Vol XVI
No. 4

OCTOBER-
DECEMBER
1961

ISSN 0019-5014

# INDIAN JOURNAL OF AGRICULTURAL ECONOMICS

INDIAN SOCIETY OF
AGRICULTURAL ECONOMICS,
BOMBAY

of the problem for further refinement would of course be desirable by studying separately the effects of change in coverage and change in method of estimation of yields in areas reporting for earlier years and separately in areas which have become reporting more recently. This study can also be extended to individual states and to individual districts within states. This however is the question of availability of facilities.

We would conclude that once again the adjustment of production data carried out by us is a definite improvement, though there can still be further scope for refinement. Dr. Shah's paper does not, however, contain any concrete suggestion for a further development of study on this problem.

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## STATISTICAL MEASUREMENT OF VARIANCES FOR MARGINAL PHYSICAL PRODUCT AND MARGINAL RATE OF SUBSTITUTION

In production function studies the measurement of variances can be an aid in predicting the input combinations resulting in profit-maximizing and cost-minimizing. The usefulness of production function analysis lies in marginal product estimates, the variances of which are, more or less, cumbersome to estimate. The variances of marginal physical products thus derived are used for setting confidence limits and carrying out tests of significance.

The sampling variability, that is so characteristic in experimental data, seems to be a hindrance in the application of predicted optima to actual farm situations. In order to remedy this, the number of controlled variables included in the experiments can be increased, or probability estimates can be placed on uncontrolled variables. Even when this is done the predictions made in terms of point estimates are not truly reliable.

Interval estimates or confidence limits can be used to improve the measure of reliability. Confidence limits indicate a range of values within which the expected or average value of an estimate may lie, given some probability level, and are based upon the variance of the prediction.

*Variance Estimates for Marginal Physical Products*

The regression equation or production function to be used is :

$$\hat{Y} = aX_1 - bX_1^2 + cX_2 - dX_2^2 + eX_1X_2 \quad \ldots \quad (1)$$

where $\hat{Y}$ is the predicted yield of a crop measured in maunds per acre and $X_1$ and $X_2$ are the pounds of inputs (say fertilizer) applied per acre.

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The coefficients of the marginal products equations are derived from the coefficients of the above regression equation. The first partial derivative of the production function gives the following equations for the marginal products of $X_1$ and $X_2$.

$$\frac{\delta Y}{\delta X_1} = a - 2bX_1 + eX_2 = MPP_{X1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\frac{\delta Y}{\delta X_2} = c - 2dX_2 + eX_1 = MPP_{X2} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

An expression for estimating the variance of the marginal products, a linear function, can be obtained by applying standard methods from statistical theory.[1] For example, in the regression equation (1) the expression for the variance of the marginal physical product of $X_1$ input is :

$$\sigma MPP^2_{X1} = \sigma^2_2 - 4X_1\sigma_{ab} + 2X_2\sigma_{ae} + 4X_1^2\sigma^2_b - 2X_1X_2\sigma_{be} + X_2^2\sigma^2_e \quad .. \quad ..(4)$$

The estimated variance can be found by substituting sample values into (4). The standard errors are the square roots of variances.

The estimated variance of the marginal physical products of nitrogen (as linear function of N and $P_2O_5$) is given by Doll *et. al.*[2] These researchers have discovered that the estimated standard error is the smallest at the means of the input levels, and that the standard error increases as the input levels deviate from the means. This is inherent in the method of least squares, which minimizes the variance at the mean. In other words, the variation in the magnitude of the estimated standard errors is due to the method of estimation used, the model selected, and the level of fertilizer used.[3]

The small size of standard error indicates that the predicted marginal products are, oftentimes, quite reliable. Thus, even when the variance of the predicted yield is large, the variance of the slope of the regression equation is relatively small.

*Significance Tests in Production Function Research*

The use of variance estimates can be made in testing the significance of the marginal product. To test whether the average marginal product, of say nitrogen (N), is significantly different from zero at a point with a given level of probability and degrees of freedom, the appropriate t-test is : $t = \frac{MPP_n - O}{\sigma MPP_n}$ where t is the calculated Student's t value, $MPP_n$ is the marginal physical product of nitrogen and $\sigma MPP_n$ is the standard error of the marginal physical product of nitrogen.

1. B. Ostle: Statistics in Research, Iowa State University Press, Ames, 1954, pp. 214-217. Also, R. L. Anderson and T. A. Bancroft: Statistical Theory in Research, McGraw Hill Book Co., Inc., New York, 1952.

2. J. P. Doll, E. H. Jebe and R. D. Munsen, "Computation of Variance Estimates for Marginal Physical Products and Marginal Rates of Substitution," *Journal of Farm Economics*, Vol. XLII, August, 1960, pp. 596-607.

3. *Ibid.*, p. 599.

In the study of production function there are three important types of applications of tests of significance[4] :

1. To investigate about a given factor, say type of labour or kind of capital that affect the amount produced. In this case, the significance of a given marginal productivity is tested. The null hypothesis is that the specific marginal productivity is zero in the unknown hypothetical infinite population. If the probability that this should happen by pure chance is very small, say less than 5 per cent or 1 per cent, it is said that the marginal productivity of the specific factor is significant.

2. To compare the estimates of various marginal productivities. Moreover, investigation can be made about the null hypothesis that they are really equal in the unknown hypothetical infinite population from which the observations are a sample. If the probability is small (as in 1 above) that the empirical deviations from the null hypothesis could have happened by pure chance, then the two marginal productivities are significantly different at the arbitrary chosen level of significance.

3. The null hypothesis, that the production function is a linear homogeneous function, or that an equal proportional increase or decrease in the amounts of all the various factors used, will bring about the same proportional increase or decrease in the total products, can also be tested. If the probability that it is so is very low, say less than 5 per cent or 1 per cent, a significant deviation from the null hypothesis will be noted. It implies that then it will be very unlikely that the production function should be linear and homogeneous and that there would be no economies of scale.

The usefulness of the significance tests for the marginal products, is apparent when "independent" variables in the regression equation are highly correlated. When such a correlation is high, it may indicate that the total effect of an input variable is distributed among several coefficients, especially when interaction (or quadratic terms) are included in the equation. Even the quadratic terms implicitly contain a linear component. For these reasons, individual tests of each coefficient could show some to be significant and others to be non-significant. Therefore, a test of the marginal product equation will indicate the importance of the total marginal effect of the input variable on yield.

*Variance Estimates for Marginal Rates of Substitution*

The marginal rate of substitution is a ratio between two linear functions. (See equations (2) and (3)). The marginal rate of substitution of input $X_1$ for input $X_2$ is:

$$MRS_{X_1, X_2} = \frac{dX_1}{dX_2} = - \frac{MPP_{X_1}}{MPP_{X_2}} = - \frac{a - 2bX_1 + eX_2}{c - 2dX_2 + eX_1} \quad \cdot\cdot \quad \cdot\cdot \quad (5)$$

4. G. Tintner, "Significance Tests in Production Function Research," in Resource Productivity, Returns to Scale and Farm Size, Ed : E. O. Heady, Glenn Johnson and L. S. Hardin, Iowa State College Press, pp. 126-128.

From equation (5) it is obvious that the marginal rate of substitution is a ratio of marginal products.[5]

$$-\frac{MPP_{X_1}}{MPP_{X_2}} = \frac{\mu T}{\mu B} \quad \dots \quad (6)$$

where T represents the numerator and B represents the denominator. The variances of the marginal rates of substitution V(R), are estimated by:[6]

$$V(R) = \frac{\mu T^2}{\mu B^2}\left(\frac{\sigma T^2}{\mu T^2} + \frac{\sigma B^2}{\mu B^2} - \frac{2\sigma TB}{\mu T \mu B}\right) \quad \dots \quad (7)$$

The computation of estimates of $\sigma T^2$ and $\sigma B^2$ was given in the section on Variance Estimates for Marginal Physical Products. Estimates of $\mu T$ and $\mu B$ are :

$$\hat{\mu}T = MPP_{X_1}$$

$$\hat{\mu}B = MPP_{X_2}$$

Also, $\sigma TB$ is computed by :

$$\hat{\sigma}TB = E(TB) - \hat{\mu}T\hat{\mu}B$$

where $E\left(TB\right) = \Gamma\left[\left(MPP_{X_1}\right)\left(MPP_{X_2}\right)\right]$.

By substituting these estimates in equation (7), an expression for the variance of the marginal rate of substitution is derived. It is obvious that V(R) is a function of $X_1$ and $X_2$ inputs. These are the estimates of the parameter in the production function and the error mean square.

It can be assumed that the numerator and denominator of the expression for a marginal rate of substitution might have a bivariate normal distribution. From this assumption it follows that[7]

$$t = \frac{T - RB}{\left(\hat{\sigma}_T^2 + R^2\hat{\sigma}_B^2 - 2R\hat{\sigma}_{TB}\right)^{\frac{1}{2}}} \quad \dots \quad (8)$$

where T and B denote the numerator and denominator as in equation (7), R is the true unknown marginal rate of substitution, and t is Student's t. The equation (8) follows probability distribution of t because (T—RB) is normally distributed ; the denominator contains a proper sample estimate of the standard deviation of this difference. Now, having specified the t-value for the desired confidence limits, squaring both sides and solving the resulting quadratic for the unknown R, it yields values of R that may be used to set confidence limits for the marginal rate of substitution.[8]

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5. Doll, *et. al. Op. cit.*, Appendix, pp. 605-606.
6. W. G. Cochran, Sampling Techniques, John Wiley & Sons, New York, Inc., 1953, p. 117.
7. Doll, *et. al. Op. cit.*, Appendix, pp. 606-607.
8. See Cochran, *Op. cit.*, p. 121.

The variances and covariances have been calculated by Doll *et. al.*[9] These authors have estimated variances and standard errors for selected points on the corn yield isoquants of 54, 90 and 130 bushels per acre. Coefficient of variance (CV), the ratio of the standard errors and the absolute values of the estimated substitution rates, are derived to enable better evaluation of each marginal rate of substitution to its standard error. In a table presented by Doll *et. al.*,[10] it can be observed that estimated variances increase in size relative to the absolute value of the marginal rate of substitution towards the ends of the isoquants. Again, this effect is a result of fertilizer levels selected, the method of estimation employed, and the models specified.

In another table there are 95 per cent confidence intervals for marginal rates of substitution on the 90 bushel isoquant. Since in least squares analysis, the independent variables are assumed fixed, the limits do not represent movements along the isoquants ; they instead represent limits within which the slope of the isoquant might fall with a given combination of nitrogen and phosphorous.

The estimated variances increase as one moves on points farther on the isoclines ; that is, for points denoting larger inputs of N and $P_2 0_5$. Also, the confidence limits are widest on the extremes of the isoclines.

*Tool for Farm Economic Theory and Policy*

The procedures presented above that can be used to derive variance estimates for marginal physical products and marginal rates of substitution, are based on techniques developed in statistical theory. This appears, decidedly, to be a useful addition to many production function analysis. Furthermore, the estimates of economic optima can be used by research and extension workers as "bench-marks" for making recommendations to farmers.

As a tool for economic theory it is important to know which specific factors influence the production process. It is also important to know if the influence of one input is larger than the other. For example, if the government (or other agency) plans to promote agricultural production, it may want to encourage the use of the factors which in all probability increase the total product. In case of tests of significance which compare two empirical marginal productivities, it may influence the particular factors of production and types of enterprise which the government may want to encourage and subsidize in view of the goals of economic policy. Tests of significance relating to the homogeneity of the product may influence policies relating to the size of enterprise. It may help to determine whether the agricultural policy should favour small or large farms.

There may, of course, be other social or political reasons for the government to formulate policies, but this must seemingly be done apart from purely economic policy considerations. To an agricultural economist the policy considerations are only concerned with the productivity of agriculture.

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9. Doll, *et. al.*, *Op. cit.*, pp. 602-604.
10. Doll, *et. al.*, *Op. cit.*, p. 602.

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