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A SIMULATED STUDY OF AN AUCTION MARKET

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A simulated model of an auction market is developed showing the relationship between the variation in valuations, the price variation and the number of independent bidders in the market. Average prices paid in a market with two or three bidders are less than average valuations. Average prices are progressively greater than average valuations as the number of bidders increases beyond four. Some applications of this model in the Australian wool market are discussed.

This article is an attempt to explain some aspects of the mechanism of an auction market in which bidding opens at a low bid and rises to the maximum price buyers are prepared to pay. A general discussion of such a market is followed by the development of a rigorous statistical model of an auction sale. The implications of relaxing some of the assumptions in the model are then examined and finally some applications of the model to the Australian wool auction market are considered.

An element of price uncertainty is a feature of all auction markets. Indeed an auction sale is based on the existence of differences between different buyers' estimates of value on the same lots. Potential buyers in a commodity auction are invited to estimate values for each sale lot. These estimates are usually based on the physical properties of sale lots and a set of price limits which apply to various grades specified in terms of the physical properties of the commodity. If the value estimates for each lot are based on subjective examination then the buyer inspects the sale lots prior to the sale. The buyer records a value for each lot and this value represents the maximum price the buyer is prepared to pay for the lots. These price limits form the basis of his bidding during an auction.

The price paid for identical lots sold at one auction sale can vary over the sale period. This variation arises because factors other than the technical characteristics of wool determine the price paid at auction. These factors are:

(i) *Variations in demand throughout a sale period.* These variations occur when buyers fill orders and then withdraw from the sale, or if buyers employ a strategy that requires a temporary withdrawal from the market [1]. In other cases a buyer may withdraw from bidding simply to balance his purchases against orders.

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(ii) *Differences in price limits between buyers.* These differences reflect variations in the competitive position of individual buyers. In a commodity market variations in price limits for specified grades will arise from differences in the efficiency of the buyer, in processing or retailing the commodity, differences in stocks held, and the level of orders. In a large market involving day to day transactions with effective methods of communicating market intelligence it is unlikely that price limits would vary over a large range.

(iii) *Errors in specification.* These frequently occur when the composite properties of a grade are estimated subjectively. Such a situation is common in auction markets for primary products such as livestock, cotton, tobacco or wool. If all buyers in such a market have identical price limits then the successful bidder will be the buyer whose estimate of value contains the largest positive error. The difference between the buyers' estimate of value and the price paid will vary according to the number of buyers in the market and the distribution of valuations.

In the particular case of a single lot sold at an auction in which successive bids rise from lowest to highest values, the price will be determined by the second highest valuation in the market and the lot purchased by the bidder with the highest valuation, either at, or one bid above, the second highest valuation.

These considerations suggest the restraints that would have to be placed on a rigorous statistical model of an auction market. The model relates to a market in which all buyers have the same price limits (i.e. all buyers have the same maximum price for specified grades of a commodity).

Model 1—Identical Price Limits for All Buyers

Restraints placed on this model relate to price variations arising from variations in demand throughout a sale period and differences in price limits between buyers. The main concern is the price variations arising from errors in specification. The model is based on the following assumptions:

(a) All buyers bid independently of each other with no collusion or price fixing agreements.

(b) There are N bidders in the market whose estimation error is normally distributed about a mean of zero ($\bar{x} = 0$) and with unit standard deviation ($S^e = 1 \cdot 0$).

(c) All sale lots are homogeneous in respect to the important commercial properties.

(d) That all of the N bidders remain active throughout the sale.

(e) That all bidders have identical price limits that cannot be varied during a sale period.

Using these assumptions valuations for each bidder will be distributed normally around the same mean. Differences between the prices paid for similar lots are a consequence of differences in estimation. In the progressive or British auction the purchaser of a sale lot will be the bidder with the highest valuation. Bids are progressively increased by different bidders until the second highest valuation is reached. The price paid will be either the second highest valuation or one more bid above that valuation.

Price distributions can be constructed from the population of valuations for a given value of *N* bidders. The second highest valuation can be selected as a satisfactory approximation for the price paid at auction.

Methods Used to Generate Price Distributions

Two methods were used to generate the price distributions, the first involved the use of the binomial expansion and the second was a computer simulation method using random normal numbers. The procedure using the binomial expansion is set out in the appendix.

Simulated distributions were generated by splitting 100,000 random numbers into groups of *N* for values of *N*; 2, 5, 10, 15 and 20. The second highest number was drawn from each group and the distributions for each value of *N* were constructed by normalizing the selected numbers[1]. Histograms were then constructed and means and variances calculated for each distribution.

The Price Distributions: Price distributions for prices generated from the binomial calculations are illustrated in Figure 1. In addition to the prices paid at auction by varying numbers of buyers the curve showing the distribution of valuations for individual buyers is set out in Figure 1. Means and variances for the price distributions generated using the binomial and random number techniques are set out in Table 1. The skewness and kurtosis of each distribution were tested and in all cases deviations from normality were not significant at the 5% level.

TABLE 1

The Means and Variances for Price Distributions Obtained Using Binomial and Computer Simulation

	Displacement of Mean Price Around Expected Mean ($\triangle$P)			Price Variance S'^2		
Number of Bidders (N)	Binomial	Computer	Difference	Binomial	Computer	Difference
2	—0·66	—0·56	0·10	0·67	0·68	0·01
3	—0·10			0·46		
4	0·20			0·37		
5	0·40	0·50	0·10	0·32	0·31	—0·01
10	0·90	1·00	0·10	0·22	0·22	0·00
15	1·15	1·25	0·10	0·18	0·18	0·00
20	1·31	1·41	0·10	0·16	0·16	0·00

[1] Normalization of the random numbers can be left until this stage because of the monotonic nature of the probability function, i.e. the function rises throughout its length.

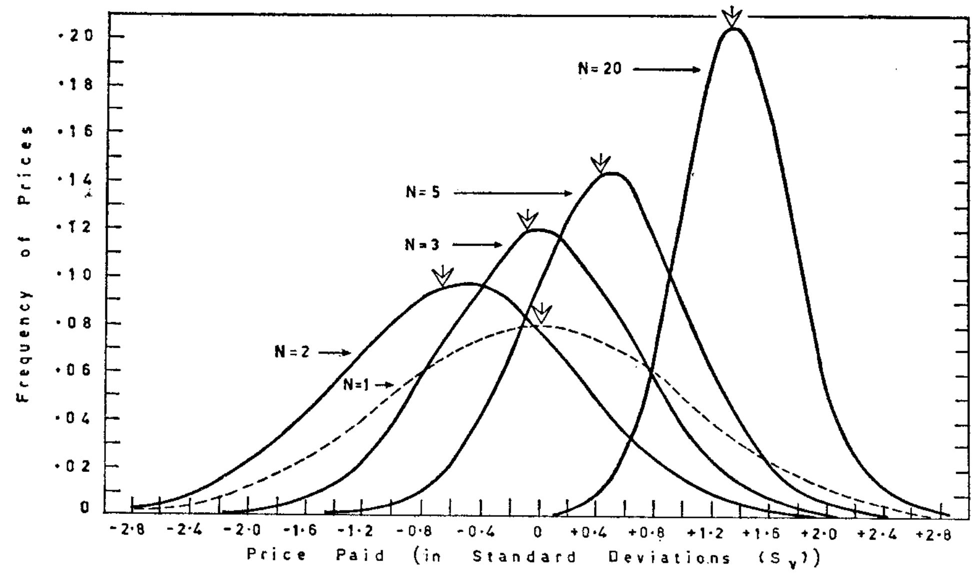

FIG. 1—Distribution of prices paid for varying numbers of buyers (N)
Note: The broken line for N = 1 illustrates the distribution of valuations for each buyer.

An inspection of Table 1 indicates that the means and variances calculated from the two methods give similar results. Average prices generated from random normal numbers were 0·01 standard deviation units higher than those obtained from the binomial method. Variances for the binomial prices were either the same or marginally different for low values of N.

At low levels of competition (i.e. two and three bidders) the average price paid is less than the average valuation. Increasing competition in a market (or increasing numbers of bidders) produces a shift in the price distribution towards higher valuations (i.e. to the right of the mean of the valuation distribution) there is also a reduction in the variance of prices paid as the number of bidders increases.

As a reduction in competition to two or three bidders results in an average price lower than the average valuation, the auction system is self-stabilizing. In relation to the average valuation, prices fall when two or three bidders are involved and rise in a positive response to the number of bidders above three. These price movements could develop into longer term effects if the valuations in any given sale were based on the average prices paid at a previous sale. The relationship between the shift in prices, the variation of prices paid and the number of bidders is set out in the next section of this article.

Interrelation Between Variables

Equations have been developed to show the association between the following variables:

N_B, the number of independent buyers in the market.

$\Delta p = (\overline{P} - \overline{V})/S_v$, where

$\bar{P}$ = the average price paid in cents per pound;
$\bar{V}$ = the average valuation in cents per pound;
S_v = the standard deviation of valuations in cents per pound.
$S' = S_p/S_v$,

where S_p = the standard deviation of prices paid in cents per pound.

(1) $N_B = 0{\cdot}53\ [(1+p)/S'] + 1{\cdot}83 \qquad R^2 = 0{\cdot}997.$

(2) $\Delta p = 6{\cdot}25 \log S' - 1{\cdot}1713, \qquad R^2 = 0{\cdot}999.$

(3) $\Delta p = 1{\cdot}88 \log N_B - 1{\cdot}0333, \qquad R^2 = 0{\cdot}975.$

The Effect of Relaxing the Assumptions

(i) In the calculations set out here N_B is given as the number of independent bidders. If assumption (*a*) is relaxed and bidders form price fixing combinations the effect is to reduce the value of N_B. Each combination would then be considered as an independent bidder.

(ii) Variations may occur between different bidders' estimates of value even when they have the same price limits. These variations will reflect differences in the bidders' estimation skill and in the bias of estimators. When the values of (S_e) the standard deviation of bidders' estimates and the bias in estimation are normally distributed the determination of S_v requires a further component of variation (i.e. S_B the standard deviation of the bias in estimates). The variance for valuations on identical lots (S_v^2) now becomes:

$$S_v^2 = S_e^2 + S_B^2.$$

(iii) Differences in the homogeneity of sale lots classified as 'identical' lots would increase the size of S_e and S_B.

(iv) The effect of relaxing assumption (*d*) is important in that it is unrealistic to expect a constant number of bidders throughout a sale period; as orders are filled bidders will withdraw or as new orders are received bidders enter the sale. Order sizes vary and consequently the time required to fill orders will vary. Under these circumstances the concept of N_B as a measure of the exact number of bidders in the market must be abandoned, but N_B may still provide a useful index of competition in the market.

When the number of bidders varies over a sale period the accuracy of N_B as a competition index will be reduced. If the number of bidders remains constant during a sale period even though individual bidders may vary their bidding tactics then the value of N_B as a competition index would be enhanced. Such a situation would be reflected in the trend of prices paid for identical lots. If the number of bidders remained constant during a sale, prices would follow a random walk over the sale period.

In the case of a large-scale commodity market, where buyers' orders were fixed at the commencement of each sale, the optimal buying policy would be a system of price limits that allocated the supply in such a manner that maximized buyer satisfaction. The optimal price limit for any given buyer would be that price that enabled the buyer to fill his orders in the time required to conduct the sale. A price limit higher than the optimal price would result in the buyer filling his orders early in the sale. A price limit lower than the optimal limit would result in a failure to meet all orders.

(v) In a commodity market based on day to day transactions with good market intelligence the general price levels for specified grades will be well known. Under these circumstances the price limits for the majority of bidders is likely to be close.

Variations in price limits will arise from differences in the time available for individual buyers to fill orders, from differences in bidding tactics or in differences in the incentive for buyers to purchase the commodity.

Where the price limits for different buyers are normally distributed around a common mean the variances for valuations will include a component due to difference in price limits (S_L^2).

The General Model

A statistical model can now be presented in which the only assumption is that all variables incorporated in the model are normally distributed. This equation is:

(4) $N_B = 0{\cdot}53\ [(S_v + \overline{P} - \overline{V})/S_p]^2$

where $S_v = (S_e^2 + S_B^2 + S_L^2)^{\frac{1}{2}}$.

$S_B^2 =$ the variances of a single bidder's estimates of value on identical sale lots.

$S_e^2 =$ the variance of different bidder's estimates on identical sale lots with the same price limits.

$S_v^2 =$ the variance of price limits between bidders for the same lots.

Application to the Australian Wool Auction Market

In the Australian wool auction market buying agents generally receive orders from clients which specify the type of wool required and a clean price limit. A wool type (or grade) is usually defined in terms of a series of subjective properties that are estimated by the buying agent or his employee.

A selling broker displays all sale lots on a show floor where they are inspected by prospective buyers.

Each agent examines the sale lots which interest him and estimates a type and yield for each lot. These estimates are used to convert the clean price limits of the client into greasy price limits, which become the maximum prices that the bidder is prepared to pay for the lots. As the estimates of both type and yield are based on visual and tactile appraisal there are errors of estimation which increase the variability of valuations [2].

During a selling season there is usually one sale series each week in some part of Australia. Each series covers from two to four days and market trends are carefully observed. Consequently it is unlikely that price limits would differ greatly between bidders at one sale but variations may arise from differences in the time periods available to fill orders [3].

Although as many as 80 bidders may be present in the auction room during a sale, only a fraction of this number would be bidding on individual sale lots. The clean prices paid for sale lots containing the same type of wool were examined for approximately 100 single sale days. In all cases the sequence of prices within sale days appeared to describe a random walk. This indicated that the factors influencing the strength of bidding during a sale have random effects and the value of N_B would give a useful indication of bidding strength. A measure of the strength of the bidding in a sale could be obtained from equation 4 if estimates of S_v, S_p, $\overline{P}$ and $\overline{V}$ can be made.

Estimating the Variance of Valuations (S_v^2). Of the three components of S_v^2 only one, S_e^2, has been estimated but reasonable approximations can be made for S_B^2 and S_L^2.

Estimates of the error of valuation of the same sale lots of wool are available for seven experienced appraisers [4]. The value of S_c ranged from 1·8 to 3·3 cents per pound greasy for good topmaking wools containing some vegetable fault. A value of 2·5 cents per pound greasy has been used in the present study.

The results of yield comparisons between different appraisers on the same lots [5] indicates that the value of S_B would be approximately 1·5 cents per pound greasy.

In general the differences between the clean price limits held by different firms for the same types of wool appear to range from zero to 4·0 cents per pound [6]. In the present study the value of S_L has been set at 1·0 cent per pound greasy.

Using these approximations the estimated value of S_v is:

$$S_v^2 = (2\cdot 5)^2 + (1\cdot 5)^2 + (1\cdot 0)^2 = 9\cdot 50$$

giving a value of $S_v = 3\cdot 08$ cents per pound greasy.

Estimating the Variance of Prices Paid for Identical Lots (S_p). Some caution must be exercised in defining 'identical lots'. Clearly the classification of sale lots by types will be subject to the same errors inherent in the estimation of greasy price limits, but in spite of this the individual type classifications are regarded as homogeneous by the wool trade. One estimate for the price variation of identical lots is based on the prices paid for pairs of sale lots, each pair being produced from a single classed line within a clip.

After classification by the classer the fleeces within a line (or grade) were randomized and pressed into bales. Two sale lots were prepared from these bales by placing alternate bales into two separate lots. Both sale lots were displayed side by side and sold one immediately after the other. Prices paid for these identical lots were then compared and an estimate of the within sale price variation obtained using an analysis of variance.

Two sets of comparisons were made; one involved 33 pairs of big sale lots (i.e. 66 lots); the other was based on 30 pairs of small lots (i.e. 60 lots). These pairs were sold in various sales over a period of 18 months. The random price variation within sales for big lots was $S_p^2 = 1\cdot 9$ or $S_p = 1\cdot 4$ cents per pound; for star lots the estimated variance was $S_p^2 = 3\cdot 9$ or $S_p = 2\cdot 0$ cents per pound greasy.

Eight pairs of big lots containing good topmaking fleece wool were sold in one sale. The within sale price variance for these lots was $S_p^2 = 2{\cdot}2$ (i.e. $S_p = 1{\cdot}5$) cents per pound. In another sale there were twelve pairs of average spinners sale lots sold and the within sale price variance for these wools was $S_p^2 = 1{\cdot}2$ (i.e. $S_p = 1{\cdot}1$) cents per pound greasy.

These estimates of S_p range from $1{\cdot}1$ to $2{\cdot}0$ cents per pound greasy, a value of $S_p = 1{\cdot}5$ cents has been selected for substitution in equation 3.

Estimating the Value of $\bar{P} - \bar{V}$. In the simulation study reported in this article the population of valuations in a sale have been normalized around $\bar{V} = 0$ and the mean of the prices paid $\bar{P}$ has been determined by selecting prices from the normalized valuations. In practice it would be difficult to estimate the mean valuation $\bar{V}$ in a wool auction market but the mean price paid for a given wool type could be quickly calculated.

Given the values of S_p and S_v the value of $\bar{P} - \bar{V}$ can be obtained by substitution in equation 2. A second alternative is to make use of the fact that woolbuyers tend to base their valuations on the most recent price information. Thus the population of valuations on any given day are generally distributed around the mean prices paid at the most recent sale. For example if $\bar{P}$ is set at $\bar{P}n$ then $\bar{V}$ could be set at $\bar{P}(n - 1)$ and:

$$\bar{P} - \bar{V} = \bar{P}n - \bar{P}(n - 1)$$

In the current illustration the value of $\bar{P} - \bar{V}$ has been calculated by substitution in equation 2.

Estimating the Value of N_B. Using the assumptions set out in this paper an estimate can be made of the average number of bidders competing at a wool auction sale. Equation 2 can be expressed in the form

$$\bar{P} - \bar{V} = [-6{\cdot}25 \log (S_p/S_v) - 1{\cdot}17]\ S_v$$

Substituting $S_v = 3{\cdot}8$ and $S_p = 1{\cdot}5$ in this equation gives a value of $\bar{P} - \bar{V} = 4{\cdot}9$ cents per pound. The value of N_B can now be estimated by substitution in equation 3. These values give an estimate of $N_B = 11$ as the average number of bidders usually bidding at wool auction sales in Australia.

Implications of the Model for Wool Auctions.

A reduction of the value of S_v has a marked effect on the competition required to produce a predetermined value of S_p or $\bar{P} - \bar{V}$, e.g. for $S_v = 2{\cdot}0$ in the above equations the number of bidders would be reduced to $N_B = 2$.

The introduction of objective measurement as a basis for selling wool would have the effect of reducing the value of S_v. This in turn would reduce the number of bidders required to maintain a constant variation of prices paid or alternatively reduce price variation and increase average prices paid with the same number of buyers.

In view of the general tendency for woolbuyers to use the average prices paid at a previous sale as a basis for valuation long term stability of prices will be achieved when $\overline{P} - \overline{V} = \overline{P}n - \overline{P}(n-1) = 0$. This condition occurs when:

$$\triangle p = 1{\cdot}88 \log N_B - 1{\cdot}033 = 0,$$
$$\text{i.e. } N_B = 3{\cdot}5 \text{ buyers.}$$

The implication of this in practice is that an auction held with less than four bidders does not provide enough competition to force buyers to pay their predetermined valuation. Under these conditions the average price paid $\overline{P}n$ will be less than $\overline{P}(n-1)$ and in the longer term prices will fall. When there are four or more bidders in the auction $\overline{P}n > \overline{P}n - 1$ and prices in the long term will rise.

The refinement of the relationships set out in this paper may provide both buyers and sellers with useful indicators of the strength of competition in the market.

A seller could base his decision to sell on both a reserve price and the strength of competition. A bidder may be able to use the shift between average valuations and average prices to estimate an optimal price limit (e.g. that price limit that would allow the bidder to fill all orders at the minimum price).

A Wool Marketing Authority could base its buying and selling operations on the level of competition in the market. It may be sufficient for the Authority to bid in the market when there are only two or three bidders operating. Any intervention at that point would have the short term effect of stabilizing prices and maximizing returns to growers by preventing a fall in prices. The short-term price reductions produced by the Authority's selling operations would be minimized if selling was only carried out when a large number of bidders were active in the market (i.e. say 10 or more).

Appendix

The Calculation of Price Distributions Using the Binomial Expansion

Let the probability that price x' is greater than x be:

$P_x =$ probability $(x' > x)$

and $P_{x_1} =$ probability $(x' < x_1)$,

then $P_x - P_{x_1} =$ probability $(x < x' < x_1)$.

Supposing there are N_B buyers in the market and the distribution of their valuations is $N(0, \sigma)$. If the event 'y' of $x' > x$ is regarded as a success,

then $y = B(n, P_x)$,

where $y = 0, 1, 2, 3 \ldots\ldots n$,

i.e. $y = 0$ means that no bid exceeded the price x,
$y = 1$ means that one bid exceeded the price x,
$y = 2$ means that two bids exceeded the price x,
up to
$y = n$ means that n bids exceeded the price x.

The probability P_x can be obtained from tables of normal probabilities for any value of x_1. As we have assumed that y is binomially distributed the probabilities of $y = 0$, $y = 1$, $y = 2$ etc. can be obtained by expanding the binomial equation.

$$(P_x + q_x)^n \text{ where } q_x = 1 - P_x$$

which gives,

$$(P_x + q_x)^n = P_x^n + nP_x^{n-1}q + \ldots + nP_xq^{n-1} + q^2.$$

The terms of this expansion have the following meaning,

P_x^n = probability ($y = n$) that all bids exceeded x.
$nP_x^{n-1}q$ = probability ($y = n-1$) that all bids except one exceed x.
. . . . .
nP_xq^{n-1} = probability ($y = 1$) that only one bid exceeds x.
q^n = probability ($y = 0$) that no bid exceeds x.

As we are only interested in the probability of $y > 1{\cdot}0$ this can be calculated by summing the first $n-1$ terms or by subtracting the last two terms from one. The last was the method used to calculate the probability of at least two bids exceeding the price x.

The probability of at least two bids falling in the range $(x < x' < x_1)$ was calculated from $P_x - P_{x1}$. Price distributions were constructed by calculating the probabilities of at least two bids falling in successive ranges of 0·2 cents over the interval +3 to —3 standard deviations (cents per pound).

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