ADVICE FROM ESTIMATED MARGINAL PRODUCTIVITIES

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In a recent major contribution to the statistical analysis of farm production, its author concludes that "management advice given to farmers which is based upon calculated 'Marginal Productivities' will only touch the fringe of management problems."[1]

This conclusion is based upon an analysis of components of variance of "efficiency", as indicated by gross return relative to total costs. In Denmark the "scale of farming" and "allocation of resources at a given scale of farming" accounted for only 11 per cent of the variation in "efficiency" whereas "management" (*i.e.*, the choice of products, technologies, and day to day supervision) accounted for 72 per cent of this variation.

It is the purpose of this note to suggest that even were these components of variance correctly identified,[2] this conclusion does not follow, at least in the short-run. By way of illustration, reference is made to an Australian study which indicates that, without deprecating the importance of "management" decisions, management advice based on marginal productivities, albeit taking the distribution of fixed assets into account when they are estimated, may make significant contributions to net farm incomes. The discussion is also relevant to the continuing controversy about the usefulness of farm standards comparisons.

This criticism is made on two fronts. The first is that the ratio of gross product to total costs is not an adequate concept for making comparisons of short-run production efficiency within or between farms. The second is that even if farms which maximize gross production relative to total cost were most efficient, a small between farm variation in this statistic which is attributable to the allocation of resources does not imply that a more rational allocation of resources will not result in significant increases in net farm profits.

The Appropriate Criteria of Efficiency

The theory of the competitive firm honours only one criterion for long run efficiency, namely, that returns to each factor of production be diminishing and equal at the margin to unit factor cost. If total expenditure available for production is limited, the criterion must be modified. Returns to factors of production must be diminishing and each equal at the margin to a common value not less than unit factor cost. Thus for a production function of the general form

$$Y = f(X_i) \quad i = 1, \ldots, m$$

where Y is the value of gross output and X_i are the values of the m factor

[1] Knud Rasmussen, "*Variance and Production Function Analyses of Farm Accounts*" (Basil Blackwell, Oxford, 1962), p. 133.

[2] See R. G. Mauldon, Review of Knud Rasmussen, *op. cit.* and Knud Rasmussen with M. M. Sandilands, 'Production Function Analyses of British and Irish Farm Accounts', *Aust. J. Agric. Econ.,* this issue.

inputs at constant unit factor cost, if working capital is not limited, economically efficient production occurs when

$$(1)\qquad \frac{\partial Y}{\partial X_i} = 1 \quad i = 1, \ldots, m$$
$$\frac{\partial X_i^2}{\partial^2 Y} < 0.$$

When working capital is limited to C, profit is maximized where

$$(2)\qquad \frac{\partial Y}{\partial X_i} = \lambda \quad i = 1, \ldots, m$$
$$\frac{\partial^2 Y}{\partial X_i^2} < 0.$$
$$\sum_{i=1}^{m} X_i = C.$$

Where $\lambda > 1$ is a constant determined by $f(X_i)$ and C. When working capital is limited to C, this solution does in fact maximize the ratio Y/C. That is, production for a given cost is most efficient (profit is maximized) when the ratio of the value of gross production to total variable cost is greatest. It does not follow, however, that maximizing gross production relative to total cost will guarantee production efficiency when production expenditure is not a given. $Y/\Sigma X_i$ is maximized where

$$(3)\qquad \frac{\partial}{\partial X_i}\left(f(X_i)/\Sigma X_i\right) = 0, i = 1, \ldots, m.$$

A non-trivial optimizing solution is only possible where $f(X_i)$ possesses regions of both increasing and decreasing returns to scale. Where, in fact, a function is characterized throughout by decreasing returns to scale, as indeed are most cross-section farm production functions of the Cobb-Douglas type, at least in the short-run, $Y/\Sigma X_i$ is a decreasing function of X_i. *i.e.*

$$\frac{\partial}{\partial X_i}\left(f(X_i)/\Sigma X_i\right) < 0, i = 1, \ldots, m.$$

Is, then, the ratio $Y/\Sigma X_i$ a useful criterion of efficiency? It is if the total expenditure on production is fixed but this sum can be freely allocated between the various factor inputs. In fact the opposite of this situation most frequently characterizes production. Some levels of factor input are fixed while working capital is available for expenditures on other input over quite wide ranges. This being the case, $Y/\Sigma X_i$ may be, at least in the short-run, a poor indicator of production efficiency.[3]

An Example

The usefulness, or otherwise, of such a concept, may best be examined by reference to an example. The results of a study of 51 whole-milk farms between Perth and Bunbury in 1955 have been reported elsewhere.[4] A production relationship was estimated in this study as

[3] Using the criterion $Y/\Sigma X_i$ is, of course, merely a variant of the practice of evaluating interfarm efficiency by calculating farm standards. A recent critical discussion of this practice has been given by W. Candler and D. Sargent, "Farm Standards and the Theory of Production Economics", *Journal of Agricultural Economics*, 15: 282-290. 1962.

[4] H. P. Schapper and R. G. Mauldon, "A Production Function from Farms in the Whole-Milk Region of Western Australia," *Econ. Record*, 33: 52-59. 1957.

(*4*) $X_6 = 25{\cdot}716\, X_1^{\,0.2339} X_2^{\,0.1333} X_3^{\,0.0679} X_5^{\,0.5508}$

Where

X_1 = number of farm workers (1·7 man years)
X_2 = Expenditure on purchased feed (£776·4)
X_3 = Expenditure on fertilizer (£181·2)
X_5 = Other expenditure directly referable to level of fixed investment (£865·6)
X_6 = Gross farm income (£4,153)

Figures in parenthesis are the geometric means for the sample. In this example the sum of the indices in equation *4* is only slightly less than *1*. Assuming a wage rate of £640 per annum the ratio of value of production to total cost, E, declines only slowly as optimum production expenditure increases. Normally, however, the production function on which a farmer must base his short-run decisions exhibits strictly diminishing returns. Many investment items and employment commitments cannot be readily changed, and expenditure of working capital must be allocated with an inherited pattern of fixed resources in mind. If E' denotes the ratio of the value of output to total variable costs for a given pattern of fixed resources, the rate of decline of E' is more rapid as production expenditure is increased than is the rate of decline of E over the same expenditure range, where input levels of all factors are freely variable. For the short-run function the ratio of the value of output to total costs (variable and fixed), E^*, will at first rise as increasing production covers the same fixed costs, but it eventually declines at a slower rate than E' as increasing variable costs give rise to a diminishing rate of output. This, of course, is the inverse of the classical U-shaped average total cost curve.

The example again illustrates the point. Suppose that labour(X_1) and fixed capital-servicing expenditure (X_5) are fixed at levels equal to their geometric means for the sample. Assuming that equation *4* represents the firm's long run production function, the firm's short run function becomes

(*5*) $X_6 = 1{,}204{\cdot}8 X_2^{\,0.1333} X_3^{\,0.0679}$.

Then for an expenditure on X_2 and X_3 which is involved at the geometric means of the sample (£957·6), the maximum gross farm return is £4,216 yielding an E' ratio of 4·40 and E^* of 1·45, with X_2 = £634·3 and X_3 = £323·1. The levels of X_2 and X_3 which maximize short run profits, however, are £547·3 and £278·8, with E' = 4·97 and E^* = 1·48. The behaviour of these ratios at various levels of total expenditure is given in Table 1.

TABLE 1

Ratios of gross value of production to expenditure for a farm with given labour and capital-servicing expenditure use.

Ratio	Σb_i	Total production expenditure (fixed plus variable)				
		£2,000	£2,444⁺	£2,779⁺⁺	£3,000	£3,500
E	0·9859	1·69	1·68	1·68	1·68	1·67
E'	0·2012	49·42	7·49	4·97	4·10	3·00
E^*	0·2012	1·15	1·51	1·48	1·43	1·33

⁺ Use of X_2 and X_3 which maximizes E^*
⁺⁺ Optimum use of X_2 and X_3

Table 1 demonstrates how the profit maximizing ratios of value of production to expenditure on inputs may vary for a given farm under various resource-use restrictions. It indicates that in the short run neither E' or E^* provides a useful criterion of efficiency for changes in resource use *within* the farm. The comparison E *v.* E^* at a given expenditure level does, however, indicate the scope for long run improvement which may be made by modifying the current fixed resource structure. The Cobb-Douglas type function implies that the long run optimum resource proportions do not change with the scale of input expenditure, though the E ratio does not indicate what the optimum level of expenditure should be.

Table 1 does not tell us anything about the usefulness of either E' or E^* as a criterion for comparing short run efficiency between farms, where the distribution of fixed assets changes from farm to farm. Nor does it demonstrate the usefulness of the ratios as indicators of the deviation from optimum short run resource allocation at given levels of expenditure within the farm. For the first of these purposes, farms in the example study were divided into groups according to numbers of labour units employed (X_1), and these groups were subdivided into three classes depending on whether low, medium, or high levels of capital-servicing expenditures (X_5) were made. Using equation *4* the optimum levels of variable inputs X_2 and X_3 were computed for each class of farms at its average level of capital-servicing expenditure use. The E^* values for optimum resource use for each class are given in Table 2. These range from 1 to 1·5 over the sample classes of farms, clearly demonstrating the inadequacy of this criterion in comparisons of short run efficiency between farms.

TABLE 2

Ratios of gross value of production to optimum expenditure for farms classified according to labour and capital-servicing expenditure use.

Number of Labour Units.	Level of capital-servicing expenditure (X_5) £		
	0-800	801-1600	more than 1601
1	1·49 (14)	1·42 (9)	(0)
2	1·14 (4)	1·26 (7)	1·27 (2)
3	0·99 (5)	1·19 (5)	1·23 (3)
5	(0)	(0)	1·19 (2)

Figures in parenthesis are the numbers of sample farms located in each class.

To demonstrate the usefulness of the E^* ratio as an indicator of short-run efficiency within the farm, the optimum levels of inputs X_2 and X_3, and the optimum output X_6 at the given levels of input X_1 and X_5 were computed for each of the farms in the sample. The estimates of optimum X_6 were then "corrected" for the "management" characteristic of each farm by multiplying by the observed X_6 and dividing by the estimated X_6 at the original levels of X_2 and X_3.[5]

[5] This implies that each short run farm function is a constant "height" above or below the equivalent short run average function over all levels of feed and fertilizer inputs. Perhaps a more realistic model would imply that the short run farm function is the average function multiplied by the "management" (error) term throughout. Estimates were made using such a model, *i.e.* the short run average function was multiplied by the observed X_6 and divided by the estimated X_6 prior to estimating optimum input and output levels. The results of both models were very similar, but in this latter model some of the farms characterized by large management correction factors gave quite unrealistic estimates for optimum input and output adjustments.

This, of course, makes no allowance for a strictly "random" deviation which in the single period analysis, cannot be separated from the "management" deviation.

Values for E were then computed for each farm from the observed outputs and inputs and also from the estimated short run optimum levels of inputs and corrected outputs. The average of the optimum E was indeed greater than that of the observed E ($1 \cdot 41$ *v.* $1 \cdot 25$) but the standard errors of both were high ($\cdot 33$ for each) and a t test of paired differences indicated no significant within farm difference between the two ratios at the 25 per cent level.

It does not follow, however, that short run adjustments in resource use would not make significant contributions to net farm incomes. This will depend upon the extent to which input levels deviate from their optimum and the rate of decline in the net revenue function in the region of the optimum. The net revenue surfaces derived from most short run cross-section production function estimates are typified by a broad plateau in the area of the optimum, and deviations of 10 or 15 per cent from optimum input levels frequently make little difference in net farm income. This is so for the net revenue function in the example study. At the geometric means for X_1 and X_5 a 10 per cent decrease or increase in both X_2 and X_3 from their optimum levels caused gross income net of feed and fertilizer expenditure to decline by only $0 \cdot 03$ per cent. Yet if the function, as adjusted by the "management" error term, is a valid basis for within farm planning, then adjustments in the levels of expenditures on fertilizer and feed would increase the values of farm production net of cash costs other than labour by an average of 33 per cent per farm. The percentage increases in income would be distributed very unevenly around this average, however. A third of the farms in the sample would receive on average only a 2 per cent net income rise. For another third it would average 10 per cent. The percentage increase within the remaining farms would be very high, averaging 100 per cent. Management advice, then, given on the basis of estimated marginal productivities may not be of much value to a large group of farmers by virtue of the flatness of the net revenue surface in the vicinity of the optimum. Yet for many who are operating with a very inefficient use of resources such advice may be of very great value, particularly if these values are computed at levels of fixed resources which characterize such farms. It might be argued that the value of advice given on the basis of marginal productivities is further limited in the short run due to the sample variation of parameter estimates. This is a problem which goes beyond the intent of this note, but one which has received the critical attention of other authors.[6]

Conclusion

Two conclusions follow. First, in the short run the ratio of total value of production to expenditure on inputs is not a useful criterion to evaluate the efficiency of changes of resource use within the farm.

[6] See, *e.g.*, H. O. Carter and H. O. Hartley, "A Variance Formula for Marginal Productivity Estimates Using the Cobb-Douglas Function," *Econometrica,* 26: 306-313, 1958, and W. A. Fuller, "Estimating the Reliability of Quantities Derived from Empirical Production Functions," *J. Farm Econ.* 44: 82-99, 1962.

Second, since the distribution of fixed assets changes from farm to farm, in the short run the same ratio fails to be a useful criterion for evaluating relative efficiency with respect to resource allocation between farms. It therefore does not follow that because the allocation of resources at given levels of expenditure contributes only a small component of the observed variation of the value of production relative to expenditure, the advice given to farmers on the basis of marginal productivities will only touch the fringe of management problems. Indeed, this conclusion may not follow even if the ratio were a good indicator of efficiency. The analysis of variance technique analyses observed deviations and not deviations from an *a priori* optimum. It is quite possible, and in many cases presumably probable, that all farmers in a region may be over or under spending on whole categories of resources.

The usefulness of such advice will depend upon the curvature of the net revenue function on the one hand and the distribution of deviations from optimum resource use among farms on the other. In many cases the net revenue surface is fairly flat in the region of its optimum, and farmers operating within certain quite wide limits may gain only little from resource adjustments. Yet the distribution of resource use in most farming groups is probably such that a sizable number of farmers could receive very significant increases in income by acting on advice given on the basis of marginal productivity estimates, computed at levels of fixed resources which typify these farms.