# LINEAR PROGRAMMING - AN EVALUATION

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Linear programming is no longer regarded as a glamorous newcomer to our methodological armoury. Indeed, recent discussion suggests that the current attitude of the profession toward the technique is one of review and stocktaking. For example, Swanson[1] has remarked on the limited use of programming in the field of applied farm management while Edwards[2] has discussed some shortcomings of the programming model and of programmers.

In this paper a further review is undertaken with particular attention being paid to Australian experience with the method.

There are not many Australian contributions discussing experience with linear programming and the majority of these are essentially non-analytical. A review would show that most research applications have been devoted to problems of farm adjustment or development. Australian agricultural economists have yet to employ the demonstrated potential of programming in macro-economic problem areas such as supply analysis[3] and interregional competition.[4] In many respects this is probably more due to a shortage of data and research personnel than to any other reason.

Although an essay in re-appraisal of the method implies experience which has led to dissatisfaction, it is hard to detect such a reaction in the Australian literature. Despite this silence an examination of the local scene seems warranted by the considerable level of discussion and criticism elsewhere. The following sections will review those problems of specification and computation which are most commonly associated with linear programming. It will be suggested that many of these objections have been exaggerated and that, while they are common to both research and extension, it is in extension that they are most acute.

## *The Theoretical Adequacy of the Programming Model*

It has been adequately demonstrated elsewhere[5] that the programming model is a logical extension of linear economic theory which is itself a restatement of the conventional theory of competitive equilibrium.

* The author is indebted to his colleagues at the University of New England. The paper also benefited from discussion of a draft at the 1963 Conference of the Australian Agricultural Economics Society.

[1] Swanson, E. R., "Programmed Solutions to Practical Farm Problems", *Journal of Farm Economics,* Vol. 43, No. 2, May, 1961.

[2] Edwards, C., "Shortcomings in Programmed Solutions to Practical Farm Problems", *ibid.*

[3] McKee, D. E. and Loftsgard, L. D., "Programming Intra-Farm Normative Supply Functions", *Agricultural Supply Functions,* Iowa State University Press, 1961.

[4] Hassler, J. B., "Principle Forces, Normative Models, and Reality", *Journal of Farm Economics,* December, 1959.

[5] Dorfman, R., Samuelson, P. A. and Solow, R. M., *Linear Programming and Economic Analysis,* McGraw Hill, 1956 Chs. 13 and 14.

Indeed, "Linear programming is marginal analysis, appropriately tailored to a finite number of activities".[6]

Of course, if the assumption of a homogeneous production function of degree one which is the basis of this theory is acceptable,[7] it is difficult to quarrel with the linearity assumption employed in programming. However, because of institutional rigidities or resource indivisibilities, variable proportions may have to be acknowledged in some cases. That is, it may be desirable to admit non-linearities. Fortunately, such a situation may be approximated by means of a series of linear segments.[8]

A further and oft-quoted objection to linear approximation is that it increases the computational burden. Actually, the validity of this objection depends upon the original size of the problem, the computing facilities available, and the number of activities which must be added in order to achieve satisfactory approximation of the curve. With modern computers this requirement would seldom get out of hand. This is particularly true in agricultural economics where many farm problems become relatively small linear programming problems.

Integer programming and linear approximation may be used to study non-convex situations. However, it must be remembered that the shadow prices in the optimum plan resulting from integer programming are largely meaningless.[9]

Loftsgard and Heady[10] have demonstrated how problems of resource allocation through time may be handled in a programming framework. The development of procedures for handling large matrices[11] suggests that the problems posed by using such "dynamic" linear programming may not be insuperable.

As Edwards[12] has pointed out, the problem of handling uncertainty seems to be the outstanding challenge to programming theorists now that it has been demonstrated that discontinuities, non-linearities, and non-convex curves can be handled.[13]

The development of quadratic risk programming[14] means that the

[6] *Ibid*, p. 133.

[7] For a discussion of the philosophical respectability of the assumption of constant returns to scale see, Samuelson, P. A., *Foundations of Economic Analysis*, Harvard University Press, 1953, pp. 84-85.

[8] Candler, W. V. and Musgrave, W. F., "A Practical Approach to the Profit Maximization Problems in Farm Management", *Journal of Agricultural Economics*, Vol. 14, December, 1960.

[9] Musgrave, W. F., "A Note on Integer Programming and the Problem of Increasing Returns", *Journal of Farm Economics*, Vol. 44, No. 4, November, 1962.

[10] Loftsgard, L. D. and Heady, E. O., "Application of Dynamic Programming Models for Optimum Farm and Home Plans", *Journal of Farm Economics*, Vol. 41, No. 1, February, 1959.

[11] Dantzig, G. B. and Wolfe, P., "Decomposition Principles for Linear Programmes", *Operations Research*, Vol. 8, January-February, 1960.

[12] Edwards, *op. cit.*

[13] Integer programming can be used in such situations, See Dantzig, G., "On the Significance of Solving Linear Programming Problems with Some Integer Variables", *Econometrica*, Vol. 28, January, 1960. Specific algorithms have also been developed, e.g. Candler, W. V., and Manning R. C., "Modified Simplex Solution for Problems with Decreasing Average Costs", *Journal of Farm Economics*, Vol. 43, No. 4, November, 1961. The above remarks should not be interpreted as denying the usefulness of developments in the field of non-linear programming, see Dorn, W. S., "Non-Linear Programming—A Survey", *Management Science*, Vol. 9, No. 2, January, 1963.

[14] Heady, E. O., and Candler, W. V., *Linear Programming Methods*, Iowa State College Press, 1958, Ch. 17.

variability of the coefficients of the objectives function can be included in the analysis. The duality property of programming suggests that we should be able to study the variance of resource supplies in a similar fashion. However, an algorithm capable of doing this has yet to be evolved.

Similarly a satisfactory procedure for handling the variance of input-output coefficients has yet to be developed.[15] In some instances the variance of input-output coefficients of resource supplies can be imputed to coefficients in the objective function and can then be handed by conventional risk programming. An alternative method would be to attach probabilities to the outcomes of variable input-output coefficient programming. This last has considerable appeal as computation is relatively straightforward and estimates of the stability of plans in the light of the variability of certain key coefficients could be readily obtained.[16]

However, it must be remembered of both these informal non-rigorous suggestions that only a few coefficients could ever be studied and that analysis may become impossibly complicated in the presence of covariances very much different from zero.

As a counsel of last resort Monte Carlo methods of simulated sampling could be employed.[17] However, due to the very large computing burden involved, even Monte Carlo seems to offer no way out when more than a few coefficients are to be studied.

### *The Problem of Specification*

The actual specification of the problem is the most difficult stage in any linear programming study. Little will be said about this here except that efficiency in specification is largely a function of training and experience on the part of programmer. The number of people in Australia, who qualify on both counts is small.

In brief, it could be said that the dangers involved in specification are the assumption of rigid joint relationships which are not justified over the whole range of production possibilities, and of flexibility in the face of change which is not feasible due to asset fixity.

The search for, and estimation of, coefficients is rather a different matter. The problems encountered here are common to most of the synthetic techniques of analysis. Farm management workers in this country are notoriously short of reliable information. Doubtless, the growing demand for such information will lead to applied research in those areas where data deficiencies are most severe. Linear programming, because of its formal nature and because of its ability to give recognisably absurd answers to incompletely specified questions, should

[15] But see Babbar, M. W., Tintner, G. and Heady, E. O., "Programming with Considerations of Variations in Input-Output Coefficients", *Journal of Farm Economics*, Vol. 37, No. 2, May, 1955, where confidence interval estimates are developed for revenue as determined by the variable behaviour of selected coefficients in the matrix. This has little to recommend it above the *ad hoc* procedures mentioned above.

[16] Results of such programming could be presented in a relatively simple fashion by mapping. Such plans may be more stable than the data from which they are devised, see Renborg, U., *Studies in the Planning Environment of the Agricultural Firm*, Almquist and Wiksells, Boktryckeri, Uppsala, 1962.

[17] Renborg, *ibid*, employs this approach.

greatly help in building up this demand.[18] In the interim it is surprising what useful information can be gathered from farmers and local research and extension personnel.

### *Computational Problems*

Programme solution is such a tedious process that a computer must generally be used. If programming is regarded is being esoteric so also is the use of a computer. A combination of the two could understandably be discouraging to the uninitiated. For the initiated the application of programming may be hampered by inaccessibility of the computer or unavailability of suitable routines. On the other hand, it is unlikely that computing costs alone would provide a serious impediment to programming. For example, very few runs on the computer at the University of Sydney, SILLIAC, cost more than £20 (at the full commercial hiring rate) and the majority cost considerably less.

Our experience at the University of New England has been that isolation from a computer cannot be regarded as seriously limiting programming work. It must be admitted that this is so only because SILLIAC has a fairly comprehensive library of computing routines. Should a routine have to be written before a problem could be tackled then our isolation would prove serious. This last point may be important to programmers in the field as the most accessible computer laboratory may not possess suitable routines.[19]

Lack of training in linear programming, together with the above problems limit the use of programming in extension. This seems to have been accepted as a matter for concern in Europe and the United Kingdom where there are reports of efforts to find procedures which are more formal than budgeting but which do not require the use of a computer.[20]

### *Programme Planning*

A variety of techniques have been developed which in the words of McFarquhar: " . . . improve on the trial and error of budgeting without requiring the help of a computer".[21] The most important of these have been dubbed programme planning though it seems that there are definite differences between them. However, they all rely to some extent on judgment and therefore lack the power and rigour of conventional programming. It has been demonstrated elsewhere[22] that such methods are imperfect substitutes for linear programming.

However, if the programme planner can handle small problems (say

[18] Hildreth points out that "Programming formulations have distinct advantages over traditional production functions in enabling the economist to directly use technical information and to pose questions in a language more like that of technical scientists" in Hildreth, C., "Some Problems and Possibilities of Farm Programming". *Fertilizer Innovation and Resource Use,* Iowa State College Press, 1957.

[19] In particular it should be possible to carry out variable resource programming if a study is to be conducted reasonably thoroughly.

[20] Clarke, G. P. and Simpson, I. G., "A Theoretical Approach to the Profit Maximization Problems in Farm Management", *Journal of Agricultural Economics,* Vol. 13, No. 3, June, 1959. McFarquhar. A. W. M., "Research in Farm Management Planning Methods in Northern Europe", *Journal of Agricultural Economics,* Vol. 15, No. 1, May, 1962.

[21] *Ibid.* This article contains a review of the more significant of the methods which have been developed.

[22] Candler, W. V., and Musgrave, W. F., *op. cit.*

and 8 × 8 matrix) in less time than a skilled clerk on a desk calculating machine using the simplex method, and if even larger problems can be handled in reasonable time, there may be a place for programme planning in extension, if only as an educational device. However, there is a temptation to reserve judgment, particularly as quite a significant computing burden remains and problems of any complexity may be difficult to handle.[23]

*Some Experience at the University of New England*

Swanson[24] reporting the development of a market for linear programming analysis of farms in the United States, suggests that the cost of programming the general run of farms may be too great for an orthodox extension agency to build up a linear programming service.

Experience at the University of New England suggests that the pioneering use of programming in an area will be comparatively expensive and time-consuming. Indeed, it may well prove impossibly expensive for the farm under study. Garnering the necessary inventory of coefficients and solving the initial difficulties of problem specification will mean that, out of necessity, this investigation must be comprehensive. However, once this initial information and experience had been accumulated one would expect further studies in the area to rapidly diminish in cost.

Waring[25] undertook such a pioneering analysis of a wheat-sheep farm near Moree "operated by a particularly knowledgeable and articulate farmer". This was an exhaustive study involving all three forms of parametric programming. Given ready access to the computer, satisfactory computing routines, and full time devotion to the task, Waring estimates that this investigation would have taken six months at the very most.[26]

Waring, Fahy and Sturgess,[27] drew upon the material and experience accumulated by Waring to investigate the adjustment possibilities confronting a group of thirty-four farmers who were in difficulty some 40 miles east of the subject of the earlier work. A record of the time taken showed eight days in the field together with 40 minutes computer time. All in all variable cost of the operation would not have been more than £100 to achieve results which, at this stage, seem highly satisfactory.

Both these projects, particularly the second, amount to case or type farm studies. An advantage of using a computer is that once a matrix has been assembled and put on tape, one can very rapidly switch from one form of the problem to another by simple editing of the original tape. Therefore extrapolation from the general results to a particular farm should be both cheap and straightforward.

[23] It is interesting to note the doubts expressed by McFarquhar about the potential of programme planning; doubts with which I am inclined to agree. It certainly seems that the arithmetic involved could rapidly become tedious. See, McFarquhar, A. W. M., "The Practical Use of Linear Programming in Farm Planning", *The Farm Economist,* Vol. 9, No. 10, July, 1961.

[24] Swanson, E., *op. cit.*

[25] Waring, E. J., *Linear Programming Using Farmer-Estimated Input-Output Coefficients,* M.Ag.Ec. Thesis, University of New England, 1962, Mimeo.

[26] Variable costs of the project were estimated at no more than £400. Even if an allowance was made for Waring's time this investigation would have been economic for the particular farm under study. Furthermore, it is possible that future extension studies need not be as intensive as this was..

[27] Waring, E. J., Fahy, J. D., and Sturgess, N. H., "Farm Planning in the Graman District, New South Wales", *Review of Marketing and Agricultural Economics,* Vol. 31, No. 3, September, 1963.

Although the question still remains as to whether linear programming studies of individual farms will become an accepted part of extension, the above examples suggest that bench-mark type studies, to determine the direction in which farm adjustment lies, will be productive. Extrapolation to the individual farm may be done by the extension officer using budgeting or requesting alteration of the type-farm programming matrix.

*Conclusion*

There are a diversity of problems to which linear programming can be applied and its use as a research tool is surely beyond dispute. A review of published Australian work shows that while successful applications have been made, the apparent potential of the technique has yet to be fully realized. This is probably due to the lack of data and trained personnel which bedevils so much work in agricultural economics in this country.

Recent developments in programming mean that many of the more familiar theoretical or specification problems are much diminished in importance. The major outstanding difficulty is the inability of programming to cater for a variable environment in a satisfactory fashion. Australian agriculture is supposed to be characterized by its variability and this suggests that Australian agricultural economists should have the incentive to contribute to the study of this intractable aspect of programming methods.

The role of programming in extension certainly warrants discussion. The potential of programming in this field has been amply demonstrated by Gruen, Pearse, Parker and Waring and his colleagues.[28] Discussion elsewhere has suggested that the major drawbacks to general application are the necessity to use a computer and expense.

Experience at the University of New England has suggested that continued application of programming in a particular area may so lower costs that the generalized use of programming in extension in that area may be economic. On the other hand isolated pecks at individual problems will certainly result in high cost programming.

While individuals who are conversant with a computer and its associated routines may be able to successfully programme in isolation from the computer, it seems that the need to be in close proximity to one will, in general, hinder programming in extension. This problem will be with us until the time that people with training in economics, programming,[29] and computer use [30] become common in extension.

[28] Gruen, F. H., "Pasture Improvement—The Farmer's Economic Choice", *The Australian Journal of Agricultural Economics,* Vol. 3, No. 2, December, 1959; Pearse, R. A., *The Development of Optimum Plans for Pasture Improvement Using Linear Programming,* Section K. A.N.Z.A.A.S., 1962; Parker, M.L., "Programming for Farm Management Extension", *Journal of the Australian Institute of Agricultural Science,* Vol. 25, No. 4, December, 1959; Waring *et al. op. cit.*

[29] The practice of restricting undergraduate tuition in programming to an elective concentrating on the mathematical theory of the method may serve to minimize the number of people graduating with training in linear programming. It is interesting to note that a less esoteric course of 40 lectures at the University of New England is widely taken.

[30] Evidence of the importance of the availability of, and familiarity with, computers and computing routines is given in West, V., "Impact of Electronic Computing on Farm Economic Research", *Journal of Farm Economics,* Vol. 40, No. 5, December, 1958.

The two most important suggestions for improving on budgeting as a "Front-line" weapon in applied farm management have been the development of techniques such as programme planning or the establishment of a centralized farm management laboratory where linear programming can be done.[31]

While there is little reason for optimism concerning the first of these alternatives the second certainly offers a means by which the full power of programming analyses can be brought to play in farm management extension. However, if there is to be any demand for the services of the laboratory, programming problems must be identified in the field. Widespread identification will only take place with the build up of suitably trained farm management workers in farm advisory work.

[31] Schapper, H. P., "Farm Management Clubs for Australia", *Journal of the Australian Institute of Agricultural Science*, Vol. 25, No. 1, March, 1959. McFarquhar, *ibid*. Of course the costs of such a laboratory would be of critical importance. The New England experience suggests that the cost of programming in a unit attached to an established agricultural economics section of a department of agriculture may be driven down to a level which would certainly be attractive to farm management clubs and, quite likely, to individual farmers. There is no reason why a department should not charge for such services.