying Replacement Costs and Salvage Values of Cows

Table 3.3. Cost of Replacement and Salvage Values of Cows

Replacement Cost	Salvage Value Of Cow	Replacement Time	ANPV
$900	Low	3	$1217
	Base	2	$1275
	High	2	$1336
$1000	Low	3	$1182
	Base	2	$1223
	High	2	$1284
$1100	Low	3	$1146
	Base	**3**	**$1188**
	High	2	$1233
$1200	Low	3	$1111
	Base	3	$1152
	High	3	$1193
$1350	Low	3	$1058
	Base	3	$1099
	High	3	$1140

1. Sensitivity To Cost of Replacement

Decreasing the cost of replacements increases the value of the ANPV of the residual cashflows and moves the point of replacement at the end of lactation two. Increasing the cost of the replacements does not affect the replacement cycle while decreasing the ANPV of the residual cashflows.

The range of the ANPV of the residual cashflow is from a high of $1275 to a low of $1099 for a range of $176 with the higher value being associated with the lower priced replacements.

2. Sensitivity To The Salvage Value of the Cow

Changing the salvage value of the base cow results in a range of ANPV of the residual cashflow of $87 with the higher salvage value cows having the low value and the lower salvage value cows the highest value. The salvage value of the cow is directly related to the age and body weight of the cow.

The higher salvage value cows produced a range of values of the ANPV of $1336 to $1140 as the replacement cost rose from $900 to $1350. The higher cost heifers were kept one lactation longer than the lower cost heifers.

The low salvage value cows produced a range of ANPV of the residual cashflow of $1217 to $1058 as replacement cost rose from $900 to $1350. The higher cost heifers were kept the same number of lactations as the lower cost heifers.

The model is more sensitive to changes in replacement cost and body weight combined than to the variables independently.

E. Varying Production Levels and Calving Intervals

Table 3.4. Varying Levels of Milk Production and Calving Intervals

Milk Production	Calving Interval	Replacement Time	ANPV
Low 6000 kg./lact.	12.0 months	3	$ 777
	13.5 months	3	$ 748
	15.0 months	3	$ 720
Base 7401 kg./lact.	12.0 months	3	$1228
	13.5 months	3	$1188
	15.0 months	3	$1148
High 12,000 kg./lact.	12.0 months	3	$1862
	13.5 months	3	$1805
	15.0 months	2	$1760

1. Sensitivity To Levels of Milk Production.

Increases in milk production increase the value of the ANPV function. While high production increases the ANPV; it does not change the economical replacement point in time.

Decreases in milk production decreases the value of the ANPV but leaves the replacement cycle unchanged at the end of the third lactation.

2. Sensitivity To Changes In Calving Intervals.

The model is sensitive to increasing the calving interval (increasing the length of each lactation) in terms of decreasing the ANPV of residual cashflow values but the economical replacement time is unchanged at all the levels of milk production examined except at the high level of 12,000 kilograms at which point the optimal replacement time occurs at the end of the second lactation.

F. Replacement Strategies Summarized

Table 3.5 Replacement Strategies Summarized

Case	LACTATION REPLACE	ANP VALUES
Base Case	**3**	**$1188**
12.0 month calving interval	3	$1228
15.0 month calving interval	3	$1148
High Prod. - 12,000 kg.	3	$1805
Low Prod. - 6,000 kg.	3	$ 748
5% Discount Rate	3	$1359
10% Discount Rate	2	$1067
15% Discount Rate	2	$ 904
0% Genetic Improve.	3	$1107
1.0% Genetic Improve.	3	$1281
1.5% Genetic Improve.	2	$1391
2.0% Genetic Improve.	2	$1530
$900 Replacement	2	$1275
$1000 Replacement	2	$1223
$1200 Replacement	3	$1152
$1350 Replacement	3	$1099
Milk Price = $39.11	3	$ 701
Milk Price = $59.11	2	$1675
Feed Price Increase	3	$ 798
Feed Price Decrease	3	$1433
High Salvage Value of Cows	2	$1233
Low Salvage Value of Cows	3	$1146
$900 - High Salvage Value	2	$1336
$1000 - High Salvage Value	2	$1284
$1200 - High Salvage Value	3	$1193
$1350 - High Salvage Value	3	$1140
$900 - Low Salvage Value	3	$1217
$1000 - Low Salvage Value	3	$1182
$1200 - Low Salvage Value	3	$1111
$1350 - Low Salvage Value	3	$1140

1. Summary.

Over all the optimum replacement time was insensitive to changes in individual variables. Replacement time was, however sensitive as model variables were changed jointly. Not only is the optimum replacement time stable at lactation 3 there is very little economic difference, in terms of net present value, between replacement at lactation 2 through lactation 5. Thus economic penalties associated with sub-optimality are not great.

While the optimum replacement time was found to be relatively stable over a broad range of lactations, it must be pointed out that total, and consequently also annual net present values, did vary considerable in magnitude with respect to variations in model parameters. For example increases or decreases in milk prices cause income levels to vary but do not cause shifts in optimum replacement times.

Therefore, as long as the cow is healthy, calving regularly, and producing at a reasonable level, there is no need for her replacement. This appears consistent with conventional producer behavior where cows can be allowed to "cull themselves" through reproductive failure, low production, sickness, injury or death.

IV. Conclusions and Summary

A. Summary

Dairy extension officers and the Alberta Dairy Seminar Committee have identified the economic life of a dairy cow as an important area of concern to the dairy industry. The purpose of this research was to develop an explanatory economic model to shed light on this question especially as it relates to the situation where cows are replaced by genetically superior ones.

Specifically, the objective of the study was to identify economically optimum replacement strategies for Alberta dairy cows where replacement occurs with genetically superior animals, and subsequently to determine differences between the calculated strategies and current Alberta practice.

Production data were collected from four co-operating dairy producers. A milk yield curve across lactations was developed from these data. This lactation curve, along with other data was used to specify the present value of net annual residual cashflows from the dairy cow. Included in the cashflows were the original value of the cow, her salvage value at the end of each lactation, the value of the calf produced plus the annual net operating revenue from milk production. The net present values for each feasible replacement pattern were evaluated using a finite Markov process to account for involuntary replacement of cows. This resulted in specification of a steady state herd composition in which cows would be replaced in such a manner as to maximize the net present value of the herd through time.

The model was tested for sensitivity to variations in basic parameters and although there was some sensitivity to variations in parameters, especially when varied jointly, the optimum replacement time remained at stable at lactation number three. Not only was the optimum replacement time stable, penalties for deviation from the optimum were economically insignificant.

The calculated economic replacement strategy was compared to the current Alberta situation as revealed in Alberta Dairy Herd Services data. No major differences were found between the replacement age predicted and that which is occurring in the industry. Furthermore, the average age of the herd calculated from the optimal strategy is almost identical to that observed in the Alberta dairy industry. It was concluded that there are no major differences between the calculated economic optimum replacement strategy and the current replacement practices being practiced by Alberta dairymen.

B. Further Research

Certain restrictive assumptions were made in this research. Because of data limitations knowledge about likely performance in subsequent lactations gained during the current lactation was notincorporated into the model. Restrictions exist also because reliable health care costs, particularly as the cow ages, and data on involuntary culling rates are lacking for Alberta conditions. As a result these data were obtainable only from published secondary literature sources. Further investigation into veterinary costs relative to age and lactation number as well as production level is recommended. The rate of change in veterinary costs as the cow ages can be of importance in the replacement decision process.

While herd demography should be studied more deeply, there appears to be a lack of adequate data to facilitate such studies. Effort could profitably be directed towards developing a more adequate data base for farm level dairy research.

Knowledge about the lifetime lactation curve of dairy cows is a deficient area in the animal science literature and yet is the single most important determinant of replacement strategies. The data available are confounded because dairy farmers are operating in an economic environment making voluntary culling decisions while involuntary culling is present at the same time. Further controlled experimentation is suggested to resolve this issue.

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VI. APPENDIX I

A. Ordinary Least Squares Results - All Cows.

Observations: 2640 Degrees of freedom: 2624
R-squared : 0.519 Rbar-squared : 0.516
Residual SS :4.373E+009 Std. error of est. : 1290.940
Total SS :9.085E+009 F(16 ,2624)=188.4971 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	1703.4361	0.0000	321.8398	5.2928
AGE	-35.2192	-0.0386	47.0279	-0.7489
INT	-100.6928	-0.0966	17.8130	-5.6527
MILK	24.6382	0.7370	0.5790	42.5465
BFAT	-540.9538	-0.1181	63.1496	-8.5662
D2	898.0042	0.2036	85.4321	10.5113
D3	1461.6843	0.2955	124.8124	11.7110
D4	1498.1974	0.2559	170.3822	8.7931
D5	1430.8752	0.2012	218.9325	6.5356
D6	1234.0803	0.1402	272.6154	4.5268
D7	1253.8706	0.1063	335.1883	3.7407
D8	1593.2653	0.1036	402.1491	3.9618
D9	1495.3037	0.0698	485.6216	3.0791
D10	1268.5747	0.0398	615.2029	2.0620
D11	2218.8230	0.0402	895.2204	2.4785
D12	1232.3630	0.0129	1399.5943	0.8805

B. Ordinary Least Squares Results - 4 or More Lactations.

Observations: 1620 Degrees of freedom: 1604
R-squared : 0.496 Rbar-squared : 0.491
Residual SS : 2.502E+009 Std. error of est. : 1248.944
Total SS : 4.960E+009 F(16 ,1604)=105.0461 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	1104.0647	0.0000	406.5746	2.7155
AGE	-88.8451	-0.1110	60.2356	-1.4749
INT	-83.3103	-0.0810	23.5897	-3.5316
MILK	24.0205	0.6950	0.8047	29.8501
BFAT	-417.8504	-0.0932	81.2215	-5.1445
D2	968.9205	0.2093	122.4543	7.9125
D3	1669.3501	0.3642	163.9484	10.1821
D4	1948.3320	0.4308	216.9615	8.9800
D5	1931.9964	0.3584	276.4489	6.9886
D6	1797.3016	0.2720	341.7042	5.2598
D7	1887.7776	0.2148	414.5106	4.5542
D8	2277.0302	0.1994	490.6084	4.6412
D9	2214.2481	0.1397	577.8765	3.8316
D10	2036.3653	0.0865	704.7815	2.8893
D11	3000.9394	0.0737	964.0102	3.1129
D12	2118.4518	0.0300	1430.8936	1.4805

C. Ordinary Least Squares Results - 5 or More Lactations.

Observations: 1219
R-squared: 0.489
Residual SS: 1.910E+009
Total SS: 3.735E+009

Degrees of freedom: 1203
Rbar-squared: 0.482
Std. error of est.: 1260.055
F(16 ,1203)=76.6263 P-value=0.00

Var	Coef.	Std. Coef.	Std. Error	t Stat
CONST	824.4844	0.0000	498.4302	1.6541
AGE	-20.5963	-0.0272	73.1703	-0.2814
INT	-87.2005	-0.0813	27.8222	-3.1342
MILK	24.1789	0.6658	0.9514	25.4121
BFAT	-415.8598	-0.0873	101.3691	-4.1024
D2	835.8940	0.1732	149.9851	5.5731
D3	1405.3713	0.2887	200.5257	7.0084
D4	1767.2721	0.3670	264.4506	6.6828
D5	1757.0593	0.3664	333.9928	5.2607
D6	1541.4569	0.2652	411.9913	3.7414
D7	1560.2684	0.2031	497.3623	3.1370
D8	1877.6796	0.1887	584.6997	3.2113
D9	1745.5008	0.1266	678.9043	2.5710
D10	1497.8618	0.0732	811.5690	1.8456
D11	2383.9375	0.0674	1065.9481	2.2364
D12	1431.9580	0.0234	1520.7792	0.9415

D. Frequency Distribution of Sample Lactations.

Lact	N	Mean	St Dev	Variance	Minimum	Maximum
1	833	6214.1	1870.5	.34989E+07	475	14503
2	609	6729.7	1804.1	.32549E+07	583	12642
3	451	7188.4	1942.9	.37748E+07	388	13199
4	299	7289.7	1809.2	.32733E+07	1770	12925
5	197	7094.5	1842.9	.33962E+07	195	12124
6	125	6977.2	1688.7	.28518E+07	2510	13748
7	68	6992.3	2092.9	.43801E+07	2462	13950
8	40	7056.5	2183.0	.47654E+07	3000	11023
9	20	7406.7	1431.9	.20503E+07	4186	10355
10	10	6611.1	2460.9	.60561E+07	1064	10015
11	3	6372.7	2694.7	.72612E+07	3272	8148
12	1					

1. Distribution Of Commercial Cow Disposals.

Commercial Herd Disposals		
Lactation Number	Number Of Disposals	Cummulative Disposals in %
1	224	26.89%
2	158	45.86%
3	152	64.11%
4	102	76.35%
5	72	84.99%
6	57	91.84%
7	28	95.20%
8	20	97.60%
9	10	98.80%
10	7	99.64%
11	2	99.88%
12	1	100.00%

2. Distribution Of ADHIS Disposals

ADHIS Herd Disposals		
Lactation Number	Number Of Disposals	Cummulative Disposals as %
1	12329	26.34%
2	12238	52.50%
3	8456	70.56%
4	5776	82.90%
5	3760	90.95%
6	2153	95.55%
7	1157	98.02%
8	536	99.17%
9	240	99.68%
10+	150	100.00%

E. Dairy Ration to Production Relation

Grain Feeding Guide With Good Forage[7]						
		Concentrate per Day				
		Butterfat				
Milk per Day		3.5%		4.0%		5.0%
lb.	kg.	lb.	kg.	lb.	kg.	lb.
20	9.1	3	1.4	5	2.3	7
25	11.3	6	2.7	7	3.2	10
30	13.6	8	3.6	9	4.1	12
35	15.9	11	5	12	5.4	15
40	18.1	13	5.9	14	6.4	18
45	20.4	16	7.3	18	8.2	22
50	22.7	19	8.6	21	9.5	26
55	25.0	23	10.4	25	11.3	30
60	27.2	27	12.2	29	13.2	34
65	29.5	31	14.1	33	15.0	
70	31.7	35	15.9	37	16.8	
75	34.0	39	17.7			
80	36.3					
85+	38.6+					

7 adapted from Alberta Agriculture Farm Mamagement Guide. 1976. page 200.

F. Grain Feeding Guide With Good Forage

Grain Feeding Guide With Good Forage Used In The Model	
Milk per Day	Concentrate per Day
kg.	kg.
5	0
9	2
11	3
14	4
16	5
18	6
20	8
23	10
25	11
27	13
30	15
32	17
34	19

VII. Appendix II
The Empirical Model and Background Data

A. Lifetime Milk Yield Curve

A sample of 833 cows representing 2656 lactations was collected from four dairy producers.

An Ordinary Least Squares Regression was carried out on the data using the independent variables: Age, Calving Interval, Days In Milk, Butterfat, and a series of Dummy Variables to represent Lactation Number and a Constant while Milk Production was the dependent variable. The results of this OLS Regression are as indicated in Appendix I.

The Predicted Milk Yields by lactation from the OLS Regression, the Mean Yields and Standard Deviations associated with the samples are reported in Table A2.2.

The sample data used in determining the Regression Coefficients are biased in that the sample data have been selected by virtue of production and longevity. Only those cows with superior production and longevity characteristics are represented in the later lactation numbers. This is evidenced by the continuously declining number of records represented at each successive lactation (See Figure A2.3). As such, the sample data is prescreened and biased towards higher production in the latter lactations than would be the case with unselected data.

Two further OLS Regressions were carried out. The same independent and dependent variables were used but the sample was adjusted to include firstly, only those cows completing four or more lactations; and secondly, only those cows completing five or more lactations. The sample of cows completing four or more lactations was comprised of 1620 observations. The results are reported in Appendix I.B and the Predicted Milk Yields are reported in Table A2.1.

For the sample representing cows completing five or more lactations, a sample of 1219 observations was analyzed. The results are reported in Appendix I.C and the Predicted Milk Yields are reported in Table A2.1.

These predicted results are depicted graphically in Figure A2.1.

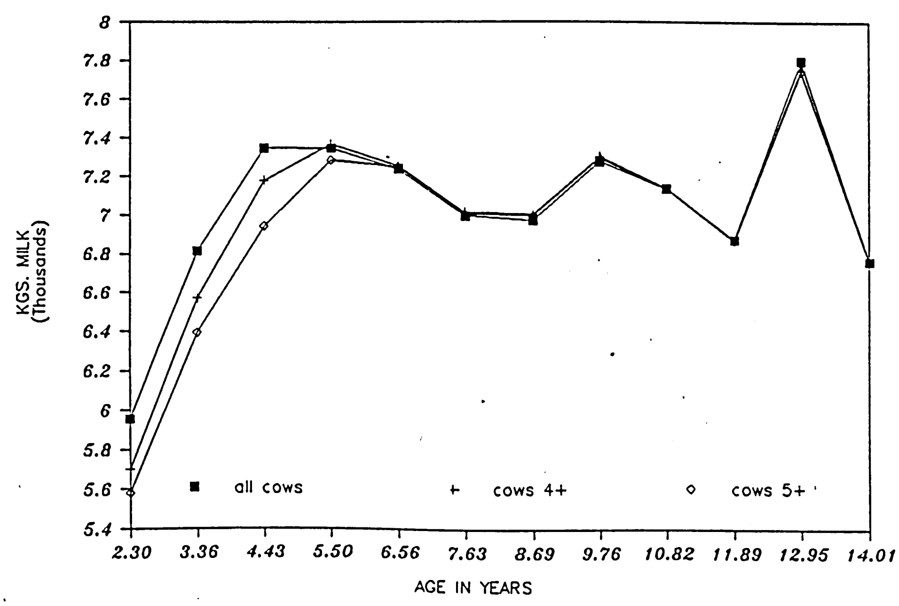

FIGURE A2.1 PREDICTED LIFETIME PRODUCTION

Of interest, is the observation that the cows with the greater longevity, produce initially at a lower level than the population in total and reach maximum production one lactation later (in both cases for 4 plus and 5 plus lactation cows) than the total population.

Because of the bias displayed in the latter lactations; the production for lactations eight, nine and ten were extrapolated from the predicted values determined in the OLS regressions by use of the age correction factors found in Table A2.4. The milk production levels used in the model are set forth in Table A2.1 and A2.3 and are displayed graphically in Figure A2.2.

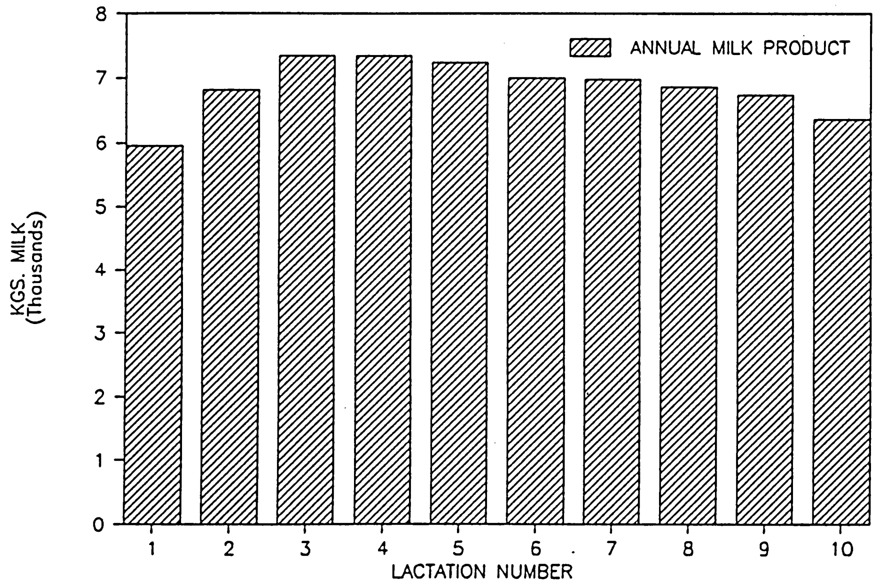

Figure A2.2 Milk Production By Lactation Used In Model

The graphed values depicted in Figure A2.2 are set forth in column 2 of Table A2.1.

Table A2.1 Predicted Milk Yields

Lactation Number	All cows	4 + Lactations	5 + Lactations
1	5954	5699	5582
2	6815	6573	6396
3	7341	7179	6943
4	7340	7363	7283
5	7235	7252	7251
6	7001	7023	7014
7	6983	7019	7010
8	6864	7313	7306
9	6740	7156	7152
10	6365	6884	6882

Column 2 of Table A2.2 sets forth the mean milk production while column 3 sets forth the standard deviation in milk production for each lactation. Column 4 sets forth the predicted milk production for each lactation using the OLS coefficients. Column 5 sets forth the predicted milk production for each lactation using the age correction factors of Table A2.4 applied to the maximum predicted milk production from column 4.

Table A2.3 compares the milk production curves used by Van Arendonk (1985) and Beaudry and Cassel (1988) as well as the standard deviations of production from Beaudry and Cassel's (1988) predicted production. These values are compared to the predicted production used in this study in column 5.

Table A2.2 Production Mean, and Standard Deviations of Sample Data				
Lactation Number	Mean Product.	Standard Deviation	Predicted Product. (regress.)	Predicted Production (age-correct)
1	6214	1871	5954	5982
2	6730	1804	6815	6674
3	7188	1943	7341	7341
4	7290	1809	7340	7917
5	7095	1843	7235	8075
6	6977	1689	7001	8075
7	6992	2093	6983	7995
8	7057	2183	6864	7917
9	7407	1432	6740	7764
10	6611	2461	6365	7340
11	6373	2695		

Table A2.3 Lifetime Production Estimates[8]				
Lactation Number	Mean Milk[9] Production	Mean Milk[10] Production	Standard[11] Deviation	Milk[12] Production
1	4970	5945	1705	5954
2	5761	6969	1904	6815
3	6447	7425	2060	7341
4	6769	7336	2312	7340
5	6937	6788	2626	7235
6	6972	6536	2683	7001
7	7000	6260	2683	6983
8	6951	5835	2649	6250
9	6902	5445	2544	5700
10	6762	4428	2502	5000

Table A2.4 depicts the age correction factors to correct maximum production to various ages as adapted from Schmidt and Van Vleck (1974).

Table A2.4 Age Correction Factors[13]										
Lact	2	3	4	5	6	7	8	9	10	11
	1.35	1.22	1.10	1.02	1.00	1.00	1.00	1.01	1.02	1.04

The data that Figure A2.3 is derived from are set forth in Tables A2.5, A2.6. and A2.7. Table A2.5 indicates the frequency distribution of lactations from the collected data sample with the related descriptive statistics.

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8 Adapted from: a) Van Arendonk (1985).
b) Beaudry and Cassel (1988).

9 Van Arendonk (1985) Agricultural Systems, Vol. 16:180.

10 Beaudry and Cassell. (1988). Journal of Dairy Science, Vol. 71:206.

11 Beaudry and Cassel (1988) pp. 206.

12 As determined from Prediction equation using Regression coefficients calculated as previously described.

13 Adapted from Schmidt and Van Vleck (1974)

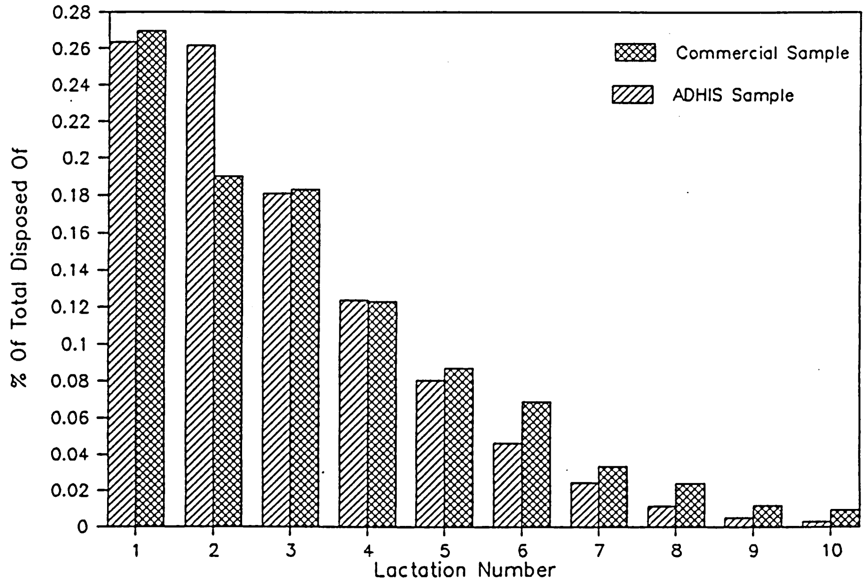

Figure A2.3. Cow Disposals By Lactations

Table A2.5. Frequency Distribution of Lactations.

Lact	N	Mean	St Dev	Variance	Minimum	Maximum
1	833	6214.1	1870.5	.34989E+07	475	14503
2	609	6729.7	1804.1	.32549E+07	583	12642
3	451	7188.4	1942.9	.37748E+07	388	13199
4	299	7289.7	1809.2	.32733E+07	1770	12925
5	197	7094.5	1842.9	.33962E+07	195	12124
6	125	6977.2	1688.7	.28518E+07	2510	13748
7	68	6992.3	2092.9	.43801E+07	2462	13950
8	40	7056.5	2183.0	.47654E+07	3000	11023
9	20	7406.7	1431.9	.20503E+07	4186	10355
10	10	6611.1	2460.9	.60561E+07	1064	10015
11	3	6372.7	2694.7	.72612E+07	3272	8148
12	1					

Table A2.6. Frequency Distribution Of Commercial Disposals.

Commercial Herd Disposals		
Lactation Number	Number Of Disposals	Cumulative Disposals in %
1	224	26.89%
2	158	45.86%
3	152	64.11%
4	102	76.35%
5	72	84.99%
6	57	91.84%
7	28	95.20%
8	20	97.60%
9	10	98.80%
10	7	99.64%
11	2	99.88%
12	1	100.00%

Table A2.6 sets forth the frequency distribution of lactations and the cumulative disposals of the collected commercial data sample as a percentage. Table A2.7 sets forth the same information for the data from the ADHIS data on cow disposals.

The two samples follow similar patterns but the ADHIS sample has 70.56% of the cows disposed of by the end of lactation three while the commercial sample has 64.11% of the cows disposed of by the same point. This is indicative of the cows in the commercial sample being kept longer than the cows in the ADHIS sample.

Table A2.7. Frequency Distribution Of ADHIS Disposals

ADHIS Herd Disposals		
Lactation Number	Number Of Disposals	Cumulative Disposals as %
1	12329	26.34%
2	12238	52.50%
3	8456	70.56%
4	5776	82.90%
5	3760	90.95%
6	2153	95.55%
7	1157	98.02%
8	536	99.17%
9	240	99.68%
10+	150	100.00%

B. Determination Of Cash Flows For the Model

Many factors interact in the milk production process. The level of milk production is influenced by nutrition, age of the cow, size of the cow and the health of the cow. Whether the cow continues in the herd can be a factor of genetics, health, reproductive failure or even the failure (death) of the cow. These interactions are depicted in Figure A2.4.

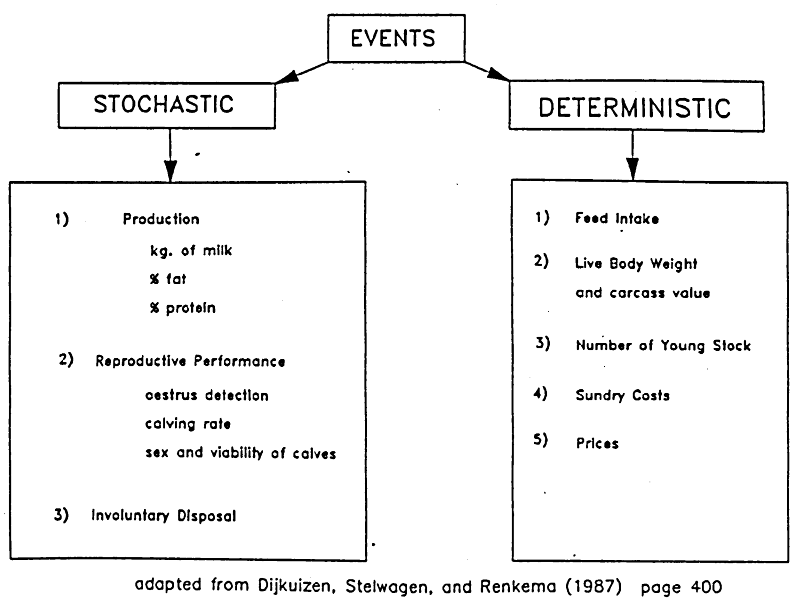

FIGURE A2.4 SUMMARY OF FACTORS AFFECTING CASH FLOW[14]

The economic data included in the model are set forth in Table A2.8:

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14 Van Arendonk (1988) in the Journal of Dairy Science. Vol.71:1051.

Table A2.8 Economic Data Used In The Base Case	
heifer cost/head	$1100
young cow $/kgs	$1.32
medium cow $/kgs	$1.10
canners and cutters $/kgs	$0.99
calf sale $/kgs	$1.54
Genetic Impovement/yr	0.50
milk price/$/hl	$49.11
Mastitis treatment $/yr	$20
veterinary costs $/yr	$20
Mastitis cost increase/yr	$5
veterinary cost inc/yr	$3
roughage cost $/kgs	$0.11
grain $/kgs	$0.25

Veterinary costs are included in the model. Mastitis treatment costs and the ensuing lost milk production and sales are a cost to the dairyman. These costs have predictable values. The mastitis costs increase with age and lactation. These facets of the milk production process and costs are portrayed in Table A2.9 and Table A2.10.

Table A2.9 sets forth the veterinary costs and value of unsalable milk associated with mastitis for each lactation. As indicated, there is a constant increase in the value of unsalable milk with each lactation as well as a constant increase in mastitis treatment costs per lactation.

Table A2.9 Veterinary Costs Associated With Mastitis[15]

Lact Number	Value of Unsalable Milk	Increase per year of age	Treat costs	Increase in Treat. Costs/year	Total Costs /Year[16]
1	32.28	3.11	17.94	1.33	59.10
2	32.28	3.11	17.94	1.33	63.54
3	32.28	3.11	17.94	1.33	67.98
4	32.28	3.11	17.94	1.33	72.42
5	32.28	3.11	17.94	1.33	76.86
6	32.28	3.11	17.94	1.33	81.30
7	32.28	3.11	17.94	1.33	85.74
8	32.28	3.11	17.94	1.33	90.18
9	32.28	3.11	17.94	1.33	94.62
10	32.28	3.11	17.94	1.33	99.06

15 adapted from:

a) Congleton. (1984). Journal of Dairy Science. Vol. 67:640

b) Hanson, Touchberry, Young, and Miller. (1979). Journal of Dairy Science, Vol. 62:1939.

16 Associated with Incidence of Mastitis only.

Table A2.10 Total Veterinary Costs			
Lactation Number	Mastitis Costs[17]	Other vet Costs[18]	Total Vet Costs
1	59.10	18.13	77.23
2	63.54	18.13	81.67
3	67.98	18.13	86.11
4	72.42	18.13	90.55
5	76.86	18.13	94.99
6	81.30	18.13	99.43
7	85.74	18.13	103.87
8	90.18	18.13	108.31
9	94.62	18.13	112.75
10	99.06	18.13	117.19

The physical data and setup of the model can be seen by examining Table A2.11 which immediately follows. Table A2.12 shows the cashflows from the model utilizing the base case data. Table A2.13 shows the NPV and ANPV of the perpetual replacement sequences with no involuntary replacement.

Table A2.14 shows the WNPV and WANPV of the perpetual replacement sequences incorporating involuntary replacement.

Table A2.11 PHYSICAL DATA										
Lactation	1	2	3	4	5	6	7	8	9	10
cow wt	485	528	559	603	603	603	600	590	580	559
calf wt	50	50	50	50	50	50	50	50	50	50
milk prod	5954	6815	7341	7340	7235	7001	6983	6864	6740	6365
kg/day	20	22	24	24	24	23	23	23	22	21
hay kgs	5311	5782	6121	6603	6603	6603	6570	6461	6351	6121
grain kg	1952	2501	2898	2898	2898	2898	2898	2501	2501	2501

Table A2.12 CASHFLOWS FROM THE MODEL										
Lactation	1	2	3	4	5	6	7	8	9	10
milk sales	2837	3247	3498	3497	3447	3336	3327	3270	3211	3033
total $ in	2837	3247	3498	3497	3447	3336	3327	3270	3211	3033
feed costs	1072	1261	1398	1451	1451	1451	1447	1336	1324	1299
vet costs	91	99	107	115	123	131	139	147	155	163
misc costs	255	292	315	315	310	300	299	294	289	273
total $ out	1418	1652	1819	1880	1883	1881	1885	1777	1767	1734
surplus	1419	1595	1679	1617	1564	1454	1442	1494	1444	1299
salvage $	640	697	738	663	663	663	594	584	574	553
calf sales	77	77	77	77	77	77	77	77	77	77
RESIDUAL	2136	2369	2494	2358	2304	2195	2113	2155	2095	1929

17 From Table A2.9.

18 Hanson, Touchberry, Young, and Miller. (1979). pp.1939.

Table A2.13 NPV and ANPV Without Involuntary Replacement										
Lactation	1	2	3	4	5	6	7	8	9	10
NPV	11351	14000	14157	13261	12360	11354	10374	9563	8789	8021
ANPV	962	1187	1200	1124	1048	962	879	811	745	680

Table A2.14 WANPV and WNPV WITH INVOLUNTARY REPLACEMENT										
Lactation	1	2	3	4	5	6	7	8	9	10
WNPV	11351	13825	14011	13331	12674	11998	11387	10915	10505	10145
WANPV	962	1172	1188	1130	1074	1017	965	925	890	860

C. Choice Of A Discount Rate

A discount rate reflects the relative value of future earnings versus the present earnings of an asset and can, thus be thought of as the relative time value placed on money. Furthermore, the discount rate can be used to account for the inherent risk in the investment (project).

For the purposes of this research project, the discount rate will be based on the Market Rate of Return on Investments of Similar Risk approach (Andrews, 1987. page 12.) wherein the appropriate discount rate will be based on a base risk-free rate[19] plus a risk premium that is based on the degree of risk experienced by the investor (the dairy operator) in comparison to the market. Since all flows of cash used in this study are measured in real terms, the discount rate chosen will also be an inflation-free rate.

In Alberta, Fluid Milk is priced by a formula that includes the following components in the relative proportions indicated.[20]

Table A2.15 Fluid Milk Pricing Formula Components	
Component	Proportion
1) Price of 16% dairy feed $/tonne (Alberta)	14%
2) Price of Alfalfa hay $/ton (Alberta)	14%
3) Index of Farm Wages (Western Canada)	12%
4) Index of Farm Inputs (Western Canada)	20%
5) Index of Consumer Prices (Canada)	10%
6) Average Weekly Industrial Wage (Alberta)	16%
7) Per Capita Sales of Milk litres/month (Alberta)	14%
Composite	100%

This formula incorporates the major components of market risk into the price of milk. Thus, the dairy producer faces fewer market risks than would producers of commodities that are not priced according to a pricing formula. The major risks faced by the producer are in the area of production variation due to health disorders, disease, weather and the natural variability of production. The dairy producer will also face the normal risks associated with the cattle market when he sells cull animals into that market. The market value of replacements can be influenced by the commercial cattle market

19 Based on the return to 3 year to five year Guaranteed Term Deposits and similar Government of Canada and Alberta bonds.

20 Adapted from Alberta Agriculture. 1986. Economics of Milk Production In Alberta. Page 48.

to the degree that salvage value enters the value of the replacement.[21] As such, the discount rate chosen should reflect a risk-free rate that is free of inflation plus a modest premium for risk of 1/2% to 1%.

At present Government of Canada and Government of Alberta Bonds are paying 9 1/2 % and 9% respectively with inflation estimated to be in the 3% range. Therefore, an appropriate inflation-free rate would be in the range of 6% to 6 1/2%. With an allowance for risk incorporated, the appropriate discount rate would be in the range of 7% to 7 1/2%. A base discount rate of 7 1/2% will be used in this study.

D. Involuntary Disposals

As outlined in Chapter 2, the involuntary replacement of dairy cows can be incorporated into the model by the use of the Markov process. By inserting the probability of involuntary disposal from column two of Table A2.17 into the matrix T from Chapter 2, the following matrix is obtained:

Table A2.16 Markov Matrix

	1	2	3	4	5	6	7	8	9	10
1	0.124	0.135	0.143	0.152	0.170	0.169	0.174	0.177	0.189	1.000
2	0.876									
3		0.865								
4			0.857							
5				0.848						
6					0.830					
7						0.831				
8							0.826			
9								0.823		
10									0.811	0.000

Table A2.17 Marginal Probability of Replacement[22]

Lactation Number	Invol Replace	Vol. Replace	Total[23] Replace	Prob.[24] Occur
1	12.4	11.0	23.4	76.6
2	13.5	3.6	17.1	82.9
3	14.3	4.1	18.4	81.6
4	15.2	5.3	20.5	79.5
5	17.0	8.0	25.0	75.0
6	16.9	11.9	28.8	71.2
7	17.4	21.6	39.0	61.0
8	17.7	34.9	52.6	47.4
9	18.9	51.1	70.0	30.0
10	20.1	62.6	82.7	17.3
11	21.5	73.5	95.0	5.0
12	24.1	75.9	100.0	0.0

The steady state matrix which denotes the herd composition under various voluntary replacement strategies is obtained from the transition matrix. The adjusted matrix T is obtained first:

21 For the purposes of this study replacements are valued at the cost of raising the replacement from birth to first calving. The costs used are based on those costs determined by Alberta Agriculture's Rudy Susko in an unpublished study of the Costs of Raising Replacement Dairy Heifers In Alberta.

22 adapted from Van Arendonk (1985) Livestock Production Science Vol. 13:340

23 Defined as the sum of Involuntary and Voluntary Replacement

24 Defined as 100 minus the total probability of replacement occurring and represents the probability of a particular lactation being completed.

Table A2.18 Adjusted Markov Matrix

	1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.876	-1.000								
3		0.865	-1.000							
4			0.857	-1.000						
5				0.848	-1.000					
6					0.830	-1.000				
7						0.831	-1.000			
8							0.826	-1.000		
9								0.823	-1.000	
10									0.811	-1.000

Solution of the system results in matrix which contains the herd composition "vector" for each replacement strategy.

The matrix c, from which the herd composition, given various voluntary replacement strategies is determined, follows:

Table A2.19 Herd Composition Matrix

C	1	2	3	4	5	6	7	8	9	10
1	1.000	.5330	.3797	.3046	.2608	.2331	.2141	.2006	.1907	.1834
2		.4670	.3326	.2668	.2285	.2042	.1876	.1757	.1671	.1607
3			.2877	.2308	.1976	.1766	.1622	.1520	.1445	.1390
4				.1978	.1694	.1513	.1390	.1303	.1239	.1191
5					.1436	.1283	.1179	.1105	.1050	.1010
6						.1065	.0979	.0917	.0872	.0838
7							.0813	.0762	.0724	.0697
8								.0629	.0598	.0575
9									.0493	.0474
10										.0384

From the herd composition matrix, the average number of lactations per cow and the average age in years of the cows in the herd given a particular voluntary replacement strategy can be derived. These values follow immediately.

Table A2.20 Predicted Herd Statistics

Lact.	1	2	3	4	5	6	7	8	9	10
ave lact	1.00	1.46	1.90	2.32	2.70	3.05	3.37	3.66	3.93	4.16
ave yrs	3.58	4.16	4.71	5.23	5.71	6.15	6.55	6.91	7.24	7.53
Annual Replace Rate %	100	53.3	37.97	30.45	26.06	23.30	21.39	20.06	19.10	18.87

The vectors Q which follows as Table A2.21, denotes the proportion of cows in each replacement pattern under different replacement strategies. For example, with a voluntary replacement strategy of replacing at the end of lactation three, there are 4.71% of the herd involuntarily replaced at the end of lactation one, 8.98% of the herd involuntarily replaced at the end of lactation two, and 86.31% of the herd replaced at the end of lactation three where the cycle is truncated by choice (i.e. voluntary truncation of the cycle).. This would result in an overall per lactation replacement rate of 37.97%.

The steady state proportion of cows in a given replacement pattern are obtained from the vector Q:

Table A2.21 Proportion Of Cows In A Particular Replacement Pattern

Q	1	2	3	4	5	6	7	8	9	10
1	1.000	.0661	.0471	.0378	.0323	.0289	.0265	.0249	.0237	.0227
2		.9339	.0898	.0720	.0617	.0551	.0506	.0475	.0451	.0434
3			.8631	.0990	.0848	.0758	.0696	.0652	.0620	.0596
4				.7912	.1030	.0920	.0845	.0792	.0753	.0724
5					.7182	.1091	.1002	.0939	.0893	.0859
6						.6391	.0992	.0930	.0884	.0850
7							.5692	.0928	.0882	.0849
8								.5035	.0847	.0815
9									.4433	.0806
10										.3841
Sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000