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# NOTES

## ECONOMIC INTERPRETATION OF MAIZE FERTILIZER EXPERIMENT

During the last few years the use of fertilizers has increased many-fold. Fertilizer recommendations are generally made without giving full consideration to economic aspects. The profits from the use of fertilizer are increased as long as the cost of using additional fertilizer is covered by the added returns. The optimum level of fertilizer use is made when the marginal revenue equals marginal cost. The farmers are raising such questions as : how much fertilizer to use and how much it will pay by using fertilizer ? This study was made to answer such practical questions related to fertilizer use : (1) How much fertilizer to use ? (2) Least cost combinations of fertilizer for specified yield level at given fertilizer prices. (3) Optimum level of fertilizer when capital outlay is fixed.

### *Material and Methods*

The data for this analysis were used from the fertilizer experiment conducted at the Indian Agricultural Research Institute, New Delhi on maize for five years (1949-53). Various levels of nitrogen, phosphorus and potash were tried with $3 \times 3 \times 2$ confounding design with two replications. The levels of N, $K_2O$ and $P_2O_5$ used were :

$N_0$ = No nitrogen

$N_1$ = 40 lbs. nitrogen/acre

$N_2$ = 80 lbs. nitrogen/acre

$P_0$ = No phosphorus

$P_1$ = 40 lbs. $P_2O_5$/acre

$P_2$ = 80 lbs. $P_2O_5$/acre

$K_0$ = No potash

$K_1$ = 60 lbs. $K_2O$/acre

The data were subjected to analysis of variance which showed that N and P were significant at 1 per cent whereas their interaction was significant at 5 per cent level. Since $K_2O$ was not significant, this analysis was confined to two factors, *i.e.*, N and $P_2O_5$.

Production function approach was used for economic interpretation of the fertilizer response data. The quadratic surface function was used as this function gave combination of increasing as well as decreasing returns and also accounted for the interaction of the factors. The form of the function used was :

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_1^2 + b_4 X_2^2 + b_5 X_1 X_2$$

The polynomial function was preferred to the exponential as well as power function because the latter functions did not explain for the interaction and also did not yield combination of increasing and decreasing returns.

Calculated yield at these levels of N and P = 11.9693 quintals. Response over control = 5.7407 quintals.

The net returns over fertilizer cost at different prices of maize are presented in Table III.

TABLE III—NET RETURNS OVER FERTILIZER COST AT DIFFERENT PRICES OF MAIZE

| Price of maize per quintal | Response over control (quintal) | Fertilizer* cost (Rs.) | Gross returns (Rs.) | Net returns over fertilizer cost (Rs.) |
|---|---|---|---|---|
| 50.00 | 5.740725 | 144.04 | 287.04 | 143.00 |
| 55.00 | ,, | ,, | 315.74 | 171.70 |
| 60.00 | ,, | ,, | 344.44 | 200.40 |

*Price of Ammonium Sulphate = Rs. 360.00 per ton.
Price of Superphosphate (single) = Rs. 275.00 per ton.

*Yield Isoquants*

Iso curves were worked out for a yield range of 7, 9 and 11 quintals and are shown in Figure 4. These levels were calculated from the fitted response equation by setting it at the level for which the isoquant was intended to develop. The equation was then arranged as quadratic of one factor and at various levels of the other factor—the corresponding values were obtained by using the formula :

$$N = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The various levels of N and $P_2O_5$ were worked out for specified yields levels (7, 9 and 11 quintals) and are presented in Table IV.

TABLE IV—VARIOUS LEVELS OF NITROGEN AND PHOSPHATE FOR DIFFERENT ISOQUANTS

| | | | | | | |
|---|---|---|---|---|---|---|
| Y = 7.00 | N | 9.824 | 2.963 | .682 | 1.970 | 6.415 |
| | P | 0 | 20 | 40 | 60 | 80 |
| Y = 9.00 | N | 44.518 | 30.692 | 25.523 | 25.576 | 29.810 |
| | P | 0 | 20 | 40 | 60 | 80 |
| Y = 11.00 | N | 73.981 | 65.384 | 61.619 | 60.494 | 64.025 |
| | P | 30 | 40 | 50 | 60 | 80 |

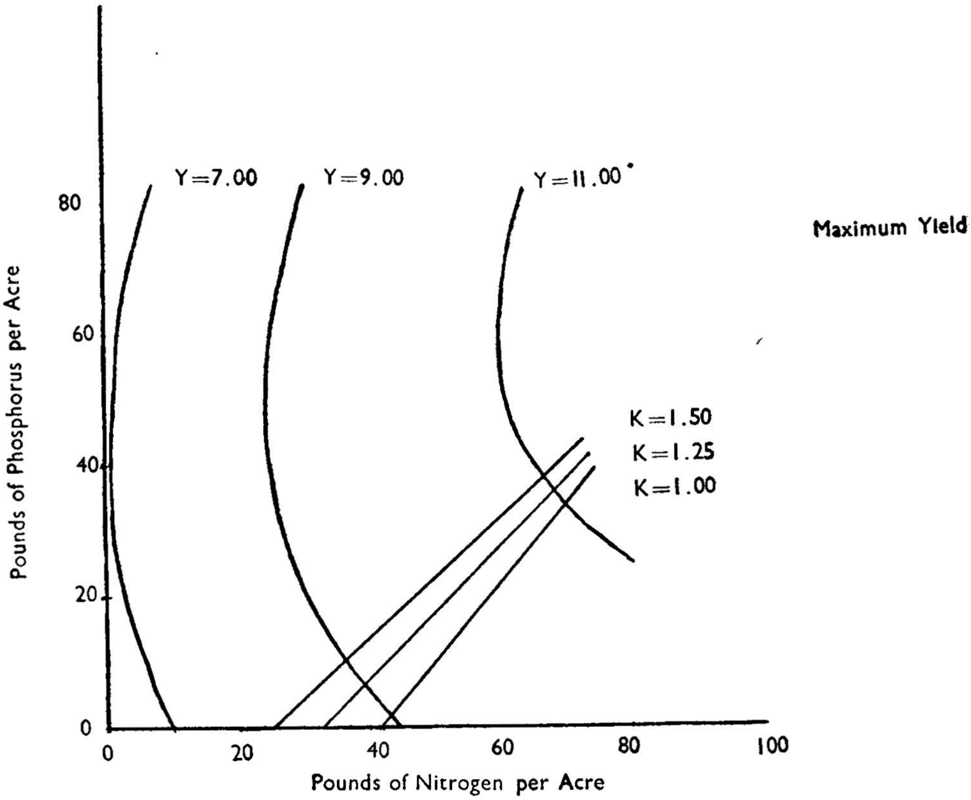


Figure 4—Yield Isoquants and Isoclines

It is apparent from Table IV and Figure 4 that the specified yield levels could be produced at various combinations of N and $P_2O_5$. The shape of isoquants showed that the two factors were substituted in a very limited range. This is due to the reason that the marginal products due to $P_2O_5$ began to diminish even at lower levels of its application.

*Marginal Rates of Substitution*

The shape of the various yield isoproduct curves at various combination of nitrogen and phosphorus was estimated by calculating the marginal rates of substitution. The marginal rates of substitution were calculated as :

$$\frac{\partial P}{\partial N} = - \frac{\dfrac{\partial Y}{\partial N}}{\dfrac{\partial Y}{\partial P}}$$

Numerically : $$\frac{\partial P}{\partial N} = - \frac{.083125 + .0002418\ P - .0009378\ N}{.033764 + .0002418\ N - .0008026\ P}$$

The marginal rates of substitution for specified yield and for various combinations of N and $P_2O_5$ are presented in Table V.

TABLE V—MARGINAL RATES OF SUBSTITUTION AT SPECIFIED YIELD LEVELS

| Yield quintals/acres | Pounds of nitrogen per acre | Pounds of phosphorus per acre | Marginal rates of substitution |
|---|---|---|---|
| 7.00 | 9.824 | 0 | — 2.0452 |
| | 5.718 | 10 | — 2.9564 |
| | 2.963 | 20 | — 4.6225 |
| | .682 | 40 | —49.8146 |
| 9.00 | 44.518 | 0 | — 0.9292 |
| | 36.035 | 10 | — 2.5021 |
| | 30.692 | 20 | — 2.3546 |
| | 25.523 | 40 | —37.6089 |
| 11.00 | 73.960 | 30 | — .7616 |
| | 65.384 | 40 | — 2.3079 |
| | 61.619 | 50 | — 4.3864 |
| | 60.494 | 60 | —174.0511 |

*Yield Isoclines*

The isoclines were estimated by setting the equation of marginal rate of substitution equal to their price ratios : $\left(\frac{\text{Price of N per unit}}{\text{Price of } P_2O_5 \text{ per unit}}\right)$. Under perfect competition, these isoclines were estimated at different price ratios (R), *i.e.*, 0.75, 1.00, 1.25 and 1.50 because these price ratios gave a practical range of the price variations of the factors :

$$\frac{\partial P}{\partial N} = \frac{.083125 + .0002418\ P - .0009378\ N}{.033764 + .0002418\ N - .0008026\ P} = \frac{CN}{CP} = R$$

The isoclines estimated at different price ratios are :

| Price ratio | Isocline |
|---|---|
| 0.75 | $P = 1.32640\ N - 68.506074$ |
| 1.00 | $P = 1.129452\ N - 47.26543$ |
| 1.25 | $P = .995984\ N - 32.866150$ |
| 1.50 | $P = .899564\ N - 22.465933$ |

These isoclines yielded the expansion path on which specified yield level could be produced at minimum cost.

*Minimum Cost Combination*

Minimum cost combination at given price ratios and for specified yield levels were calculated by solving isocline equations for the given price ratio along with the isoquant equation for the specified yield. The minimum cost combinations of N and $P_2O_5$ for different price ratios and for various yield levels are given in Table VI.

TABLE VI—MINIMUM COST COMBINATIONS OF N AND $P_2O_5$ AT DIFFERENT PRICE RATIOS AND SPECIFIED YIELD LEVELS

| Price ratio | Yield level | Pounds of nitrogen | Pounds of phosphorus |
|---|---|---|---|
| 1.00 | 9.00 | 43.0769 | 1.3907 |
| | 11.00 | 70.8379 | 32.7455 |
| 1.25 | 9.00 | 38.9883 | 5.9656 |
| | 11.00 | 68.5029 | 35.3616 |
| 1.50 | 9.00 | 36.0580 | 9.9705 |
| | 11.00 | 66.8190 | 37.6420 |

*Profit Maximization*

The optimum levels of fertilizer use were determined by solving the following equations simultaneously :

$$\frac{\partial Y}{\partial N} = \frac{cN}{cY} \quad \ldots\ldots\ldots \quad \text{(i)}$$

$$\frac{\partial Y}{\partial P} = \frac{cP}{cY} \quad \ldots\ldots\ldots \quad \text{(ii)}$$

*i.e.*, $$\frac{\partial Y}{\partial N} = .083125 + .0002418\ P - .0009378\ N = \frac{cN}{cY} \quad \ldots\ldots\ldots \quad \text{(i)}$$

$$\frac{\partial Y}{\partial P} = .033764 + .0002419\ N - .0008026\ P = \frac{cP}{cY} \quad \ldots\ldots\ldots \quad \text{(ii)}$$

where cN = Price per lb. of nitrogen,
cP = Price per lb. of phosphorus,
cY = Price per quintal of maize.

As the prices of agricultural products experienced wide fluctuation over time, optimum levels of fertilizer use were calculated for a range of maize prices (Rs. 50.00, 55.00 and 60.00). The prices of nitrogen and phosphorus used were :

cN = .796550 per lb.
cP = .793784 per lb.

The optimum levels of N and $P_2O_5$ were worked out by solving equation (i and ii) at various prices of maize and were presented in Table VII.

TABLE VII—OPTIMUM COMBINATIONS OF N AND $P_2O_5$ FOR DIFFERENT PRICE LEVELS OF THE PRODUCT

| Price of maize per quintal (Rs.) | Optimum levels of N (lbs.) | Optimum levels of $P_2O_5$ (lbs.) | Response over control (quintals) | Fertilizer cost (Rs.) | Net return over fertilizer (Rs.) |
|---|---|---|---|---|---|
| 50.00 .. | 83.916 | 47.568 | 5.337 | 104.60 | 162.25 |
| 55.00 .. | 86.093 | 50.025 | 5.407 | 108.29 | 189.10 |
| 60.00 .. | 87.907 | 52.068 | 5.461 | 111.35 | 216.31 |

*Optimum Dose for given Capital Outlay*

In many cases the optimum doses of fertilizers may not be possible to apply due to financial handicaps and uncertainty. In such cases where capital was limited, the optimum level of fertilizer was determined from the interaction of the isocline corresponding to $\frac{cN}{cP}$ : $cN.N + cP.P = C$ (Capital).

The solution was obtained by moving along the isocline until all capital was exhausted. In the present case $\frac{cN}{cP} = \frac{.796550}{.793784}$ and the corresponding isocline was:

$$\frac{\partial P}{\partial N} = \frac{.083125 + .0002418\ P - .0009378\ N}{.033764 + .0002418\ N - .0008026\ P} = \frac{.796550}{.793784}$$

$$.0011804\ N + .0010472\ P = .049243 \quad \ldots\ldots\ldots \quad \text{(i)}$$

$$.796550\ N + .793784\ P = C \quad \ldots\ldots\ldots \quad \text{(ii)}$$

The response and net returns over fertilizer use when capital outlay (C) was fixed at Rs. 50.00, 75.00 and 100.00 were calculated by solving the above equation for various levels of C and are presented in Table VIII.

TABLE VIII—OPTIMUM COMBINATIONS OF N AND $P_2O_5$ WITH CAPITAL OUTLAY FIXED

| Capital outlay | Pounds of nitrogen | Pounds of phosphorus | Response over control | Price of maize per quintal | Net returns over fertilizer cost |
|---|---|---|---|---|---|
| 50.00 .. | 51.633 | 11.177 | 3.4087 | 50.00 | 120.44 |
| | | | | 55.00 | 137.48 |
| | | | | 60.00 | 154.52 |
| 75.00 .. | 66.412 | 27.840 | 4.5285 | 50.00 | 151.43 |
| | | | | 55.00 | 174.07 |
| | | | | 60.00 | 196.71 |
| 100.00 .. | 81.196 | 44.500 | 5.2395 | 50.00 | 161.97 |
| | | | | 55.00 | 188.17 |
| | | | | 60.00 | 214.37 |

*Summary*

This agronomic trial on maize was conducted at Indian Agricultural Research Institute, New Delhi. The levels of nitrogen and phosphorus used in this experiment were 0, 40 and 80 lbs. each. Using the Production Function approach, economic interpretation of the data gave the optimum levels of nitrogen and phosphorus at 86.09 lbs. and 50.02 lbs. respectively. Economic interpretation of the production surfaces data were therefore necessary for making fertilizer recommendation to the farmers.

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## PREDICTION OF FERTILIZER CONSUMPTION IN INDIA

There are several methods of predicting fertilizer consumption and each of these has its limitations. The more important of them are (*i*) potential method, (*ii*) regression method, and (*iii*) trend analysis method. In this note, a detailed study has been made in case of trend analysis method by using autoregressive series.

### *Trend Analysis Method*

In order to study future prospects of fertilizer consumption, past data for the period 1951-52 to 1963-64 have been analysed and trend equations are fitted. This method has its limitations in as much as it depends only on the past performance and does not include any significant change which may happen in future. Under the circumstances like food crisis, special emphasis is to be given to fertilizers. These types of efforts can hardly be impounded in any statistical model and some realistic estimate, keeping in mind the trend projections, has to be found in such cases. Trend analysis does include these factors if the problem is not a new one and if such efforts were being made in the past as well.

With a view to find trend analysis we have taken year 1951-52 as zero (base), 1952-53 as 1 and so on and this is denoted by t. Thus t takes values 0, 1, 2, 3, .... and so on, its value being 19 corresponding to the year 1970-71. Consumption of Nitrogen (N), Phosphorus (P) and Potash (K) is analysed separately as well as combined. Moving averages of order three has been taken in order to remove random fluctuations and as the result of which trend values are found. These trend values are denoted by Yt corresponding to time t and trend equation fitted is $Yt = a + b\ Y(t-1)$. These equations are fitted with the help of least squares method and are given below :

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*Results and Discussion*

The response equation developed from the five years' pooled data gave the following results :

$$Y=6.228582^{**}+.083125^{**} N+.033764 P^{*}-.0004689 N^{2*}-.0004013 P^{2*}+0002418 NP^{*}$$
$$(.288842)\ (.012507)\ (.012507)\ (.0001421)\ (.0001421)\ (.0001007)$$
$$R^2 = .9213$$

** Significant at 1 per cent level.
* Significant at 5 per cent level.

The results showed that all the coefficients in the equation were significant. As the value of the coefficient of multiple determination was quite high, it showed that the response equation gave a good fit.

*Derived Yields*

The derived yields were calculated at various levels of N and $P_2O_5$ and pre sented in a three dimensional diagram. The calculated yields as compared with the observed yields gave a close approximation which meant that the fitted function conformed very closely to the data. The calculated yields were also depicted on the graph showing the response of N and $P_2O_5$ at various levels of $P_2O_5$ and N. The derived yields are presented in Table I.

TABLE I—DERIVED YIELDS IN QUINTALS AT VARIOUS LEVELS OF N AND $P_2O_5$

| P/N | $N_0$ | $N_{20}$ | $N_{40}$ | $N_{60}$ | $N_{80}$ | $N_{100}$ |
|---|---|---|---|---|---|---|
| $P_0$ | 6.2286 | 7.7035 | 8.8032 | 9.5278 | 9.8772 | 9.8521 |
| $P_{20}$ | 6.7433 | 8.3150 | 9.5115 | 10.3327 | 10.7790 | 10·8504 |
| $P_{40}$ | 6·9370 | 8·6054 | 9·8987 | 10·8166 | 11·3597 | 11·5278 |
| $P_{80}$ | 6·9085 | 8·5747 | 9·9646 | 10·9794 | 11·6194 | 11·8840 |
| $P_{100}$ | 6·3610 | 8·2229 | 9·7096 | 10·8211 | 11·5579 | 11·9193 |

Table I as well as the Figures 1 to 3 indicated that the yields increased in the initial stages and ultimately diminished giving a concave shape to the response curve when one factor was varied and other was fixed. It was also apparent that the response curve was quite steep when the fixed factor was held high. The slices of the production surface also showed the same relationship.

*Marginal Products*

Marginal products due to N $\left(\frac{\partial Y}{\partial N}\right)$ and $P_2O_5$ $\left(\frac{\partial Y}{\partial P}\right)$ were calculated by taking partial derivative from the response curve at various levels of N and $P_2O_5$.

$$Y = 6.228582 + .083125 N + .033764 P - .0004689 N^2 - .0004013 P^2 + .0002418 NP$$

$$\frac{\partial Y}{\partial N} = .083125 + .0002418 P - .0009378 N$$

$$\frac{\partial Y}{\partial P} = .033764 + .0002418 N - .0008026 P$$

$$Y = 6.22858 + .083125\,N + .033764\,P - .0004689\,N^2 - .0004013\,P^2 + .0002418\,NP$$

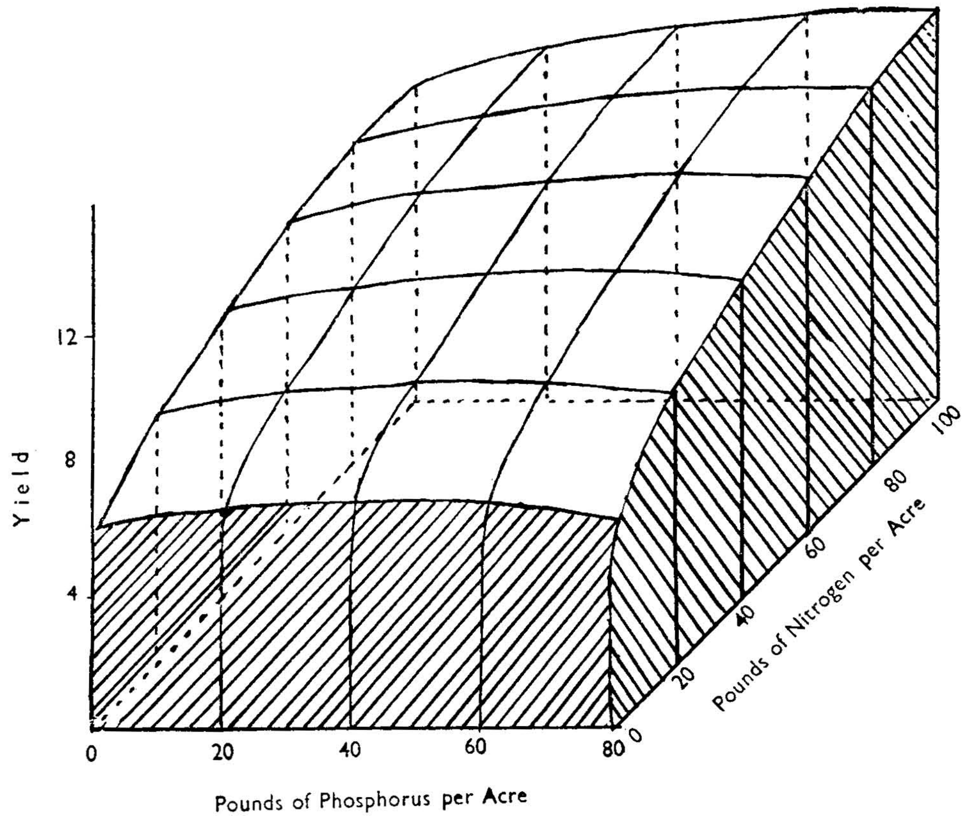


Figure 1—Predicted Yield Surface of Maize

The marginal products for different combinations of N and $P_2O_5$ were calculated and presented in Table II.

TABLE II—MARGINAL PRODUCTS FOR VARIOUS COMBINATIONS OF N AND $P_2O_5$

| P/N | $N_0$ | $N_{20}$ | $N_{40}$ | $N_{60}$ | $N_{80}$ | $N_{100}$ |
|---|---|---|---|---|---|---|
| $P_0$ .. .. .. .. | .0831 | .0644 | .0456 | .0268 | .0081 | .0106 |
| | .0338 | .0386 | .0435 | .0483 | .0531 | .0579 |
| $P_{20}$ .. .. .. .. | .0879 | .0692 | .0504 | .0317 | .0129 | — .0058 |
| | .177 | .255 | .0274 | .0322 | .0371 | .0419 |
| $P_{40}$ .. .. .. .. | .0928 | .0740 | .0553 | .0365 | .0178 | — .0009 |
| | .0016 | .0085 | .0113 | .0162 | .0210 | .0258 |
| $P_{80}$ .. .. .. .. | .0976 | .0789 | .0601 | .0414 | .0226 | .0039 |
| | — .0304 | — .0256 | — .0208 | — .0159 | — .0111 | — .0063 |

$$Y = 6.22858 + .083125\,N + .033764\,P - .0004689\,N^2 - .0004013\,P^2 + .0002418\,NP$$

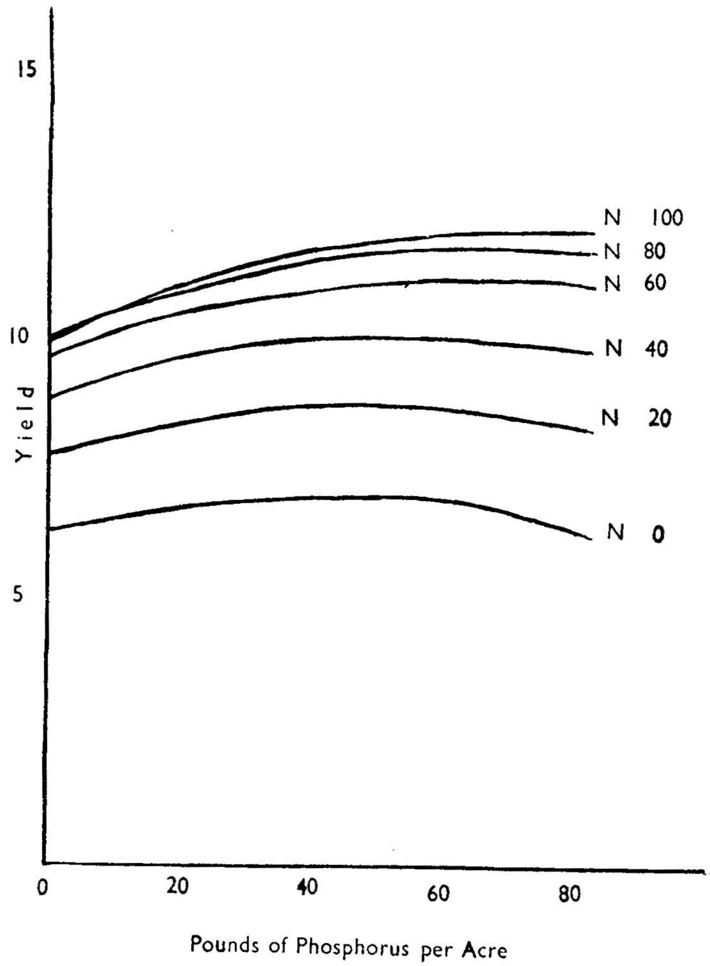


Figure 2—Maize Yield Response to Phosphorus at Various Levels of Nitrogen

Marginal products showed the rate of change in yield at different levels of N and $P_2O_5$ (Table II). Such data served as a good guide for taking a decision on the quantity of one factor to be used when other factor was fixed. It was also apparent that the marginal products due to N increased at higher levels of $P_2O_5$ and vice versa.

*Maximum Yield*

The maximum yield was obtained from the use of factors at the levels where the marginal products of the respective factor became zero. The levels were

$$Y = 6.22858 + .083125\ N + .033764\ P - .0004689\ N^2 - .0004013\ P^2 + .0002418\ NP$$

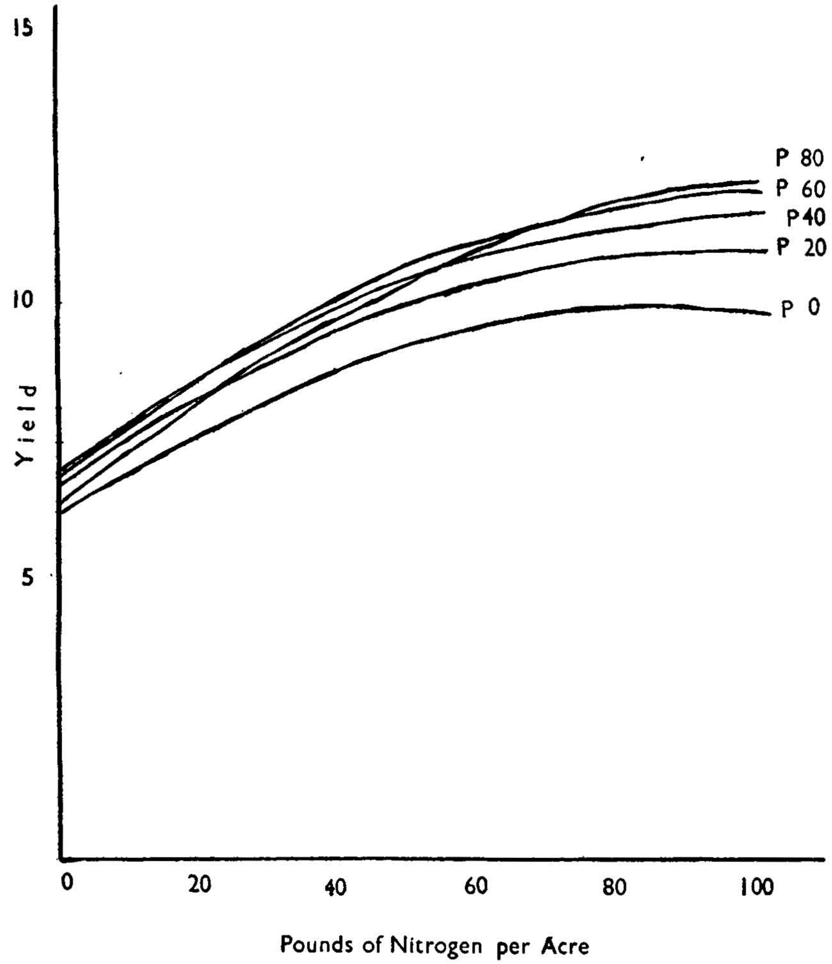


Figure 3—Maize Yield Response to Nitrogen at Various Levels of Phosphorus

calculated by solving the marginal product equations of both the factors simultaneously.

$$\frac{\partial Y}{\partial N} = .083125 + .0002418\ P - .0009378\ N = 0$$

$$\frac{\partial Y}{\partial P} = .033764 + .0002418\ N - .0008026\ P = 0$$

Solving these equations we get :

$N = 106.86$ lbs./acre

$P_2O_5 = 74.56$ lbs./acre

TABLE I—TREND EQUATIONS OF NUTRIENT CONSUMPTION IN INDIA

| Nutrient | Trend Equation Yt = a+b Y(t—1) | S.E. of b |
|---|---|---|
| Nitrogen (N) | Yt = 1.19 Y(t—1)—2.35 | 0.0038 |
| Phosphorus (P) | Yt = 1.31 Y(t—1)—1.08 | 0.0036 |
| Potash (K) | Yt = 1.23 Y(t—1)—0.61 | 0.0432 |
| Total (N+P+K) | Yt = 1.22 Y(t—1)—0.6.00 | 0.0319 |

Consumption of plant nutrients for the year 1965-66 to 1970-71 worked out on the basis of above equations is given in Table II.

TABLE II—ESTIMATED CONSUMPTION OF FERTILIZERS FOR THE YEAR 1965-66 TO 1970-71

('000 *tonnes*)

| Year | Consumption of nutrient N | P | K |
|---|---|---|---|
| 1965-66 .. .. .. | 575.8 | 189.0 | 75.0 |
| 1966-67 .. .. .. | 682.9 | 246.5 | 91.6 |
| 1967-68 .. .. .. | 810.3 | 321.8 | 112.0 |
| 1968-69 .. .. .. | 961.9 | 420.5 | 137.2 |
| 1969-70 .. .. .. | 1142.3 | 549.8 | 168.1 |
| 1970-71 .. .. .. | 1357.0 | 720.2 | 206.2 |

There is no doubt that these trend consumption figures are on low side compared to targets fixed by Government but in practice these targets were never achieved in the past and the likelihood of being achieved in future is very dim. Table III giving actual consumption and targets fixed strengthens the above point.

TABLE III—CONSUMPTION AND TARGETS OF FERTILIZERS

('000 *tonnes of nutrient*)

| | Second Five-Year Plan | | | Third Five-Year Plan | | | Fourth Five-Year Plan | | |
|---|---|---|---|---|---|---|---|---|---|
| Nutrient | Target | Actual consumption 1960-61 | Consumption as % of target | Target | Actual consumption in 1964-65 (July-June) | Consumption as % of target | Target | Trend estimates for 1970-71 | Total estimate as % of target |
| N | 370 | 210 | 56.8 | 1000 (800) | 492 | 49.2 (61.5) | 2000 | 1357 | 67.9 |
| P | 120 | 70 | 58.3 | 400 (250) | 147 | 36.8 (58.8) | 1000 | 720 | 72.0 |
| K | 30 | 26 | 86.7 | 200 (150) | 72 | 36 (48) | 350 | 206 | 58.9 |
| Total | 520 | 306 | 58.8 | 1600 (1200) | 711 | 44.4 | 3350 | 2283 | 68.1 |

*Note :* Figures in brackets refer to revised targets.

It can be seen from Table III that consumption of total nutrient was only 58.8 per cent and 44.4 per cent (59.2 per cent) of the target in the Second and the Third Five-Year Plan respectively. Trend estimates for the consumption of total nutrient in the Fourth Five-Year Plan which represent 68.1 per cent of the target, can only be surpassed if efforts without any precedent are made and the same come out to be successful.

*Conclusion*

It seems that the trend model developed above provides reasonably good estimate of the consumption of plant nutrients at the national level. Further statewise disaggregation would be desirable and shall probably give better results as area covered for estimation will be more homogeneous.

Even if the supply position is improved adequately, it will be difficult to market the quantity anticipated by targets because a lot of development work is to be undertaken which will take a long time. Moreover the factors like prices of agricultural products, irrigated and total acreage and prices of fertilizers which affect fertilizer demand are not expected to improve considerably in favour of fertilizer demand. Already prices of agricultural commodities have reached a saturated point and further rise, which would have increased fertilizer demand, is not expected to be significant. Moreover, now shift of area from cash crops to foodgrains is expected and is being emphasized because of severe shortage of foodgrains and this point will certainly go against the interest of fertilizer demand.

However, there is a wide gap between targets and trend estimates for the Fourth Plan and some sincere and strong efforts are necessary to overcome this gap.

R. C. DAHIYA*

## A NOTE ON THE ELASTICITY OF THE MARKETABLE SURPLUS OF A SUBSISTENCE CROP—A COMMENT

Raj Krishna in his note on the elasticity of the marketable surplus of a subsistence crop[1] comes to the conclusion that "the likelihood of a perverse market supply behaviour is extremely small." The purpose of this note is to show that this conclusion is too optimistic and we may have backward bending supply curve.

I shall use Raj Krishna's notation to make comparison easy. Let

$Q$ = the quantity of wheat produced.

$C$ = the quantity of wheat consumed.

$M$ = the quantity of wheat marketed.

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1. Raj Krishna, "A Note on the Elasticity of the Marketable Surplus of a Subsistence Crop," *Indian Journal of Agricultural Economics*, Vol. XVII, No. 3, July-September, 1962, pp. 79-84.

c = $\frac{C}{Q}$ = the consumption ratio.

m = $\frac{M}{Q}$ = the sales ratio.

r = $\frac{Q}{M} = \frac{1}{m}$ = the reciprocal of the sales ratio.

P = the relative price of wheat.
Y = the total income of the peasants.
e = the elasticity of the market supply with respect to P.
d = the total elasticity of home consumption with respect to P.
b = the elasticity of output with respect to P.

Abstracting from changes in inventories, we have

$$M = Q - C$$

$$\therefore \frac{dM}{dP} = \frac{dQ}{dP} - \frac{dC}{dP}.$$

In terms of elasticities, this reduces to

$$e = rb - (r-1)\, d.$$

Given that $r > 1$ and $b \geqslant 0$, e will certainly be positive if $d < 0$. Raj Krishna then goes on to derive an expression for d and hence find likely values of e given estimated values of other elasticities.

Raj Krishna specifies the home consumption function in terms of elasticities.

$$\frac{dC}{C} = g\frac{dP}{P} + h\frac{dY}{Y}. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots (1)$$

where g is the elasticity of the substitution effect and h is the elasticity of the income effect.[2] It is not quite clear what he means by the substitution effect; is it $\frac{\partial C}{\partial P}$ with income kept constant or is it $\frac{\partial C}{\partial P}$ with utility kept constant ?

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2. I suppose this is derived from a consumption function $C = F(P, Y)$. Differentiating both sides

$$dC = \frac{\partial C}{\partial p}\, dP + \frac{\partial C}{\partial Y}\, dY.$$

Dividing by C

$$\frac{dC}{C} = \frac{\partial C}{\partial p}\frac{dP}{C} + \frac{\partial C}{\partial Y}\frac{dY}{C}$$

$$\therefore \frac{dC}{C} = g\frac{dp}{p} + h\frac{dY}{Y}.$$

Let us assume by substitution effect he means $\left.\frac{\partial C}{\partial P}\right|_{\bar{y}}$. Then given that

$$Y = \text{some constant} + QP$$

by differentiating both sides we get

$$dY = QdP + P\frac{dQ}{dP}dP.$$

Raj Krishna asserts that dY=QdP—CdP, *i.e.*, he omits the term $P\frac{dQ}{dP}dP$ and further substracts CdP. Obviously, farmers' monetary income, as defined here, does not depend upon their own consumption. Substituting for dY in (1) we get

$$\frac{dC}{C} = g\frac{dP}{P} + h\frac{Q + P\frac{dQ}{dP}}{Y}dP.$$

It can be easily shown that

$$\frac{dC}{C} = g\frac{dP}{P} + h\,k\frac{dP}{P} + h\,k\,b\frac{dP}{P}$$

where $k = \frac{QP}{Y}$. It follows that

$$d = g + h\,k + h\,k\,b.$$

Compare this with Raj Krishna's result, which is

$$d = g + m\,h\,k.$$

Given that $g < 0$, $0 < m < 1$, $h > 0$, $k > 0$, $b > 0$, it is clear that Raj Krishna's result has a downward bias. As it is shown below, for some values of our parameters, this bias makes d negative and, therefore, e positive when in fact d is positive and e negative (*i.e.*, backward bending supply).

If we assume that he is dealing with the pure substitution effect $\left(\textit{i.e.}, \left.\frac{\partial C}{\partial P}\right|_{\bar{U}}\right)$ then our refutation of his conclusion is based on a different argument. We have

$$C = \Psi(P, Y)$$

$$\therefore\ dC = \left.\frac{\partial C}{\partial P}\right|_{\bar{Y}} dp + \left.\frac{\partial C}{\partial Y}\right|_{\bar{P}} dY.$$

But substituting for $\left.\frac{\partial C}{\partial p}\right|_{\bar{Y}}$,

$$\left.\frac{\partial C}{\partial p}\right|_{\bar{Y}} = \left.\frac{\partial C}{\partial p}\right|_{\bar{U}} - C \left.\frac{\partial C}{\partial Y}\right|_{\bar{P}}$$

$$\therefore dC = \left.\frac{C}{\partial p}\right|_{\bar{U}} dp - C \left.\frac{\partial C}{\partial Y}\right|_{\bar{p}} dp + \left.\frac{\partial C}{\partial Y}\right|_{\bar{p}} dY$$

$$= \left.\frac{\partial C}{\partial p}\right|_{\bar{U}} dp + \frac{\partial C}{\partial Y}\left[ dY - cdp \right]$$

$$\therefore \quad \frac{dC}{C} = g' \frac{dp}{P} + h \frac{dY - Cdp}{Y}.$$

where g′ is the elasticity of the pure substitution effect and h is the income elasticity. What Raj Krishna calculates as dY is this dY — Cdp. (Still, his error about omitting $P \frac{dQ}{dP}$ dp remains). It is easy to show that

$$\frac{dC}{C} = g' \frac{dP}{P} + m\,h\,k \frac{dp}{P} + h\,k\,b \frac{dp}{P}$$

$$\therefore \quad d = g' + m\,h\,k + h\,k\,b.$$

Again, compare this with Raj Krishna's result, which is

$$d = g' + m\,h\,k.$$

Omitting h k b is not so serious because it is small compared to the other terms. However, his error remains because he uses the wrong value for g′ which he takes as estimates of price elasticity obtained from demand studies. It is well known that such estimates are not the pure substitution elasticities but simple price elasticities, *i.e.*, $\left.\frac{\partial C}{\partial P}\right|_{\bar{Y}} \cdot \frac{P}{C}$. It can be shown that

$$g' = g + \frac{\partial C}{\partial Y} P.$$

Of course when we allow for this fact and substitute for g′ we get our original equation, which is

$$d = g + h\,k + h\,k\,b.$$

In order to compare the two results I shall reproduce Raj Krishna's table and present the corresponding results, using the corrected formulation.

CALCULATION OF PLAUSIBLE LIMITS OF e

RAJ KRISHNA'S

| Plausible Ranges of Parameters | Values Relevant for Min. e | | Values Relevant for Max. e |
|---|---|---|---|
| b = .1 to .2 | .1 | | .2 |
| g = —.2 to —.4 | —.2 | | —.4 |
| h = .5 to .8 | .8 | | .5 |
| k = .1 to .7 | .7 | | .1 |
| | m = .1 | m = .5 | m = .9 |
| Min. e | 2.30 | .12 | .08 |
| Max. e | 5.56 | .78 | .26 |
| Min. e (g = 0) | .50 | —.08 | .06 |

Plausible Limits of e (This Note's Results)

| | m = .1 | m = .5 | m = .9 |
|---|---|---|---|
| Min. e | —2.74 | —.216 | .07 |
| Max. e | 5.56 | .74 | .56 |
| Min. e (g = 0) | —4.744 | —.416 | .04 |

The results need no further comments, and it is clear that we can have backward bending supply curve.

In the above discussion, I have not questioned Raj Krishna's behavioural assumptions. Of course we can have different assumptions which would make the situation better or worse, but that is a question to be decided empirically.

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