VALUE OF PREDICTORS OF UNCONTROLLED FACTORS IN RESPONSE FUNCTIONS

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A method is developed for assessing the monetary value of additional information in the framework of a response process which involves interaction between controlled and uncontrolled factors. The method is illustrated through an analysis of the value of a rainfall predictor in determining optimal applications of nitrogen to wheat in an environment with variable rainfall. The analysis defines the conditions under which the predictor is most valuable and suggests types of predictors which may be of greater value.

Introduction

In recent years economists have favoured decision theory as a normative tool for decision making under uncertainty.[1] However, relatively little attention has been explicitly focused on the economics of obtaining additional information to reduce uncertainty. In this paper a method is developed and illustrated for evaluating information in the context of a micro-production function which we denote as a response function.

Theoretical Considerations

In the classical theory of the firm, a micro-production function is used to describe the production process. A key assumption of the usual theory is the existence of perfect knowledge about future events. By relaxing this assumption we can examine the physical characteristics of a process which determine whether or not information will have economic value.

In its simplest form, the response function may be written as

$$y = f(x_i), \quad (i = 1, 2, \ldots, n),$$

where y is output and the x_i are perfectly divisible, homogeneous inputs under the control of the decision maker. Profit is defined by the objective function,

$$\pi = p_y y - \Sigma p_i x_i,$$

which is maximized when

$$\partial y / \partial x_i = p_i / p_y,$$

and the required second-order conditions are satisfied.

This simple response function is readily partitioned to a more realistic and comprehensive model of production by specifying the relationship

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[1] Several authors (e.g. [5, 7, 8]) have reviewed modern developments and agricultural applications of such aspects of decision theory as utility analysis and Bayesian statistics.

in a form which explicitly considers uncertainty and controllability[2], namely

$$y = f(x_i, x_j, x_k), \ (i = 1, 2, \ldots, n; j = n + 1, n + 2, \ldots, m; \\ k = m + 1, m + 2, \ldots, r),$$

where x_i is the level of the i-th controlled input, x_j is the level of the j-th uncontrolled input known at the time of the decision and x_k is the level of the k-th uncontrolled input which is unknown (uncertain) at the time of the decision. The prices of the uncontrolled inputs, the p_j and p_k, here and in general are zero.

If the x_i and x_k have independent effects in the production process, then in general

$$\partial y / \partial x_i = f'(x_i, x_j)$$

and

$$\partial^2 y / \partial x_i \partial x_k = 0.$$

That is, the marginal products of the x_i are independent of the levels of the x_k. Since the x_j are known or can be estimated prior to making the decision, the optimal levels of x_i, denoted $x_i{}^*$, can be determined. Alternatively, if the x_i and x_k interact in the production process,

$$\partial y / \partial x_i = f'(x_i, x_j, x_k).$$

In this case, the marginal products of the x_i cannot be equated directly with the appropriate price ratios to determine the optimal levels of the x_i. In the absence of perfect knowledge,[3] the decision maker is unable to choose the levels of x_i which will necessarily be optimal *ex post* and the decision maker may incur a loss—a cost of uncertainty. This cost arises from the interaction between the controlled and uncertain factors in the process. Thus, interaction between controlled and uncertain factors is a necessary condition for additional information on the uncertain factors to have economic value. Information is defined here as anything which causes revision of the prior probability of an event.

An Illustrative Application

The relationship among input factors for a large number of agricultural response processes can be viewed in the theoretical framework outlined. Here we will consider one such process. While concentration on a particular process will not enable any empirical generalizations to be drawn concerning the whole area of information in a production economics setting, we hope our example will suggest approaches which may be useful in other situations.

Our illustrative example relates to the questions of the application of nitrogen to wheat and the prediction of rainfall one year ahead. Russell [9] has recently reported an extensive series of investigations into the response of wheat to nitrogen in the South Australian wheat belt—an area where use of applied nitrogen is not part of the traditional technology. Bowen has recently suggested in a confidential publication that

[2] Without being too pedantic about the degree of precision of control over inputs, we believe inputs can usefully be considered in the simple dichotomy of controlled and uncontrolled. An analogous model of an agricultural response process has been discussed in [3, pp. 194-205].

[3] Here we only consider the case where the prices are known with certainty. In practice, p_y will often be uncertain (p_i less frequently) and it will then be most appropriate to use the expected prices computed with the decision maker's subjective probabilities [1, p. 50].

it may be possible to make long-term predictions of rainfall *trend* as distinct from rainfall *amount*.[4]

The factors[5] considered were: applied nitrogen, growing season (May to October) rainfall, 15 atmospheres soil moisture, total soil nitrogen (0-6 inches) and initial soil nitrate (0-36 inches). The dependent variables considered were grain yield, and response in grain yield (i.e. $y = Y_N - Y_{zero\ N}$). We used conventional least-squares regression to fit various second-order and some third-order polynomials using several combinations of dependent and independent variables. The final equation, chosen on the basis of a subjective consideration of tests of statistical significance, agronomic principles and computational feasibility, was

$$\begin{aligned} y = \underset{(1\cdot 51)}{0\cdot 05274N} - \underset{(2\cdot 46)}{0\cdot 00156N^2} + \underset{(11\cdot 25)}{0\cdot 01755NG} - \underset{(3\cdot 80)}{0\cdot 00498NS_m} \\ - \underset{(4\cdot 26)}{0\cdot 00119NGS_n}, \end{aligned}$$

$$\bar{R}^2 = 0\cdot 54$$

where

y = response in grain yield (bushels per acre),
G = growing-season rainfall (inches),
N = nitrogen applied (pounds of nitrogen per acre),
S_m = soil moisture content at 15 atmospheres (per cent),
S_n = initial soil nitrate (p.p.m. of nitrogen).

Respective values of Student's t are shown in parentheses. For the purpose of our expository example we abstract from the difficulties related to the algebraic specification of, and variance not explained by, the chosen response function. The assumed relevant prices of y and N are \$1.10 per bushel and 12·5 cents per pound respectively.[6]

A feature of the function is the significant interaction between response to nitrogen and rainfall. This interaction is a necessary condition for additional information on rainfall to have any significant economic value. The third-order term (NGS_n) apparently accounts for the significant interaction between nitrogen response and site and season reported by Russell [9, p. 459].

In this response process, the x_i are represented by N, the x_j by S_m and S_n, and the x_k by G. Thus for any combination of values of N, S_m and S_n, y is determined by the variable G.

4 We are grateful to Dr E. G. Bowen, Chief of the Division of Radio Physics, C.S.I.R.O. for allowing us to use data from the predictor he is developing.

5 Data [9, pp. 455-457] on nitrogen response were recorded at 16 widely scattered locations in South Australia, varying in average annual rainfall from 11·5 to 19·9 inches. Fifty-two site-season combinations are represented in the experiments in which nitrogen was applied at 0, 11·5, 23 and 46 lb. per acre. Soil measurements were taken at sowing time and rainfall recorded over the growing season of May to October.

6 These prices were approximately the present value of wheat net of harvest and selling costs and the marginal cost of applied nitrogen respectively at the time of the study. Subsequently, nitrogen price has declined substantially and will probably continue to do so for some time.This price fall implies slight reductions in the computed values of the predictions reported herein.

Historical rainfall data for a specific location can be used to estimate a continuous probability distribution for G, which is denoted as the prior distribution, $h_o(G)$. Using this distribution we can compute the level of nitrogen, $N_o{}^*$, which maximizes expected profit. In decision theory jargon, $N_o{}^*$ is the prior optimal action. Expected profit is maximized when

$$\begin{aligned} E(\partial y/\partial N) &= E(p_N/p_y) \\ &= p_N/p_y, \end{aligned}$$

where, for the prior case,

$$E(\partial y/\partial N) = \int (\partial y/\partial N) h_o(G) dG.$$

As an example of this computation, for the location[7] considered in this paper, when $S_m = 3$ per cent and $S_n = 1$ p.p.m., expected profit is maximized when $N = N_o{}^* = 18 \cdot 1$ lb. per acre.

We turn now to considering how additional information on rainfall can influence optimal rates of fertilizer, and, through measurement of the consequent effects on profits, we will show how the value of additional information can be assessed. Additional probabilistic information is combined with the prior distribution to give posterior distributions.

Posterior Distributions of Annual Rainfall

Additional information on rainfall is provided by a predictor of trends (i.e. upward or downward) in annual rainfall from one year to the next. The amount of additional information on rainfall in any prediction is determined by the direction of the predicted trend and the rainfall in the previous year. For example, a prediction of an upward trend following a year of very low rainfall is not very informative.

The posterior distributions of annual rainfall were determined by comparing observed rainfall with the prediction for each year over a period of years. Six predictions denoted by $[P_k;\ k = 1, 2, \ldots, 6]$, are considered in this analysis. These include three predictions of a downward trend when rainfall in the preceding year is in the first $(k = 1)$, the second or third $(k = 2)$, or the fifth $(k = 3)$ deciles respectively; and three predictions of an upward trend when rainfall in the preceding year is in the fifth $(k = 4)$, the eighth or ninth $(k = 5)$, or the tenth $(k = 6)$ deciles respectively. The first decile is the lower end of the range of the distribution of rainfall. Relative frequency distributions of rainfall conditional on each type of prediction were then estimated. Since there was no evidence that the accuracy of the predictions varied significantly over the Australian continent, rainfall data expressed in deciles for 90 centres were pooled to estimate posterior distributions of annual rainfall measured in deciles. These posterior distributions were then converted to annual rainfall measured in inches for the specific location, and are denoted by $p_k(A_j)$ and the subscript k indicates that each distribution is conditional on a prediction P_k.

Posterior Distribution of Growing-Season Rainfall

Although the rainfall predictor is directly relevant to annual rainfall, it is also relevant to several other variables. In this response process, because growing-season rainfall is both the logical measure of rainfall and the measure recorded by Russell, posterior distributions of growing-

[7] Eurelia, S.A.

season rainfall were required. To establish a quantitative relationship between growing-season rainfall, G, and annual rainfall, A, these were each classified into six intervals to give the sets $[G_i;\ i = 1, 2, \ldots, 6]$ and $[A_j;\ j = 1, 2, \ldots, 6]$. A matrix of conditional probabilities $[p(G_i|A_j);\ i, j = 1, 2, \ldots, 6]$ was estimated from historical rainfall data, making necessary adjustments for finiteness [10]. If $p_k(A_j)$ are the posterior probabilities of annual rainfall, then the posterior probabilities of growing-season rainfall $p_k(G_i)$ are calculated from

$$p_k(G_i) = \sum_{j=1}^{6} p(G_i|A_j)p_k(A_j).$$

These discrete distributions were smoothed by plotting cumulative distributions, denoted $H(G)$, in a manner following that of Schlaifer [10, pp. 290-299] and are illustrated in Figure 1.

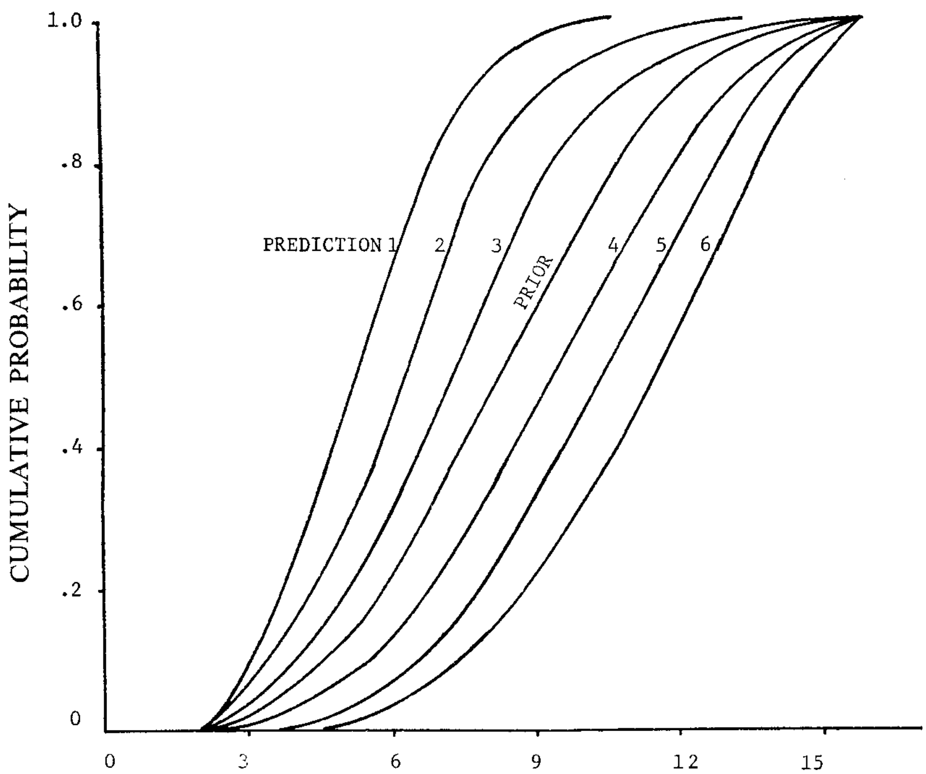

FIG. 1—Cumulative posterior probability distributions of growing-season rainfall given the predictions, compared with the prior distribution.

Fitting Functions to the Probability Distributions

Because standard distributions such as the normal and lognormal did not provide a satisfactory description of the posterior distributions, a method of polynomial approximation was developed. The cumulative distribution was segmented and each segment approximated by a polynomial function. Satisfactory approximations were obtained by segmenting the cumulative distribution, $H(G)$, about $H(G) = 0{\cdot}5$. Cubic functions were fitted to eight co-ordinates of each segment. In nearly all cases this procedure gave a coefficient of multiple determination greater than 0·99. Estimated probability density functions were obtained by differentiating the cumulative functions. A probability density function, $h(G)$, thus consists of two segments defined as

$$h_1(G) = d[H_1(G)]/dG, \ a \leqslant G \leqslant b$$

and

$$h_2(G) = d[H_2(G)]/dG, \ b \leqslant G \leqslant c.$$

This procedure was used to obtain the posterior distributions $[h_{1k}(G), h_{2k}(G); k = 1, 2, \ldots, 6]$ for the predictions $[P_k; k = 1, 2, \ldots, 6]$. In each case the area under the density function was very close to unity, indicating a satisfactory approximation.

In order to explore the potential for improvement in the rainfall predictor, we also examined the following hypothetical predictors:

(i) predictor of trends in growing-season rainfall analogous to the annual rainfall predictor;
(ii) perfect predictor of trends in growing-season rainfall;
(iii) perfect predictor of annual rainfall; and
(iv) perfect predictor of growing-season rainfall.

Computing the Value of Information

In this analysis, the decision maker is assumed to be indifferent to the riskiness of various outcomes in decisions on fertilizer use. Since the monetary returns from the use of nitrogen fertilizer form only a small proportion of total returns, this may not be an unreasonable assumption. Thus, in effect, the relevant segment of the decision maker's utility function is approximated by a linear function. This simplifying assumption enables us to ignore the variance and higher moments of y. Appropriate analysis for a decision maker strongly averse to risk would involve determining strategies which maximize expected utility—an analysis made very difficult by the general absence of data on variability over time as it relates to varying input rates.

Analogously to the prior case, expected profit for a given level of soil moisture and initial nitrate is maximized when

$$E[\partial y/\partial N] = p_N/p_y,$$

where

$$E[\partial y/\partial N] = \int_a^b (\partial y/\partial N) h_{1k}(G)\,dG + \int_b^c (\partial y/\partial N) h_{2k}(G)\,dG.$$

One solution for the prior distribution $h_o(G)$ was noted above. In a similar manner, a Bayesian strategy $[N_k{}^*; k = 1, 2, \ldots, 6]$ relating to the set of predictions $[P_k; k = 1, 2, \ldots, 6]$ can be computed. The results are displayed in Table 1.

When fixed costs are ignored, profit for any combination of the productive factors is given by

$$\pi = p_y y - p_N N,$$

and expected profit μ_k for the prediction P_k is

$$\begin{aligned}\mu_k &= E(\pi)\\ &= \int_a^b \pi h_{1k}(G)\,dG + \int_b^c \pi h_{2k}(G)\,dG,\end{aligned}$$

TABLE I

Bayesian Strategies for Six Annual Rainfall Predictions Compared with the Prior Actions

Soil nitrate	15 atmospheres soil moisture	Prior	Prediction 1	2	3	4	5	6
p.p.m.N	per cent	lb. N per acre						
1	3	18·1	4·0	8·4	13·3	25·7	31·7	36·2
3	3	11·9	0·0	3·7	7·8	18·4	23·5	27·4
5	3	6·7	0·0	0·0	2·3	11·1	13·4	18·6
1	6	13·3	0·0	3·7	8·5	20·9	26·9	31·4
3	6	7·1	0·0	0·0	3·0	13·6	18·7	22·6
5	6	1·0	0·0	0·0	0·0	6·3	10·6	13·8
1	9	8·5	0·0	0·0	3·7	16·1	22·1	26·7
3	9	2·3	0·0	0·0	0·0	8·8	13·9	17·9
5	9	0·0	0·0	0·0	0·0	1·5	5·8	9·0

where the expectation is over G since this is the probabilistic factor in π. Using these expected profit functions based on the posterior distributions, $h_k(G)$, the expected profits μ_k' and μ_k^* for inputs N_o^* and N_k^* respectively can be computed for each prediction P_k. The value, V_k, of the prediction P_k is then determined as

$$V_k = \mu_k^* - \mu_k'.$$

Values of six predictions of annual rainfall for a range of soil conditions are shown in Table 2.

TABLE 2

Values of Some Annual Rainfall Predictions

Soil nitrate	15 atmospheres soil moisture	Prediction 1	2	3	4	5	6
p.p.m.N	per cent	cents per acre					
1	3	33	15	3	9	31	56
3	3	24	11	2	7	23	41
5	3	13	7	2	4	15	28
1	6	33	15	3	9	31	56
3	6	20	11	2	7	23	41
5	6	3	2	0	4	15	28
1	9	28	15	3	9	31	56
3	9	8	5	2	7	23	41
5	9	0	0	0	0	5	14

Similarly, if q_k is the probability of the prediction P_k being made and q_o is the probability of *no* prediction being made (i.e. only prior information is used),[8] the maximum expected profit given the rainfall predictor is

$$\mu^* = \Sigma q_k u_k^*. \quad (k = 0, 1, \ldots, 6)$$

[8] For the predictions considered here,
$[q_0, q_1, q_2, \ldots, q_6] = [0{\cdot}398, 0{\cdot}011, 0{\cdot}036, 0{\cdot}254, 0{\cdot}254, 0{\cdot}036, 0{\cdot}011]$.

If μ_o^* is the expected profit given only prior information, the value, V, of the *predictor* is

$$V = \mu^* - \mu_o^*$$

Using this procedure, the values of the annual rainfall predictor and the various hypothetical predictors were computed for the same set of conditions and the results are tabulated in Table 3.

TABLE 3

Values of Various Rainfall Predictors

Soil nitrate	15 atmospheres soil moisture	Predictor of annual rainfall	Predictor of growing-season rainfall	Perfect predictor of growing-season rainfall trends	Perfect predictor of annual rainfall	Perfect predictor of growing-season rainfall
p.p.m.N	per cent	cents per acre				
1	3	5·7	7·9	16·0	21·8	36·0
3	3	4·2	5·8	11·2	15·6	26·0
5	3	2·8	3·5	7·4	10·0	15·0
1	6	5·7	7·9	15·7	21·4	34·0
3	6	4·2	5·6	10·7	14·9	22·0
5	6	2·0	2·4	5·2	7·1	10·0
1	9	5·6	7·6	15·2	20·4	31·0
3	9	3·8	4·5	8·7	11·7	17·0
5	9	0·3	0·5	1·9	2·3	5·0

Discussion

Implications for the Rainfall Predictor

Although the largest value of the annual rainfall predictor is only 5·7 cents per acre, aggregated over a large area the potential value in formulating strategies for nitrogen application may be quite substantial. How much of this potential would be realizable in practice depends on the extent to which farmers attempt to maximize expected profits in their decisions on fertilizer rates. As the predictions could apparently be derived and released at relatively small cost, there appears to be some economic justification for supplying such additional information on rainfall to farmers.

While the value of individual predictions could exceed 30 cents per acre, such valuable predictions would be made so rarely that the value of the predictor on average would be relatively low. The values for each prediction differ markedly with the measured characteristics of the soil. If regional patterns of soil nitrogen could be identified, areas of low nitrogen status could be defined in which the publicity of the predictions could most profitably be concentrated.

The values of various hypothetical predictors suggest scope for improvement of the annual rainfall predictor. Thus a significant increase in value could be obtained by a more accurate predictor of trends. However, the ultimate value of the predictor is limited by the fact that trend rather than an absolute measure of rainfall is the subject of prediction.

For example, where rainfall is high and a downward trend is predicted, very little additional information is provided. It is unlikely in such a case that all the information available to the forecaster is given in the prediction of a trend. The use of probabilistic forecasts of actual rainfall rather than trends seems feasible and would be more valuable.

The hypothetical predictor of growing-season rainfall indicates that some of the additional information on annual rainfall is lost when it is related to growing-season rainfall. It appears that in most agricultural processes, some categorization of rainfall other than annual would be a more appropriate basis for prediction.

General Implications of the Approach

For systems of prediction analogous to the one discussed here, final assessment of value can be made only after investigating the predictions *ex post* for a lengthy period. However, the decision-theoretic approach allows us to reach, *ex ante*, interim conclusions concerning the economic usefulness of a predictor.[9]

Our analysis has established that interaction between the controlled and uncertain inputs is necessary for 'uncertainty-reducing' information to have an economic value. In the nitrogen response example considered, the value of the rainfall predictor is not strongly influenced by the moisture-holding characteristics of the soil,[10] but decreases markedly as initial soil nitrate increases. These results can be explained by the fact that soil nitrate affects the interaction between rainfall and applied nitrogen through a third-order interaction term. Thus, in the generalized response function, $y = f(x_i, x_j, x_k)$, it is the interaction between the x_i and the x_k which determines whether information on the x_k has a positive value, but it is the interaction between all three types of input, along with the specification of the process, the prices, the prior distribution and the predictor which determines the magnitude of the value.

Much scope exists for integrating decision theory with a broad class of topics in production economics in which the central problem is assembling knowledge as a basis for action. Questions which might be considered in this framework range from deciding when to stop experimenting on a response process to incorporating probabilistic information on uncontrolled production factors such as weather and prices [2, Ch.4.]. We hope that the methodology developed in this paper might prove useful in considering many questions of the latter type.

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[9] A study of management strategies in the Californian raisin industry [6] provides an example of decision-theoretic analysis of short-term weather predictions.

[10] This results in part from the non-negative constraint on nitrogen inputs and in part from our simplistic model of rain-soil-plant water relations. More realistic models using a water balance approach have been used with success in analysis of crop response, e.g. [4].

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