AE 97004 **July 1997**

Regional Factor Demand in U.S. Wheat Production

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Selected Paper for Presentation at the
1997 AAEA Annual Meetings
in Toronto, Canada

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Abstract

This study applies a translog cost function to analyze factor demands in four U.S. wheat production regions. The results show that factor demands are inelastic across all regions. The interdependent relationships among inputs and the effects of price changes on the relative cost shares are substantially different across regions.

Key words: wheat production, factor demand, elasticity of substitution, regional comparisons, translog models, curvature condition.

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Many studies have analyzed demand for production factors at the regional or state level for selected commodities or regions (e.g., Weaver; Shumway; Grisley and Gitu; Dixon, Garcia, and Mjelde; Shumway and Alexander; Taylor and Monson). However, little attention has been given to factor demands in the U.S. wheat industry and comparisons among major wheat production regions of the United States. The objective of this study is to use a translog cost function to analyze the input demands for energy, capital, materials, and labor and the effects of changes in the factor prices on the relative cost shares and the factor utilization in four major U.S. wheat production regions.

The Translog Cost Function

Assume that there exists a well-behaved aggregate production function for the United States wheat industry,

$$y = F(E, C, M, L, T), \tag{1}$$

where *y*, *E*, *C*, *M*, and *L* refer to aggregate output, energy, capital, materials, and labor, respectively. *T* is the time trend and reflects technical change. According to Diewert, there exists a unique dual cost function which reflects the production technology. The dual cost function is

$$C(y, w) = \underset{x}{\mathrm{Min}}\left[w'x:\ F(x) > y\right], \tag{2}$$

where *C* is total cost, *x* is a vector of the inputs, and *w* is a vector of the corresponding input prices.

For purposes of estimation, a general and flexible functional form is used to describe the production function in empirical analyses. The flexible functional forms provide a second-order approximation to an arbitrary function (Diewert). The great advantage of flexible functional forms is that they do not impose *a priori* restrictions on the elasticities of substitution. In this study, we arbitrarily apply a translog cost function to analyze factor demand in four U.S. wheat production regions.

The Transcendental Logarithmic (translog) functional form was first proposed by Christensen, Jorgenson, and Lau. It is a second-order Taylor approximation in logarithms to an arbitrary function. The translog dual cost function with the regional dummy variables can be written as follows:

$$\ln C = \beta_0 + \sum_{i=1}^{n} \beta_i \ln w_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{ij} \ln w_i \ln w_j + \lambda_t T + \sum_{k=1}^{m} \gamma_k D_k$$
$$+ \frac{1}{2} \lambda_{tt} T^2 + \sum_{k=1}^{m} \gamma_{kt} T D_k + \sum_{i=1}^{n} \lambda_{ti} T \ln w_i + \sum_{k=1}^{m} \sum_{i=1}^{n} \gamma_{ki} D_k \ln w_i \tag{3}$$

where $\delta_{ij} = \delta_{ji}$, $\sum_{i=1}^{n} \beta_i = 1$, and $\sum_{i=1}^{n} \delta_{ij} = \sum_{j=1}^{n} \delta_{ij} = \sum_{i=1}^{n} \lambda_{ti} = \sum_{i=1}^{n} \gamma_{ki} = 0$.

This function is positive, linearly homogeneous in prices, and symmetric. However, it is not necessarily monotonically increasing in prices, nor is it necessarily concave. Monotonicity and concavity must be checked for each observation of the data after the function has been estimated (Kohli).

Given the level of output under the assumption of perfect competition in the factor market, the cost minimizing input demand functions can be simply derived by differentiation of the cost function:

$$\frac{\partial \ln C}{\partial \ln w_i} = \frac{\partial C}{\partial w_i} \frac{w_i}{C}, \qquad i = 1, \ldots, n. \tag{4}$$

Using Shephard's Lemma,

$$x_i = \frac{\partial C}{\partial w_i}, \qquad i = 1, \ldots, n. \tag{5}$$

Then, the derived input demand functions can be expressed in share form as

$$S_i = \frac{w_i x_i}{C} = \beta_i + \sum_{j=1}^{n} \delta_{ij} \ln w_j + \lambda_{ti} T + \sum_{k=1}^{m} \gamma_{ki} D_k, \qquad i = 1, \ldots, n. \tag{6}$$

Thus, the share of each input is a linear function of the logarithms of the input prices, a time trend, and regional dummies.

Following Uzawa, the Allen partial elasticity of substitution (AES) between input *i* and *j* can be calculated as

$$\sigma_{ij} = \frac{\delta_{ij} + S_i S_j}{S_i S_j}, \qquad i \neq j, \tag{7}$$

$$\sigma_{ii} = \frac{\delta_{ii} + S_i^2 - S_i}{S_i^2}, \qquad i = j. \tag{8}$$

Let ε_{ij} be the price elasticity of input demand for input i with respect to the price of input j:

$$\varepsilon_{ij} = \frac{\partial \ln x_i}{\partial \ln w_k} = \frac{\partial \ln x_i}{\partial \ln w_j} \frac{w_j}{x_i}. \tag{9}$$

Allen showed that the price elasticity of input demand for production can be directly calculated from the *AES* as: $\varepsilon_{ij} = \sigma_{ij} S_j$. Once that the estimate $\Sigma = [\sigma_{ij}]$ has been obtained, the matrix of the price elasticity of input demand E can be calculated as: $E = \Sigma S$, where $s = [S_i]$ and $S = \mathrm{Diag}(s)$. A necessary and sufficient condition for the translog cost function to be concave requires that all the eigenvalues of matrix Σ are nonpositive. That is, the matrix of Allen elasticity of substitution is negative semidefinite.

Blackorby and Russell argued that the AES provides no information about the curvature of the isoquant and the relative cost shares, and cannot be interpreted as the marginal rate of substitution, and that the AES is completely uninformative. Morishima proposed an alternative measure of the factor substitution, known as the Morishima elasticity of substitution (MES). The MES is defined as a logarithmic derivative of a quantity ratio with respect to a marginal rate of substitution or a ratio of input prices. It measures the curvature of the isoquant and the effects of changes in price ratios on relative cost shares. According to Blackorby and Russell, the MES can be written as

$$\omega_{ij} = \varepsilon_{ji} - \varepsilon_{ii} = \frac{\delta_{ij} + S_i S_j}{S_j} - \frac{\delta_{ii} + S_i^2 - S_i}{S_i}, \qquad i \neq j, \tag{10}$$

The MES can also provide complete information about relative factor cost shares in response to a change in factor prices (Huang). This measure can be written as

$$\eta_{ij} = 1 - \omega_{ij} \tag{11}$$

The relative cost share is decreasing (increasing) if the MES is greater (less) than one.

Data and Estimation Procedure

The translog cost function is applied to analyze input demand and technology in U.S. wheat production from 1975 to 1990. In this study, we assume that input allocation decisions in wheat production are independent of output allocation decisions. The input factors used in U.S. wheat production were aggregated into four input categories: energy (E), capital (C), materials (M), and labor (L). The data cover four major wheat production regions: Central and Southern Plains, Southeast and North Central, Northern Plains, and Northwest. Data on quantities of these four inputs for each planted acre were taken from the USDA budgets reported in the *Economic Indicators of the Farm Sector*: *Cost of Production--Major Field Crops and Livestock and Dairy*. The input price indexes (1977 = 100) were obtained from the USDA, *Agricultural Prices*. The trend variable, T, started from 1975, was included as an independent variable to capture the technological change. Three regional dummies are also included in the model in order to obtain the consistent estimates.

Adding the error term e_i to each equation of (6) results in an econometric system of cost share equations. This system was first estimated using the pooled data of all four wheat production regions with the symmetry and linear homogeneity restrictions imposed. Since the sum of S_is is equal to unity, the cost share equation for materials (M) was arbitrarily dropped to ensure the nonsingularity of the disturbance covariance matrix, and its price was used as the *numèraire*. The remaining system was estimated using Zellner's iterative seemingly unrelated regression (ISUR) with parameter restrictions. The parameters of the dropped cost share equation were derived from the relationships among the parameters. However, the resulting cost function failed to be concave for some observations of the data. Since the estimates which violate the necessary concavity conditions have no economic meaning, the concavity restrictions were imposed in model estimation. To ensure concavity restrictions implied by microeconomic theory, the Wiley, Schmidt, and Bramble (WSB) reparameterization procedure outlined by Kohli was used to estimate the model. However, concavity condition is a local property for translog cost function, and the WSB procedure can only impose concavity locally. It is necessary to check whether the concavity conditions are satisfied for each observation of the sample. Kohli suggested that, in practice, imposing the curvature conditions at one data point that seems to be the most serious offender may turn out to impose the conditions globally. Following the WSB procedure, we imposed concavity conditions for each year of the sample, and found that 1986 is the year for which concavity was the most apparently violated. The global concavity was ensured after we imposed the concavity restrictions in 1986. This was done by restimating the model after having renormalized all input prices and time trend for 1986. Because of the reparameterization, the model becomes nonlinear in the parameters. The nonlinear system of cost share equations with concavity restriction imposed in 1986 was estimated using the nonlinear seemingly unrelated regression procedure from *SHAZAM, Version 7.0* (White).

Empirical Results

The estimation results of the nonlinear system of cost share equations are listed in Table 1. To more easily interpret the results, we only reported the estimates of original parameters (the δ_{ij}s), rather than those of the reparameterized terms. Most estimates for the regional dummy variable are statistically significant at the 5% level, indicating the significant differences in factor demand among the four major wheat production regions in the United States.

With the parameter estimates of the translog cost function, the Allen partial elasticities of substitution (AES) were calculated according to equations (7) and (8) at the sample mean of the cost shares for each wheat production region and reported in the left panel of Table 2. The positive signs indicate substitution relationships between any pair of inputs, except for that between energy and labor; they are strong complements with elasticities ranged from about -6 for Regions 1 and 4 to about -13 for Regions 2 and 3. A strong substitution relationship was found between capital and labor across all wheat production regions, especially in Regions 2 and 3. The elasticities of substitution of the rest of the pairs of inputs are very similar across regions, but are relatively small.

The relationships among energy, capital, materials, and labor can also be shown by the price elasticities of factor demand in the right panel of Table 2. The results suggest that the demands for all four input factors are inelastic across all regions. The demand for materials has the smallest own-price elasticity, while the demand for labor is the most elastic among the four inputs with an elasticity of near -0.9 across regions, which is close to those for North Dakota (1.02) and South Dakota (0.79) as found by Weave. However, the own-price elasticities for other inputs are much smaller than those in Weave's study. In addition, the substantial complementarity among inputs in North Dakota and South Dakota are also not found in this study. The different results may originate from the different model specifications.

In contrast with the Allen elasticity, which is partial adjustment to the price of one factor, the Morishima elasticity of substitution (MES) reflects the adjustment of relative factors in response to a change of relative factor prices. The MES for all wheat production regions are shown in the left panel of Table 3. The positive MES between any pair of inputs implies that they are substitutable with each other. The elasticities of factor ratios, capital-energy, and labor-capital are greater than 1 for all regions, confirming the strong substitution relationships between capital and energy and between capital and labor indicated by the Allen elasticities. However, the complement relationship between energy and labor indicated by the Allen elasticity measure is shown only in Region 2 by Morishima elasticity. Huang suggested that this inconsistency may be caused by the different definitions of these two elasticities.

Table 1 . Parameter Estimates of the Translog Cost Functions -- U.S. Wheat Production, 1975-90

	Cost Share of			
Variable	Energy	Capital	Materials	Labor
Constant	0.1061 (0.0077)*	0.3329 (0.0147)*	0.4389 (0.0173)*	0.1221 (0.0066)*
Factor Price of				
Energy	0.0460 (0.0200)*		(**Symmetric**)	
Capital	0.0337 (0.0298)	0.0099 (0.1072)		
Materials	-0.0046 (0.0291)	-0.1104 (0.0696)	0.1085 (0.0759)	
Labor	-0.0751 (0.0185)*	0.0668 (0.0459)	0.0064 (0.0392)	0.0018 (0.0341)
Time Trend	0.0015 (0.0011)	-0.0034 (0.0016)*	0.0004 (0.0019)	0.0016 (0.0011)
Region Dummy 1	-0.0183 (0.0107)	0.0987 (0.0151)*	-0.0846 (0.0149)*	0.0042 (0.0083)
Region Dummy 2	-0.0436 (0.0099)*	-0.0588 (0.0136)*	0.1306 (0.0148)*	-0.0283 (0.0071)*
Region Dummy 3	-0.0292 (0.0097)*	0.0199 (0.0133)	0.0558 (0.0154)*	-0.0465 (0.0077)*

Note: Concavity was imposed in 1986.
Asymptotic standard errors are in parentheses and an asterisk (*) indicates significance at the 0.05 level.

Table 2. Allen Elasticities of Substitution (AES) and Estimated Price Elasticities of Factor Demand at the Sample Mean by Region, 1975-90

	Allen Elasticities of Substitution (AES)				Price Elasticities of Factor Demand with Respect to the Price of			
	Energy	Capital	Materials	Labor	Energy	Capital	Materials	Labor
Production Region (1): Central and Southern Plains								
Energy	-4.3942	1.8021	-4.3942	-5.5597	-0.4422	0.7515	0.3229	-0.6322
Capital	1.8021	-1.3413	0.2820	2.4096	0.1814	-0.5593	0.1040	0.2740
Materials	0.8759	0.2820	-0.9140	1.1536	0.0882	0.1176	-0.3369	0.1312
Labor	-5.5597	2.4096	1.1536	-7.6564	-0.5595	1.0048	0.4253	-0.8706
Production Region (2): Southwest and North Central								
Energy	-3.5632	2.9209	0.8800	-12.5982	-0.2341	0.7790	0.5135	-1.0584
Capital	2.9209	-2.6108	0.2909	3.9830	0.1919	-0.6963	0.1698	0.3346
Materials	0.8800	0.2909	-0.3949	1.1313	0.0578	0.0776	-0.2304	0.0950
Labor	-12.5982	3.9830	1.1313	-10.6498	-0.8278	1.0623	0.6602	-0.8947
Production Region (3): Northern Plains								
Energy	-4.3735	2.1846	0.8917	-12.9306	-0.3640	0.7460	0.4553	-0.8373
Capital	2.1846	-1.8439	0.3670	4.0230	0.1818	-0.6296	0.1874	0.2605
Materials	0.8917	0.3670	-0.5422	1.1947	0.0742	0.1253	-0.2768	0.0774
Labor	-12.9306	4.0230	1.1947	-14.0178	-1.0761	1.3737	0.6100	-0.9077
Production Region (4): Northwest								
Energy	-4.4034	2.0098	0.8978	-5.6714	-0.4381	0.6733	0.4062	-0.6413
Capital	2.0098	-1.8970	0.2718	2.7643	0.2000	-0.6355	0.1230	0.3126
Materials	0.8978	0.2718	-0.6801	1.1259	0.0893	0.0911	-0.3077	0.1273
Labor	-5.6714	2.7643	1.1259	-7.7035	-0.5643	0.9261	0.5094	-0.8711

Table 3. Morishima Elasticities of Substitution (MES) and Effects of Factor Price Change on Cost Shares at the Sample Mean by Region, 1975-90

	Morishima Elasticities of Substitution (MES)				Effects of Factor Price Change on Cost Shares			
	Energy	Capital	Materials	Labor	Energy	Capital	Materials	Labor
Production Region (1): Central and Southern Plains								
Energy		0.6236	0.5304	-0.1173		0.3764	0.4696	1.1173
Capital	1.3108		0.6769	1.5641	-0.3108		0.3231	-0.5641
Materials	0.6599	0.4409		0.7622	0.3401	0.5591		0.2378
Labor	0.2384	1.1446	1.0018		0.7616	-0.1446	-0.0018	
Production Region (2): Southwest and North Central								
Energy		0.4260	0.2919	-0.5937		0.5740	0.7081	1.5937
Capital	1.4753		0.7739	1.7586	-0.4753		0.2261	-0.7586
Materials	0.7440	0.4002		0.8907	0.2560	0.5998		0.1093
Labor	-0.1637	1.2293	0.9898		1.1637	-0.2293	0.0102	
Production Region (3): Northern Plains								
Energy		0.5458	0.4382	-0.7121		0.4542	0.5618	1.7121
Capital	1.3756		0.7549	2.0033	-0.3756		0.2451	-1.0033
Materials	0.7321	0.4642		0.8869	0.2679	0.5358		0.1131
Labor	0.0704	1.1681	0.9850		0.9296	-0.1681	0.0150	
Production Region (4): Northwest								
Energy		0.6381	0.5275	-0.1262		0.3619	0.4725	1.1262
Capital	1.3088		0.7266	1.5616	-0.3088		0.2734	-0.5616
Materials	0.7138	0.4306		0.8170	0.2862	0.5694		0.1830
Labor	0.2298	1.1837	0.9984		0.7702	-0.1837	0.0016	

The effects of factor price change on relative cost shares are reported in the right panel of Table 3. The high positive elasticities of energy-labor (1.12 to 1.71) indicate a significant increase in the cost share of energy to labor in response to the relatively high energy price than labor wages, with the largest increase in Region 3. On the other hand, the negative elasticity of capital-labor indicates a reduction in the cost share of capital to labor in response to the relatively high capital price to the labor wage. However, only Region 3 shows the significant reduction with an elasticity of -1.003. The elasticity of labor-energy (1.16) indicates that a marginal increase of the labor wage would also significantly increase the cost share of labor relative to energy in Region 2.

Concluding Remarks

This study applies a translog cost function to analyze the demands for energy, capital, materials, and labor in the U.S. wheat industry of four major production regions. The results show that the demands for these four inputs are inelastic across all wheat production regions. However, the demand for labor and capital service are more elastic than for energy and materials in U.S. wheat production. The price elasticities of factor demand do not vary significantly across regions, but some Allen elasticities of substitution are substantially different across regions. The substitution between capital and labor and the complement relationship between energy and labor are much stronger in the Northern Plains and Southwest and North Central regions than in the Central and Southern Plains and Northwest regions. The estimated Morishima elasticities indicate that relatively large changes in energy or capital prices would cause significant changes in cost shares across all regions, while the impact of change in the labor wage is only found to be significant in the Southwest and North Central regions. The changes in the price of materials did not show any strong effects on factor utilization in U.S. wheat production over the sample period. Further research may analyze the technological change in the U.S. wheat industry and its different effects on factor demands across all U.S. wheat production regions.

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