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Family farms are the dominant type of agricultural organization throughout the world. Though they vary in the degree of commercialization with respect to their product both from one place to another and from one period of time to another, they all share one thing in common, i.e., the feature that they have the major source of factor supply within themselves. Thus they possess both the features of the commodity-supplying and factor-employing firms and of the commodity-consuming and factor-supplying households, and are often called the "firm-household complexes" Tang [8].

At the early phases of economic development, the accumulation of capital is meagre and the opportunity for wage employment is limited. Hence the members of farm family have little if any choice other than supplying their labor to their own family farm. The resultant production can be regarded as wages in kind, a part of which is directly consumed and the rest is exchanged for other commodities or factors of production. Thus the employment of family labor therefore the farm production is so organized that the welfare of the farm family is maximized. Here, its similarity to the wage-earner's household is dominating.

As the economy develops, both the accumulation of capital and the opportunities for wage employment expand, therefore labor becomes relatively scarce. Eventually, the economy reaches the "turning point". See, e.g., Fei and Ranis [2]. Now the members of the farm family can "sell" at the market rate of wage as much labor as they want to. In other words, the opportunity cost of their labor has turned to be equal to the market rate of wage. Therefore, even the family farm is expected to pay to the members of the family a reward equal to the market rate of wage. Thus the
farm production is so organized that the residual profit (the value of product net of the value of factor inputs) is maximized. Here its similarity to the capitalist firm has come to the fore.

As the recent developments in the Japanese agriculture indicate, such a drastic transformation of family farms' behavior may have as important an impact on the production and distribution in agriculture/consequently in the economy as a whole as the "agricultural revolution" had prior to the "industrial revolution". Fei and Ranis remark a similar transformation at the turning point in their celebrated Development the of the Labor Surplus Economy [2]. They divide/farm population into two classes - landlords and cultivators and argue that landlords will start to assert their capitalist-entrepreneur capacity after the turning point has been reached. This paper concentrates upon the analysis of family farms' behavior and attempts to establish in a rigorous manner a stronger thesis, i.e., that even cultivators, whether owner-cultivators or tenant-cultivators, will start to claim the same capitalist-entrepreneur capacity after the turning point has been reached.

The following section lays out the basic assumptions made in regard to the family farms. Section Three investigates the behavior of these farms in the economy where both labor and capital service markets are limitational. Section Four attempts a similar investigation in the economy where the labor market is competitive while the capital service market is not. Several remarks on the implications of the above transformation conclude the paper.

II. THE STRUCTURE OF FAMILY FARM

(a) Preference Ordering

1) The family welfare $U$ is a continuous, twice differentiable, monotonically increasing function of the economic leisure $l$ and the quantity of a commodity $q$ consumed by the family.
ii) It is strictly quasi-concave, so that the indifference curves are/convex to the origin. See, e.g., Arrow and Enthoven (1).

iii) It possesses non-increasing marginal utilities.

iv) Both the economic leisure and the commodity are indispensable.

\[ (1.1) \quad U = U(l, q) ; \quad l, q \geq 0, \]
\[ (1.2) \quad U_1 > 0, \quad U_2 > 0, \]
\[ (1.3) \quad U_1 \rightarrow \infty \text{ (as } l \rightarrow 0), \quad U_2 \rightarrow \infty \text{ (as } q \rightarrow 0), \]
\[ (1.4) \quad U_{11} \leq 0, \quad U_{22} \leq 0. \]

(b) Technological Possibilities

i) The farm production \( Q \) is a continuous, twice differentiable, non-decreasing function of labor \( L \) and capital (including land) \( K \).

ii) It is strictly quasi-concave and concave, so that the isoquants are strictly convex to the origin.

iii) It possesses strictly diminishing marginal productivities.

iv) Both labor and capital are indispensable.

\[ (2.1) \quad Q = Q(L, K) \geq 0; \quad L, K \geq 0, \]
\[ (2.2) \quad Q(0, K) = Q(L, 0) = 0, \]
\[ (2.3) \quad Q_1 \geq 0, \quad Q_2 \geq 0, \]
\[ (2.4) \quad Q_{11} < 0, \quad Q_{22} < 0. \]

(c) Resource Endowments: \( T, K \)

\( T \) hours of time and a fixed stock of capital yielding \( \bar{K} \) units of capital service per unit of time are available to this family. Therefore,

\[ (3.1) \quad 1 + L + L_s \leq T, \quad T > 0, \]
\[ (3.2) \quad 1 \leq T, \quad T > 0, \]
\[ (3.3) \quad K + K_s \leq \bar{K}, \quad \bar{K} \geq 0, \]
where $L_S$ denotes the hours of labor supplied to other firms if positive (employed from other households if negative), and where $K_S$ denotes the units of capital service supplied to other firms if positive (employed from other households if negative).

(d) Exchange Opportunities

All commodity and factor markets are purely competitive, unless otherwise specified.

i) price of the commodity : $P = 1$ (unity)

ii) price of the farm product : $P > 0$

iii) wage rate : $r_1 > 0$

iv) rental rate of capital : $r_2 > 0$

(e) Additional Specifications

i) Stationarity: the number of family members, the capital stock, and all prices remain constant.

ii) Coincidence of payments, so that no interest payments are necessary.

iii) Perfect foresight.

(f) Budget Constraints

Under these specifications, the family expenditure on the consumable commodity is constrained by the following relations.

(4.1) $pQ(L,K) + r_1(T - L - 1) + r_2(K - K) - q \geq 0$, if the farm product is not consumable.

(4.2) $pQ(L,K) + r_1(T - L - 1) + r_2(K - K) - pq \geq 0$, if it is consumable, reflecting the assumption that the farm product and the consumable commodity are of the same quality.
III. FARMERS' SUPPLY RESPONSE IN THE LABOR-SURPLUS ECONOMY

At the early phases of economic development, the accumulation of capital is meagre and the wage employment by the capitalist firms is severely limited. To be specific, it is assumed that the wage employment available to this family is only $E$ hours, and that the procurement of capital service from other households is possible only up to $G$ units. Thus,

\[(5) \quad L_s = T - L - 1 \leq E, \quad E \geq 0,\]

\[(6) \quad -K_s = K - \bar{K} \leq G, \quad G \geq 0.\]

Furthermore, it is assumed that the product of this farm is not consumable, partly to disprove the common association of the production for market with the capitalist-entrepreneur characteristics of the family farm. Hence, the budget constraint (4.1) applies to this family. The problem facing this family is that of maximizing the family welfare (1.1) subject to the wage employment opportunity (5), the capital procurement possibility (6) and the budget constraint (4.1). The time constraint (3.2) is assumed not to be binding and is omitted to avoid trivial solutions. This is one of the quasi-concave programming problems Arrow and Enthoven (1), and is assured of an optimal solution by the assumptions (1.1)-(2.4).

(a) The Kuhn-Tucker-Lagrange conditions (or K-T-L conditions for short) associated with the optimal solutions are as follows.

\[(7.1) \quad \lambda^0 \left[ \lambda^0 P^0_1 (L,K) - \lambda^0 r_1 + \lambda^0 \right] = 0, \quad \lambda^0 P^0_1 (L,K) - \lambda^0 r_1 + \lambda^0 \leq 0,\]

\[(7.2) \quad K^0 \left[ \lambda^0 P^0_2 (L,K) - \lambda^0 r_2 - \lambda^0 \right] = 0, \quad \lambda^0 P^0_2 (L,K) - \lambda^0 r_2 - \lambda^0 \leq 0,\]

\[(7.3) \quad \lambda^0 \left[ u_1^0 (1,q) - \lambda^0 r_1 + \lambda^0 \right] = 0, \quad u_1^0 (1,q) - \lambda^0 r_1 + \lambda^0 \leq 0,\]

\[(7.4) \quad q^0 \left[ u_2^0 (1,q) - \lambda^0 \right] = 0, \quad u_2^0 (1,q) - \lambda^0 \leq 0,\]

\[(7.5) \quad \lambda^0 \left[ p^0 (L,K) + r_1 (T - L^0 - 1) + r_2 (\bar{K} - K^0) - q^0 \right] = 0, \quad p^0 (L,K) + r_1 (T - L^0 - 1) + r_2 (\bar{K} - K^0) - q^0 \leq 0,\]
\[ \lambda_2^0 [E + L^o + l^o - T] = 0, \quad E + L^o + l^o - T \geq 0. \]
\[ \lambda_3^0 [G + K - K^o] = 0, \quad G + K - K^o \geq 0, \]
\[ \lambda_1, \lambda_2, \lambda_3 \geq 0: \text{ Lagrange multipliers.} \]

For the moment, let us concentrate our analysis on the interior solutions where all relevant variables including the Lagrange multipliers take on positive values. Then the conditions (7.1), (7.3) and (7.4) imply that
\[ R = \frac{u_1}{u_2} = r_1 - \frac{\lambda_2^0}{\lambda_1^0} = P_{Q_1}^0 > 0, \quad \frac{u_3}{u_4} = P_{Q_1}^0 < r_1, \]
i.e., that at equilibrium the marginal rate of substitution of leisure for commodity or the demand price of leisure (alternatively the supply price of labor) in terms of the commodity (or income in kind) in the farm household is equal to the marginal productivity of labor (or the demand price of labor) in terms of the commodity on the family firm, which, in turn, is smaller than the market wage rate by \( \lambda_2^0/\lambda_1^0 \).

Furthermore, the marginal productivity of labor proves to be positive, since satiation is impossible both with respect to leisure and commodity. See (1.2) above.

The second order condition for the interior solutions is that the sign of the following bordered Hessian determinant \( A(1) \) is positive.
\[
A(1) = \begin{bmatrix}
\lambda_1 P_{Q_1} & \lambda_1 P_{Q_2} & 0 & 0 & P_{Q_1} - r_1 & 1 & 0 \\
\lambda_1 P_{Q_2} & \lambda_1 P_{Q_1} & 0 & 0 & P_{Q_2} - r_2 & 0 & -1 \\
0 & 0 & u_{11} & u_{12} & -r_1 & 1 & 0 \\
0 & 0 & u_{21} & u_{22} & -1 & 0 & 0 \\
P_{Q_1} - r_1 & P_{Q_2} - r_2 & -r_1 & -1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} > 0
\]
(b) **Price Responses**, or the effects of price changes on the equilibrium values of the relevant variables are estimated by the usual comparative statics method of taking the total differentials of the K-T-L conditions (7.1) - (7.7) associated with the interior solutions and solving them for selected variables in terms of the product price \( p \) while suppressing the changes in other parameters. For instance,

\[
\frac{\partial L}{\partial p} = -\frac{1}{A(1)} \left[ \lambda_1 Q_1 A_{11}^{(1)} + \lambda_1 Q_2 A_{21}^{(1)} + Q A_{51}^{(1)} \right].
\]

The first and second terms on the right-hand side represent the direct and indirect output effects Ferguson [3], respectively. The third term represents the income effect on the labor employment of the family firm. That is peculiar to the family farm but is nonexistent in the case of ordinary capitalist firms. The indirect output effect on the labor employment via the change in the capital employment vanishes since the constraint on the procurement of capital is assumed to be effective.

After some calculation, this expression is simplified as follows.

\[
(10.1') \quad \frac{\partial L}{\partial p} = \frac{1}{A(1)} U_1 \frac{Q}{q} \left[ \frac{q}{pq} - \varepsilon_q(R) \right],
\]

\[
\varepsilon_q(R) = \frac{\partial \log(R)}{\partial \log(q)} > 0.
\]

\[
(10.2) \quad \frac{\partial K}{\partial p} = 0, \quad K^o = \bar{K} + G.
\]

Hence,

\[
(11) \quad \frac{\partial L}{\partial p} \geq 0, \quad \frac{\partial Q}{\partial p} = Q_1 \frac{\partial L}{\partial p} \geq 0 \quad \text{if} \quad \varepsilon_q(R) \geq \frac{q}{pq}.
\]

The expression \( \varepsilon_q(R) \) is defined as the elasticity of the supply price of labor with respect to income and has been shown elsewhere [6] by the present author to be equal to the reciprocal of the elasticity of the demand for income in terms of effort for the wage-earner's household due to Robbins [7], which in turn is defined as follows.
(12.1) \( \eta_{qp} = -\frac{d \log(q)/d \log(p)}{q} \geq 1, \quad p_q = \frac{1}{r_1} \),

(12.2) \( \frac{\partial L}{\partial r_1} \leq 0 \) if \( \eta_{qp} \geq 1 \), \( L = q \cdot p_q \),

(12.3) \( \eta_{qp} = 1/\epsilon q (R) \).

The relations (11), (12.2), (12.3) and the fact that \( q = pQ \) if \( r_1 (T - L - 1) + r_2 (K - K) = 0 \) indicate the similarity between the labor supply by the wage-earner's household and the product supply by the family farm in the labor-surplus economy.

IV. FARMERS' SUPPLY RESPONSE IN THE LABOR-SCARCE ECONOMY

(a) Kuhn-Tucker-Lagrange Conditions for Interior Solutions

As the economy develops, both the capital accumulation and the wage employment opportunities by the capitalist firms expand. To be specific, let it be assumed that the wage employment opportunity \( E \) open to this family expands, while the capital procurement possibility \( G \) remains invariant. Eventually, the wage employment constraint (5) will become ineffective, consequently, the associated Lagrange multiplier \( \lambda_2 \) will vanish. Therefore, the K-T-L conditions for interior solutions will be rewritten accordingly.

(13.1) \( pQ^0_1 (L,K) - r_1 = 0 \)

(13.2) \( \lambda^0_1 pQ^0_2 (L,K) - \lambda^0_1 r_2 - \lambda^0_3 = 0 \)

(13.3) \( u_1^0 (l,q) - \lambda^0_1 r_1 = 0 \)

(13.4) \( u_2^0 (l,q) - \lambda^0_1 p = 0 \)

(13.5) \( pQ^0 (L,K) + r_1 (T - L - 1) + r_2 (K - K) - pq^0 = 0 \)

(13.7) \( K + G - K^0 = 0 \)

Here, it is assumed that the farm product is consumable, partly to disprove the common association of the production for family consumption with the consumer characteristics of the family farm. The conditions (13.1), (13.3) and (13.4) imply that
(14) \[ R^o = \frac{U_1^o}{U_2^o} = Q_1^o = \frac{r_1}{p} > 0, \]

i.e., that both the supply price and the marginal productivity of labor turn out to be equal to the market wage rate.

Furthermore, it is noted that the K-T-L conditions (13.1) - (13.7) are quasi-decomposable in the sense that the equilibrium rates of factor employment \( L^0 \) and \( K^0 \) are independent of the equilibrium rates of consumption \( l^0 \) and \( q^0 \). The equilibrium rate of capital employment \( K^0 \) is determined by the condition (13.7) and is substituted into the production function elsewhere in the K-T-L conditions. Then, the equilibrium rate of labor employment \( L^0 \) is determined by the condition (13.1). Since the condition (13.2) is rewritten as

(13.2') \[ p^0_2 (L,K) - r_2 - \mu = 0, \mu = \frac{\lambda_1^0}{\lambda_1^0} \text{ or } p^0_2 (L,K) - r_2 > 0, \]

it should be clear that the conditions (13.1), (13.2) and (13.7) coincide with the K-T-L conditions associated with the interior solutions to the problem of maximizing the profit to the firm sector \( \pi = pQ (L,K) - r_1L - r_2K \) subject to the capital procurement constraint (6).

The resultant profit is passed to the household sector. Then the equilibrium rates of consumption \( l^0 \) and \( q^0 \), and the equilibrium value of the Lagrange multiplier \( \lambda_1^o \) are determined by the conditions (13.3), (13.4) and the budget constraint (13.5), which can be conveniently rewritten as

(13.5') \[ \pi^o + r_1 (T - l^o) + r_2 K - pq^o = 0. \]

Finally, the equilibrium value of the Lagrange multiplier \( \lambda_2^o \) are determined to satisfy the condition (13.2), which is always possible, since it is free to change independently of the value of other variables, or alternatively since any internal imputation can be given to the limited capital. Thus, the profit maximization on the part of the firm sector has proved to be consistent with the
welfare maximization for the firm-household complex as a whole. For a similar notion of consistency, see, e.g., Jorgenson and Lau [41].

(b) **Price Responses** are estimated in the same way as in the previous section to endorse the capitalist-entrepreneur characteristics of the family farm in the labor-scarce economy.

\[
\frac{\partial L}{\partial p} = \frac{1}{A^{(2)}} \left[ -Q_1A_{11}^{(2)} - \lambda_1Q_2A_{21}^{(2)} + \lambda_1A_{41}^{(2)} - (Q - q) A_{51}^{(2)} \right],
\]

where \( A^{(2)} \) represents the bordered Hessian determinant associated with the K-T-L conditions (13.1) - (13.7), and which may also be formed by deleting from the determinant \( A^{(1)} \) the row and column corresponding to the Lagrange multiplier \( \lambda_2 \) and by making such mutatis mutandis changes as are made necessary by the fact that the farm product is now assumed to be consumable. The first and second terms on the right-hand side represent the direct and indirect output effects, respectively. The third term represents the commodity-factor (cross) substitution effect which is peculiar to the family farm, and which comes into existence because the farm product is now made to be consumable. The fourth term represents the income effect. The indirect output effect vanishes because the capital procurement constraint is effective. The commodity-factor (cross) substitution and the income effects vanish.
because the K-T-L conditions are quasi-decomposable. After some calculation, this expression is simplified as follows.

\[(15.1') \frac{\partial L}{\partial p} = -\frac{1}{pQ_{11}} Q_1 > 0.\]

\[(15.2) \frac{\partial K}{\partial p} = 0, \quad K^* = K + G.\]

Hence, \[\frac{\partial Q}{\partial p} = Q_1 \frac{\partial L}{\partial p} > 0,\]

i.e., the supply response is positive as expected of the capitalist firms.

(c) Supplementary Remarks

It should be noted that the consistency between the profit maximization and the welfare maximization has been obtained even though the procurement of capital is still limited and that the major part of farm product is consumed by the farm family itself. In the economy where both the wage employment and the capital service are readily available, it has been shown elsewhere \[5\] by the present author that the associated K-T-L conditions become fully decomposable and the family farm reinforces its capitalist-entrepreneur characteristics. On the other hand, where the wage employment is limited while the capital service is readily available, it has also been shown by the present author \[5\] that the associated K-T-L conditions restore nondecomposability and the family farm remains in its worker-consumer status. Thus, the importance of the labor market as compared with the capital service market should be obvious in determining the behavior of the family farm.

The preceding discussions have an implication that is of interest in the development context. The dominance of the worker-consumer characteristics of a family farm in the labor-surplus economy, in which many LDC's are currently found, leads to an inelastic or even inverse response to a product price rise, technological progress and other economic incentives. In fact, if the family welfare function is of the Cobb-Douglas type, it has been verified by the present author \[6\] that it will
lead to a completely inelastic supply curve. If it is of the CES type with the elasticity of substitution less than unity, it will lead to a backward-sloping supply curve. If it is a homogeneous polynomial of degree two, the associated supply curve will be an inverted sigmoid. These supply responses are often ascribed erroneously to the lack of rationality on the part of these farms. One purpose of this paper is to reveal the hidden rationality on their part and to defend their inelastic or backward-sloping supply curves.

This dominance has a further implication. It reduces the relevance of the "production economics" to the family farms in the LDC's, most of which supposedly remain in the labor-surplus stage. Here, the "production economics" is understood to represent the theory of firm as applied to agriculture. So far many technical advisers have been sent from the DC's to the LDC's and they have "produced" much advice based on the production economics. Unfortunately, such advice appears not to have been accepted very willingly. This is not without good reasons. Their advices instead should have been based on the "theory of firm-household complex". If they force their advice based on the production economics, they are forcing their own value judgement or welfare function to the farmers in the LDC's. If they instead simply present to them the relevant income-leisure efficiency frontier just as the portfolio analysts present to their clientele, possible investors, the E-V efficiency frontier and let them choose a program represented by a point on it that is most desirable in the light of their welfare function, they would have been by far more popular.

V. CONCLUDING REMARKS

In concluding this paper, the following remarks may be in order.

i) The capitalist-worker dichotomy is not necessarily required for the modernization of the traditional agricultural economy which supposedly
requires the maximization of profit and welfare.¹)

ii) The competitiveness of the labor market is more important than that of the capital service market in transforming the behavior of farm families.²)

iii) The common association of the production for market with the capitalist-entrepreneur characteristics or alternatively of the production for family consumption with the consumer characteristics of the family farms can be fallacious and needs further justifications.

iv) This paper concentrates on the analysis of family farms which will continue farming after the wage employment has become abundant. However, there may be a greater number of farm families which will quit farming as the wage employment opportunities increase. For these families, the K-T-L condition (13.1) must hold in inequality. Furthermore, there is a third group of farm families, notably those with aged members or members lacking adequate education or training, for which the wage employment is not readily available and which will remain in their worker-consumer status after the labor market in general has become competitive.

v) Finally, the results of this paper apply, mutatis mutandis, to other types of firm-household complex, notably many "family firms" in service trades, as well.
REFERENCES


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(1.2) Fei and Ranis [2] are not clear on these points.