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importance of the agricultural sector in the Indian economy needs no emphasis. Thus, from the agricultural planning point of view, the input-output table should distinguish as many sectors as possible within the agricultural sector and include or distinguish such other sectors which provide inputs to or processed products from agriculture. The classification of sectors for use in the construction of the table is, of course, conditioned by availability of adequate and reliable statistical information. The coverage of statistical data and their availability is at present more widespread and reliable in the agricultural sector than in animal husbandry, forestry and fisheries. As such, more production activities within the agricultural sector need to be included in the construction of any input-output table that might be attempted now and which is to be bigger than the present one. For instance, the production activities that might be considered are production of major food crops, commercial crops like cotton, sugarcane and oilseeds and plantation crops like tea, coffee and rubber.

A. V. K. SASTRI*

A SIMPLE QUANTITATIVE MODEL FOR THE ALLOCATION OF LAND TO MORE THAN ONE CROPS IN RELATION TO SIZE†

It has been observed that the allocation of cultivated land for different crops varies with the size of the total land holding,[1] and in particular that cash crop acreage forms a larger proportion of the total land holding in the larger holdings than in the smaller holdings. In general the net profitability per acre is greater in the case of cash crops than in the case of food crops. A number of explanations can be put forward for this difference in the allocation-pattern in such a situation. Here the allocation of land between two main groups of crops only, namely, cash crops and food crops is considered.

(1) Domestic demand for food crops both for consumption, kind wages, etc., wherever wages are given in kind and there are other kind transactions, has to be taken into account while studying the allocation-pattern of land between cash crops and food crops.

Domestic demand for food crops which is for consumption and which forms the major part of the total domestic demand is proportional to the family size and not to the size of the holding. To the extent, per capita (or per consumption unit) land is larger in a holding, a greater percentage of the land becomes available for profit-cultivation (cultivation for non-consumption purposes). Again a large holder may be less averse to monetary transactions and hence he may not mind going in for the most profitable crop allocation even though it may entail his buying some food crops for his own use. Similarly, a small holder may go in for more kind wage and other non-monetary transactions than a larger holder and hence

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1. Sapre, Seminar paper entitled "Changes in the Cropping Pattern of Some Farmers in Two Irrigated Villages in the Nasik District during 1940-60," Gokhale Institute of Politics and Economics, Poona 4, 1961.

the domestic demand for food crops is relatively greater for the small farmer. This type of demand for food crops may be termed the transactions demand or the demand with a transaction-motive behind it.

(2) There could be a precautionary or a risk covering motive also in that the farmer would go without money rather than without food in case of emergencies arising out of price rises, crop failures, etc. This tendency may be found to a greater extent in a small farmer than in a large farmer.

(3) There may also be a psychological inertia factor resisting a change to cash crops. One would expect this factor to be more powerful in the case of small farmers than in the case of large farmers. This is only a hypothesis which needs to be tested. Some of the reasons for this behaviour pattern are : The small farmer may be characterised by (1) less monetization, (2) less finances needed for the change over, and (3) less risk-bearing capacity, than a large farmer.

(4) There may be a feasibility factor which has to be taken into account if there are important size-differentials in land such as a larger percentage of the larger holdings are irrigated and so on. These feasibility factors could of course be taken account of in the profitability factor by proper evaluation of the cost of production.

(5) The profitability factor or the profit maximization-motive could also lead to different crop-allocations for holdings of different sizes. In what follows only the last one is taken up for study. The subsistence or domestic-demand factor could, however, be easily incorporated into the simple quantitative models that are considered below for the profit maximizing crop-allocations.

For the sake of simplicity let us suppose that there are only two groups of crops under study. The analysis can easily be generalized to more than two crops. These two groups may be taken to be the group of cash crops and the group of food crops. Let x_1 and x_2 stand for the acreages under the two food and cash crops respectively and let the total land available be k acres. Let p_1 and p_2 be the food and cash value of crops on one acre of land each and let $C(x_1, x_2)$ be the joint cost functions of cultivating the food and cash crops on x_1 and x_2 acres of land. Then the farmer can be supposed to choose x_1 and x_2 so that his net profit

$$\Pi = p_1x_1 + p_2x_2 - C(x_1, x_2)$$

is maximized subject to $x_1 + x_2 \leqslant k$, $x_1 \geqslant 0$, $x_2 \geqslant 0$.

The other inputs on land are supposed to be in optimum proportion to the land. As mentioned before, it is assumed here that other non-economic considerations do not enter into the picture. One economic consideration that may be easily taken into account is the tendency of the farmer to grow his own food. This may not at all be as unscientific and as sentimental as it appears, for considerations of risk due to uncertain prices, considerations of economy of transactions and considerations of the satisfactory quality of the food may influence the farmer's decision regarding allocation of land. This could be done by setting aside enough land to produce food to satisfy the farmer's domestic demand, that is by replacing the inequality $x_1 \geqslant 0$ by $x_1 \geqslant x_{10}$ or replace x_1 by $x_1' = x_1 - x_{10}$.

The farmers considered are small farmers and hence they do not influence the prices and hence they do not influence p_1 and p_2, the productivity per acre being assumed to be constant.

This is a particular case of joint production when some of the resources that are available are fixed. The problem as posed above is one of programming and further as the intention is to investigate how the allocation-proportions change as k the size of the landholding changes, it becomes a problem of parametric programming[2] (not necessarily linear) where the constraint-constants such as k in this problem change. As the interest in this Note lies more in the study of changes in the allocation-proportions as k changes than in the methodology of parametric programming, only explicit solutions are worked out with the assumption of some plausible cost-function and no attempt is made to study or develop general methods of parametric programming when the constraint-constants change, in the linear or non-linear case.

The plausible cost-functions considered are :

(1) linear and continuous independent cost-function;
(2) linear and discontinuous independent cost-functions;
(3) discontinuous and quadratic joint cost-functions; and
(4) independent and quadratic cost-functions.

(1) This is the simplest case to be considered. Here the cost-function is given by

$$C(x_1, x_2) = C_1(x_1) + C_2(x_2)$$

$$\text{where } C_1(x_1) = c_0 + c_1 x_1$$

$$\text{and } C_2(x_2) = c_0' + c_2 x_2.$$

c_0 and c_0' are evidently the fixed costs and c_1, c_2 are the constant marginal costs.

The net profit-function is then given by

$$\Pi = p_1 x_1 + p_2 x_2 - (c_0 + c_1 x_1) - c_0' + c_2 x_2)$$

and this has to be maximized subject to $x_1 \geqslant x_{10}$, the minimum domestic demand and $x_2 \geqslant 0$ and $x_1 + x_2 \leqslant k$.

It can easily be shown that the net profit is maximized when

$$\left.\begin{array}{l} x_1 = k \\ x_2 = 0 \end{array}\right\} \text{ if } (p_1 - c_1) > (p_2 - c_2) \qquad \text{(i)}$$

and

$$\left.\begin{array}{l} x_2 = k - x_{10} \\ x_1 = x_{10} \end{array}\right\} \text{ if } p_2 - c_2 > p_1 - c_1 \qquad \text{(ii)}$$

(provided the profit Π is positive at the maximizing point).

2. For a brief discussion of parametric programming see S. Vajda: Mathematical Programming, Chapter on Parametric Programming.

p_1-c_1 and p_2-c_2 are the marginal net returns per acre for the food and cash crops respectively and the results (1) and (2) can be interpreted as that x_{10} should be allotted for food crops and the rest of the land should be allotted to that crop for which the marginal net return is larger. One would expect the marginal net return to be higher for cash crops than for food crops and hence one would except result (2) to hold. The proportion of land allotted to food crops to total land in this case is given by $\frac{x_{10}}{k}$ and this diminishes as k, the total size of landholding increases, a result which is consistent with the observed tendency (p. 56). x_{10} largely depends upon the size of the family and its food consumption pattern and hence it need not be constant from farmer to farmer. If x_{10} is zero, then from (1) and (2) it follows that all the land is allotted to that crop which has a higher marginal net return. x_{10} itself, however, may not be independent of k. The size of the family and k may be positively correlated and to that extent as k increases x_{10} may on the average increase; on the other hand as k increases the income of the farmer increases and he may substitute part of the home-grown food by processed and other food so that x_{10} may decrease and to that extent $\frac{x_{10}}{k}$ would decrease still further as k increases. Though no verification is attempted here, one would expect that the overall effect would be one of $\frac{x_{10}}{k}$ decreasing as k increases.

(2) The cost-functions in (1) are unrealistic in that even when x_1 (or x_2) is exactly zero, that is, even when food (cash) crops are not at all produced the cost of production is c_0 (or c_0') and not zero. This shortcoming in the cost-function can be corrected by taking the cost-functions to be given by

$$
\begin{aligned}
C_1(x_1) &= c_0 + c_1 x_1 && x_1 > 0 \\
&= 0 && x_1 = 0 \\
\text{and} \quad C_2(x_2) &= c_0' + c_2 x_2 && x_2 > 0 \\
&= 0 && x_2 = 0
\end{aligned}
$$

Here x_{10} is assumed to be zero.

These are linear cost-functions with discontinuity at x_1 (or x_2) $= 0$.

The net profit is then given as follows :

$$
\begin{aligned}
\Pi &= p_1x_1 + p_2x_2 - (c_0 + c_1 x_1) - (c_0' + c_2 x_2) \\
&\qquad \text{when } x_1 \neq 0,\ x_2 \neq 0 \\
&= p_1 x_1 - c_0 - c_1 x_1 \\
&\qquad \text{when } x_1 \neq 0,\ x_2 = 0 \\
&= p_2 x_2 - c_0' - c_2 x_2 \\
&\qquad \text{when } x_1 = 0,\ x_2 \neq 0 \\
&= 0 \qquad \text{when } x_1 = 0,\ x_2 = 0.
\end{aligned}
$$

Π is to be maximized subject to

$$x_1 \geqslant 0,\ x_2 \geqslant 0 \text{ and } x_1 + x_2 \leqslant k.$$

It can be easily shown by the same elementary methods[3] as for continuous linear profit functions that the optimizing solution is given as follows :

3. For an exposition of these see R. G. D. Allen: Mathematical Economics, Chapter on Linear Programming.

(1) No production takes place if $k < \min. \left(\frac{c_o'}{p_2 - c_2}, \frac{c_o}{p_1 - c_1}\right)$. $\frac{c_o}{p_1 - c_1}$ can be interpreted as the amount of land necessary to recover the fixed cost c_o by growing food crops and similarly for $c_o'/(p_2 - c_2)$.

(ii) If $k > \min. \left(\frac{c_o'}{p_2 - c_2}, \frac{c_o}{p_1 - c_1}\right)$ then all the land is allotted to the food or cash crops according as k is less than or greater than $(c_o' - c_o)/[(p_2 - c_2) - (p_1 - c_1)] = \lambda$.

If $c_o' - c_o > 0$ and $p_2 - c_2 > p_1 - c_1$ as one would expect then λ can be interpreted as the amount of land for which the net profit with either of the crops grown is the same. If $k > \lambda$, then the disadvantage of larger fixed cost c_o' for cash crops is more than met by the advantage of larger marginal net return $p_2 - c_2$ and hence only cash crops are grown. If $k < \lambda$ this is not true and only food crops are grown. If $k = \lambda$ then either of the crops but only one crop can be grown.

These results are consistent with the observed trend mentioned on p. 56.

When $x_{10} \neq 0$, these results get modified and we get the solutions as follows.

x_{10} of land by the restriction imposed has to be allotted for food crops. All the rest of the land $k - x_{10}$ is allotted for cash crops or food crops according as

$$k - x_{10} \gtrless \frac{c_o'}{p_2 - c_2 - (p_1 - c_1)} = \mu > 0 \text{ when } p_2 - c_2 > p_1 - c_1$$

μ can be interpreted as the land required to cover the fixed cost c_o' of cash crops with the per acre gain $p_2 - c_2 - (p_1 - c_1)$ in marginal net return the cash crops have over the food crops. Since food crops have to be grown on x_{10} acres of land, their fixed cost has anyway to be incurred. Hence if the rest of the land $(k - x_{10})$ is more than what is necessary for the fixed cost c_o' to be covered by the gain in marginal net return, then cash crops can be grown on all of it and if $k - x_{10}$ is not enough to cover the fixed cost c_o', then food crops are to be grown on it. If $k - x_{10} = \mu$ or the fixed cost c_o' can be just met then either cash crops or food crops but only one crop and not both can be grown on the rest of the land $k - x_{10}$.

(3) In both the cases (1) and (2), the cost-functions for the two crops are independent. The benefit or otherwise of growing the crops together if any is not taken into account. Such a joint effect may be introduced in the cost-function in a simple manner as follows.

Let
$$C(x_1, x_2) = c + c_1 x_1 + c_2 x_2 - c_3 x_1 x_2, \quad x_1 \text{ and } x_2 \neq 0 \text{ simultaneously}$$
$$= 0, \quad x_1 = x_2 = 0.$$

$c_3 > 0$ when there are benefits of joint production, and $c_3 \leqslant 0$ otherwise. Then

$$\Pi = (p_1 - c_1)\, x_1 + (p_2 - c_2)\, x_2 + c_3\, x_1\, x_2 - c,$$
$$\text{when } x_1 \text{ and } x_2 \neq 0 \text{ simultaneously}$$
$$= 0 \quad \text{when} \quad x_1 = x_2 = 0.$$

Again this has to be maximized subject to

$x_1 \geqslant x_{10},\ x_2 \geqslant 0$ and $x_1 + x_2 \leqslant k$.

First let us consider the case of joint benefits, that is $c_3 > 0$.

Also as before let $p_2 - c_2 > p_1 - c_1$ and let $x_{10} = 0$.

Then the solution can be shown to be given by :

(i) $x_1 = \frac{k}{2} - \frac{p_2 - c_2 - (p_1 - c_1)}{2\, c_3} < \frac{k}{2}$

and $x_2 = \frac{k}{2} + \frac{p_2 - c_2 - (p_1 - c_1)}{2\, c_3}$

provided $k \geqslant \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$, for otherwise x_1 becomes negative and provided the net profit at this point is positive. If the net profit is negative at this point then evidentally $x_1 = 0 = x_2$ is the solution or no production takes place.

(ii) If $k \leqslant \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$ then (o, k) is the solution or all the land is allotted to cash crops provided $k > \frac{c}{p_2 - c_2}$, for otherwise the net profit is negative at (o, k). If $k < \frac{c}{p_2 - c_2}$ then (o, o) is the solution or there is no production.

Results in (i) and (ii) can be summarized as follows:

When $k < \frac{c}{p_2 - c_2}$ there is no production; when $k > \frac{c}{p_2 - c_2}$ but $\leqslant \left[\frac{p_2 - c_2 - (p_1 - c_1)}{c_3}\right]$ only cash crops are to be grown and when k increases further, food crops are to be grown on less than half of the land and cash crops on the rest. The proportion of food crops land to total land increases to $\frac{1}{2}$ as k increases to infinity. These results are contrary to what is usually expected for they imply that as the size of the total land increases, the proportion allotted to food crops increases.

If $x_{10} \neq 0$ then x_{10} is anyway allotted for food crops and the above argument can be applied to the allotment of the rest of the land. Results (i) and (ii) become

(iii) If $k - x_{10} \leqslant \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$,

$$x_1 = x_{10}$$

and $x_2 = k - x_{10}$.

(iv) and when

$$k - x_{10} > \frac{p_2 - c_2 - (p_1 - c_1)}{c_3},$$

$$x_1 = \frac{k - x_{10}}{2} + x_{10} - \frac{(p_2 - c_2) - (p_1 - c_1)}{2\,c_3} \text{ and } x_2 = k - x_1.$$

Then x_1/k is fixed and equal to $\frac{x_{10}}{k}$ till $k \leqslant x_{10} + \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$ and then as k increases from $x_{10} + \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$ to ∞, $\frac{x_1}{k}$ diminishes from a little over $\frac{1}{2}$ to $\frac{1}{2}$ if $x_{10} > \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$ and x_{10} increases from a little less than $\frac{1}{2}$ to $\frac{1}{2}$ if $x_{10} < \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$ and is equal to $\frac{1}{2}$ if $x_{10} = \frac{p_2 - c_2 - (p_1 - c_1)}{c_3}$. All these results simplify if $c_3 > 0$, that is, there are disadvantages of growing both the crops simultaneously. Then the results can be shown to be as follows :

x_{10} is by assumption allotted to food crops. The rest of the land $k - x_{10}$ is allotted to cash crops or food crops according as $(p_2 - c_2 - c_3 x_{10})$ which is the marginal net return for cash crops corrected for the disadvantage of having to grow both the crops simultaneously is greater than or less than $p_1 - c_1$ provided the profit is positive at these points.[4]

(4) If instead of the independent linear cost-functions, independent U-shaped cost-functions given by

$$C_1(x_1) = a_1 - 2\,b_1 x_1 + c_1 x_1^2 \quad , x_1 \neq 0$$
$$= 0 \quad , x_1 = 0$$
$$\text{and } C_2(x_2) = a_2 - 2\,b_2 x_2 + c_2 x_2^2 \quad , x_2 \neq 0$$
$$= 0 \quad , x_2 = 0,$$

(a_r, b_r, and c_r being > 0, $r = 1, 2$)

$\Pi = [p_1 x_1 + p_2 x_2 - C_1(x_1) - C_2(x_2)]$

has to be maximized subject to

$x_1 + x_2 \leqslant k$, $x_1 \geqslant 0$ and $x_2 \geqslant 0$, x_{10} being assumed to be zero.

Let Π_{rm} be the maximum of Π_r attained at x_{rm}, $r = 1, 2$. It is easy to see

$$\left.\begin{aligned} &\text{that } x_{rm} = \frac{p_r + 2\,b_r}{2\,c_r} \\ &\text{and } \Pi_{rm} = c_r \left(\frac{p_r + 2\,b_r}{2\,c_r}\right)^2 - a_r \end{aligned}\right] r = 1, 2.$$

4. By changing the variable x_1 to $x_1' = x_1 - x_{10}$, Π reduces to $\Pi = (p_2 - c_2 - c_3 x_{10}) x_2 + (p_1 - c_1) x_1' - c_3 x_1' x_2 + (p_1 x_{10} - c - c_1 - x_{10})$ which has to be maximized, subject to $x_1 + x_2 \leqslant k$ or $x_1' + x_2 \leqslant k'$ where $k' = k - x_{10}$, and $x_1' \geqslant 0$, $x_2 \geqslant 0$. Since joint production is disadvantageous only that crop for which the coefficient in Π above is larger need be grown on all k' and hence we get the above result.

One would expect

$$a_2 > a_1$$
$$\Pi_{2m} > \Pi_{1m}$$
$$\text{and} \quad x_{2m} > x_{1m},$$

that is fixed costs, maximum net profit and the size of holding at which the maximum occurs are all greater for cash crops than for food crops. Even with these plausible assumptions about Π_1 and Π_2, the solutions are not easy to write as $\Pi_r(x_r)$ is discontinuous at $x_r = 0$, $r = 1, 2$. Then as k increases from 0 to ∞ it can be seen[5] that the solution is given by :

(i) (o,o) or no production is undertaken when k is not large enough to make $\Pi_1(x_1)$ positive.

(ii) then as k increases, so long as k is not large enough to make $\Pi_2(x_2) > \Pi_1(x_1)$, (k,o), (x_{1m},o), (x_{10}', k—x_{10}') or (x_{1m}, k$-x_{1m}$) is the solution depending upon the actual values of a_r, b_r, c_r, $r = 1, 2$ and k. That point at which Π is the largest has to be chosen.

(iii) Then as k increases further, so long as $k < x_{1m} + x_{2m}$ (o,k), (o, x_{2m}), (x_{10}', k—x_{10}') or (k — x_{2m}, x_{2m}) is the solution, the solution again depending upon the actual values of a_r, b_r, c_r, $r = 1, 2$ and k. That point at which Π is the largest has to be chosen. In (ii) and (iii) x_{10}' maximizes Π when $x_1 \neq 0$ and $x_2 \neq 0$ and is given by

$$\frac{\partial \Pi_1(x_1)}{\partial x_1} = \left[\frac{\partial \Pi_2(x_2)}{\partial x_2}\right]_{x_2 = k - x_1}$$

or by

$$x_{10}' = \frac{2c_2k - (p_2 - p_1) - 2(b_2 - b_1)}{2(c_1 + c_2)}$$

and (iv) When $k \geqslant x_{1m} + x_{2m}$, $x_1 = x_{1m}$, $x_2 = x_{2m}$ is the solution. The rest of the land $k - x_{1m} - x_{2m}$ is not utilized.

On the whole one could say that the tendency exhibited here is one of allotting a larger proportion of land to food crops when k is relatively small and a smaller proportion of land to food crops when k is relativelly large. Some land being left uncultivated as is the case here for some values of k is not entirely unrealistic, for fallow land is not unusual. These results have to be slightly modified when $x_{10} \neq 0$.

Other cost-functions could also be brought in but the ones considered above appear to be the simpler and more plausible of the cost-functions that can be considered.

(5) In the above cases, no restrictions on the availability of resources other than land are imposed. Restrictions on the irrigation water available, or the capital equipment available or the amount of fertilizers available or the total credit available to a farmer with a given land holding should really be imposed to make the analysis more realistic. The amounts of these various physical or fiscal re-

5 With the assumptions made above, Π_1 and Π_2 can be represented graphically. The above solutions are written by taking into account the shapes and relative positions of $\Pi_1(x_1)$ and $\Pi_2(x_2)$.

sources available may not always be proportional to k but may be other functions of k. The analysis to be worked out in general terms becomes rather complicated. As an illustration the case of an additional restriction on irrigation water with linear cost-function which appears to be more plausible than others in agriculture is considered in detail below.

The profit function is now

$$\Pi = (p_1 \quad c_1)\, x_1 + (p_2 - c_2)\, x_2 - c_o - c_o'$$

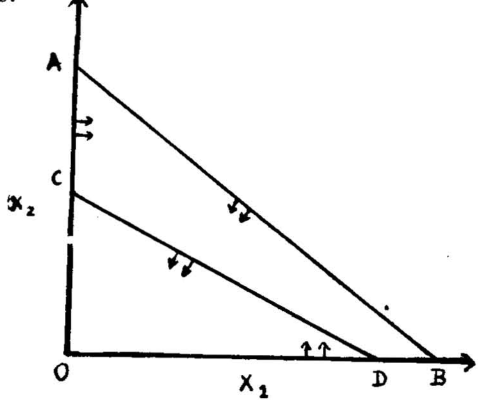
which has to be maximized subject to

$$x_1 + x_2 \leqslant k$$
$$w_1 x_1 + w_2 x_2 \leqslant W(k)$$

and $x_1 \geqslant 0$, $x_2 \geqslant 0$, x_{10} being put equal to zero. w_1 and w_2 are the amounts of water needed per acre for food and cash crops respectively and W(k) is the amount of water available to the farmer which is taken to be a function of k. w_2 may be expected to be greater than w_1. If the water available is too little and $W(k) < \min. (w_1 k, w_2 k)$, then only the water constraint need be taken into account.[6]

Then all the land for which water is available (which is $\frac{W(k)}{w_1}$ in the case of food crops and $\frac{W(k)}{w_2}$ in the case of cash crops) is allotted to cash crops or food crops according as the ratio of net marginal returns $\frac{p_2 - c_2}{p_1 - c_1}$ is greater than or less than the ratio of water requirements per acre $\frac{w_2}{w_1}$. If the total amount of water available is sufficiently large and $W(k) > \max. (w_1 k, w_2 k)$ then only the land constraint $x_1 + x_2 \leqslant k$ is applicable and all the land is allotted to cash or food crops according as $p_2 - c_2 \gtrless p_1 - c_1$. If k lies between $\frac{W(k)}{w_1}$ and $\frac{W(k)}{w_2}$ then if $\frac{p_2 - c_2}{p_1 - c_1} > \max.\left(1, \frac{w_2}{w_1}\right)$ or $< \min. \left(1, \frac{w_2}{w_1}\right)$, (k, o) or $\frac{W(k)}{w_2}$

6.



(1) If the linear restrictions are represented by the regions to the left of the st. lines AB and CD as in the figure to the left and $x_1 \geqslant 0$ and $x_2 \geqslant 0$; then if as in the figure CD lies entirely below AB, the feasibility region is given by the set of points in and on the triangle OCD and hence the restriction corresponding to AB becomes inoperative.

(2) For the simple method of solution when there are two variables and two linear restrictions followed here see R. G. D. Allen: Mathematical Economics, Chapter on Linear Programming.

is the solution according as $(p_1 - c_1)\,k - c_0 - c_0'$ is greater than or less than $(p_2-c_2)\,k-c_0-c_0'$. Since $p_2-c_2>p_1-c_1$ all land gets allotted to cash crops. If $\frac{p_2 - c_2}{p_1 - c_1}$ lies between 1 and $\frac{w_2}{w_1}$ then the solution is given by the point of intersection of
$x_1 + x_2 = k$ and $w_1 x_1 + w_2 x_2 = W(k)$.

The point of intersection is given by

$$x_{1m} = (w_2 k - W(k))/\,(w_2 - w_1)$$
$$\text{and } x_{2m} = (W(k) - w_1 k)\,/\,(w_2 - w_1).$$

Since W(k) lies between w_1k and w_2k the water needed for cultivating food and cash crops respectively, x_{1m} and x_{2m} are both non-negative. When $W(k)=ak$, or the same amount of water is available for every acre, then the equations giving x_{1m} and x_{2m} reduce to

$$\left.\begin{aligned}\frac{x_{1m}}{k} &= \frac{w_2 - a}{w_2 - w_1}\\ \frac{x_{2m}}{k} &= \frac{a - w_1}{w_2 - w_1}\end{aligned}\right\}\text{ independent of } k.$$

It may, however, be that more water per acre is available for larger holdings than for smaller holdings. As a simple representation of such a situation we may take $W(k) = ak^2$. Then the water per acre $\frac{W(k)}{k}$ is proportional to k. The equations giving x_{1m} and x_{2m} then reduce to

$$\begin{aligned}\frac{x_{1m}}{k} = \frac{w_2 - ak}{w_2 - w_1} &\to 0 \text{ as } k \to \frac{w_2}{a} \text{ (from below)}\\ &\to 1 \text{ as } k \to \frac{w_1}{a}\end{aligned}$$

$$\begin{aligned}\text{and } \frac{x_{2m}}{k} = \frac{ak - w_1}{w_2 - w_1} &\to 1 \text{ as } k \to \frac{w_2}{a}\\ &\to 0 \text{ as } k \to \frac{w_1}{a} \text{ (from above).}\end{aligned}$$

Again the tendency exhibited is one of all food crops for sufficiently small k, then for some range of k, a combination of cash and food crops the proportion of cash crops rising with k, then finally with larger k cultivation of cash crops on all k.

The results can be modified when x_{10} is not equal to zero.

(6) If in the above case the cost-functions are linear but discontinuous at zero as in case (2), then the above results are modified a little. As in (5) when $W(k) >$ max. $(w_1k,\ w_2k)$ or $<$ min. $(w_1k,\ w_2k)$, only the land constraint or

only the water constraint is operative. When only the land constraint is operative the problem reduces to that considered in case (2). When only the water constraint is operative by changing the variables to $x_1' = w_1 x_1$ the water constraint can be reduced to the form $x_1' + x_2' \leqslant k$ and the treatment in case (2) can be applied and the results obtained. When W(k) lies between $w_1 k$ and $w_2 k$, the constraint-lines intersect in the first quadrant. It is still true that the maximum occurs at one of the vertices $\left(o, \frac{W(k)}{w_2}\right)$, $\left(\frac{w_2 k - W(k)}{w_2 - w_1}, \frac{W(k) - w_1 k}{w_2 - w_1}\right)$, (k, o) or (o, o), of the feasibility region. The profits at these points are given by $(p_2 - c_2)\frac{W(k)}{w_2} - c_0'$, $\left[(p_1 - c_1)\left(\frac{w_2 k - W(k)}{w_2 - w_1}\right) + (p_2 - c_2)\left(\frac{W(k) - w_1 k}{w_2 - w_1}\right) - c_0 - c_0'\right]$, $(p_1 - c_1)k - c_0$ and o respectively. That point at which the profit is maximum has to be chosen. The initial costs enter into these expression in an unsymmetric fashion and hence the solutions in case (5) where the cost-functions are linear and continuous at zero may not be applicable here.

As mentioned earlier only a few simple cases are considered above to illustrate how the proportion of land allotted to food crops changes as the size of the land-holding changes. No empirical verification is attempted here as empirically estimated cost-functions at least in the Indian context are hard to find.[7]

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WHY CROP-CUTTING SURVEYS

SOME FURTHER COMMENTS

The purpose of this Note is to clear up certain issues raised by Dr. C. H. Shah in his reply[1] to my earlier Note[2] on crop-cutting surveys[3] published in this Journal.

The validity or otherwise of the use of t and F tests can hardly be a matter for argument. There are certain well-defined criteria for their application. The principal criterion is that the two sets of observations must be statistically independent. Without attempting an explanation of what is meant by statistical independence which can be found in any good textbook, I had indicated that the fact that each pair of observations from the two sets belonged to a common year vitiated

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7. The cost-function considered being simple, the solutions obtained explicitly have rather simple forms and hence it is not felt necessary to try the solutions out with hypothetical numerical values for the parameters. Moreover unless the hypothetical numerical values are within a plausible range, the numerical solutions obtained may lead to rather misleading impressions.

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1. "Comparison of Yield Estimates Prepared on the Basis of Traditional and Crop-Cutting Methods—A Reply," C. H. Shah, this *Journal*, Vol. XVIII, No. 2, April-June, 1963.

2. "Why Crop-Cutting Surveys?—A Rejoinder." V. G. Panse, this *Journal*, Vol. XVIII, No. 2, April-June, 1963.

3. "Comparison of Yield Estimates Prepared on the Basis of Traditional and Crop-Cutting Methods." C. H. Shah, this *Journal*, Vol. XVII, No. 4, October-December, 1962.