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A LINEAR PROGRAMMING MODEL FOR THE SELECTION OF CROP ENTERPRISES ON AN AVERAGE FARM IN WESTERN UTTAR PRADESH*

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An attempt has been made in this paper to specify a linear programming model[1] for the selection of crop enterprises on an average farm in western Uttar Pradesh. The present analysis does not refer to any specific farm, but it represents the conditions expected on an average farm.

RESOURCE RESTRICTIONS

The objective of the farmer is to maximize the farm business income, given the resource restrictions, as specified below :

Land	..	12 acres
Adult workers	..	2
Pair of bullocks	..	1
Working capital	..	Rs. 1,000.00

Land :—There are three different qualities of soils on the farm, each of which is suited for different crops. The area under each of these soil types is as below :

(*a*)	Heavy clay soil (irrigated)	= 1 acre
(*b*)	Irrigated loam soil	= 7 acres
(*c*)	Unirrigated loam soil	= 4 acres

Since there are two cropping seasons, *Kharif* and *Rabi*, the farmer has 24 acres of crop-land for the whole year.

Human labour :—The two adult workers on the farm, *i.e.*, the farmer and his son, supply 22 months or 668 working days[2] of labour during the year. Out of the total of 668 days, only 85 per cent or 568 days can be used for crop production, the rest being allocated to the maintenance and upkeep of cattle. Thus, 284 human labour days are available for crop enterprises in each cropping season.

Bullock labour :—The farmer maintains one pair of bullocks which are used exclusively for crop enterprises. Assuming that the bullocks work for 11 months, or 334 days in a year, 167 bullock labour days are available in each season.

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1. For a detailed discussion of the technique of linear programming, the following publications are referred to: 1. A. Charnes, W. W. Cooper and A. Henderson : An Introduction to Linear Programming, John Wiley and Sons, Inc., New York, 1953 ; 2. D. K. Desai, "Linear Programming Applied to Problems in Indian Agriculture," *The Indian Journal of Agricultural Economics*, Vol. XIV, No. 2, April-June, 1960 ; 3. Robert Dorfman : Application of Linear Programming to the Theory of the Firm, University of California Press, Berkeley, California, 1951 ; 4. Robert Dorfman, P. A. Samuelson and R. M. Solow : Linear Programming and Economic Analysis, McGraw-Hill Book Company, Inc., New York, 1958 ; 5. E. O. Heady and Wilfred Candler : Linear Programming Methods, The Iowa State College Press, Ames, Iowa, U.S.A., 1958.

2. A working day is considered to be eight hours.

Working capital consists of that capital which the farmer needs to meet his day-to-day farm expenses, *e.g.*, the cost of seed, manure, irrigation charges, etc. The farmer has Rs. 1,000.00[3] for use in crop production, at the beginning of the year. The division of the available capital among two agricultural seasons poses a problem. It can neither be said that Rs. 500.00 would be available in each season, as the whole amount of Rs. 1,000.00 could possibly be utilised in *Kharif*, nor it is possible to assume that the amount of working capital invested in the production of *Kharif* crops would be replenished by the sale of these crops at the end of the season and hence would be available for reinvestment in *Rabi*, because some of the *Kharif* crops like sugarcane, continue in *Rabi* as well. It is, therefore, decided that the whole sum of Rs. 1,000.00 is to be made available for use in *Kharif* and that no capital is reserved specifically for *Rabi*. Those *Kharif* crops which remain in the field for one year would have a positive capital coefficient also in *Rabi*, while the crops which are harvested at the end of the *Kharif* season would become a capital supplying activity for the *Rabi* season. Such crops would have a negative capital coefficient in the *Rabi* capital restriction. In order to facilitate the transfer of capital from *Kharif* to *Rabi*, a separate activity has to be included in the programme. This activity would have a coefficient of plus one in the *Kharif* capital restriction and minus one in the *Rabi* capital equation. This point can be understood more easily after going through the whole model.

The restrictions can now be listed as below :

b_1	= *Kharif* heavy clay soil (irrigated)	= 1 acre
b_2	= *Kharif* loam soil (irrigated)	= 7 acres
b_3	= *Kharif* loam soil (unirrigated)	= 4 acres
b_4	= *Rabi* heavy clay soil (irrigated)	= 1 acre
b_5	= *Rabi* loam soil (irrigated)	= 7 acres
b_6	= *Rabi* loam soil (unirrigated)	= 4 acres
b_7	= Human labour days available in *Kharif*	= 284
b_8	= Human labour days available in *Rabi*	= 284
b_9	= Bullock labour days available in *Kharif*	= 167
b_{10}	= Bullock labour days available in *Rabi*	= 167
b_{11}	= Working capital available in *Kharif*	= Rs. 1,000.00
b_{12}	= Working capital available in *Rabi*	= 0

PROCESSES

Although a farmer in western Uttar Pradesh grows many crops and mixtures of these crops in each of the two seasons, only the most important crops have been considered as alternatives in the present analysis. It has also been recognized that almost all the farmers would like to have a few milch cattle. But, due to the insufficient data on dairy enterprises and also because of the fact that the farmer would maintain these cattle, even if they are uneconomic, dairying has not been considered as an alternative. However, to ensure that the optimal programme meets the fodder requirements of these cattle, additional restrictions have been included in the programme. These restrictions are of the following form :

b_{13} = *Kharif* fodder requirement $\geqslant$ 1.5 acres of *juar*
b_{14} = *Rabi* fodder requirement $\geqslant$ 28 *maunds* of wheat *bhusa*

3. The sum of Rs. 1,000.00 may look to be too high. However, all of it is not cash. It includes farm-grown products, like seed, farmyard manure, etc., used on the farm.

Juar is the only *Kharif* crop to be considered in the programme, which can supply fodder for cattle. The *Rabi* fodder requirement can be met either by one acre of irrigated wheat or two acres of unirrigated wheat.

The following crops, listed by seasons, have been considered as alternatives :

Kharif	*Rabi*
Sugarcane	Sugarcane
Paddy	Wheat (irrigated)
Cotton	Wheat (unirrigated)
Maize	Gram
Juar	

Since, in the linear programming technique it is assumed that the resource requirements for each process must be independent of the level of production of the other processes in the model, an important problem in the application of this technique to the selection of crop enterprises is the existence of complementary relationship among some crops, with respect to fertility input. For example, wheat taken after some *legume* crop in *Kharif* requires less nitrogen per acre than wheat grown after *juar*, so as to give the same yield per acre. This difficulty can, however, be handled by considering rotation as an individual process. But, because of the lack of sufficient input-output data for the various rotations as a whole, it has been assumed that there is no complementarity among the different crops.

An additional problem is encountered due to the differing soil-type requirements and harvesting periods of some of the crops. Paddy can only be grown on heavy clay soil, which is suitable for gram alone in *Rabi*. Thus, either the heavy clay soil would have to be left fallow in *Rabi* or gram could be taken after paddy. Hence instead of gram and paddy taken as single crops, the following two processes are considered :

Paddy	Fallow
Paddy	Gram

Cotton is grown on irrigated loamy soil, but since it is harvested late in November, the only alternatives available in this case are :

Cotton	Fallow
Cotton	Gram

In general, maize, *juar*, and gram do not require any irrigation and hence it is customary in western Uttar Pradesh to grow the above crops as dry crops. However, these crops can be grown on loamy soils with or without facilities for irrigation. Even if facilities for irrigation exist they are not used to irrigate any of the above crops grown on that portion of the farm land which has such facilities. Since the loamy soils have been divided into two categories, *i.e.*, the portion of the loamy soils on farm having facilities for irrigation (7 acres) and the portion of loamy soils having no such facilities (4 acres), each of the above-mentioned crops has to be classified into two processes : (*a*) unirrigated crop grown on that portion of loamy soils on the farm which has facilities for irrigation, and (*b*) unirrigated crop grown on that portion of loamy soils on the farm having no irrigational facilities. The non-land resource requirements and the output of each of the two processes representing maize, *juar*, and gram separately, are exactly the same, since in both cases no irrigation is actually given to these crops.

The following processes have been considered as alternatives in the programme :

		Kharif	*Rabi*
x_1	=	acre of sugarcane	continued
x_2	=	acre of paddy	fallow
x_3	=	acre of paddy	gram
x_4	=	acre of cotton	fallow
x_5	=	acre of cotton	gram

x_6 = acre of unirrigated maize grown on irrigated loamy soil
x_7 = acre of unirrigated maize grown on unirrigated loamy soil
x_8 = acre of unirrigated *juar* grown on irrigated loamy soil
x_9 = acre of unirrigated *juar* grown on unirrigated loamy soil
x_{10} = acre of unirrigated gram grown on irrigated soil
x_{11} = acre of unirrigated gram grown on unirrigated soil
x_{12} = acre of wheat (irrigated)
x_{13} = acre of wheat (unirrigated)
x_{14} = activity for transferring *Kharif* capital to *Rabi* capital

THE MODEL

The programming model employed to maximize the farm business income of the farmer is of the following general form :

$$\text{Max } P = \sum_{j=1}^{14} x_j p_j$$

Subject to

$$\sum_{j=1}^{14} a_{ij} x_j \leqslant b_i \quad (i=1, \ldots, 12)$$

$$\sum_{j=1}^{14} a_{ij} x_j \geqslant b_i \quad (i=13, 14)$$

and $x_j \geqslant 0, (j=1, \ldots, 14)$, where

P = the total farm business income in rupees
x_j = the level of the process j $(j=1, \ldots, 14)$
p_j = the farm business income from one unit of the jth process
b_i = the quantity available of the ith resource $(i=1, \ldots, 12)$
b_i = fodder requirements to be met $(i=13, 14)$

The specific form of the model, with the input-output coefficients, is given in the opposite page.

MODEL

Maximize	P=	$336x_1$	$+75x_2$	$+118x_3$	$+67x_4$	$+110x_5$	$+45x_6$	$+45x_7$	$+55x_8$
		1	2	3	4	5	6	7	8
Subject to									
b_1	1≥	$0x_1$	$+1x_2$	$+1x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_2	7≥	$1x_1$	$+0x_2$	$+0x_3$	$+1x_4$	$+1x_5$	$+1x_6$	$+0x_7$	$+1x_8$
b_3	4≥	$0x_1$	$+0x_2$	$+0x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+1x_7$	$+0x_8$
b_4	1≥	$0x_1$	$+1x_2$	$+1x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_5	7≥	$1x_1$	$+0x_2$	$+0x_3$	$+1x_4$	$+1x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_6	4≥	$0x_1$	$+0x_2$	$+0x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_7	284≥	$21x_1$	$+37.4x_2$	$+37.4x_3$	$+18x_4$	$+18x_5$	$+45x_6$	$+45x_7$	$+25x_8$
b_8	284≥	$40x_1$	$+0x_2$	$+14.8x_3$	$+0x_4$	$+14.8x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_9	167≥	$7x_1$	$+8.8x_2$	$+8.8x_3$	$+5x_4$	$+5x_5$	$+8x_6$	$+8x_7$	$+6x_8$
b_{10}	167≥	$10x_1$	$+0x_2$	$+5.9x_3$	$+0x_4$	$+5.9x_5$	$+0x_6$	$+0x_7$	$+0x_8$
b_{11}	1,000≥	$160x_1$	$+80x_2$	$+80x_3$	$+51x_4$	$+51x_5$	$+78x_6$	$+78x_7$	$+40x_8$
b_{12}	0≥	$+130x_1$	$-80x_2$	$-36x_3$	$-51x_4$	$-7x_5$	$-78x_6$	$-78x_7$	$-40x_8$
b_{13}	1.5≤	$0x_1$	$+0x_2$	$+0x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+0x_7$	$+1x_8$
b_{14}	28≤	$0x_1$	$+0x_2$	$+0x_3$	$+0x_4$	$+0x_5$	$+0x_6$	$+0x_7$	$+0x_8$

Maximize	P=	$+55x_9$	$+43x_{10}$	$+43x_{11}$	$+81x_{12}$	$+30x_{13}$	$+0x_{14}$		
		9	10	11	12	13	14		
Subject to									
b_1	1≥	$+0x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_2	7≥	$+0x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_3	4≥	$+1x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_4	1≥	$+0x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_5	7≥	$+0x_9$	$+1x_{10}$	$+0x_{11}$	$+1x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_6	4≥	$+0x_9$	$+0x_{10}$	$+1x_{11}$	$+0x_{12}$	$+1x_{13}$	$+0x_{14}$		
b_7	284≥	$+25x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_8	284≥	$+0x_9$	$+14.8x_{10}$	$+14.8x_{11}$	$+33x_{12}$	$+26x_{13}$	$+0x_{14}$		
b_9	167≥	$+6x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$		
b_{10}	167≥	$+0x_9$	$+5.9x_{10}$	$+5.9x_{11}$	$+20x_{12}$	$+18x_{13}$	$+0x_{14}$		
b_{11}	1,000≥	$+40x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+1x_{14}$		
b_{12}	0≤	$-40x_9$	$+44x_{10}$	$+44x_{11}$	$+160x_{12}$	$+88x_{13}$	$-1x_{14}$		
b_{13}	1.5≤	$+1x_9$	$+0x_{10}$	$+0x_{11}$	$+0x_{12}$	$+0x_{13}$	$+0x_{14}$	$-1x_{15}$	
b_{14}	28≤	$+0x_9$	$+0x_{10}$	$+0x_{11}$	$+28x_{12}$	$+14x_{13}$	$+0x_{14}$		$-1x_{16}$

TABLE I

Crops	Acres	Farm business income (Rs.)	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
x_1	1.45	487.2	0	1.45	0	0	1.45	0	30.45	58.00	10.15	14.50	232.0	188.5
x_3	1.00	118.0	1.0	0	0	1.0	0	0	37.40	14.80	8.80	5.90	80.0	0
x_5	4.55	500.5	0	4.55	0	0	4.55	0	81.90	67.34	22.75	26.84	232.0	0
x_8	1.00	55.0	0	1.00	0	0	0	0	25.00	0	6.00	0	40.0	0
x_9	4.00	220.0	0	0	4.0	0	0	0	100.00	0	24.00	0	160.0	0
x_{11}	4.00	172.0	0	0	0	0	0	4.0	0	59.20	0	23.60	0	176.0
x_{12}	1.00	81.0	0	0	0	0	1.00	0	0	33.00	0	20.0	0	160.0
Total requirement		1,633.7	1.0	7.0	4.0	1.0	7.0	4.0	274.75	232.34	71.7	90.84	744 0	524 5
Total availability			1.0	7.0	4.0	1.0	7.0	4.0	284.0	284.0	167.0	167.0	1,000 0	524 5[c]
Surplus			0	0	0	0	0	0	9.25	51.66	95.3	76.16	—	—
Rabi capital supply from sale of *Kharif* crops													—	268.5[a]
Capital transfer from *Kharif* to *Rabi* through x_{14}													—256.0	256.0[b]

[a] The sum of Rs. 268.5 relates to the sale of *Kharif* crops (see the discussion on working capital) as detailed below:

Process	Level (acres)	Capital supplied for *Rabi* crops per unit (see the specific form of the model) Rs.	Total capital supplied for *Rabi* crops Rs.
X_3 (Paddy-Gram)	1.00	36.00	36.00
X_5 (Cotton-Gram)	4.55	7.00	31.85
X_8 (Unirrigated *Juar*)	1.00	40.00	40.00
X_9 (Unirrigated *Juar*)	4.00	40.00	160.00
Total			267.85*

* The difference of Rs. 0.65 (268.5—267.85) is due to rounding errors.

[b] A sum of Rs. 256.00 has been transferred from *Kharif* to *Rabi* as the total capital supply in *Kharif* is Rs. 1,000.00, the requirement being Rs. 744.00 (256.00=1,000.00—744.00).

[c] The total capital supply in *Rabi* equals to Rs. 524.5 (268.5+256.0), as is evident from [a] and [b] above.

In order to handle the last two inequalities, artificial variables have been introduced into the model. A very high negative return figure is attached to the slacks of these artificial variables. The detailed procedure of the use of artificial variables in solving the minimization problems is given in Chapter IV of "Linear Programming Methods" by Heady and Candler.[4] No price has been attached to the slack or disposal variables, which indicate the surplus quantities or amounts of the different resources in the final solution.

RESULTS AND THEIR INTERPRETATION

The final solution was obtained by the use of the simplex method in fifteen iterations. The levels of each of the crops that are in the solution, the farm business income yielded by these crops, and the amount of each resource utilised by these crops are given in Table I.

It can be seen from Table I that all types of soils are cropped in both *Rabi* and *Kharif* seasons. As one can expect, all the capital supply has been used. Because of the capital transfer activity and a separate *Rabi* capital equation, the farmer could use Rs. 1,268.50, while the initial supply was only Rs. 1,000.00. A substantial percentage of the total bullock labour supply has been left unused, which indicates that given the other resource restrictions, the farm can provide employment to bullocks only for about six months, even if all the resources are put to their best uses. Some off-farm work should, therefore, be found for the bullocks.

The story of human labour utilisation is, however, different. Out of the total supply of 668 human labour days, 75.9 per cent has been used on crop production. Fifteen per cent of the total labour supply was put aside for cattle maintenance. Thus, more than 90 per cent of the total available family labour has been used on the farm.[5] It can thus be seen that an efficient allocation of resources among crops provides more days of gainful employment to the family labour on an average farm in western Uttar Pradesh than is the case of non-optimal allocation.

Table II gives the ($z_j - c_j$)[6] figures for crops not in the solution and for the resources which have completely been utilised.

TABLE II

Particulars	$z_j - c_j$		Particulars	$X_j - c_j$	
Kharif heavy clay soil	b_1	77.58	Paddy—F	x_2	2.58
Kharif loam soil (irrigated)	b_2	55.00	Cotton—F	x_4	2.58
Kharif loam soil (unirrigated)	b_3	55.00	Unirrigated Maize	x_6	10.00
Rabi heavy clay soil	b_4	0	Unirrigated Maize	x_7	10.00
Rabi loam soil (irrigated)	b_5	14.58	Unirrigated Gram	x_{10}	12.00
Rabi loam soil (unirrigated)	b_6	2.58	Wheat (unirrigated)	x_{13}	13.14
Kharif capital	b_{11}	0.92			
Rabi capital	b_{12}	0.92			

4. *Op. cit.*

5. In an investigation conducted by the author it was found that only 61.8 per cent of the total family labour was used on farm operations. See Jai Krishna : A Family Farm Study, unpublished report submitted in partial fulfilment of the requirements of M.Sc. (Ag.) degree in Agricultural Economics, Government Agricultural College, Kanpur, 1954, p 24.

6. The $z_j - c_j$ figures for those restrictions, whose slacks are not in the solution, *i.e.*, the resources which have completely been utilised, represent the marginal value products of such resources. The $z_j - c_j$ figure of the process not in the solution represents the amount by which the farm business income from one unit of such a process will have to be increased, if it were to come in the solution.

The above table demonstrates that the marginal value product of an acre of *Kharif* land is much higher than the marginal value product of an acre of *Rabi* land of the same quality. The marginal-value product of working capital is very high, which points out that it is a very limiting resource. Each additional rupee of working capital invested in farming would bring a return of Re. 0.92.

Linear programming yields a unique solution in maximizing or minimizing the given objective function. However, one very interesting feature of the solutions of the present programme is that they allow a greater flexibility to the farmer to choose among the five alternative combinations of crop enterprises, without much sacrifice in the total business income. The levels of the various crop activities and the total farm business income yielded in the last five iterations are given in Table III.

TABLE III

		I	II	III	IV	V
Farm business income (Rs.)		1,610	1,617	1,626	1,628	1,633
Acres of :						
Sugarcane	x_1	2.90	2.34	1.63	1.45	1.45
Paddy—F	x_2	1.00	1.00	1.00	—	—
Paddy—Gram	x_3	—	—	—	1.00	1.00
Cotton—F	x_4	3.10	—	—	—	—
Cotton—Gram	x_5	—	3.66	4.37	4.55	4.55
Unirrigated Maize	x_6	—	—	—	—	—
Unirrigated Maize	x_7	.25	.33	.44	.46	—
Unirrigated *Juar*	x_8	1.00	1.00	1.00	1.00	1.00
Unirrigated *Juar*	x_9	3.75	3.67	3.56	3.54	4.00
Unirrigated Gram	x_{10}	—	—	—	—	—
Unirrigated Gram	x_{11}	—	—	4.00	4.00	4.00
Wheat (irrigated)	x_{12}	1.00	1.00	1.00	1.00	1.00
Wheat (unirrigated)	x_{13}	—	—	—	—	—

It is evident from the above table that a greater acreage under sugarcane does not contribute to any increase in the farm business income, as is the common belief in western Uttar Pradesh. Similarly, it can also be seen that gram is a better alternative for resource use as compared to wheat, both irrigated and unirrigated.

CONCLUSIONS

The primary concern of this article has been to develop a linear programming model for the selection of crop enterprises on an average farm in western Uttar Pradesh. The data used and the results obtained have, therefore, been of secondary importance. However, in order that linear programming solution be meaningful, it is of utmost importance that the input-output data should be reliable and the processes should be representative of the action alternatives that can be followed on farms whose organizations are being programmed.

The model constructed in this paper may need to be modified to fit a specific case. In addition to the processes included in the programme, one may need to add some other processes. The high $z_j - c_j$ figure for capital suggests the addition of a capital borrowing activity. Human labour and bullock labour supplies have been divided only into two categories in this model. However, one may wish to follow a different classification.