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Firm Theory Incorporating Growth and Risk: Integration Into Farm Management Research

by

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Recent developments in the theory of the firm, specifically those involving growth and risk, have not yet been fully integrated into research in farm management and agricultural production economics. The classical analysis of Irving Fisher is introduced to show the interrelationship of production and consumption decisions over time, from which it is deduced that adequate theoretical and empirical analysis in farm management requires explicit treatment of (1) the entrepreneur's time preference for consumption, and (2) his risk preferences. Thus, the paper emphasizes integration of recent development in utility theory into empirical applications; illustrations are drawn from recently published and unpublished work.

Conceptual Framework

The traditional static theory of the firm with its concepts of "short run" and "long run" provides an unsatisfactory framework for analyzing the most important decision problems of the farm firm--e.g. capital investment or "capital budgeting" decisions--which are taken within a dynamic context. Moreover, the usual investment criteria such as to "maximize the present value" may have limited relevance within the decision context of the farm firm. Hirschleifer (in Solomon, 1959, p. 205-228), for example, shows that the "correct" investment decision can be attained only by recognizing that consumption is the ultimate objective of investment.

Hrschleifer's argument, drawing on Irving Fisher's (1930) classic analysis, is briefly sketched for the two-period case in Figure 1. In Figure 1, the horizontal axis K_0 represents income in period zero; the vertical axis K_1 represents income in period one. Assume an investor at point Q. The line QQ' represents the investor's "market opportunity line;" i.e., a line representing all combinations of current and future income possible by lending current income at the interest

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rate i (slope of QQ' = 1 + i). The curve QSR'T represents the "production opportunities curve" available to the investor starting at Q; i.e., the locus of points attainable as he sacrifices more current income K_0 by productive investment yielding income K_1 in period 1. It is concave to the origin, indicating diminishing returns to investment.

The investor's objective is to reach his highest indifference curve. Moving along the production opportunities curve QSR'T, the highest indifference curve U_1 is attained at point S. But he could reach still a higher indifference curve by moving along QSR'T to the point R', then moving in the reverse direction (borrowing) along PP' to arrive at point R on the indifference curve U_2. Line PP' is a "market opportunity line" parallel to QQ'. The solution, therefore, proceeds in two steps. The investor reaches the "productive" solution at R', then moves among the "market line" PP' to a point R, satisfying his time preference for income consumption. That is, he makes the best investment from a productive point of view and then "finances" it in the loan market.1/

(FIGURE 1)

If the conditions underlying the above formulation were commonly met in practice, the farm management analyst's job would be comparatively simple, since the production and consumption decisions could be considered separately. The analyst would find the optimum multiperiod production plan given the market rate of interest, prices, and the multiperiod production function (e.g., by using a standard multiperiod linear programming formulation in which the objective function is defined as the maximum present value of income over an n-period planning horizon). The farmer would then (theoretically) take the stream of income from the "production solution" as a lifetime budget constraint and, by appropriate borrowing and lending at the market interest rate, convert it into the stream of income which would maximize his utility. That is, maximizing the present value of the future production income stream automatically guarantees that the entrepreneur has the opportunity of reaching his maximum utility.

Unfortunately, the above analysis rests on assumptions not ordinarily met in practice. The most critical are (1) there is no capital rationing, (2) the borrowing rate equals the lending rate, and (3) risk is unimportant. Baumol and Quandt (1965) show that an explicit utility function for time preference in consumption must be introduced in the case of capital rationing since the market interest rate is then no longer the relevant discount rate. Baumol and Quandt's formulation is as follows:

$$\max \sum_t U_t W_t$$

subject tp $-\sum_j a_{jt}x_j + W_t \leq M_t$ 2/

$$x_j \geq 0$$

$$W_t \geq 0$$

$$C_t \geq 0$$

where
W_t = withdrawal of income from the firm for consumption in year t,
U_t = (fixed) utility per dollar of income consumed in year t,
X_j = level of project j,
a_{jt} = net cost (negative value) or net income (positive value) of project X_j in year t, and
M_t = budget constraint in year t.

The ratio $\frac{U_t}{U_{t'}}$ for period t and t' specifies a subjective discount rate between the two periods. The duality of linear programming is employed to show that the optimum investment plan is achieved when the subjective discount rate $\frac{U_t}{U_{t'}}$ equals the ratio $\frac{P_t}{P_{t'}}$ of the shadow prices P_t associated with the budget constraint M_t in period t; i.e., $\frac{U_t}{U_{t'}} = \frac{P_t}{P_{t'}}$. 3/ Baumol and Quandt also show that this formulation is equivalent to maximizing the present value of the firm's net income where the utility values U_t serve as the discounting device. That is, in general, the first restriction in the above formulation will hold as an exact equality. Therefore,

$$W_t = M_t + \sum_j a_{jt}x_j$$

The objective function can then be rewritten:

$$\sum_t U_t W_t = \sum_t U_t (M_t + \sum_j a_{jt}x_j)$$

or

$$\underbrace{\sum_t U_t W_t}_{\text{Present value of withdrawals}} = \underbrace{\sum_t U_t M_t}_{\text{Constant}} + \underbrace{\sum_t \sum_j U_t a_{jt} x_j}_{\text{Present value of firm's net income}}$$

Hence, the present value of the firm's net income differs from the present value of withdrawals only by a constant, and using either as the objective function would give the same solution.

The Baumol and Quandt formulation is quite satisfactory _if_ the value of U_t can be specified. However, the authors offer no suggestions or empirical evidence on this point. Neither do they address the question of risk--obviously a central issue for decisions involving long planning horizons, such as capital investments and growth. The remainder of the paper, therefore, indicates some operational approaches to incorporating the principal concepts from the above discussion into empirical farm management and production economics analysis.

Operational Approaches

The above analysis suggests that the standard objective function of the firm (profit maximization in the short run, maximization of present value of income in the long run) needs to be replaced by a more general utility function which considers multiple goals and risk. Thus, the utility function is defined as $U = f(Z, Z_2, \ldots, Z_n)$, where the Z_i represents n goals, and utility is theoretically maximized by assigning specific weights to the various goals. Alternatively, tradeoffs among goals can be derived and the decision-maker allowed to make his choice directly (presumably by applying subjective weights). In practice, these procedures are unsatisfactory, particularly where more than two goals are involved. The assignment of weights is highly arbitrary and presentation of tradeoffs in more than two dimensions is confusing and unwieldy.

One special case of the multiple-goal framework which concentrates on risk, and for which a well-developed theory has evolved in recent years is von Neumann-Morgenstern (N-M) utility theory. In this case $U = f(Z_1, Z_2)$ are the expected value and variance of net income, respectively. Standard interviewing procedures (standard lottery questions) are used to derive the entrepreneur's utility function (e.g., see Halter and Dean, 1971, Chap. 3). If the individual's utility function is quadratic, for example, the expected utility E(U) of a farm plan with expected income $E(Z_1)$ and variance Z_2 is estimated by the following equation:

$E(U) = b\,E(Z_1) + c\,Z_2 + c\,(E(Z_1))^2$, where $b>0$ and $c<0$ are constants. Cast in an E-V (expectation-variance) framework the utility can be plotted as a series of indifference curves. Tangency of one of these curves with the E-V production frontier gives the cropping plan which maximizes utility. Figure 2 shows an example from California agriculture, where the E-V (or E-$\sqrt{V}$ in this case) production frontier is derived by quadratic programming. A quadratic utility function gives utility curves such as U_1, U_2, U_3, showing an optimum at cropping system a_9. This framework typically is applied to short-run decisions such as annual cropping patterns. However, the same approach also has been proposed in the area of capital budgeting by such authors as Linter (1965), Adelson (1965), and van Horne (1966). In this case, the net cash flow data for a sequence of years are discounted back to the present at a riskless or bank discount rate. Since the cash flows for a particular project depend on stochastic components (e.g., yields and prices), the present value figure will have a probability distribution with expected value E and variance V. The choice among projects then proceeds in the framework outlined above.

A more promising approach to incorporating the utility function in the multiperiod problem of production and consumption may be the approach suggested by Encarnación (1964) and elaborated further by Ferguson (1965). The typical entrepreneur (or corporate firm) is assumed to have multiple goals or objectives, and a multidimensional utility function is required to characterize the relationship among these objectives. Encarnación assumes that the entrepreneur has a lexicographic utility function which ranks a hierarchy of objectives where goal Z_1 is "more important than Z_2," Z_2 is "more important than Z_3," etc. Assume an entrepreneur with n goals (Z_1, Z_2, ..., Z_n). Consider two alternative solutions or "plans" Z^o and Z^1 which provide the following levels of attainment of each of the goals:

$$Z^o = (Z_1^o,\ Z_2^o,\ \ldots.,\ Z_n^o)$$

$$Z^1 = (Z_1^1,\ Z_2^1,\ \ldots.,\ Z_n^1)$$

Then by lexicographic ordering, $U(Z^o) > U(Z^1)$ if $Z_1^o > Z_1^1$, irrespective of the relationship between Z_i^o and Z_i^1 for $i > 1$. If $Z_1^o = Z_1^1$, then the choice between Z^o and Z^1 is based on the relative value of the second components Z_2^o vs. Z_2^1. If $Z_2^o = Z_2^1$, the choice is made by reference to the third component, etc. Now let Z^*_i be a "satisfactory" or "saturation point" for objective Z_i. That is, assume $\frac{\partial U}{\partial Z_i}\Big|_{Z_i > Z^*_i} = 0$, or, in words, that the marginal utility of over-achievement of goal Z_i is zero. Applying this argument to the two-goal case, let:

$$Z_1^o = (Z_1^o, Z_2^o)$$
$$Z^1 = (Z_1^1, Z_2^1)$$

If $Z_1^o \geq Z_1^*$ and $Z_1^1 \geq Z_1^*$, then the decision is made by reference to the maximum value of Z_2. That is, find the plan which maximizes the value of Z_2 subject to $Z_1 \geq Z_1^*$. Extending the argument to n goals, the objective is to maximize the least important goal, subject to "satisfactory" levels for all other goals. That is: max Z_n

subject to $Z_i \geq Z_i^*$ for i = 1,..., n-1.

If no feasible solution exists, drop goal Z_n and formulate a new problem:

max Z_{n-1}

subject to $Z_i \geq Z_1^*$ for i = 1,..., n-2.

Proceed in this way until a feasible solution is reached. The result will be consistent with the lexicographic utility function specified.

The logic of this procedure can be simply demonstrated by reference to Baumol's hypothesis of sales maximination subject to a minimum profit constraint. Here the lexicographic utility function is a two-component vector (Z_1, Z_2) where Z_1 is profit and Z_2 sales. Let Z_1^* be the minimum satisfactory profit. Then sales Z_2 are maximized subject to a minimum level of profit Z_1^*. A recent study (Lin, 1973) suggests that "satisfactory" levels of goals can be specified meaningfully by most decision makers.

A study of small land-reform farm development in Southern Italy (Dean and De Benedictis, 1964) used a lexicographic utility of the following type:

$$U = f\ (\underline{Z}_1, Z_2, Z_3),$$

where Z_1^* represents a 1 x n vector of minimum "satisfactory" levels of annual consumption withdrawals from the firm for the n-year horizon of the planning model, Z_2^* represents a firm survival goal represented in this case by a series of restrictions forcing diversification among high-risk crops, and Z_3 represents a growth goal, in this case represented by maximizing the present value of the income stream of the firm using a "riskless" discount rate of 8 percent. The problem was solved within the multiperiod linear programming framework where $\underline{Z}_1$ and Z_2 enter as restrictions and Z_3 is maximized. 4/

We might consider now the possibilities of combining the advantages of the E-V framework (including V-N utility) and lexicographic utility. The E-V framework by itself appears to be incomplete; extreme variability in and of itself may be considered undesirable, but an additional disavantage of high variability for a given expected income is that it increases the probability of "going broke." With this distinction in mind, consider a lexicographic utility function with four components:

$$U = (\underline{Z}_1, Z_2, Z_3, Z_4)$$

where $\underline{Z}_1^*$ = "satisfactory" consumption levels in each year;

Z_2^* = a "firm survival" goal specified as prob. (percentage equity in any year $\leq$ x percent) $\leq$ p;

Z_3 = expected net income

Z_4 = variance of net income

An E-V production frontier could then be derived showing the tradeoff goals Z_3 and Z_4 subject to the specified "satisfactory" levels of the consumption $(\underline{Z}_1)$ and firm survival (Z_2) goals. In practice, this would probably need to be done by simulation (for a relevant empiriral example see Eidman, et al., 1968)

Implications for Positive Analysis

The above discussion has concentrated on the use of utility theory to provide improved advice or recommendation to decision makers. However, there is some evidence that utility theory also explains and predicts farmer behavior more accurately than the traditional behavioral assumption of profit maximization. Officer and Halter (1968) found that fodder reserve decisions of Australian farmers were more accurately predicted by V-N utility (and modification thereof) than profit maximization (cost minimization). Lin (1973) found that both lexicographic and V-N utility explain decisions by large California farmers more accurately than profit maximization. If these preliminary results are borne out in wider testing, it suggests that the poor predictions of farmer supply response using LP methods might be improved substantially by substituting utility maximization for profit maximization in the objective functions. Likewise, large regional LP models based on profit maximization might provide more "realistic" results if the objective function were recast in utility terms.

Other Uses of Utility Theory

Implicitly the above analysis is based on the traditional individually-owned and operated farm unit in a capitalist society. While this model is relevant for an important segment of the world's agriculture, it ignores newly developing forms of ownership and management in the socialist countries (labor management, cooperatives, state farms, etc.). Still, elements of the above models are directly relevant for decision making in socialist settings. Experience with group utility functions of the V-N type is unfortunately extremely limited (e.g., see Officer <u>et al.</u>, 1967) and probably inpractical at this time. But the multiple-goal lexicographic framework would appear to be practical for group situations--in this case the ordering and "satisfactory" goal levels would be specified by the relevant decision unit, such as the government agency managing the farm or the elected peasant's management committee. To illustrate, the utility function specified may be to maximize income per worker (Z_3) subject to minimum number of workers employed (Z_1^*) and minimum production of an export crop (Z_2^*).

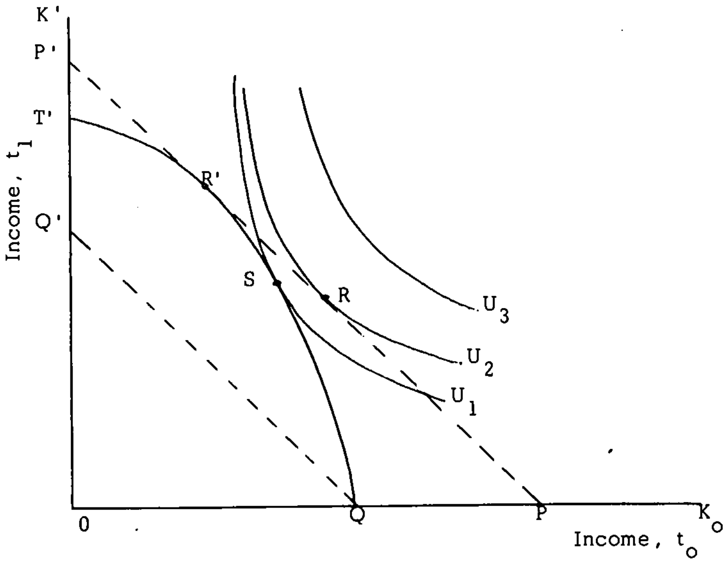

FIGURE 1: Optimum Investment Decision

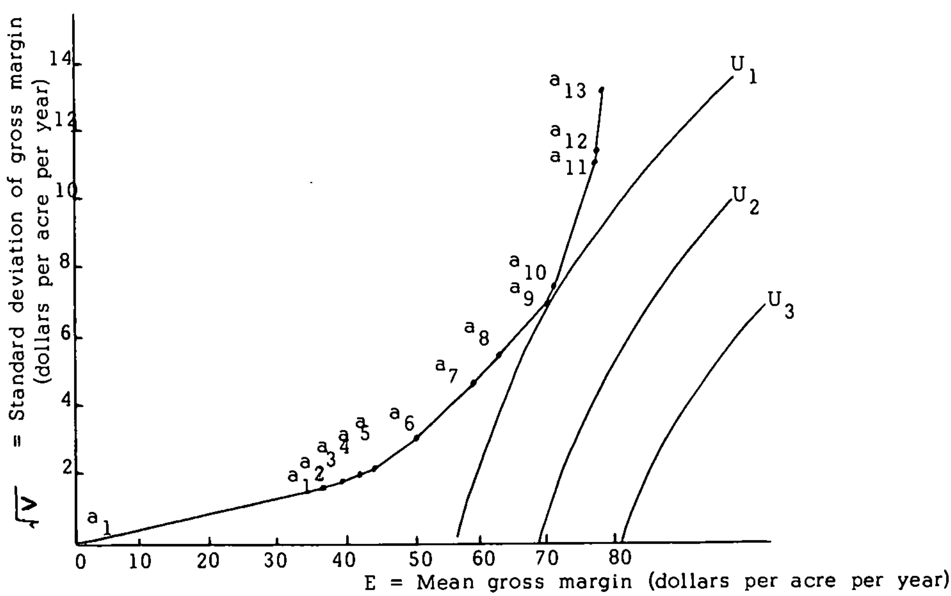

FIGURE 2: Optimum Cropping System for California Farm Using E-V Framework

FOOTNOTES

1/ The above two-period argument can be generalized by maximizing a multiperiod consumption function subject to a multiperiod production function. Lin (1973) has shown that the first-order conditions imply that the entrepreneur must equate, between every pair of periods (1) his marginal rate of time preference in consumption, (2) the compound market rate of interest between periods, and (3) the marginal internal rate of return on investment between periods.

2/ In another formulation, Baumol and Quandt include the carryover of funds C_t from period t to the subsequent period. Hence, this restriction becomes:

$$-\sum_j a_{jt}x_j + C_t - C_{t-1} \leq M_t$$

3/ This equality assumes all $W_t \geq 0$ in the optimum solution. If $W_t = 0$ but $W_{t'} > 0$, then $\frac{U_t}{U_{t'}} \leq \frac{P_t}{P_{t'}}$.

4/ The linear programming shadow prices show the sacrifice in goal Z_3 required to meet the "satisfactory" goal levels Z_1 and Z_2. If the sacrifice is "too high" the entrepreneur may decide to change the "satisfactory" levels. Optimization would then proceed iteratively until the entrepreneur is content with the solution.

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