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An Economic Analysis of Peasant Agriculture Under Risk

by

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Conventional wisdom about the economic behaviour of peasant farmers and the markets in which they operate is based largely on deterministic microeconomic theory. Consequently, it is not at all certain that policies formulated on the basis of such wisdom are the most relevent given the considerable risk in production which characterizes much peasant agriculture.

This paper contains results from microeconomic models of the peasant farmer and his markets which do take account of risk. Results are not developed to the level of policy prescription, but rather, the more modest goal is pursued of providing a better understanding of the implications of risk to economic behavior.

The paper comprises two parts. The first part presents a theory of sharecropping and semi-subsistence farming under the assumption that individual farmers are risk averse in the expectation and variance of returns. The second part is concerned with the equilibrium of markets under risk, and results are presented about the price and welfare characteristics of market equilibrium for a wide range of assumptions about the ways in which farmers form price expectations over time.

I. Analysis of the Peasant Farm Under Risk

The analysis is confined to peasant farmers who engage in some market transactions for their products and inputs. These may be cash or barter transactions of surplus produce above family subsistence requirements, in which case implicit prices are involved, or, as will be shown, they may be payments in kind for services rendered by landlords under sharecropping arrangements. The model to be presented is applicable to both cases, but it is developed in a notation appropriate to sharecropping farms.

The literature contains a number of models of barter trade relationships in subsistence agriculture [1, 3, 4, 8, 9, 12,] , and a class of model for sharecropping subsistence farms has been developed by Cheung [2], together with an analysis of the land market.

In introducing risk we assume farmers to maximize a utility function

$$U = U(C, L, V_c) \tag{1}$$

where U indicates utility, C and V_c are the subjective expectation and variance of a random variable representing total family consumption, and L is total family labor employed on the farm.

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Consumption is assumed to depend on the production of a homogenous good ξ , which in turn is a random variable with expectation X and variance V_x;

$$E\xi = X , \qquad E [\xi - E\xi]^2 = V_x \tag{2}$$

where E is the expectation operator. Expectation and variance of consumption are linked to production by the following identities:

$$C = (1 - r) X - W' Z \tag{3}$$

$$V_c = (1 - r)^2 V_x \tag{4}$$

where r is the share of product paid to the landlord for land, Z is ah $nx1$ vector of inputs, and W is an $nx1$ vector of wages in kind paid to the n factors of production.

Assuming that the behaviour of the product depends only on random disturbances exogenous to the nature of the production function, we can write:

$$\xi = F (L , Z , u) \tag{5}$$

where u represents the random component of production, and is assumed to have a finite mean and variance.

Given the above specifications, the first order conditions for the maximization of utility (1) over the production set $(L : Z)$ are:

$$(1 - r) X_\ell = - \frac{U_\ell}{U_c} - \frac{U_v}{U_c} [2 (1 - r)^2 \text{ Cov } (\xi , \xi_\ell)] \tag{6}$$

$$(1 - r) X_z = W - \frac{U_v}{U_c} [2 (1 - r)^2 \text{ Cov } (\xi , \xi_z)] \tag{7}$$

where ξ_ℓ and X_ℓ denote, respectively, the marginal productivity of labor and its expectation, ξ_z and X_z are corresponding vectors for the other factors of production, and U_c , U_ℓ and U_v are marginal utilities (partial derivatives) with respect to expected consumption, labor and variance of consumption, respectively.

Given a production function with conventional properties of decreasing marginal productivity, the terms Cov (ξ , ξ_ℓ) and Cov (ξ , ξ_z) will be positive, and equations (6) and (7) indicate that factors of production will be used at levels at which their marginal value productivities exceed their unit costs by some risk factor. (In the case of family labor the factor cost is the marginal disutility of work, U_ℓ.) This means that the demand for factors is less under risk than in a deterministic environment, and that the 'optimal' size of farms will be

smaller. Similar results have been obtained in the literature for the analysis of the more general firms operating under risk [5, 7, 11] .

Another implication of the first order conditions concerns the existence of a market for land and other inputs even in the absence of cash transactions. Suppose that the landlord supplies the sharecropper with the inputs Z , and for which he collects some further proportion α of the sharecropper's total production in addition to r , the rent for land. Then equation (7) can be rewritten as:

$$(1 - r)\, X_z - \alpha X = -\frac{U_v}{U_c} \left[2\,(1 - r)^2 \operatorname{cov}(\xi , \xi_z) \right]$$

This can be interpreted as the sharecropper's implicit supply function of the amount of product he will offer the landlord in return for land and other inputs. Further, this supply is stochastic because the actual amount offered, $(r + \alpha)\, \xi$, is stochastic with total production. If we also postulate that the landlord behaves so as to maximize some function of total rent collected over all his sharecropping farms, then it would also be possible to derive his demand for the sharecropper's product, and for which he would offer land and other inputs. Thus, not only does a market exist for the exchange of products for land and other inputs, but the actual share of production paid to the landlord will depend on the equilibrium conditions of the market.

The above model readily encompasses non-sharecropping peasant farmers who sell their products and purchase their inputs in more competitive cash or barter markets. It is only necessary to redefine $1 - r$ as product price, and to retain the vector W as the vector of factor market costs.

Comparative Statics

Equations (6) and (7) can be rearranged in vector form. Assume, for simplicity, that the marginal utilities U_c , U_ℓ and U_v are constant. Then $-U_\ell / U_c$ corresponds to a given shadow wage rate for family labor, and can be incorporated in the vector of factor wages W . Family labor L can then be included in the vector of inputs Z , and the system written as:

$$\overset{*}{X} = E\,[F(\overset{*}{Z} , U)] = G(\overset{*}{Z})$$

and

$$(1 - r)\, \frac{\partial X}{\partial Z}\, [\overset{*}{Z}(r, W)] = W - \frac{U_v}{U_c} \left[2(1 - r)^2 \operatorname{Cov}(\overset{*}{\xi} , \xi_z) \right]$$

where the stars indicate that the functions are evaluated at the optimum point $\overset{*}{\xi}_z = \partial F / \partial Z$. By differentiating the system, the following comparative static results can be obtained.

$$\frac{\partial X}{\partial r} = \frac{1}{1-r} \left[\left(\frac{\partial G}{\partial Z}\right) H^{-1} \left(\frac{\partial G}{\partial Z}\right)' + 4\frac{U_v}{U_c} \left(\frac{\partial G}{\partial Z}\right) H^{-1} S' \right] \tag{8}$$

$$\frac{\partial Z}{\partial r} = \frac{1}{1-r} H^{-1} \left[\left(\frac{\partial G}{\partial Z}\right)' + 4 \frac{U_v}{U_c} S' \right] \tag{9}$$

$$\left(\frac{\partial X}{\partial W}\right)' = \frac{1}{1-r} \frac{\partial G}{\partial Z} H^{-1} \left[I - 2 \frac{U_v}{U_c} (1 - r)^2 \sum \right] \tag{10}$$

$$\frac{\partial Z}{\partial W} = \frac{1}{1-r} H^{-1} \left[I - 2 \frac{U_v}{U_c} (1 - r)^2 \sum \right] \tag{11}$$

where H is the Hessian matrix, and S and $\sum$ are, respectively, the nxl and nxn matrices:

$$S = (1 - r) \left[\frac{1}{2} (1 - r) \operatorname{Cov} \left(\frac{\partial F}{\partial r}, \frac{\partial F}{\partial Z} \right) + \operatorname{Cov} \left(\xi, \frac{\partial^2 F}{\partial Z \partial r}\right) - \operatorname{Cov} \left(\xi, \frac{\partial F}{\partial Z}\right) \right]$$

$$\sum = \operatorname{Cov} \left(\xi , \frac{\partial^2 F}{\partial Z \partial W} \right) + \operatorname{Cov} \left(\frac{\partial F}{\partial W} , \frac{\partial F}{\partial Z} \right) .$$

It is evident from the above expressions that if risk aversion is strong enough, the comparative static results of a deterministic model may be reversed. For example, if we regard $1-r$ as the equivalent of price, then equations (8) and (9) indicate the possibility of a downward sloping supply function, and upward sloping factor demand functions.

The results also point to the possibility of the covariance terms exactly outbalancing the deterministic terms, causing, as a consequence, null or quasi-null sensitivity of the equilibrium solution to price variation.

II. Analysis of the Market Under Risk

In this section we explore the nature of the equilibrium of a market given risk in production. The analysis assumes competitive behaviour and deterministic demand. While this is compatible with many cash and barter markets in a peasant economy, its relevance to landlord-sharecropper transactions depends on the existence of competitive behaviour between landlords so that monopoly exploitation of sharecroppers does not occur.

In order to simplify the analysis it is further assumed that market supply and demand functions can be approximated by the following linear forms:

$$S_t = \lambda \varepsilon_t H(P) \tag{12}$$

$$D_t = a - b P_t \tag{13}$$

where a , b and λ are positive constants, ε_t is a multiplicative

stochastic yield term, P denotes price, and $H(P)$ is some function of price.

It is not necessary to make specific assumptions about the risk behaviour of individual farmers, beyond that λ, the slope of market supply, is greater than zero. This need not of course exclude the possibility of some individual farmers being sufficiently risk averse as to have negatively sloped supply curves, providing these are more than offset by classically behaved producers.

What is necessary to the market model (12) and (13) are assumptions about the nature of $H(P)$. In the peasant farm model, expectations of price, or its equivalent, were implicitly assumed to be given, and the problem of how these might change over time was ignored. This turns out to be a crucial issue in the market model, since the dynamics of the way in which price expectations are formed determines not only the adjustment behaviour of the market, as in a deterministic environment,but also the nature of an equilibrium if attained.

A Naïve Stochastic Cobweb Model

Typically, agricultural production involves lagged supply responce, and producers must make input decisions on the basis of what they anticipate the ensuing market price will be. The price they anticipate will be based on some sort of learning procedure on past prices, and we begin our analysis with the most naïve of learning models in which last year's price is thought to be a good estimate of the expectation of current years price.

The appropriate market model is:

$$S_t = \lambda \varepsilon_t P_{t-1}$$

$$D_t = a - b P_t$$

This is nothing more than a classical but stochastic cobweb model, and the market clearing price in year t is

$$P_t = {}^{a}/_{b} - {}^{\lambda}/_{b}\, \varepsilon_t P_{t-1} \qquad (14)$$

An equilibrium will exist if (14) converges, but it is readily apparent that $\lim_{t\to\infty} P_t$ cannot converge to a unique value, because ε_t is stochastic. Rather, P_t has a cumulative probability distribution $F_t(P)$, and we can say an equilibrium exists for the market if $F_t(P)$ has a limiting probability distribution $\bar{F}(P)$.

If we assume

$$E(\varepsilon_t) = \mu \quad , \text{ all } t$$

$$E[\varepsilon_t - E(\varepsilon_t)]^2 = \sigma^2 \quad , \text{ all } t$$

$$E(\varepsilon_t, \varepsilon_{t-1}) = 0$$

$$\text{with} \quad 0 < \mu, \sigma^2 < \infty$$

then conditions for the convergence of $F_t(P)$ can be developed from the theory of Markov processes. Without presenting the necessary algebra, we simply state that $F(P)$ does indeed exist providing $\lambda/b < 1/\mu$ is true.

It can also be shown that the mean and variance of the price distribution converge, and in particular that

$$\lim_{t\to\infty} E(P_t) = \frac{a}{b+\lambda\mu}$$

and

$$\lim_{t\to\infty} \text{Var}(P_t) = \frac{a^2 \lambda^2 (\mu_2 - \mu^2)}{(b+\lambda\mu)^2 (b^2-\lambda^2\mu_2)} \qquad (15)$$

where μ_2 is the second moment of ε around zero. Necessary and sufficient condition for the convergence of $E(P_t)$ and $\text{Var}(P_t)$ are $\frac{\lambda}{b} < \frac{1}{\mu}$ and $\frac{\lambda^2}{b^2} < \frac{1}{\mu_2}$ respectively. The latter condition is stronger, and in fact implies the first.[1/] It is interesting to compare these conditions with the necessary and sufficient condition for convergence of a deterministic cobweb model; this is $\frac{\lambda}{b} < 1$.

We therefore have a result, which is in fact characteristic of all models of market equilibria under risk, that price never stabilizes to a unique level, but can only converge in its probability distribution. However, the naïve cobweb model says much more than this. Because P_t does not converge, but only $E(P_t)$, then producers will make different input decisions each year, even when the market is in equilibrium. Consequently the demand for inputs is stochastic. Put another way, this means that producers operate on a different supply schedule each year, where the actual schedule in year t is conditional on P_{t-1}.

Another very interesting result is that the expected price corresponds to the intersection of demand and expected supply. This can

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1/ It can also be shown that, provided the limits in (15) exist, and under the assumptions made, the covariance of $P_t P_{t-1}$ will tend to a definite limit, so that the weak law of large numbers can be applied. This can be expressed as follows:

$$\text{plim}_{T\to\infty} \left| \sum_{t=1}^{T} P_t - \sum_{t=1}^{T} E P_t \right| = 0 ,$$

where plim indicates the probability limit.

be shown as follows:

$$E(S_t) = \lambda E(\varepsilon_t P_{t-1}) = \lambda E(\varepsilon_t) E(P_{t-1}) ,$$

because $E(\varepsilon_t \varepsilon_{t-1}) = 0$ implies $E(\varepsilon_t P_{t-1}) = E(\varepsilon_t) E(P_{t-1})$.

Now, equating expected supply and demand, and solving for price $\overset{*}{P}_t$, gives :

$$\overset{*}{P}_t = \frac{a}{b} - \frac{\lambda}{b} E(\varepsilon) E(P_{t-1})$$

However, taking the expectation of actual market clearing price in equation (14) , one obtains:

$$E(P_t) = \frac{a}{b} - \frac{\lambda}{b} E(\varepsilon) E(P_{t-1})$$

so that $E(P_t) = \overset{*}{P}$, and this result holds whether or not the market attains an equilibrium.

The nature of the equilibrium for the naïve cobweb model can be conveniently portrayed in geometry when ε has a lower and upper bound, (Figure 1). Let $\varepsilon_M < \varepsilon < \varepsilon_X$, then it can be shown that the equilibrium price distribution has the range

$$\frac{a(b - \lambda \varepsilon_X)}{b^2 - \lambda^2 \varepsilon_X \varepsilon_M} < P < \frac{a(b - \lambda \varepsilon_M)}{b^2 - \lambda^2 \varepsilon_X \varepsilon_M}$$

In Figure 1, $\dot{P}_M$ and $\dot{P}_X$ denote, respectively, these convergent lower and upper bounds on price, and $S_{|\varepsilon}$ denotes a conditional supply curve given a fixed ε . Hence $S_{|\varepsilon_m}$ and $S_{|\varepsilon_x}$ define the conditional supply schedules given ε_m and ε_x occur, and they also define a funnel which contains all other possible conditional supply curves. $S_{|\mu}$ denotes the expected supply curve given $\varepsilon = E(\varepsilon) = \mu$. The supply schedule in period t after input decisions have been made is a horizontal line, equal to P_{t-1} , and extending from $S_{|\varepsilon_m}$ to $S_{|\varepsilon_x}$; for example, line QR in Figure 1 . The actual supply is of course only given once ε_t is observed, but the choice of a horizontal line depends on input decisions.

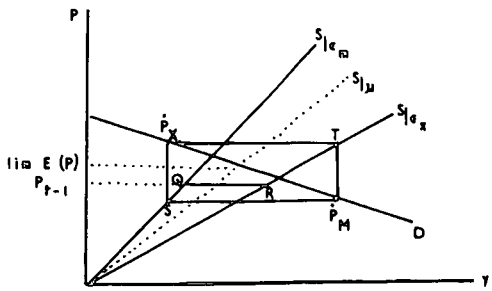


Figure 1.

The meaning of the equilibrium price distribution is that the market will converge to a state, regardless of the starting point, in which actual price each year lies in the range P_M to P_X, for all possible outcomes of ε. Geometrically, such an equilibrium exists precisely when a rectangle can be drawn in the positive P,y quadrant which has two opposite corners lying on the demand curve, and one of each remaining corners lying on the $S|_{\varepsilon_m}$ and $S|_{\varepsilon_x}$ curves; for example, rectangle P_X T P_M S in Figure 1. Once the market finds its way into this rectangle, the dynamics of the model postulated do not enable it to get out again in the absence of an external shock; for example, a shift in the demand schedule.

So far we have assumed a very naïve producer learning model in which $H(P) = P_{t-1}$. Now because this leads to stochastic input decisions, even at market equilibrium, the price anticipation each year represents a single observation on a conditional market clearing equilibrium given the previous year's input decisions of producers. In the absence of more complete information about the price distribution associated with fixed input decisions, producers therefore readjust their input decisions each year according to price signals which arise in part from a random observation on ε. This is inefficient as we shall subsequently show, and more rational models of producer learning should cause them to react only to price signals arising from more fundamental disequilibrium of previous year's inputs. There are of course a wide range of models of possible producer learning procedures, and which vary in the degree to which they are efficient in the above sense. However, one particular model, which we shall call the "conditional expectation" model, turns out to be the limiting case, and all other formulations lie between the naïve cobweb and this model in their degree of efficiency. We propose now to discuss the conditional expectation model, and then a fairly general class of intermediate models, which includes a Nerlove type adaptive expectation model.

The "Conditional Expectation" Model

Suppose that producers, or more likely, some forecasting agency, were to provide price anticipations each year based on fuller information about price, given the input decisions of the previous year. More specifically, let us assume that the conditional expectation of price given the inputs decisions of the previous year is calculated, and that producers adopt this as their anticipation of expected price in the current year. The market model is then:

$$S_t = \lambda \varepsilon_t E(P_{t-1})$$

$$D_t = a - b P_t$$

and market clearing price in period t is:

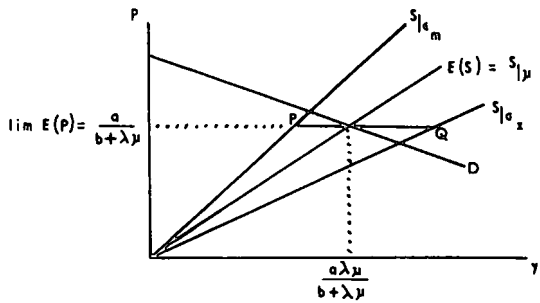
$$P_t = \frac{a}{b} - \frac{\lambda}{b} \varepsilon_t E(P_{t-1})$$

It is easy to show that expected price $E(P)$ converges to an equilibrium

$$\lim_{t\to\infty} E(P) = \frac{a}{b+\lambda\mu}$$

providing the necessary and sufficient condition $\frac{\lambda}{b} < \frac{1}{\mu}$ is satisfied. This is exactly the result obtained for the naive cobweb model, and price in equilibrium is still stochastic, and has a limiting probability distribution. However, there is an important difference. Input decisions in equilibrium are now deterministic, because the anticipated price, $E(P)$, becomes a constant over time. Geometrically, when ε is bounded, we have that producers operate on their expected supply curves, and in equilibrium fix inputs so that output lies on the horizontal line between the $S|_{\varepsilon_m}$ and $S|_{\varepsilon_x}$ curves which passes through the intersection of demand and expected supply; line PQ in Figure 2 .

Figure 2.



The limiting variance of price in equilibrium is:

$$\lim_{t\to\infty} \mathrm{Var}\,(P) = \frac{a^2 \lambda^2 (\mu_2 - \mu^2)}{b^2 (b + \lambda\mu)^2} \qquad (16)$$

and this is smaller than the limiting variance of price for the naive cobweb model (15) because $0 < b^2 - \lambda^2\mu_2 < b^2$.

The nature of the equilibrium is that producers no longer react to price signals arising from random observations on yields ε , but only readjust their input decisions on the basis of calculated price signals arising from disequilibria of inputs.

Weighted Cobweb Models

While it is quite possible for some agency to provide farmers

with conditional price expectations, this is not a very descriptive model of the real world. A class of more descriptive models can be defined by setting

$$H(P) = \sum_{i=1}^{m} \alpha_{t-i} P_{t-i}$$

where $\sum_{i=1}^{m} \alpha_{t-i} = 1$. That is, producers take some weighted average of past prices as an anticipation of expected price in year t . The Nerlove type adaptive expectation model.

$H(P) = P_{t-1} + \alpha (P_{t-1} - P_{t-2})$ is an interesting special case of this class of models.

The appropriate market model is now:

$$S_t = \lambda \epsilon_t \sum_{i=1}^{m} \alpha_{t-i} P_{t-i}$$

$$D_t = a - b P_t$$

and market clearing price is

$$P_t = \frac{a}{b} - \frac{\lambda}{b} \epsilon_t \sum_{i=1}^{m} \alpha_{t-i} P_{t-i}$$

It can be shown that the limiting expectation and variance of price in equilibrium are:

$$\lim_{t\to\infty} E(P_t) = \frac{a}{b+\lambda\mu}$$

$$\lim_{t\to\infty} \mathrm{Var}(P_t) = \frac{\lambda^2}{b^2} \{ \mu_2 \lim_{t\to\infty} E \left[\left(\sum_{i=1}^{m} \alpha_{1-i} P_{t-i} \right)^2 \right] - \frac{a^2 \mu^2}{(b+\lambda\mu)^2} \} \qquad (17)$$

The expectation of price for the weighted cobweb therefore converges to the same value as that for the conditional expectation model. However, since the limiting variance of the conditional expectation model can be written as:

$$\frac{\lambda^2}{b^2} \left[\mu_2 \lim_{t\to\infty} [E(P_t)]^2 - \frac{a^2 \mu^2}{(b+\lambda\mu)^2} \right] , \qquad (18)$$

comparison of equations (17) and (18) shows that the limiting variances coincide only when

$$\lim E \left| \left(\sum_i \alpha_{t-i} P_{t-i} \right)^2 \right| = \lim [E(P_t)]^2$$

It can be shown that, providing $\alpha_{t-i} \geq 0$ for all i , then

$$\lim E\left[\left(\sum_i \alpha_{t-i} P_{t-i}\right)^2\right] \geq \lim \left[E(P_{t-1})\right]^2 ,$$

so that in general, the limiting variance of price for the weighted cobweb is at least as great as for the conditional expectation model. In other words, price is more variable at the equilibrium. This reflects the fact that input decisions do not necessarily converge to a unique level in a weighted cobweb model, hence a weighted average of past prices, with one observation for each year, necessarily leads to some readjustment of inputs decisions following price signals which reflect variations in yields rather than disequilibrium on inputs. It follows that the weighted cobweb model will be more efficient the closer

$$\lim E\left[\left(\sum_{i=1}^{m} \alpha_{t-i} P_{t-i}\right)^2\right] \text{ is to } \lim \left[E(P_{t-1})\right]^2$$

This depends entirely on the estimating properties of $\sum_{i=1}^{m} \alpha_{t-i} P_{t-i}$ as an estimator of $E(P_{t-1})$, which in turn depends upon the choice of α weights and the number of years taken in the price calculation.

Finally, we compare the weighted cobweb results with the naïve cobweb. The limiting expectations of price are the same, but the limiting price variances differ. The price variance equation (15) for the naive cobweb can be written as:

$$\lim \operatorname{Var}(P_t) = \frac{\lambda^2}{b^2}\left[\mu_2 \lim E(P_{t-1}^2) - \frac{a^2\mu^2}{(b+\lambda\mu)^2}\right]$$

Comparison with equation (17) shows that the weighted cobweb has the smaller limiting price variance if

$$\lim E\left[\left(\sum_{i=1}^{m} \alpha_{t-i} P_{t-i}\right)^2\right] < \lim E(P_{t-1}^2)$$

This is true whenever $\alpha_{t-i} \geq 0$ for all i, so that the naïve cobweb provides the limiting case of the most inefficient model in terms of limiting variance of price.

Some Welfare Aspects of Market Equilibrium

The results for the wide range of models considered indicate that in equilibrium, price will be stochastic with a limiting expectation of $\frac{a}{b+\lambda\mu}$ and a limiting variance which reflects the stability of demand for inputs. If we use the sum of consumer and producer surpluses (net social product) in equilibrium as an approximate measure of social welfare, then the welfare characteristics of the equilibria can be explored. We note that the producer surplus so measured will no longer be surplus profits, but will be surplus utility when decisions are made with respect to some risk utility function. A similar proviso is necessary for consumer surplus when consumers are risk averse.

An immediate consequence of a stochastic equilibrium price is that net social product is also stochastic. Social criteria of market equilibria must therefore involve assumptions about aggregate behaviour towards risk. In as much as aggregate wishes are reflected through Government or other policy institutions, then this means assumptions about the risk criterion to be used by such decision making bodies. For the purpose of this paper we assume risk neutrality at the aggregate level, so that the expected value of net social product can be taken as a measure of social gains. This assumption is usually made in price stabilization studies [6, 10, 13].

On this basis we have derived the expected net social product for the naïve cobweb and conditional expectation models at equilibrium. These results are;

a) for the naïve cobweb model:

$$E(NSP) = \frac{\frac{1}{2} a^2 \lambda\mu \left(1 + \frac{1}{b}\right)}{b + \lambda\mu} - \frac{\frac{1}{2} a^2 \lambda \left(\lambda\mu_2 + b\mu\right) \left(b - \lambda\mu\right)}{b \left(b + \lambda\mu\right) \left(b^2 - \lambda^2\mu_2\right)}$$

b) for the conditional expectation model:

$$E(NSP) = \frac{\frac{1}{2} a^2 \lambda\mu \left(1 + \frac{1}{b}\right)}{b + \lambda\mu} - \frac{\frac{1}{2} a^2 \lambda \left(\lambda \mu_2 + b\mu\right)}{b \left(b + \lambda\mu\right)^2}$$

Since the required condition for convergence of the variance of price is $\frac{\lambda^2}{b^2} < \frac{1}{\mu_2}$, and since this implies $b^2 > \lambda^2\mu^2$ and $b^2 > \lambda^2\mu_2$, it is easy to show that the conditional expectation model provides tha largest social welfare. Thus if farmers were induced to base price expectations on calculated expectations of previous years price, this would lead to the equilibrium with smallest price variance and largest social gain.

We have also explored the expected consumer and producer surpluses separately for the two models, and found the very interesting result that consumer welfare is actually greater with a naïve cobweb than a conditional expectation model, but the reverse holds for producers. This suggests an interesting source of conflict of interests between the agricultural and non-agricultural sectors in a cash economy, or between landlord and tenants in a subsistence economy.

Conclusions

One purpose of this paper has been to show that a number of important results obtained from deterministic models of peasant economies

do not necessarily hold under risk. In particular, it has been shown that the peasant farmer can be quite rational in using less inputs and a smaller scale size of farm than classical marginal analysis would suggest, and that the slope of his product supply and factor demand curves may even be perverse in sign. At the market level it has been shown that the nature of an equilibrium in terms of price variability, input demands and social welfare depends largely on the way in which farmers form price expectations over time.

These, and other results, suggest interesting policy considerations which cannot be explored here. However, we consider this an important area for further investigation, and one which has been rather neglected in the literature considering the importance of risk to vast numbers of peasant farmers.

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