UWA

The University of Western Australia

AGRICULTURAL ECONOMICS

SCHOOL OF AGRICULTURE

Nedlands, Western Australia 6009

GRAZING MANAGEMENT DECISION MAKING IN THE PASTORAL ZONE OF WESTERN AUSTRALIA: AN APPLICATION USING CONTROL THEORY

K M WANG*, G S RICHMOND*, R B HACKER**,
G HERTZLER* and R K LINDNER*

Agricultural Economics
Discussion Paper 6/89

* School of Agriculture, The University of Western Australia, Nedlands, Western Australia 6009, Australia.

** W A Department of Agriculture, Baron-Hay Court, South Perth, Western Australia 6151, Australia

Paper presented to the Thirty-Third Annual Conference of the Australian Agricultural Economics Society, Lincoln College, Christchurch, New Zealand, 7-9 February 1989.

This work was supported by a grant from the Wool Research and Development Fund.

GRAZING MANAGEMENT DECISION MAKING IN THE PASTORAL ZONE OF WESTERN AUSTRALIA: AN APPLICATION USING CONTROL THEORY

ABSTRACT

Rangeland degradation within the arid zone of Western Australia has occurred as a consequence of sheep overstocking. Optimum grazing management strategies and rangeland rehabilitation techniques are needed to maintain the resource base for future use. In this paper an optimal control framework is developed for the derivation of grazing management decisions. "An integrated model of an arid grazing ecological system (IMAGES)" is used to derive the rangeland dynamics. The state of the grazing ecological system is summarized into four variables: the population of mature desirable perennial plants, young desirable perennial seedlings, old desirable perennial seedlings and total forage biomass. The controls are a set of different seasonal stocking rates within the year. Optimal decision rules are derived for both a deterministic and stochastic case study. Generally, the optimum stocking rate increase with increasing value for the 4 state variables. In the deterministic case it combines both uniform and varied stocking rates while in the stochastic case only the uniform stocking rate prevails. Compared to the stochastic case, the net present value is higher for most cases under deterministic climatic sequence, although there are some exceptions. The differences in the deterministic and stochastic cases can be dramatic. The reason appears to be highly variable rainfall combined with nonlinear production functions and adjustment costs. An over-optimistic expectation about the weather can be very expensive. Work to verify the stochastic results is continuing.

WANG,K.M., RICHMOND,G.S., HACKER,R.B., HERTZLER,G. and LINDNER,B.K.

Introduction

Historically, sheep overstocking has occurred in the pastoral areas of Australia. This has caused a decline in range productivity. This trend is likely to continue unless long term management strategies which facilitate an improvement in range carrying capacity are devised. This study is an extension to the "Computer Modelling of Rangeland Regeneration in the Arid Zone of W. A. (Wang *et al* 1988)" and "IMAGES:an Integrated Model of an Arid Grazing Ecological System (Hacker *et al* 1989)". The problem of long term grazing management decision making at the station level has been focused upon by adopting a stochastic optimal control approach. The station is split into a number of paddocks. Each paddock on the station can be allocated to a pasture type based on dominant vegetation within the grazing area. Decision making at the station level is composed of a number of decisions at the paddock level. Rangeland dynamics within the paddock was described by a sheep grazing simulation model IMAGES (Wang *et al* 1988, Hacker *et al* 1989), which consists of a set of difference equations. The state of the grazing ecosystem within the paddock is summarised into four state variables, although ideally at least nine state variables would be required. The control variable is the pattern of stocking rates over a time interval of one year which comprises of three four-monthly rainfall seasons. Each season has a distinctive rainfall pattern. Optimal management decisions are derived by using stochastic dynamic programming.

A Decision Framework of Rangeland Management

The decision-making processes under uncertainty in the rangeland environment can be described by the following framework:
At the beginning of each decision period, based on a decision rule, the range manager makes management decisions after observing the state of the paddock grazing ecosystem to achieve the management objectives. The management decisions together with climatic sequences affects the evolution of the paddock ecosystem which will generate the returns to the manager as well as the period-ending state. The period-ending state in turn will affect future management decisions. Thus the decision cycle is repeated.

Stochastic Optimal Control Model

The above range management decision framework can be closely represented by a stochastic optimal control model. The formulation of such model involves the following components: an objective function; sets of state; stochastic and control (decision) variables; and a set of state transition equations.

In the application of this decision model to rangeland management, the objective function is assumed to represent the objectives of the decision-maker. In the study, it is assumed that the range manager is risk neutral. Therefore, the objective function can be specified as maximisation of expected present value of the stream of net profits received. The state variables must be observable, and capture as closely as possible those aspects of past history which influence evolution of the rangeland condition and economic returns, and upon which the manager bases his decision. In the study, we assume that the state of range condition can be jointly described by the availability of forage biomass and desirable perennial plant density. The stochastic variables are those which can capture the major stochastic variation in the evolution of the rangeland ecosystem. Rainfall and evaporation are used as basic stochastic driving variables. The control variables include those variables which can be manipulated by the manager to influence range condition and future profits. Stocking rate is used as the control variable. The set of state transition equations make up the model of range dynamics which describes the development of the rangeland ecosystem over time. Therefore, the model predicts the future range condition by using current range condition for given sequences of grazing management decisions and climatic patterns.

The formulation of the decision problem is to discover the optimal decision rule to maximize the expectation of the objective function subject to the transition probabilities which are derived from state transition equations. Mathematically, the formulation of the stochastic optimal control model can be specified as follows:

$$\text{Maximize } E_0\sum_{t=0}^{\infty} \alpha^t g(\mathbf{x}_t,\mathbf{u}_t,\mathbf{w}_t) \qquad (1)$$

Subject to

$$\mathbf{x}_{t+1}=f(\mathbf{x}_t,\mathbf{u}_t,\ \mathbf{w}_t) \qquad (2)$$

$$\mathbf{x}_0 \text{ is given}$$

where

E_0 is the expectation held at initial period, t=0;

α is the discount factor, $\alpha=1/(1+r)$, r is real annual interest rate;

$g(\mathbf{x}_t,\mathbf{u}_t,\mathbf{w}_t)$ is the annual net return function;

$\mathbf{x}_t,\mathbf{u}_t,\mathbf{w}_t$ are the vector of state, control and random variables, respectively;

$f(\mathbf{x}_t,\mathbf{u}_t,\mathbf{w}_t)$ is a set of transition equations which represent range dynamics.

The constraint of a set of state transition equations (2) can be replaced by the transition probabilities $P_{ij}(\mathbf{u}_k)$. A transition probability is defined as the probability that the next period state will be j given that the current state is i and control $u=\mathbf{u}_k$ is applied. It can be specified as a conditional probability

$$P_{ij}(\mathbf{u}_k)=P(\mathbf{x}_{t+1}=j|\mathbf{x}_t=i,\mathbf{u}_t=\mathbf{u}_k).$$

Therefore, state transition probabilities can be calculated from the transition equations (2) because the state in period t+1 is a random variable, its conditional distribution depends on the current state and control as well as the distribution of random variable $\mathbf{w}$.

Simulation Model IMAGES

In order to derive the state transition probabilities a simulation model which consists of the state transition equations has to be constructed and used as a fundamental instrument to generate transition probabilities. The model used in this study is a modified version of IMAGES which integrates the evolution of arid grazing ecosystem in the winter rainfall pastoral zone of Western Australia. The main functional components within the plant-animal-climate interface in a single paddock on a four monthly basis were simulated. The essence of IMAGES is illustrated in Figure 1 (see page 21). The stochastic driving variables for the simulation model are daily rainfall and evaporation which affect the soil store water. A soil water balance submodel (WATBAL) developed by Fitzpatrick *et al* (1967) was used to derive the number of wet pentads i.e. 5 day growth periods, over a four monthly season. Wet pentads together with the management decisions of stocking rate and treatment drive the vegetation dynamics through three related components: ephemeral forage biomass, perennial forage biomass and desirable perennial plant density. The desirable perennial plant density consists of 6 four monthly age-cohorts seedlings, i.e. 0-4, 4-8, ... , 20-24 months and one adult class i.e. 24+ months. These three components in turn influence sheep performance through sheep intake, wool production and sheep mortality and lambing rate, which are the main factors determining economic returns to the woolgrower.

The model IMAGES is expressed mathematically by a set of nine difference equations, one for each of the state variables: ephemeral and perennial forage biomass, and seven age-cohorts of desirable perennial plants. State transitions are functional on the state of the paddock grazing ecosystem, management decisions and the number of wet pentads. The general form of the difference equations is

$$S_{t+1}=S_t+R_t-D_t \qquad (3)$$

where R_t is the inflow of biomass/recruitment to the state variable S_t, and D_t is outflow of biomass/disappearance from the state variable over the four month period.

The inflow of forage biomass is modelled by a first-order negative feedback control system such that the rate process adjusts towards an equilibrium. The rate of inflow is assumed proportional to the discrepancy between the environmental carrying capacity and the current forage biomass level. This system may also be called a goal-seeking mechanism, in which the goal is the level of environmental carrying capacity. The proportional rate is controlled by a number of factors. They consist of the product of a series of scaling factors. For example, the inflow of perennial forage biomass is given by the following form:

$$R_t=(K_t^f-S_t^f)*SMI(NWP_t)*GCI(S_t^f,K_t^f) \qquad (4)$$

where K_t^f is the environmental carrying capacity of perennial forage and S_t^f is the initial perennial forage biomass. The scaling factors SMI and GCI ranging from 0 to 1 are soil moisture index and growth capacity index which are functions of the number of wet pentads NWP_t, initial forage biomass and environmental carrying capacity, respectively. The environmental carrying capacity is an increasing function of the density of mature desirable perennial plants. With some variation, the same mathematical form is utilized for the inflow of ephemeral forage biomass.

The outflow of forage biomass is made up of two pools: sheep intake and other non-consumptive losses. Sheep intake is also modelled by a goal-seeking mechanism. Sheep potential intake which acts as the goal of the system is regulated by a series of scaling factors. For example, within the limits, sheep ephemeral forage intake SI_t^h is calculated by

$$SI_t^h=PI*AI(S_t^h+R_t^h)*QI(DG_t^h) \qquad (5)$$

where PI is sheep potential intake and scaling factors AI and QI are ephemeral forage availability index and quality index which are functional on the available ephemeral forage $S_t^h+R_t^h$ and ephemeral forage digestibility figure DG_t^h, respectively. With some variation the same functional form was applied to sheep perennial forage intake except that an additional scaling factor was used to account for sheep diet preference.

Other non-consumptive losses, within the limits, are modelled by a donor controlled system. Where a flow rate is proportional to the amount of the available forage biomass, the proportional rate is derived by a maximum rate parameter and a series of scaling factors.

Desirable perennial plant numbers are treated separately from forage biomass since this plant group directly influences rangeland productivity, through the provision of desirable forage in the long term. The population dynamics of desirable perennial plants are modelled by a system similar to the Leslie system (1945, 1948). In the system, the simulation time step is equal to the length of the age interval. Therefore, the population of all age-cohorts are shifted every time-step one place and are diminished at the same time by the mortality rate. The survival rate of each cohort is made a function of a maximum rate parameter and a series of scaling factors. For example, survival rate of seedlings of six cohorts $sR_{t,k}$ is modelled as follow:

$$sR_{t,k}=1*SMI(NWP_t,k)*GPI(G_t)*ECI((S_t^h+R_t^h)/K_t^h,k) \quad k\in[1,6] \qquad (6)$$

where 1 is the maximal survival rate and scaling factors SMI, GPI, ECI, of values between 0 and 1, are soil moisture index, grazing pressure index and ephemeral forage competition index which are functions of number of wet pentads, cohort age k, grazing pressure G_t, relative ephemeral forage density $(S_t^h+R_t^h)/K_t^h$ and cohort age, respectively.

The survival rate of the mature desirable perennial plant is modelled differently to those of the seedlings. The natural mortality rate of the plants is first calculated from the life span of dominant species, and then environmentally induced mortality rate is calculated by a function of a maximal rate parameter and a scaling factor. The scaling factor is made a function of grazing pressure, number of wet pentads and relative desirable perennial plant density.

The flow from the mature plant to the first age-cohort seedlings is modelled by a goal-seeking mechanism similar to that of forage inflow. The recruitment is proportional to the gap between the environmental carrying capacity S_m^p and the number of current mature desirable perennial plants $S_{t,7}^p$. The proportional rate consists of the product of two scaling factors: replacement capacity index RCI and germination index GMI which are functions of mature desirable perennial plant density, and number of wet pentads and seasons, respectively. Thus

$$S_{t,1}^p=(S_m^p-S_{t,7}^p)RCI(S_{t,7}^p)GMI(NWP_t,s) \qquad (7)$$

where $S_{t,1}^p$ is first age-cohort seedling population and s is seasonal index.

A detailed description of IMAGES is given by Wang and Hacker *et al* (1988,1989).

Methods

Because of "the curse of dimensionality" in the optimization algorithm (Bellman 1957), it is necessary to condense the model into a manageable size. The process of condensation are in two directions. First, the state of grazing ecosystem originally characterized by nine state variables was condensed into four state variables. Second, the time step was enlarged to one year for the optimization framework.

Model condensation

For combating the problem of dimensionality, the original nine state variables was condensed into four state variables. Forage biomass of ephemeral and perennial were aggregated into a single compartment, total forage biomass. Seedling cohorts of age 0-4, 4-8, 8-12 months were condensed into a single state, young seedlings and those of 12-16, 16-20, 20-24 months into an old seedling state. The mature plants still remain as a state variable. However, there are still problems in initiating the simulation of IMAGES because the model uses initial values of nine state variables to start simulations. In order to retain IMAGES as the fundamental transition probability generator, it was decided to use a set of assigning coefficients to allocate the values of aggregated state into each individual state rather than change the parameters in IMAGES. In other words, the condensation process works as a mechanism to determine the initial values for IMAGES simulations. The assigning coefficients for perennial forage biomass and ephemeral forage biomass are 1,0, respectively, since simulation starts at season 1 by then all the above-ground green tissue dies back for ephemerals. The assigning coefficients for seedling cohorts 0-4/4-8/8-12 months are 0/1/0 or 0/0.5/0.5 depending on the level of the young seedling population. The ratio of 0/1/0 was used for two reasons: first, the mean age is possibly the most representative statistic for an unknown age distribution, which involves less error in calculating transition probabilities comparing to the use of other statistics. Second, the climatic pattern of the winter rainfall pastoral region of Western Australia can be classified into three four monthly seasons: January-April; May-August; September-December. This corresponds to the three recognizable periods of unreliable summer rainfall, reliable winter rainfall and reliable summer drought, respectively. Therefore, the most possible distribution of young seedling population at beginning of season 1 is 0/1/0 under the above climatic regime. The ratio of 0/0.5/0.5 are used to account for extraordinary wet years which was represented by a dense population of seedlings. The same set of ratios were used for old seedlings based on the same rationale.

Time Intervals in the Simulation and Optimization

Two time steps were involved in the study. First, a four monthly interval was used in the simulation model, in which vegetation dynamics were characterized by nine state variables. Number of wet pentads were generated by a given probability density distribution function (see Table 1, page 24) which was derived from the output of simulating WATBAL submodel by using a past 50 year daily rainfall and evaporation figures. The second time interval resulted from one year simulations with IMAGES, and calculates annual transitions from initial state described jointly by four state variables to its terminal state.

An objective function was also used to calculate expected annual net profits for a given initial state and policy. These annual net profits, along with yearly transitions formed the data base for the optimization program.

Objective Function

The objective function based on the concept of net profit margin, NPM, was used to calculate expected annual profits. Net profit margin is defined as the gap between price of one unit output and its average total costs. In the application of a hypothetical station with wether paddocks in the pastoral zone of W.A., net profit margin can be calculated by the following rule:

NPM=net value of wool-value of death losses±value of changes in the flock size-stocking rate adjustment costs-average variable costs-average fixed costs (8)

Gross value of wool produced per sheep per season is calculated by multiplying average wool production per sheep WC_s and net average greasy price which is obtained by deducting wool marketing costs Mw from greasy price Pw. This value subtracting the costs of shearing and wool harvesting consists of the net value of wool produced per sheep per season. The net value of wool is then converted into per hectare basis by multiplying the stocking rate at the end of season $u_s*(1-SM_s)$, where SM_s is seasonal sheep mortality rate. Value of death losses is imputed by multiplying the market price of sheep Ps_s and the number of deaths. Value of changes in the flock size per hectare is calculated by multiplying the market price of sheep and the change in the stocking rate. This value can be positive or negative. Stocking rate adjustment costs are calculated by multiplying the adjustment costs per sheep and the change in the stocking rate. According to the direction of adjustment, this cost may or may not include selling costs. If the direction of adjustment is to build up the flock the adjustment costs only include freight costs. Otherwise, it consist of two components: selling costs and freight costs. A steady state assumption was used to calculate the change in the stocking rate at the beginning of every year, i.e. $u_0=u_3$. Although this assumption is valid for the deterministic case, it may underestimate/overestimate the stocking rate at the beginning of next period for the stochastic case if the state of grazing ecosystem at the beginning of next year is better/worse than that of the current year. This is because the state of the grazing ecosystem in the next year is stochastic and is currently unknown to the range manager.

The average variable cost per sheep includes the cost of following items:
(a) direct labour which varies with sheep numbers.
(b) fuel and oil.
(c) repair and maintenance of the farm equipment.
(d) interest paid.
The average variable cost per season per hectare is calculated by multiplying stocking rate and the average variable cost.
The average fixed costs per hectare include the costs for:
(a) the salaries of operator and family labour.
(b) depreciation of farm improvements, plant and equipment.
(c) land rent.
(d) the expenses of administration.

Since the exclusion of average fixed costs from net profit margin function will not affect the derivation of optimal decision rule, the costs were taken out in the optimisation algorithm.
Mathematically, annual net profit margin is specified as follows:

$$NPM=\sum_{s=1}^{3}\ ((Pw-Mw)*WC_s-SC)*u_s*(1-SM_s)-Ps_s*u_s*SM_s-Ps_s*(u_s-u_{s-1})$$

$$-|u_s-u_{s-1}|*SAC_s-AVC*u_s-AFC \qquad (9)$$

where

$$SAC_s=FC+SLC_s \qquad \text{if } u_s<u_{s-1} \qquad (10)$$

$$SAC_s=FC \qquad \text{if } u_s>u_{s-1} \qquad (11)$$

and

NPM is annual net profit margin, \$/ha;
Pw is annual average greasy price, \$/kg;
Mw is wool marketing and distribution costs to woolgrowers, \$/kg;
WC_s is greasy wool production per sheep per season, kg/sheep;
SC is shearing and other wool harvesting costs on per sheep basis per season, \$/sheep;
u_s is stocking rate at beginning of season s, and $u_0=u_3$, sheep/ha;
SM_s is seasonal sheep mortality rate;
Ps_s is seasonal price of sheep, \$/sheep;
SAC_s is stocking rate adjustment costs, \$/sheep;
FC is freight costs for selling or buying a sheep, \$/sheep;
SLC_s is seasonal sheep selling costs, \$/sheep;
AVC is average variable cost per season per sheep, \$/sheep;
AFC is average fixed costs per season, \$/ha.

The value of the parameters used in the net profit margin function are presented in Appendix A.1.

Wool production and sheep mortality are nonlinear to the state and control variables and adjustment costs are asymmetric. Therefore, the optimal stocking rates will differ under deterministic and stochastic rainfall.

Partition of State and Control Space

Optimization algorithm can be applied only if the dimension of state and control spaces is very small because of the problem of computer storage. Therefore, it is necessary to partition the state and control spaces into finite sets. For the state space, the number of grids for a state variable should be positively related to its importance in terms of its shadow price. In addition, the partition can not be too coarse to lose the important features of model behaviour. With results from preliminary simulation runs under different conditions, the following partitions were decided:

Adult desirable perennial plants, #/ha:	0	400	1600	3200	6400
Young seedlings, #/ha:	0	2000	5000		
Old seedlings, #/ha:	0	1000	2500		
Total forage biomass, kg d.m./ha:	0	80	160	320	800

Therefore, the total number of state in the state space is 5*3*3*5=225. Although, some of the combinations may not exist, they are still retained for the purpose of facilitating the computer programming.

The decision space was partitioned into 40 strategies consisting of various combinations of stocking rates over a one year time interval. Table 2 (see page 25) presents a description of all 40 grazing strategies. As illustrated by Table 2, strategy 1 is total destocking. Strategies 2-15 are the pattern of fixed stocking rates over a year and strategies 16-40 are the pattern of stocking rates of different levels corresponding to three rainfall seasons.

Simulations of IMAGES were run under the 40 grazing strategies, by using initial values obtained from combinations of partition points of four state variables. The output from the simulations was organized into a data set with records consisting of number of distinct transitions, initial state index, policy index, average return, frequency of transitions and terminal state index. The resulting data set was subsequently used by optimization algorithm to determine optimal strategies. The complete operational sequence for the simulation-optimization procedure is shown in Figure 2 (see page 22).

Optimization Algorithm

The solution of the stochastic optimal control problem presented in (1)-(2) can be derived by Howard's Dynamic Programming with the Markov chain approach (Bertsekas 1976). Defining V(i) as the maximum expected value of the discounted stream of returns, given the initial state $X_0=i$, the necessary and sufficient condition for optimality is that V(i) satisfies the following functional equation of dynamic programming:

$$V(i)=\max_{u\in U} E_{w}\{g(i,u,w)+\alpha V(j)\} \qquad i,j\in S \tag{12}$$

where
i,j are initial and terminal state, respectively;
u is policy;
E is expectation with respect to random variable w;
g(i,u,w) is annual net profit function;
S is state space;
U is control space;
α is discount factor;
V(j) is optimal value function given initial state is j.

For the discrete-time model with finite sets for state and control space, functional equation (12) can be rewritten as

$$V(i)=\max_{u\in[1,M]} \{\bar{g}(i,u)+\alpha\sum_{j=1}^{N} P_{ij}(u)V(j)\} \qquad i,j\in[1,N] \tag{13}$$

where
$\bar{g}(i,u)$ is annual expected return;
P_{ij} is the transition probability from state i to state j under a given policy u;
N, M are the dimensions of state and control spaces, respectively.

There are three methods for solving this functional equation: successive approximation, linear programming and policy iteration. Successive approximation is essentially the dynamic programming algorithm. It starts with an arbitrary Nx1 vector V(j) and computing Nx1 vector V(i), and substituting V(j) by V(i), then repeats these computations iteratively until V(i) and V(j) converge. Thus, it generates, in the limit, the optimal value function and an optimal policy. Policy iteration and linear programming can determine the optimal policy in a finite number of iterations. However, they require solution of linear equations system with dimension of state space for each iteration, or of a linear programme of dimension as large as the state space*control space. Therefore, when the dimension of the problem is very large both methods are not practical. However, they are attractive when the size of the problem is small because accurate optimal solutions can be derived within a finite number of iterations. Since the problem of rangeland management has potentially a large dimension to account for in the dynamics of grazing ecosystem, successive approximation method was chosen for the derivation of optimal management strategies. The programme sequence for the algorithm is shown in Figure 3 (see page 23).

Empirical Results and Discussion

The optimal grazing decision rule is derived from the model containing four state variables, and is useful for the following reasons.The rules derived from both deterministic and stochastic cases are also stationary over time, since rangeland dynamics are formulated as a stationary system, i.e. the same parameters and probability density distribution function of wet pentads are used in every period. In addition, the decision rule is a function of the current state of range condition, therefore the range manager can decide the optimal stocking rates over the year at the beginning of each period after observing the state of the range condition.

Deterministic Case Study

Under the deterministic case, the number of wet pentads was set by using the mode of the wet pentads distribution within each season i.e. season 1 NWP=2, season 2 NWP=11, season 3 NWP=0. Since there are four state variables, a four-dimensional table was needed to illustrate the optimal stocking rules and their corresponding net present values.

Table 3.a (page 26) shows the derived optimal decision rules. As indicated by the table, the optimal stocking rules can be subdivided into 15 sections. Each section gives the decision rules corresponding to the combination of 5 levels of total forage biomass, 3 levels of young seedlings,one adult plant and one old seedling level. Each section consists of 5 columns and 3 rows corresponding to the different levels of forage biomass and young seedlings, respectively.

As illustrated by Table 3.a, the optimal decision rules in each section is constant with respect to different levels of young seedlings. This shows that the optimal stocking rules are not sensitive to the population of young seedlings under the deterministic climatic regime. Thus, young seedlings are not important in the decision making process under the deterministic case.

Table 3.b (page 27) presents the net present values under the deterministic case. As indicated by this Table, the value generally increases with increasing value of four state variables. The highest value \$65.26/ha occurs at the state of highest value for all four state variables. The lowest value \$0/ha occurs at the states of zero forage biomass.

Shadow price (or marginal value) of the state variables is given by the partial derivatives of the optimal value function with respect to the state variables. The discrete approximations of shadow price of 4 state variables is calculated by using Table 3b and presented by Table 3.c to 3.f. As indicated by Table 3.c (page 28), shadow prices are zero in all situations except in the states of the sections of zero and low (400/ha) adult plant density, combined with zero and medium density (>1000/ha) of old seedling population. This implies that young seedlings can contribute profit to the management only when the population of adult plants and old seedlings is lower. When abundant adult plants or old seedlings exist, young seedlings are not important to the decision making process or to the increase of net present value of the land.

Contrary to the young seedlings, a marked difference in the pattern of stocking rates with respect to the forage biomass can be observed. Generally speaking, the stocking rates (in Table 3.a) are positively related to the level of forage biomass although the pattern of stocking rates will change between uniform and varied stocking rates. For example, the optimal stocking rules for section 9 which corresponds to the state of a combination of 1600 adult plants, 2500 old seedlings, 5 level of forage biomass and 3 level of young seedlings, indicate that the pattern of stocking rates change from total destocking (policy 1) to varied stocking rates: (.1/1.0/.1, policy 33), (.4/1.0/.2, policy 39) and then to uniform stocking rates: (.2/.2/.2, policy 7), (.6/.6/.6, policy 15). The total destocking policy is optimal for forage biomass at zero level in all stuations. This implies that if the paddock has been totally defoliated or after serious drought the optimal policy is to destock under the deterministic case, no matter how many desirable perennial plants or seedlings exist. Within sections 3, 7, 8, 10, 11, and 12, some patterns of optimal stocking rate changes from high to low when the forage biomass increases. For example, in section 8 the pattern of stocking rates corresponding to state 99 and 100 change from policy 15 (.6/.6/.6) to policy 14 (.55/.55/.55) when the forage biomass increases from 320 kg d.m./ha to 800 kg d.m./ha. This seems inconsistent with the intuitive reasoning that the optimal stocking rates should increase with a rise in total forage biomass. The reason for such inconsistencies is that high stocking rates which affects the evolution of the grazing ecosystem generate higher current profits but results in a lower ending state; while the low stocking rate which gives lower current profits can end with a higher terminal state. The loss of current profit due to the low stocking rate is then outweighed by the increase of net present value from lower ending state to higher ending state. Thus the optimal policy for these states is to use a lighter stocking rate so as to achieve a higher ending state with a greater net present value. For example, for state 100, it has been indicated that with low uniform stocking rates of policy 14 (.55/.55/.55) the ending state is 94 with a lower current profit of $3.42; adopting higher stocking rates i.e. policy 15 (.6/.6/.6) ended in state 93 with a higher current net profit margin of $3.71. Although the current profit is higher for the higher stocking rate which is outweighed by the change in the net present value from $42.04 to $42.89. Theoretically, if the state and control spaces are continuous sets, i.e. both state and control space can be partitioned into infinite number of infinitesimal intervals, this inconsistent phenomenon will disappear since it is always possible to use at least the same stocking rate for the higher state without resulting in a lower ending state. Therefore, in the empirical implication, we can still keep the concept that higher stocking rate should apply to the greater forage biomass.

Table 3.d (page 29) presents the shadow price of total forage biomass. As illustrated by this Table, shadow price of forage biomass is positive everywhere and generally decreases with the increasing forage availability although there are some exceptions. This indicates that the forage biomass is more important when the forage availability is very low. Comparing the shadow prices row by row in each section, the shadow price generally does not change with the young seedling levels. Therefore, the value of forage biomass is not influenced by young seedlings. Comparing the shadow price of forage biomass with respect to adult plants, it generally increases with the higheradult population. Thus the value of forage biomass is relatively more important at higher adult plant density. Comparing the shadow price with respect to old seedlings, it generally increases with a rise in density of old seedling population although there are some exceptions. Therefore, the value of forage biomass increases if the old seedling population increases, other things being equal.

Optimal grazing strategies with respect to desirable perennial adult plants can be analysed by comparing section by section and row by row in Table 3.a. Generally, the stocking rates alter between uniform and varied levels for different adult populations. However, it can be concluded that stocking rate generally increases with increasing adult population though there are some exceptions. These exceptions can be explained by the similar reasons to that of forage biomass. For example, optimal stocking rate of policy 15 i.e. 5/.5/.5 was chosen for state 184 (in section 13)instead of the policy 15 i.e. 6/.6/.6 which is optimal for state 94 (in section 7). This is because with low stocking rate: .5/.5/.5 a much higher increase in the net present value can be achieved due to the higher ending state (state 184). However, if high stocking rate: .6/.6/.6 was applied, a lower ending state (state 183) with a lower net present value will be generated. Thus the increase in the current profit due to high stocking rate is compensated by the reduction of net present valued for the lower ending state. On the other hand, the reason for state 94 to take a high stocking rate: .6/.6/.6 rather than:.5/.5/.5 is because both stocking pattern ended with same ending state (state 93) while high stocking rate can generate higher current profits. The low stocking rate for state 94 can not maintain itself as an ending state, unlike state 184. This is because high adult plant densities can produce a greater volume of forage biomass, of which a large proportion remains at the end of year. However, the way of partition of the state variable also affects this phenomenon.

As presented by Table 3.e (page 30), shadow price of adult plants changes with different adult plant densities. Generally, the shadow price is high when the adult plants ranges between 0-400/ha, other things being equal. The lowest shadow prices occur in section 15 which is the combination of both high adult and old seedling populations. Thus, the marginal value of adult plants can only increase slightly when there are abundant adult and old seedlings available. Comparing the shadow price with respect to young seedlings, the shadow prices are generally insensitive to the young seedlings though there are some exceptions. Comparing the shadow price with respect to the forage biomass, it shows no systematic pattern of change with increasing forage biomass. However, at zero forage biomass level, the shadow price of adult plants is zero everywhere. This is because under deterministic climatic regime once forage were totally defoliated even total destocking can not get the ending state out of zero forage biomass level. Thus, the value of adult plants is zero no matter how many adult or seedlings exist at the beginning of the year. These states consists of a chain of irreversible states under the deterministic case.

Optimal grazing strategies with respect to old seedlings can be analysed by comparing section by section and column by column in Table 3.a. The optimal grazing strategies seems insensitive to the change from 0 to 1000 seedlings/ha. However, it does show differences when old seedlings increase to 2500 seedlings/ha. Generally, higher stocking rate will correspond to higher old seedling populations, although there are few exceptions such as states 99 & 104, 114 & 119, 129 & 135 (in section 8 and 9). These exceptions can be explained by the similar reason to those of adult plants.

Shadow price of old seedlings are presented by Table 3.f (page 31). As illustrated by the Table, the price generally increases with higher densities although there are some exceptions. The very high values occur when adult plants are scarce (less than 400 plants/ha). Zero shadow price occurs at the combinations of adult plants of above 1600 plants/ha and the old seedlings 1000 plants/ha. Also zero marginal value of old seedlings occurs at zero forage biomass.

Stochastic Case Study

Under stochastic climatic regime, the number of wet pentads was generated by using the probability density function of wet pentads (see Table 1). Table 3.g (page 32) presents the optimal grazing strategies. Like the deterministic case, the Table of optimal stocking rules can be subdivided into 15 sections. Among them, only 4 sections i.e. 1, 2, 4, and 5 show different stocking rate pattern. The others all have uniform stocking rates of policy 6 (i.e. .15/.15/.15) as the optimal grazing strategy. The total destocking policy is optimal for the combinations of zero adult plant, zero old seedlings and zero to 2000 young seedlings. Also, it is optimal for the state of 400 adult plants/ha, zero old seedling, zero young seedlings and three lower forage biomass levels. In addition to the above two grazing policies, there are optimal policies ranging from policy 2 to policy 5(i.e. .02/.02/.02 to .1/.1/.1) corresponding to the different level of adult plants, seedlings and forage biomass.

As illustrated by Table 3.h (page 33), the net present value under stochastic climatic regime increases with increasing value of the four state variables. The highest value $6.99/ha occurs at the state of highest density of adult plants combining with 800 kg d.m./ha forage biomass. The lowest value occurs at the states where no adult and no seedlings exist.

Comparing the optimal grazing policies with respect to the young seedlings, the stocking rates generally increases with increasing young seedling population provided that the adult population and old seedling population are lower. When the population of adult and old seedlings are higher i.e.≥1600 adults/ha and ≥2500 seedlings/ha, respectively, the optimal grazing decisions are insensitive to the young seedling population.

The net present value also increases with the increasing young seedling population, especially at the low level of adult and old seedling populations. The shadow price of young seedlings are presented by Table 3.i (page 34). As indicated by the Table, all the shadow prices are greater or equal to zero. For the zero value of shadow price it means that the net present value can not change with more young seedlings. Therefore, in the case of zero shadow price, they are redundant resource. The most important contribution of young seedlings occurs at the combinations of 0 adult plants and 1000 old seedlings or 400 adult plants and 0 old seedlings.

Comparing the optimal policies with respect to forage biomass, it seems that the optimal grazing policies are not very sensitive to the different level of forage biomass. It implies that under the stochastic climatic regime the optimal long term grazing policies can not take advantage of short term increase in the initial forage biomass. The net present value, though increases with forage biomass, is also insensitive to the different level of forage biomass. Table 3.j (page 35) indicates that the shadow price of forage biomass generally declines with the increasing level of forage biomass. At the highest level i.e. 800 kg d.m., the value of shadow price are very close to zero.

In terms of analysing the optimal grazing decisions with respect to the adult plants, the optimal stocking rate generally increases with a rise adult population when adult plant and old seedlings are less than 1600 and 1000, respectively. Above which the optimal stocking rate are constant. The net present value also follows the same manner as the optimal policies.

The shadow price of the adult plants are presented by Table 3.k (page 36). As indicated by the Table, the shadow price is very close to zero when adult plant and old seedlings are greater than 1600 and 1000 respectively. The maximum value occurs when the combined population of adults, old seedlings and young seedlings are 400, 1000, 0, respectively.

Comparing the optimal policies with respect to old seedlings, the optimal stocking rate show increasing trend with increasing old seedling population provided that adult population is lower than 1600 plants/ha. Above which, there are no changes in the optimal policies corresponding to the old seedlings. The net present values also follow the same pattern as optimal decisions.

Table 3.1 (page 37) presents the shadow price of old seedlings. The maximal value occurs at the combination of 400, 1000, 2000 for adult, old seedling and young seedlings, respectively. Generally, when the population of adult plants are greater than 1600, the value of shadow price for old seedlings is very close to zero. This is consistent with the pattern of shadow price for adult plants. Since optimal net present values are insensitive to the adult plant when adult population is greater than 1600 more old seedlings occurring at this level are redundant.

With regards to the deterministic climatic regime, the optimal stocking rates generally are lower under the stochastic case, although they are some exceptions. Most of total destocking grazing policy for the zero forage biomass in the deterministic case disappeared under the stochastic climatic regime. The destocking policy under stochastic climatic regime occurs on two occasions. In the first situation, it is the state where no adults and old seedlings, combined with zero or 2000 young seedlings will exist. Second, it is in a situation where adults, old seedlings, young seedlings are: 400, 0, 0 per ha respectively, as well as forage biomass levels consisting of the following values: 0, 80, 160 kg d.m./ha. For these states, by applying only grazing management decisions the range condition is irreversible under the stochastic climatic regimes. However, treatment policies such as reseeding or ponding techniques may improve the range condition for these irreversible states. Therefore, there is a need to further the study by including treatment techniques in the decision making process.

Another important finding is the disappearance of all the varied stocking rates under the stochastic case. This implies that under stochastic climatic regime the uniform stocking rate is better than the varied one. In the arid zone, the climatic pattern is very unreliable, varied stocking rates which mainly try to capture the probability of rainfall pattern will loss profitability in the long term due to high cost of stocking rate adjustment and inaccurate rainfall prediction.

The net present value are also dramatically reduced under stochastic climatic regime for most states. For example, the highest value of $65.26/ha corresponding to the last state (state 225), i.e. highest value of all 4 state variables, in the deterministic case is reduced to only $6.99/ha under a stochastic climatic regime. The difference between these two values can represent the value of perfect weather forecast under this state. For most irreversible states in the deterministic case the net present value increases under stochastic climatic regime. This is due to the possibility that these states can be transmitted by the corresponding optimal grazing policies into other states under stochastic climatic regime. Also, there are other states, such as state 7, 8 (in section 2) which the stochastic climatic sequence can produce higher profitability.

The stochastic and deterministic results were quite different. We are currently verifying the stochastic results. If they withstand scrutiny, the implication is that deterministic decision rules are not applicable. The dramatic differences between the deterministic and stochastic cases cannot be attributed to risk aversion because risk neutrality is assumed. The differences must be attributed to the highly stochastic weather and non-linearities in the objective function.

Summary and Conclusion

This paper uses stochastic optimal control framework to solve the range regeneration management problem with respect to decisions about stocking rate. A simulation model IMAGES developed earlier was used to investigate the vegetation response to different stocking rate levels. Annual transition probabilities were derived by combining 9 state variables into 4 aggregated variables: desirable perennial adult plants, desirable perennial old seedlings, desirable perennial young seedlings and total forage biomass.

The optimal decision rules were derived for both deterministic and stochastic climatic case studies. These decision rules are useful because they are stationary and functional on the above 4 observable state variables. Thus, each year, the range manager can monitor the value of these 4 state variables and make grazing decisions based upon the optimal decision rules.

Generally, the optimum stocking rate increase with increasing value for the 4 state variables. In the deterministic case it combines both uniform and varied stocking rates while in the stochastic case only the uniform stocking rate prevails. The net present value also increases with the increasing value of 4 state variables though there are some exceptions in the deterministic case where irreversible states occur. The optimum grazing policies and net present values are not sensitive to the high densities of adult plants and old seedlings under a stochastic case. The magnitude of stocking rate and net present value are generally higher in the deterministic case, though there are some exceptions.

Further research is needed to include the treatment decisions into the control space and to decide optimal timing for the application of rehabilitation treatment.

REFERENCES

Bellman, R. (1957). *Dynamic programming*. Princeton Univ Press, Princeton, New Jersey.

Bertsekas, D.P. (1976). *Dynamic programming and stochastic control*. New York: Academic Press.

Derman, C. (1970). *Finite state Markovian decision processes*. Academic Press, New York.

Fitzpatrick, E.A., Slatyer,R.O and Krishnan A.I. (1967). Incidence and Duration of Periods of Plant Growth in Central Australia as Estimated from Climate Data. *Agr.Meteorol*, **4**,389-404.

Hacker, R.B., Wang, K.M., Lindner,R.K and Richmond G.S.(1989). *I.M.A.G.E.S: an Integrated Model of an Arid Grazing Ecological System*. Unpublished paper. Western Australian Department of Agriculture, South Perth.

Leslie, P.H. (1945). On the use of matrices in certain population mathematics. *Biometrika*, **33**, 183-212.

Leslie, P.H. (1948). Some further notes on the use of matrices in population mathematics. *Biometrika*, **35**, 213-45.

Wang,K.M., Hacker,R.B., Lindner,R.K and Richmond G.S. (1988). *Computer modelling and rangeland regeneration in the arid zone of Western Australia*. Contributed paper presented to the 32nd Annual Conference of the Australian Agr. Econ. Society, La Trobe University, February 1988.

Appendix A.1 The Value Used for the Parameter in the Objective Function

Imputed net value of wool produced:
Average greasy price: $3.61/kg
Marketing and distribution costs: $0.52/kg

(a) The costs are charged as percentages of gross greasy price
wool tax: 4%
market support levy: 4%
broker's commission: 1.4628% (1.59% of gross proceeds less market support levy and wool tax)
insurance:.04%
subtotal: 9.5028%
(b) The costs are charged on per bale/kg basis (1 bale=171.46 kg)
wool packs: 3.52 cents/kg
freight: 3.02 cents/kg ($5.18 per bale)
warehousing: 7.59 cents/kg ($13.01 per bale)
coretest certificate: 1.89 cents/kg ($3.27 per bale)
interlotting: 0.36 cents/kg
rehandling: 1.53 cents/kg
subtotal: 17.91 cents/kg
Marketing and distribution costs: 3.61*.095028+.1791=0.52

Shearing and other wool harvesting costs per season: $1.18/sheep

shearing: 152.87 cents/sheep
crutching: 87 cents/sheep
classing & shedhands: 88.37 cents/sheep
pressing, branding & weighing: 27.20 cents/sheep
subtotal: 355.44 cents/sheep

Shearing and other wool harvesting costs per season: $3.5544/3=$1.18

Sheep price at season 1: $6.14
at season 2: $9.84
at season 3: $9.93

Stocking rate adjustment costs:
selling costs at season 1: $0.53/sheep (6.14*.05+.22=.53)
at season 2: $0.71/sheep (9.84*.05+.22=.71)
at season 3: $0.72/sheep (9.93*.05+.22=.73)
commission: 5% of sale price
saleyard fees: 22 cents/sheep
freight costs:$1.15/sheep (assuming distance in 100 kms and double deck sheep)

Average variable costs:
veterinary: $.18/sheep
mustering: $.20/sheep
mulesing, mark, drench, vaccinate & tag:$.50/sheep
fuel & oil: $1.03/sheep
repair & maintenance:$1.49/sheep
general, water supply, fences, plant, vehicles, aircraft and

others
interest paid: $.37/sheep
subtotal: $3.77/sheep

Average fixed costs:

rent: 2 cents/ha
depreciation: $.18/ha
farm improvements, plant and equipment
operator and family labour: $.35/ha
administration: $.02/ha
subtotal:$.57/ha

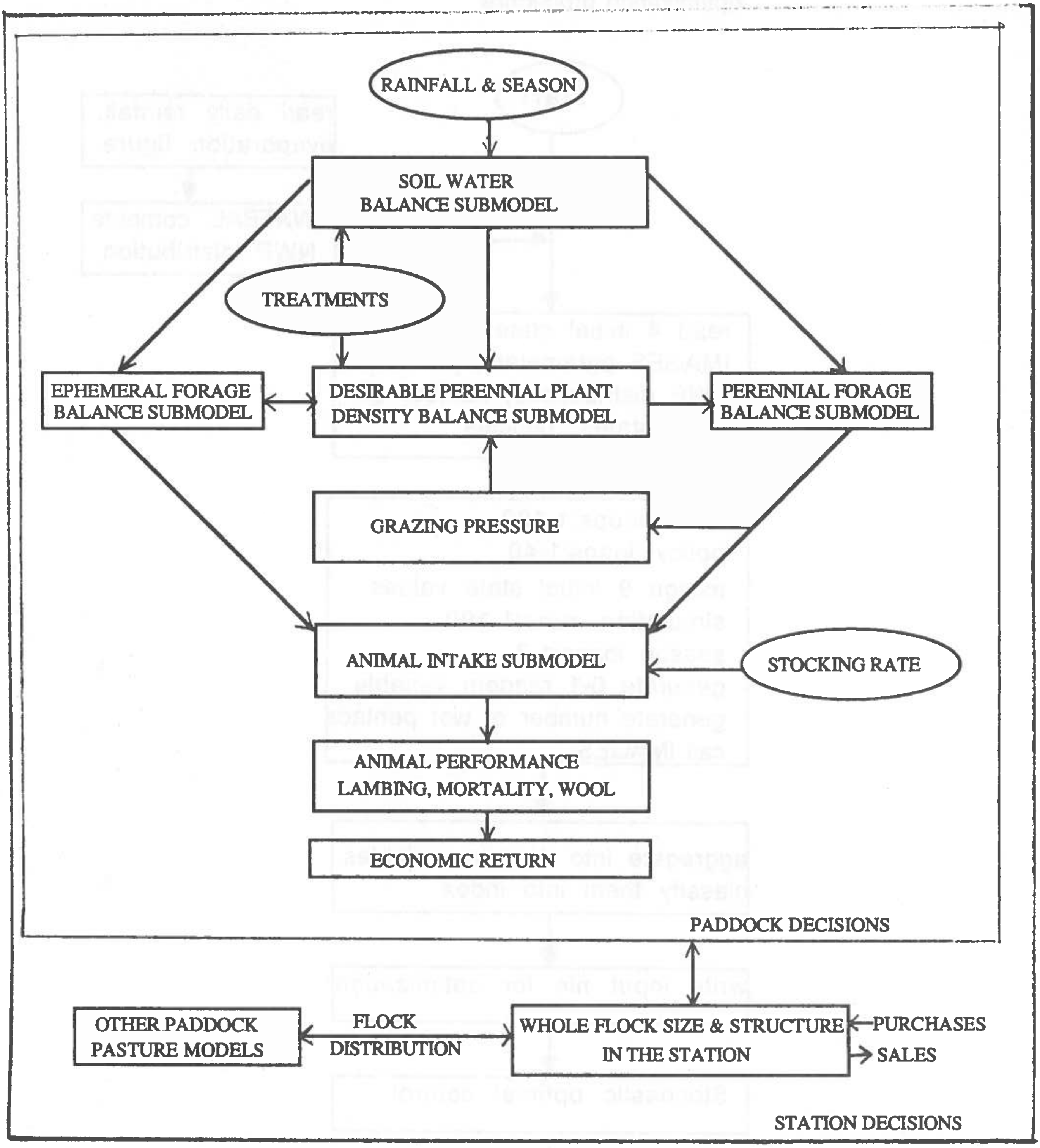

FIGURE 1. IMAGES: A SIMULATION MODEL OF RANGELAND ECOSYSTEM

Figure 2: Operational sequence for combining simulation and optimization procedure

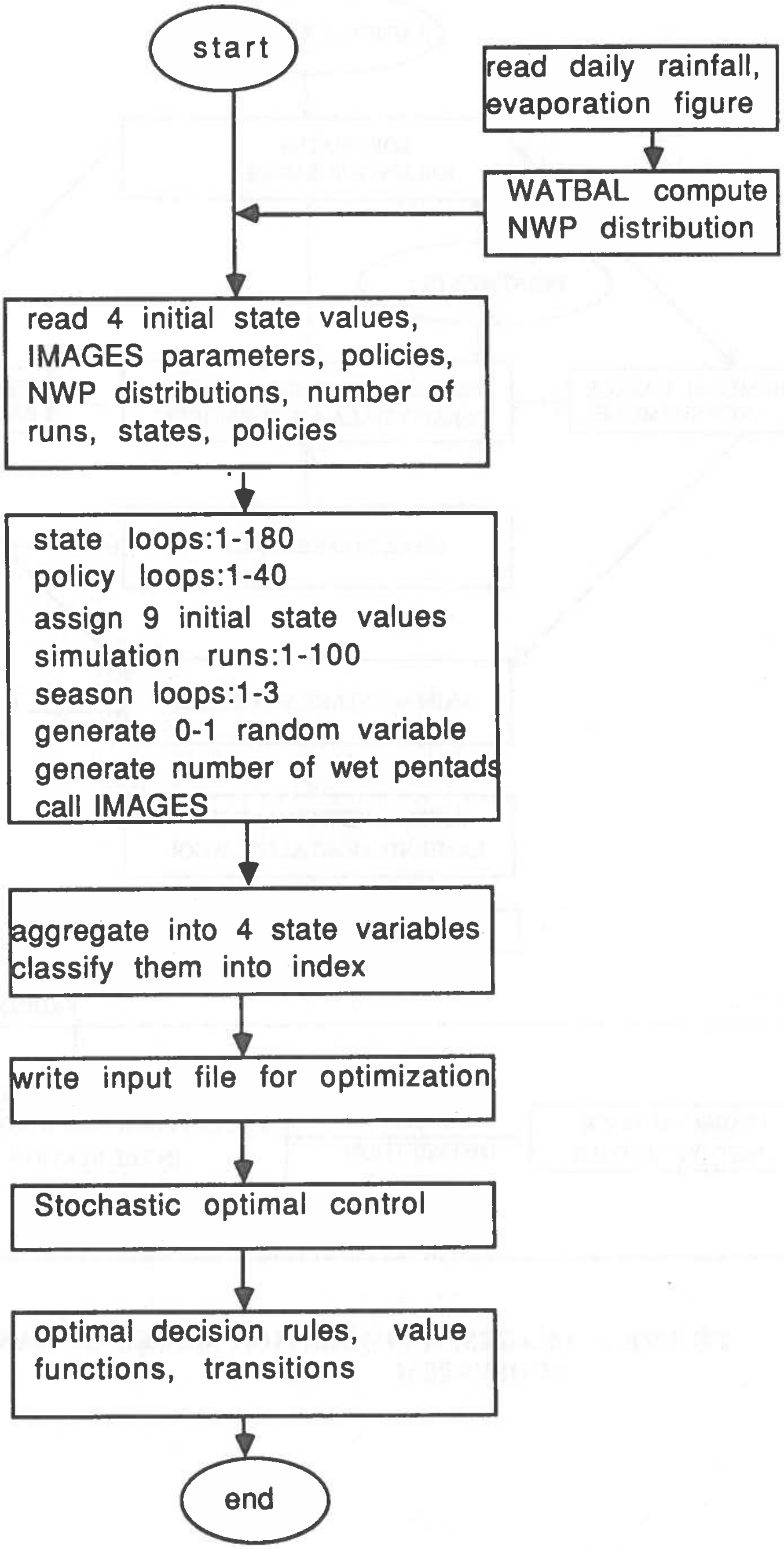

Figure 3: Optimization algorithm-successive approximation method

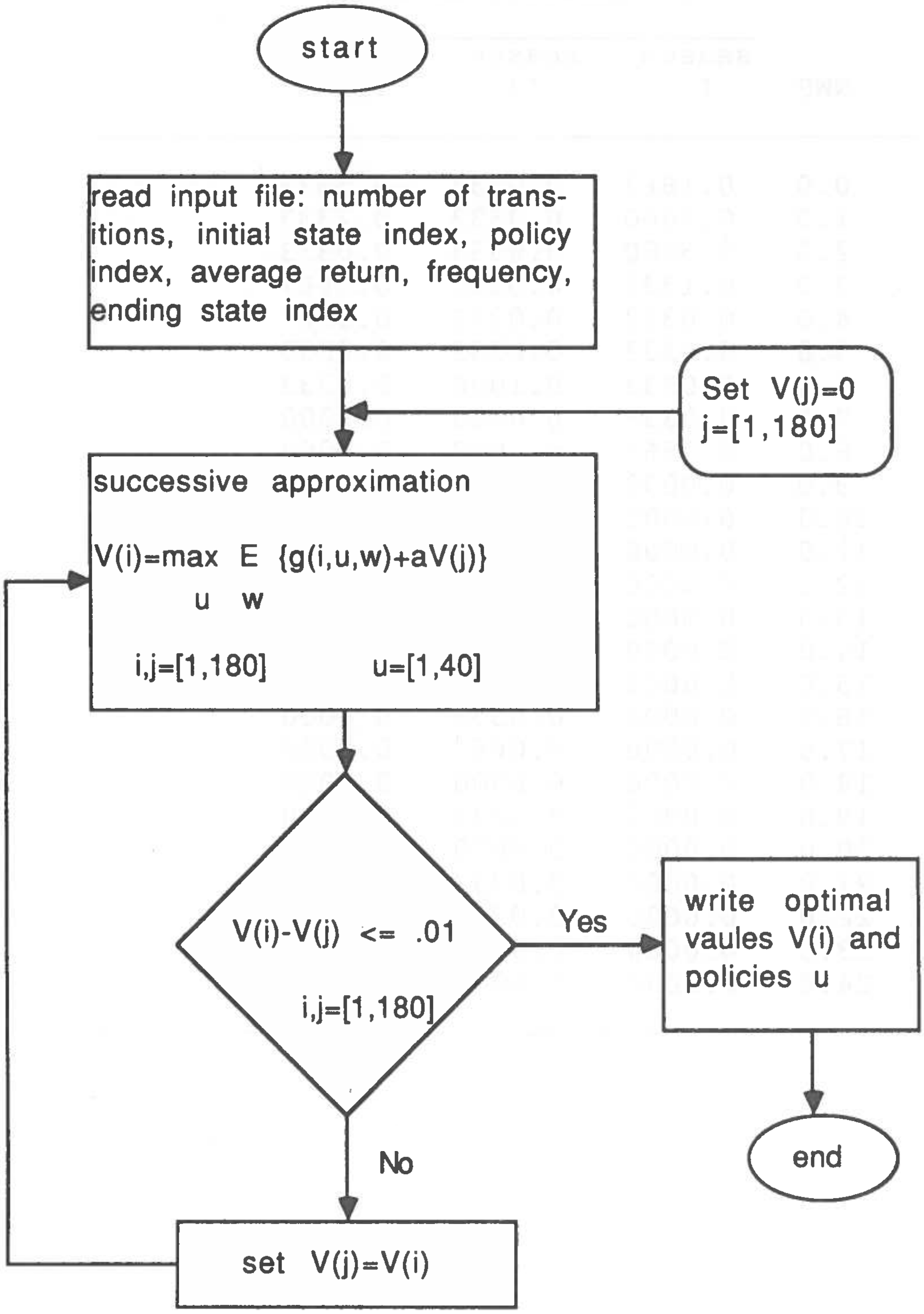

Table 1. Probability density function of NWP distribution

	probability density		
NWP	season I	season II	season III
0.0	0.1667	0.0333	0.5333
1.0	0.2000	0.0333	0.2333
2.0	0.3000	0.0333	0.0333
3.0	0.1333	0.0000	0.0667
4.0	0.0333	0.0333	0.0667
5.0	0.0333	0.0333	0.0333
6.0	0.0333	0.1000	0.0333
7.0	0.0333	0.0000	0.0000
8.0	0.0667	0.1000	0.0000
9.0	0.0000	0.1000	0.0000
10.0	0.0000	0.0333	0.0000
11.0	0.0000	0.1667	0.0000
12.0	0.0000	0.0667	0.0000
13.0	0.0000	0.0667	0.0000
14.0	0.0000	0.0000	0.0000
15.0	0.0000	0.0000	0.0000
16.0	0.0000	0.0333	0.0000
17.0	0.0000	0.0667	0.0000
18.0	0.0000	0.0000	0.0000
19.0	0.0000	0.0333	0.0000
20.0	0.0000	0.0000	0.0000
21.0	0.0000	0.0333	0.0000
22.0	0.0000	0.0333	0.0000
23.0	0.0000	0.0000	0.0000
24.0	0.0000	0.0000	0.0000

Table 2 Grazing Management Strategies

policy index	season I	season II	season III	policy index	season I	season II	season III
	-----stocking rate------ (sheep/ha)				-----stocking rate----- (sheep/ha)		
1	.0	.0	.0	21	.05	.7	.0
2	.02	.02	.02	22	.05	.7	.05
3	.05	.05	.05	23	.05	1.0	.0
4	.08	.08	.08	24	.05	1.0	.05
5	.1	.1	.1	25	.1	.4	.0
6	.15	.15	.15	26	.1	.4	.05
7	.2	.2	.2	27	.1	.4	.1
8	.25	.25	.25	28	.1	.7	.0
9	.3	.3	.3	29	.1	.7	.05
10	.35	.35	.35	30	.1	.7	.1
11	.4	.4	.4	31	.1	1.0	.0
12	.45	.45	.45	32	.1	1.0	.05
13	.5	.5	.5	33	.1	1.0	.1
14	.55	.55	.55	34	.4	.7	.0
15	.6	.6	.6	35	.4	.7	.1
16	.0	.4	0	36	.4	.7	.2
17	.0	.7	.0	37	.4	1.0	.0
18	.0	1.0	.0	38	.4	1.0	.1
19	.05	.4	.0	39	.4	1.0	.2
20	.05	.4	.05	40	.4	1.0	.4

TABLE 3.A

Optimal Grazing Management Strategies Under Deterministic Climatic Regime

ADULT PLANTS #/ha	Total Forage Biomass kg d.m. /ha															YOUNG SEEDLINGS #/ha
	0	80	160	320	800	0	80	160	320	800	0	80	160	320	800	
	optimal policy index															
0	1a					2					3					0
0	1	34	34	37	37	1	34	31	37	15	1	37	31	38	35	2000
0															45b	5000
400	4					5					6					0
400	1	31	37	37	37	1	31	37	37	37	1	31	33	7	39	2000
400															90	5000
1600	7					8					9					0
1600	1	31	37	15	14	1	31	37	15	14	1	33	39	7	15	2000
1600															135	5000
3200	10					11					12					0
3200	1	8	39	35	15	1	8	39	35	15	1	9	40	39	15	2000
3200															180	5000
6400	13					14					15					0
6400	1	34	6	13	15	1	34	6	13	15	1	34	7	13	15	2000
6400															225	5000
Old Seedlings #/ha	0					1000					2500					

a. Section index. Section index ranges from 1 to 15.

b. State index. State index ranges from 1 to 225. It starts from top left-hand corner, moving from left to right, row by row and ends with bottom right-hand corner.

TABLE 3.B

Optimal Net Present Value Under Deterministic Climatic Regime

		Total Forage Biomass kg d.m. /ha																
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800		
		net present value $/																
ADULT PLANTS #/ha	0					5.88			5.74	7.90	10.54						0	YOUNG SEEDLINGS #/ha
	0	0	0.36	1.81	3.05	5.88	0	0.36	5.74	7.90	10.54	0	38.47	40.36	42.51	43.37	2000	
	0					10.46			36.62	38.78	41.42						5000	
	400		5.51	7.20	7.93	10.49											0	
	400	0	5.51	7.20	7.93	10.49	0	38.24	39.93	40.67	43.47	0	43.31	45.43	45.99	52.15	2000	
	400		36.39	38.08	38.81	41.37											5000	
	1600																0	
	1600	0	40.21	42.04	42.89	43.88	0	40.21	42.04	42.89	43.88	0	45.27	47.15	50.25	52.62	2000	
	1600																5000	
	3200																0	
	3200	0	45.47	47.34	51.79	52.68	0	45.47	47.34	51.79	52.68	0	58.46	60.54	63.30	64.01	2000	
	3200																5000	
	6400																0	
	6400	0	59.25	61.02	63.63	64.74	0	59.25	61.02	63.63	64.74	0	59.23	61.36	63.97	65.26	2000	
	6400																5000	
		0					1000					2500						
		Old Seedlings #/ha																

TABLE 3.C

Shadow Price of Young Seedlings Under Deterministic Climatic Regime

ADULT PLANTS #/ha	YOUNG SEEDLINGS #/ha	Total Forage Biomass kg d.m. /ha														
	Old Seedlings #/ha	0					1000					2500				
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800
		cents/seedling														
0	0															
0	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	5000	0	0	0	0	0.23	0	0	1.54	1.54	1.54	0	0	0	0	0
400	0															
400	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
400	5000	0	1.03	1.03	1.03	1.03	0	0	0	0	0	0	0	0	0	0
1600	0															
1600	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1600	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3200	0															
3200	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3200	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6400	0															
6400	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6400	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 3.D

Shadow Price of Forage Biomass Under Deterministic Climatic Regime

Total Forage Biomass kg d.m. /ha

cents/kg d.m

ADULT PLANTS #/ha	YOUNG SEEDLINGS #/ha	Old Seedlings #/ha: 0					1000					2500				
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800
0	0					0.59			6.73							
0	2000		0.45	1.81	0.77	0.59		0.45	6.73	1.35	0.55		48.1	2.36	1.34	0.2
0	5000					1.54			45.32							
400	0		6.89													
400	2000		6.89	2.11	0.46	0.53		47.8	2.11	0.46	0.58		54.1	2.65	0.35	1.28
400	5000		45.5													
1600	0															
1600	2000		50.3	2.29	0.53	0.21		50.3	2.29	0.53	0.21		56.6	2.35	1.94	0.49
1600	5000															
3200	0															
3200	2000		56.8	2.34	2.78	0.19		56.8	2.34	2.78	0.19		73.1	2.6	1.72	0.15
3200	5000															
6400	0															
6400	2000		74.1	2.21	1.63	0.23		74.1	2.21	1.63	0.23		74	2.66	1.63	0.27
6400	5000															

TABLE 3.E

Shadow Price of Adult Plants Under Deterministic Climatic Regime

Total Forage Biomass kg d.m. /ha

cents/plant

ADULT PLANTS #/ha	YOUNG SEEDLINGS #/ha	Old Seedlings #/ha: 0					1000					2500				
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800
0	0															
0	2000															
0	5000															
400	0		1.29	1.35	1.22	1.15			8.55	8.19	8.23					
400	2000	0	1.29	1.35	1.22	1.15	0	9.47	8.55	8.19	8.23	0	1.21	1.27	0.87	2.17
400	5000		9.01	9.07	8.94	7.73			0.83	0.47	0.51					
1600	0		2.89	2.9	2.91	2.78										
1600	2000	0	2.89	2.9	2.91	2.78	0	0.16	0.18	0.18	0.03	0	0.16	0.14	0.35	0.04
1600	5000		0.32	0.33	0.34	0.21										
3200	0															
3200	2000	0	0.33	0.33	0.56	0.55	0	0.33	0.33	0.56	0.55	0	0.82	0.84	0.82	0.71
3200	5000															
6400	0															
6400	2000	0	0.43	0.43	0.37	0.38	0	0.43	0.43	0.37	0.38	0	0.02	0.03	0.02	0.04
6400	5000															

TABLE 3.F

Shadow Price of Old Seedlings Under Deterministic Climatic Regime

		Total Forage Biomass kg d.m. /ha																
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800		
										cents/seedling								
ADULT PLANTS #/ha	0							0	0.39	0.48	0.47			2.31	2.31	2.20	0	YOUNG SEEDLINGS #/ha
	0						0	0	0.39	0.48	0.47	0	2.54	2.31	2.31	2.20	2000	
	0							0	3.48	3.57	3.10			0.25	0.25	0.14	5000	
	400							3.27	3.27	3.27	3.30						0	
	400						0	3.27	3.27	3.27	3.30	0	0.34	0.37	0.35	0.58	2000	
	400							0.19	0.19	0.19	0.21						5000	
	1600																0	
	1600						0	0	0	0	0	0	0.34	0.34	0.49	0.58	2000	
	1600																5000	
	3200																0	
	3200						0	0	0	0	0	0	0.87	0.88	0.77	0.76	2000	
	3200																5000	
	6400																0	
	6400						0	0	0	0	0	0	0	0.02	0.02	0.03	2000	
	6400																5000	
				0					1000					2500				
									Old Seedlings #/ha									

TABLE 3.G

Optimal Grazing Management Strategies Under Stochastic Climatic Regime

ADULT PLANTS #/ha	Total Forage Biomass kg d.m. /ha															YOUNG SEEDLINGS #/ha
	0	80	160	320	800	0	80	160	320	800	0	80	160	320	800	
	optimal policy index															
0	1	1	1	1	1	3	4	4	4	4					3a	0
0	1	1	1	1	1	3	4	4	4	4	6	6	6	6	6	2000
0	4	4	4	4	5	5	6	6	6	6					45b	5000
400	1	1	1	2	2	5	5	5	5	5	6					0
400	4	4	4	4	4	6	6	6	6	6	6	6	6	6	6	2000
400	3	4	5	5	5	6	6	6	6	6					90	5000
1600	7					8					9					0
1600	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	2000
1600															135	5000
3200	10					11					12					0
3200	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	2000
3200															180	5000
6400	13					14					15					0
6400	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	2000
6400															225	5000
			0					1000					2500			
	Old Seedlings #/ha															

a. Section index. Section index ranges from 1 to 15.

b. State index. State index ranges from 1 to 225. It starts from top left-hand corner, moving from left to right, row by row and ends with bottom right-hand corner.

TABLE 3.H

Optimal Net Present Value Under Stochastic Climatic Regime

		Total Forage Biomass kg d.m. /ha																
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800		
		net present value $/ha																
	0	0	0	0	0	0	1.12	1.15	1.22	1.23	1.23	6.91	6.94	6.95	6.96	6.96	0	Y
A	0	0	0	0	0	0	5.83	5.97	5.98	5.99	5.99	6.92	6.95	6.96	6.96	6.96	2000	O
D	0	0.14	0.16	0.17	0.18	0.19	6.68	6.73	6.74	6.82	6.83	6.93	6.95	6.96	6.97	6.97	5000	U
U																		N
L	400	0	0	0	0	0	6.60	6.62	6.63	6.63	6.64	6.93	6.95	6.96	6.96	6.97	0	G
T	400	0.14	0.15	0.16	0.16	0	6.72	6.82	6.83	6.84	6.84	6.93	6.96	6.96	6.97	6.97	2000	
	400	5.31	5.39	5.40	5.47	5.47	6.83	6.92	6.93	6.94	6.94	6.94	6.96	6.97	6.97	6.98	5000	S
P																		E
L	1600	6.82	6.85	6.85	6.86	6.86	6.93	6.95	6.96	6.96	6.97	6.94					0	E
A	1600	6.90	6.93	6.94	6.94	6.95	6.93	6.96	6.97	6.97	6.97	6.95	6.97	6.98	6.98	6.98	2000	D
N	1600	6.92	6.95	6.96	6.96	6.96	6.94	6.96	6.97	6.97	6.98	6.95					5000	L
T																		I
S	3200	6.94		6.97	6.97								6.97			6.98	0	N
	3200	6.94	6.97	6.97	6.98	6.98	6.95	6.97	6.98	6.98	6.98	6.95	6.98	6.98	6.98	6.98	2000	G
	3200	6.95		6.98	6.98								6.98			6.99	5000	S
#/ha																		
	6400					6.98											0	#/ha
	6400	6.95	6.98	6.98	6.98	6.99	6.95	6.98	6.98	6.99	6.99	6.95	6.98	6.98	6.99	6.99	2000	
	6400					6.99											5000	
				0					1000					2500				
		Old Seedlings #/ha																

TABLE 3.i

Shadow Price of Young Seedlings Under Stochastic Climatic Regime

ADULT PLANTS #/ha	YOUNG SEEDLINGS #/ha	Total Forage Biomass kg d.m. /ha														
		0	80	160	320	800	0	80	160	320	800	0	80	160	320	800
		cents/seedling														
0	0															
0	2000	0	0	0	0	0	0.24	0.24	0.24	0.24	0.24	0	0	0	0	0
0	5000	0	0.01	0.01	0.01	0.01	0.03	0.03	0.03	0.03	0.03	0	0	0	0	0
400	0															
400	2000	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0
400	5000	0.17	0.17	0.17	0.18	0.18	0	0	0	0	0	0	0	0	0	0
1600	0															
1600	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1600	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3200	0															
3200	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3200	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6400	0															
6400	2000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6400	5000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Old Seedlings #/ha	0					1000					2500				

TABLE 3.J

Shadow Price of Forage Biomass Under Stochastic Climatic Regime

ADULT PLANTS #/ha	Total Forage Biomass kg d.m. /ha															YOUNG SEEDLINGS #/ha
	0	80	160	320	800	0	80	160	320	800	0	80	160	320	800	
	cents/kg d.m.															
0		0	0	0			0.04	0.09	0.01			0.04		0.01		0
0		0	0	0	0		0.17	0.01	0.01	0		0.04	0.01	0	0	2000
0		0.02	0.01	0.01			0.06	0.01	0.05			0.03		0.01		5000
400		0	0	0			0.03		0			0.03	0.01	0		0
400		0.01	0.01	0	0		0.13	0.01	0.01	0		0.04	0	0.01	0	2000
400		0.01	0.01	0.04			0.11		0.01			0.02	0.01	0		5000
1600			0	0.01			0.03					0.04				0
1600		0.04	0.01	0	0		0.04	0.01	0	0		0.02	0.01	0	0	2000
1600			0.01	0			0.02					0.02				5000
3200		0.04	0.00	0.01								0.02	0.01			0
3200		0.04	0.01	0	0		0.02	0.01	0	0		0.04	0	0	0	2000
3200		0.02	0	0								0.04	0			5000
6400																0
6400		0.04	0	0	0		0.04	0	0.01	0		0.04	0	0.01	0	2000
6400																5000
	0					1000					2500					
	Old Seedlings #/ha															

TABLE 3.K

Shadow Price of Old Seedlings Under Stochastic Climatic Regime

Total Forage Biomass kg d.m. /ha

cents/seedling

ADULT PLANTS #/ha	Old Seedlings #/ha: 0					1000					2500					YOUNG SEEDLINGS #/ha
	0	80	160	320	800	0	80	160	320	800	0	80	160	320	800	
0						0.11	0.11	0.12	0.12	0.12	0.39	0.39	0.38	0.38	0.38	0
0						0.58	0.60	0.60	0.60	0.60	0.07	0.07	0.07	0.06	0.06	2000
0						0.65	0.66	0.66	0.66	0.66	0.02	0.01	0.01	0.01	0.01	5000
400						0.66	0.66	0.66	0.66	0.66	0.02	0.02	0.02	0.02	0.02	0
400						0.66	0.67	0.67	0.67	0.67	0.01	0.01	0.01	0.01	0.01	2000
400						0.15	0.15	0.15	0.15	0.15	0.01	0	0	0	0	5000
1600						0.01	0.01	0.01	0.01	0.01						0
1600						0	0	0	0	0	0	0	0	0	0	2000
1600																5000
3200																0
3200						0	0	0	0	0	0	0	0	0	0	2000
3200																5000
6400																0
6400						0	0	0	0	0	0	0	0	0	0	2000
6400																5000

TABLE 3.L

Shadow Price of Old Seedlings Under Stochastic Climatic Regime

ADULT PLANTS #/ha	Total Forage Biomass kg d.m. /ha															YOUNG SEEDLINGS #/ha
	0	80	160	320	800	0	80	160	320	800	0	80	160	320	800	
						cents/seedling										
0						0.11	0.11	0.12	0.12	0.12	0.39	0.39	0.38	0.38	0.38	0
0						0.58	0.60	0.60	0.60	0.60	0.07	0.07	0.07	0.06	0.06	2000
0						0.65	0.66	0.66	0.66	0.66	0.02	0.01	0.01	0.01	0.01	5000
400						0.66	0.66	0.66	0.66	0.66	0.02	0.02	0.02	0.02	0.02	0
400						0.66	0.67	0.67	0.67	0.67	0.01	0.01	0.01	0.01	0.01	2000
400						0.15	0.15	0.15	0.15	0.15	0.01	0	0	0	0	5000
1600						0.01	0.01	0.01	0.01	0.01						0
1600						0	0	0	0	0	0	0	0	0	0	2000
1600																5000
3200																0
3200						0	0	0	0	0	0	0	0	0	0	2000
3200																5000
6400																0
6400						0	0	0	0	0	0	0	0	0	0	2000
6400																5000
	0					1000					2500					
	Old Seedlings #/ha															