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A STOCHASTIC MODEL FOR THE OPTIMAL REPLACEMENT OF RUBBER TREES

by

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Recognizing the lack of realism in optimal replacement analyses that assume constant prices and yield patterns over time, a stochastic model appropriate to rubber production is developed. Data drawn from Peninsular Malaysia are used to implement the model. The results suggest that efforts should be directed towards establishing bench mark maximum annuities as guides to more economic replanting decisions rather than emphasizing earlier replanting *per se*. The significance of price stabilization policies also becomes evident.

Introduction

Amid the very considerable literature on the optimal replacement of assets [7, 11, 12, 13, 14, 15, 23, 25] the inclusion of stochastic elements has generally concentrated on unintentional replacement [15]. This is a situation where an asset, be it a light bulb, a machine, a cow or a coconut tree, has unexpectedly 'died'. It is rare indeed to find a stochastic formulation of the *intentional* replacement problem. [But see 27]. Here the concern is with an asset which continues to produce, but an uncertain amount at an uncertain price and the question posed is: when should it be replaced by a new asset whose future income stream is also uncertain?

This is a very real problem in the world's natural-rubber industry with its vast area (Table 1), often operated by smallholders, widely fluctuating prices (Figure 1) and continuously changing technology. With replanting undertaken approximately every thirty years some three per cent of the rubber area shown in Table 1 should be replanted each year. Thus total replantings amount to at least one hundred thousand hectares per annum.

This paper starts with a discussion of certain key elements of the deterministic optimal replacement model, some of which have not been generally recognized in the literature. It then proceeds to modify the basic model by the inclusion of stochastic elements appropriate to rubber production. The stochastic model is then applied to a set of data obtained from Peninsular Malaysia. The purpose is to follow earlier work [11] to see the manner in which the conclusions of the deterministic model should be modified in the light of this attempt to mimic the real rubber production situation more closely.

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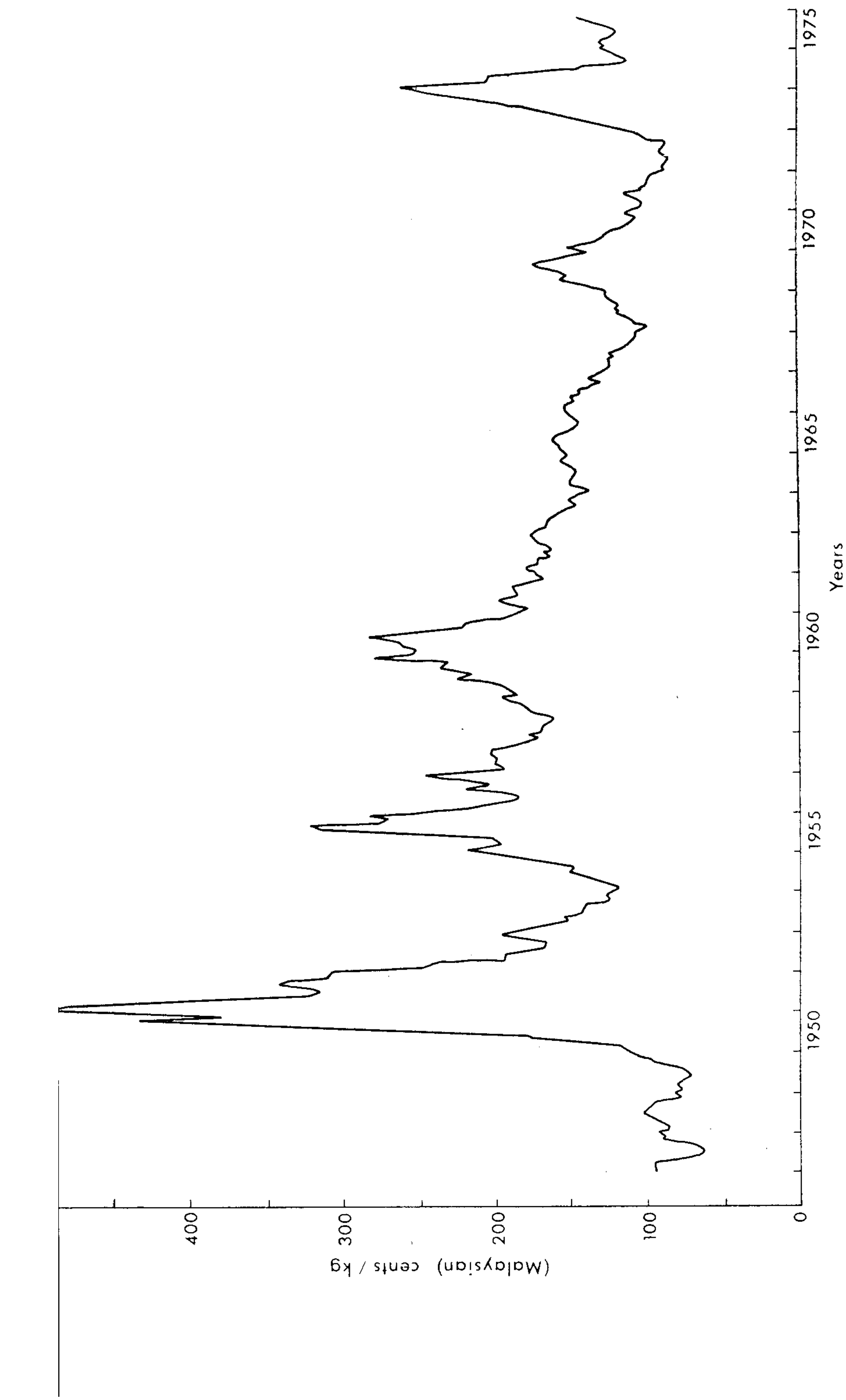


FIGURE 1.
Rubber prices, 1947-1975*

Source: Malaysian Rubber Exchange.
* The prices illustrated are for Ribbed Smoked Sheet rubber, Grade 1 (RSS1) on the Malaysian Rubber Exchange.
Sources: *World Rubber Statistics Handbook 1946-1970* (International Rubber Study Group, London 1974). *Rubber Statistical Bulletin* (International Rubber Study Group) Various issues. Table of Natural rubber prices.

TABLE 1

Area Under Plantation Rubber (Hectare)

Territory	End of	Estates†	Small holdings	Total
Peninsular Malaysia	1972	610,372	1,092,000	1,702,500
Rest of Malaysia	1972	37,105	253,425	290,530
Indonesia	1965	520,000	1,467,000	1,987,000
Sri Lanka	1971	106,525	123,334	229,859
Thailand	1965	—	735,000*	735,000
Vietnam (South)	1965	75,297	25,000*	100,000*
India	1974	57,294	163,971	221,265
Burma	1964	35,000*	27,500	62,517
Philippines	1970	21,800	—	21,800
Papua New Giunea	1970	13,740	—	13,740
Zaire	1959	67,586	25,450	93,036
Liberia	1973	76,700	43,100	119,800
Nigeria	1965	31,895	207,500*	240,000*
Cameroon Fed. Rep.	1965	20,500	600	21,100
Ivory Coast	1970	12,050	—	12,050
Others‡	(all 1965)	70,346	7,170	77,566

* Figures are estimated or partly estimated.

† Estate areas refer to holdings of 40 hectares (100 acres) and over except for Vietnam where they refer to holdings of over 500 hectares.

‡ Excludes mini producers or countries whose data refer to only the 1940s or early 1950s. e.g. Fiji is listed as having 300 hectares in 1950.

Sources: Adapted from *Rubber Statistical Bulletin,* Sept. 1975, Table 54, p. 48.

The Deterministic Model

The principles of optimal asset replacement are most clearly stated by Perrin [23, 1972] and for ease of exposition and comparison we shall follow his notation and terminology. With the initial planting of a perennial crop on new ground we consider simply this 'challenger' for the use of the resources. For a single cycle of the asset the present value of the stream of earnings is given by

$$C(o,s,1) = \int_0^s R(t)e^{-\rho t}dt + M(s)e^{-\rho s} \tag{1}$$

where $C(o,s,m)$ is the present value of the stream of residual earnings from a Challenger to be initiated in year 'o' and replaced at age 's' by a series of 'm' challengers. The income stream is made up of a flow of net earnings $R(t)$ which, in the case of most perennials, is negative during the early years, and a salvage value $M(s)$. ρ is the rate at which these earnings are discounted. This is the most general way of stating the replacement problem since it allows for *multipoint* outputs (apples, wool, milk, coconuts, etc) in $R(t)$, *point* outputs (forests, wine storage, beef) in $M(s)$ or a combination of both. Thus in the specific case considered in this paper we are concerned with the multipoint flow of earnings from rubber latex and not with the potential point output of the salvage value of the wood. The actual flow of earnings from a given type (clone) of rubber tree is a complex function of age, the tapping system used, product prices and the costs of production. This is discussed fully in a later section of the paper.

For the case of a series of identical cycles of an asset the 'self replacement' problem is to find the date 's' which maximizes the value of the entire income stream.

The stream is:

(2) $C(o,s,\infty) = C(o,s,1)+e^{-\rho s}C(o,s,1)+e^{-\rho 2s}C(o,s,1)+ \dots .$

This series reduces to (3) as the number of cycles tends towards infinity.[1]

$$(3) \qquad C(o,s,\infty) = \frac{1}{1-e^{-\rho s}}C(o,s,1)$$

which is the expression for the present value of a perpetual annuity received every 's' years.

Maximising (3) with respect to the replacement date 's' we get the following first order condition

$$(4.1) \qquad [R(s)+M'(s)]e^{-\rho s} = \left[\int_0^s R(t)e^{-\rho t}dt+M(s)\right]\frac{\rho}{e^{\rho t}-1}$$

where $M'(s)$ is the first derivative of the salvage function which continues to be included for completeness. Equation (4.1) states that the discounted marginal returns must equal the annuity formed from the discounted total flow of earnings from the asset plus the salvage value. This general principle of replacement is therefore equivalent to saying that we seek that date at which the a.verage annual earnings (i.e. the annuity) are maximised (which is where marginal and average earnings are equal).

Using the discrete interest rate 'r', the annually compounded analog to the sinking fund annuity formulae on the right-hand side of (4.1) is possibly more familiar[2]:

$$(4.2) \qquad [R(s)+\Delta M(s)](1+r)^{-s} = \left[\sum_0^s R(t)(1+r)^{-t}+M(s)\right]\frac{r}{(1+r)^s-1}$$

Collecting all the discount terms onto the right-hand side we have the requirement that the annual returns plus the *change* in the salvage value must equal the annuity formed by the sum of the discounted annual earnings plus the salvage value, but since in reality the equality condition is unlikely to hold, the decision rule is best expressed in the form of an inequality:

$$(4.3) \qquad [R(s)+\Delta M(s)] > \left[\sum_0^s R(t)(1+r)^{-t}+M(s)\right]\frac{r(1+r)^s}{(1+r)^s-1}$$
$$> [R(s+1)+\Delta M(s+1)]$$

It is best expressed in this form because there is no algorithm by which one can solve equation 4.1 for the optimal replacement date. To obtain s, one must iteratively proceed to check the inequalities in 4.3 in each time period.

Applying the decision rule in (4.3) to rubber data from Sri Lanka and using discrete parametric changes in yield curves, product prices and interest rates leads to a number of conclusions [see 11 and 15]

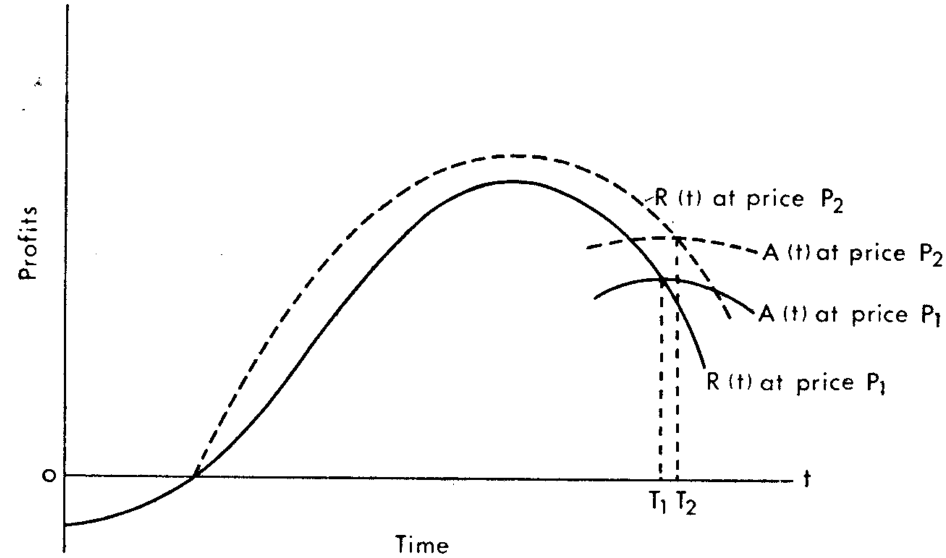
[1] There may be an objection to considering an infinite number of cycles but aside from the greatly simplified mathematics, this ruse is not unreasonable in the case of long lived crops like rubber. Here, since each cycle is some 30 years in length, the additional terms in the series become of little significance after the second cycle with any reasonable interest rate.

[2] The annuity formulae given on the right hand side of (4.1) or (4.2) is that given in most actuarial tables. Perrin [23] chose to combine all the discount terms on the right hand side of the equation and hence he has

$$-\ \rho/(1-e^{-\rho s}) \text{ which equals } \rho e^{\rho s}/e^{\rho s-1}$$

which has as its discrete equivalent the term between the inequalities in (4.3).

which can be intuitively grasped from Figures 2 and 3 which show the annual net revenue curves as R(t) and the associated annuity curves as A(t)[3] (i.e. the term between the inequalities in (4.3)). These conclusions are very general and are not tied either to the specific data or to Sri Lanka, but will apply to most commercial natural rubber producing situations and indeed to many other multipoint output systems as well.



R (t) = annual net returns

A (t) = annuity earned if replacement is made in year t

FIGURE 2.

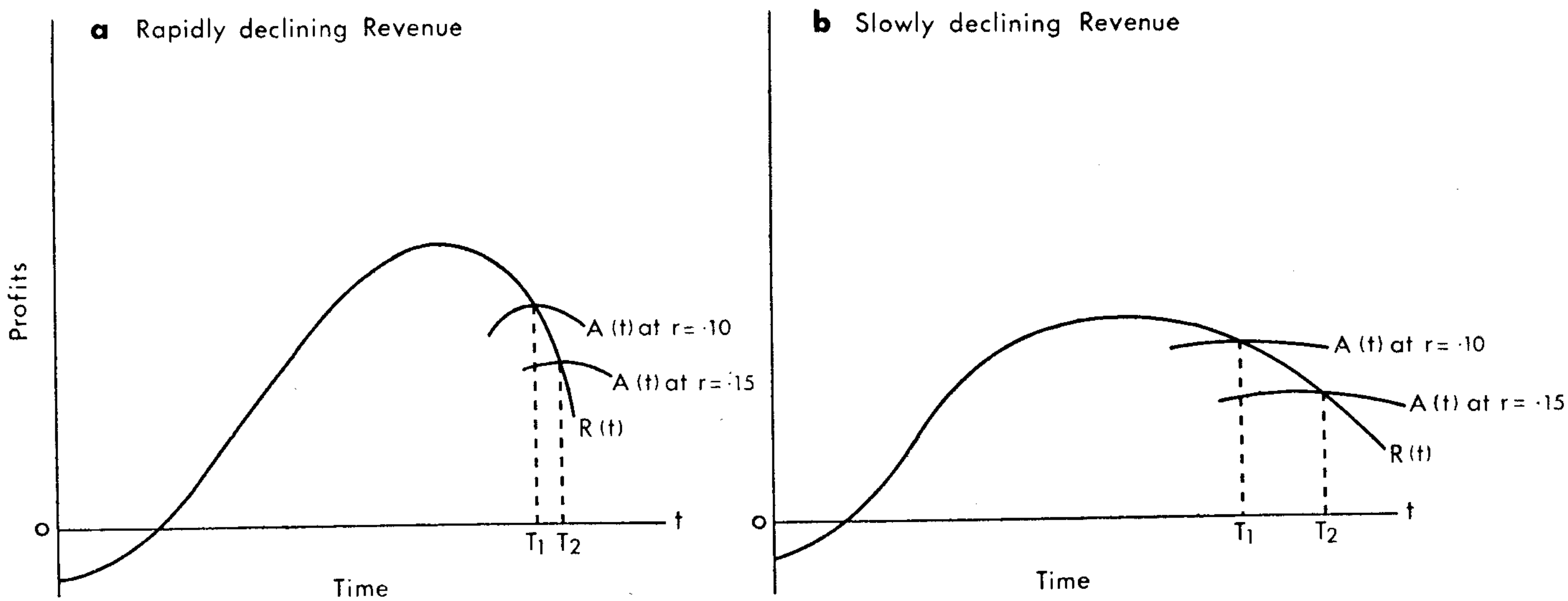
Effects of Price Changes on Optimal Replacement Dates(T)

(1) Changes in rubber latex prices have little influence on the optimal replacement age since the annual returns and annuities move up and down together. Figure 2.

(2) Increases in interest rates will lengthen the optimal cycle because although the annuity expression increases somewhat, the discount term inside the square brackets in (4.3) causes the overall value of the annuity to fall. The increase in interest rates also causes the annuity curve to be less peaked so that the choice of suboptimal dates two or three years on either side of the optimal date has a negligible impact on the value of the annuity. Figure 3(a) and (b).

(3) The sensitivity of the optimal date to interest rate changes is determined by the slope of revenue function after it starts to decline. Thus the case depicted in Figure 3(b) will be more sensitive than that in Figure 3(a). However, while Figure 3(b) with its more gradual decline in annual returns shows greater sensitivity to interest rate changes, the annuity curve is flatter

[3] In what follows salvage values are ignored. But see [11] where it is shown that the optimal replacement dates with respect to salvage values move in the opposite direction to those of annual returns.



R (t) = annual net returns
A (t) = annuity earned if replacement is made in year t

FIGURE 3.
Effects of Interest rate Changes on Optimal Replacement Dates

about the maximum and hence the penalty for a sub-optimal replacement decision will be less than in the case shown in Figure 3(a).

(4) Most forms of disembodied technological change (Ethrel stimulation, intensified tapping) not only raise yields but result in more rapid declines during the latter years hence reducing the sensitivity to interest rate changes.

(5) Embodied technical change (more precocious clones and a shorter period of immaturity) tends to raise yields in the early years and raises annuity values so that replanting dates are earlier.[4]

The Stochastic Model

In spite of the interesting results that come from incremental changes in key parameters of the deterministic model there is a serious lack of realism in the underlying assumptions of a fixed yield pattern and of constant prices over the full period of analysis. For these reasons Ward and Faris [27, 1968] developed a stochastic model for the optimal replacement of plum trees. They used a Markov Chain process with a transition probability matrix to define the move from one age-yield state to the next. Dynamic programming was used to define the optimum policy of keeping or replacing the trees each year. Ward and Faris adopted this approach because the yields of plums in any particular year are determined not only by the age of the trees and the current weather but also by the yield of the previous year. This is a feature common to many perennial crops, most notoriously in coffee, cloves, apricots and nectarines, but absent in crops such as tea and rubber which, around their age specific yield patterns, can provide a series of consecutive 'good' or 'bad' years depending on weather patterns. That is to say, for rubber there is an equal probability that a good year or a bad year will follow a good year.

The random fluctuation of rubber yields around the expected yield curve allows for a more straightforward approach to introducing stochastic elements into the optimal replacement problem for such crops. The annual net revenue $[R(t)]$ must be broken down into its component parts and the stochastic elements identified. This will first be done in general terms before modelling a specific Malaysian situation.

In the deterministic case we have:

$$R(t) = p.\ Y(t) - C(t) \tag{6}$$

where $R(t)$ is still the net annual revenue determined by the difference between gross revenue $[p.\ Y(t)]$ and total costs $[C(t)]$. $Y(t)$ is the yield of latex in year 't' as defined by the specified yield curve for a particular clone being exploited using a particular system of tapping; 'p' is the latex price (assumed constant), and $C(t)$ is the tapping and collection costs.

In real life all three variables on the right hand side vary from one time period to the next. Thus the actual yield (denoted in what follows as $Y(t)$) varies from the mean (or expected) yield pattern (shown as

[4] Thus when a new Challenger with a higher annuity comes between the inequalities in (4.3), for the condition to hold the Defender (the current stand of trees) will be replaced at a higher and earlier yield.

$\overline{Y}(t)$) by some random element. Since the expected yield changes in a particular pattern as the tree grows, we would expect the random element [$u_y(t)$] to be time specific. Thus we can write:

$$Y(t) = \overline{Y}(t) + u_y(t) \tag{7}$$

As was shown in Figure 1, rubber prices have fluctuated widely since World War II. This situation can be modelled in a number of ways. For example, it could be of the general form:

$$P(t) = a + bt + u_p \tag{8}$$

with 'a' being the current price, 'b' the price trend and u_p being the random variation in price which, in contrast to yields, will not be time specific.

Fluctuations in yields and prices have an obvious direct impact on gross revenue but, given any conventional distinction between fixed and variable costs, yield fluctuations will also cause random changes in costs. Furthermore, as will be described below, in Malaysia the wage rates in the estate sector are linked directly to the rubber price. Thus yield and price fluctuations cause fluctuations in net revenues through their direct influence on gross revenue and their indirect effects on the costs of production.

It was noted earlier that there is no algorithm for solving the deterministic model so that a solution has to be obtained using an iterative procedure. Similarly, the specification of the random element in equation (7) as being time specific and the possible introduction of floor and/or ceiling prices into (8) preclude the analytical derivation of a probability distribution of net returns.[5] Thus again, an iterative procedure must be used to derive the distribution of maximum annuities and of optimal replacement dates. A Monte Carlo procedure of running multiple iterations of a specific problem with random elements is ideal for this type of situation.

Implementation of the Model

The above description of the model presents the bare essentials of the approach. This section describes in more detail the actual functional forms used and the Peninsular Malaysian estate data upon which they are based. It deals in turn with yields, prices and costs of production.

Yields: Yield data have been taken from Lim's very detailed study (Lim, [18, 1976]).[6] A commercial rubber tree (*Hevea Braziliensis*) can be said to pass through four phases during its life: a period of immaturity (usually six years), tapping of virgin bark (ten years under standard practices), tapping of first renewal bark (another ten years) and finally the tapping of second renewal bark which can extend production a further ten years or more.[7] These phases are shown in

[5] As in Lim [18], Appendix C or, more relevantly, in Lin [21] pp 89-91.

[6] Lim's analysis of rubber yield curves is by far the most detailed available. It should be noted however that the analysis fails to make explicit distinction between time series and cross-section data. This should be done for perennial crops [10]. It is unlikely that there is any consistent bias in his estimated coefficients but there is little doubt that his regression equations would fit better if the distinction were made.

[7] The tapping system can be changed in subsequent tests but not in a functional or random fashion year by year. Evidence to date suggests negligible short run price elasticities of supply for rubber [6, 20].

Figure 4. Phase 1 is described by a quadratic function:[8]

(9.1) $\overline{Y}(t) = a + b(t-6) + c(t-6)^2 \qquad (t=6, \ldots, 15)$

Phase 2 is a linear transformation of the yields in the first phase:

(9.2) $\overline{Y}(t) = A + B\overline{Y}_{t-10} \qquad (t = 16, \ldots 25)$

Phase 3 is a linear decline from the yield in year 25:

(9.3) $\overline{Y}(t) = \overline{Y}_{26} - \beta(t - 26) \qquad (t = 26, \ldots 39)$

The lack of distinction in Lim and elsewhere (e.g. in [24]) on the variability in yields attributable to date and location for a given age

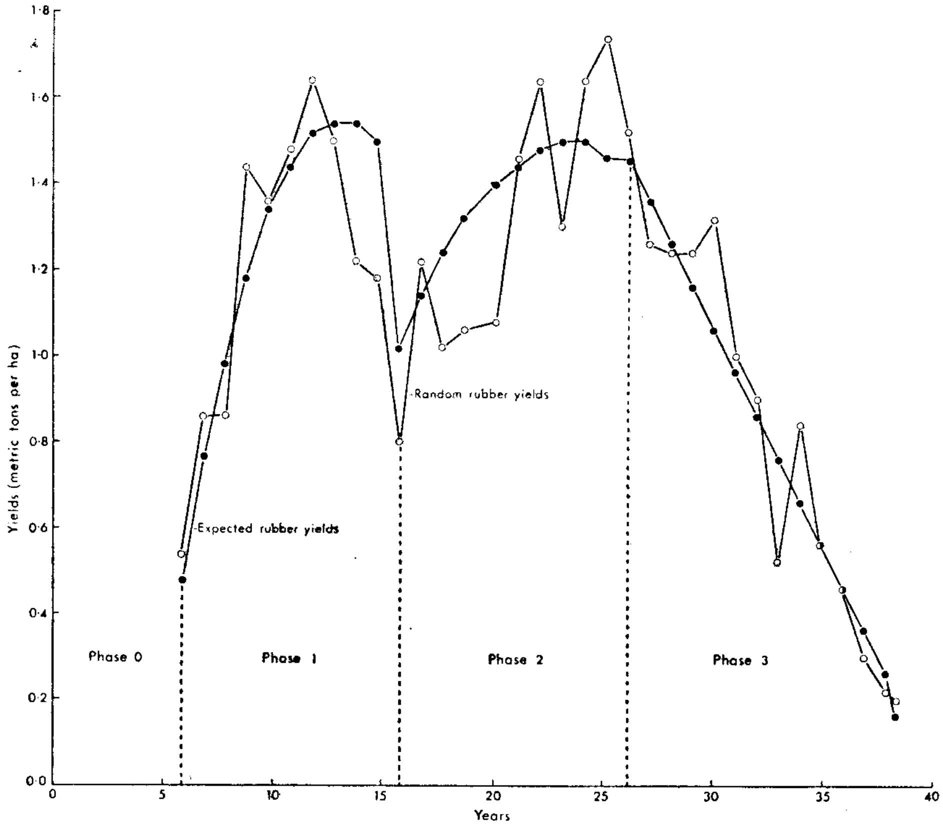


FIGURE 4.
Expected and Random Yields by Tapping Phase.

posed a problem. It is most likely that the distribution of age specific yields is positively skewed with yields dropping further from the mean in bad years than they would rise above mean yields in good years. However, until more objective data become available (e.g. along the lines of Day's analysis of field crops [9, 1965]) it was decided to proceed with an assumption that yields vary from year to year with a constant coefficient of variation and with a normal distribution about the mean. Correspondence with the Rubber Research Institute of Malaysia (RRIM) has confirmed that the standard deviation increases as yields increase and at a rate such that 'there is no evidence of the coefficient of variation being inversely related to yield'.[9]

[8] The immature period (Phase 0) runs for the six years 0 to 5.
[9] Personal communication, Rubber Research Institute of Malaysia.

Hence we assume that the random variation in equation 7 takes the form:

(10) $$u^1_y(t) = s^1_y(t).E$$

(where $E \sim N\ (0, 1)$
and $s^1_y(t)$ is the standard deviation of yields in year t. Given the assumption that the coefficient of variation is some constant percent (V^1_y) we have

(11) $$s^1_y(t) = V^1_y.\overline{Y}(t)$$

Locational variability, that is uncertainty as to whether the specific clone will perform as well on a particular block or holding as advised by the RRIM, is allowed for by making the intercept term of the quadratic equation for Phase 1 random ('a' in equation (9.1)). This provision means that the yield curve in each iteration can shift in a parallel fashion above and below the mean yield curve. This has a similar effect to allowing the opening of the first tapping panel to be random. Thus the final form of the annual yield equation has two random elements:

(12) $$Y(t) = \overline{Y}(t) + u^1_y(t) + u^2_y$$

where $u^2_y = s^2_y.E$ with a similar assumption of a constant coefficient of variation (V^2_y percent)[10], so

(13) $$s^2_y = V^2_y.a$$

where 'a' is the intercept in equation (9.1).

Prices: Two features are particularly evident from an examination of natural rubber prices for the post-war period. First, that there is some evidence of a downward trend and, secondly, that while the amplitude of the fluctuations has been considerable in the past, it has been less in recent years.[11] In his analysis of monthly rubber prices Allen [1] found a close conformity of the prices to a log-normal distribution which, he suggests, reinforces 'the point that price is determined by a large number of independent factors (market forces). It is therefore unrealistic to develop 'econometric models' which purport to predict single valued prices . . . Any realistic forecast would need to take acount of *probabilities,* which means that one must forecast not only mean prices but also the scatter. This would be a formidable exercise and it is by no means certain that there is any rational procedure for carrying it out'. [1. p. 124]. The Monte Carlo technique used here is a rational procedure for generating such a scatter of prices. Taking into account the declining trend of recent years, the somewhat reduced amplitude of fluctuations, the log-normal distribution of prices and recent average prices, the following rubber price equation was used in the computer runs of the model.

(14) $$p(t) = 1.40e^{(-0.01t\ +\ 0.30E)}$$

where $p(t)$ is the price of rubber in Malaysian dollars (M$) per kilo-

[10] E was again taken from a normal distribution although a skewed distribution in favour of yields less than research yields would probably mimic the real world more correctly.

[11] That is, the fluctuations from 1961 to the end of 1972 were more moderate compared to the period 1947 to 1960. See Figure 1. The period since 1973 needs more careful analysis of real price effects. For some comments on the impact of rubber price fluctuations on the Malaysian economy see Kasper [16, p. 13] and for a recent comment in the financial press see Davenport [8, 1976, pp. 44-47]

gram at time t and prices are assumed to be declining at the rate of one per cent per annum. $E \sim N(0, 1)$.

There is the provision for a minimum floor price. This was set for the iterations reported here at the pessimistic level of 80 Malaysian cents per kilogram and is noticeable in Figure 5.

Costs: Cost data were built up from the Rubber Research Institute of Malaysia's *Guide to Estate Management* [24] and from Lim's study [18]. As such, the data refers very specificially to commercial rubber estates in Peninsular Malaysia. Wage rates are related to the rubber stand density, the number of tapping days per annum, the size of the tapping task, the latex and scrap yield, and the Malaysian rubber price.[12] Thus wages, and tapping costs in particular, are random since they are functions of both of the initial random variables. In a similar manner, while the constant cess per kilogram to finance Malaysian rubber research is affected by yields, revenue and reserve fund taxes are step functions of the rubber price. Marketing and processing costs per hectare are determined by yield and quality of production in the form of the proportion of total rubber that is in the form of 'scrap'.

Fertilizer, weeding sprays and Ethrel stimulant applications are functions of time rather than specific yield levels. This is to say that, following RRIM recommendations, these inputs are applied at specific times during the productive phases noted earlier. In the present study the fertilizer price is fixed and can only be changed parametrically while ideally the quantity of fertilizer applied should not only be a function of time (as a surrogate for the rate of response), but also of the input/output price ratio.

Discussion of Results

Runs of the model have been made taking the general yield characteristics of RRIM clones 607, 513 and 605 with parametric changes of the coefficient of variation of yields (V^1_y) and of the rate of decline in yields in Phase 3[13], (i.e. changes in the coefficient β in equation (9.3)). The number of iterations of this Monte Carlo type model was set at 200, the discount rate at 10 per cent and the prices were determined by equation (14).

Figures 4 and 5 show the pattern of mean and random yields (for a specific clonal variety) and prices over a forty year period for one particular iteration. The resulting net revenue and annuity curves are

[12] Davenport [8] gives a recent and readable account of some of these complexities. The actual equations relating wage rates to these variables are presented in Appendix 1 of [24].

[13] There are three major output stages in the computer programming of this model. The first stage prints out yields, random yields, prices, costs (with a detailed breakdown by labour, administration, fertiliser and Ethrel stimulants, marketing and processing, and taxes), net revenues and amortised values. The key elements are plotted in Figures 4, 5 and 6. This form of output was useful in the verification of the model but is suppressed when many iterations are required. Secondly, an iteration matrix is printed: this gives the optimal year and the value of the annuity in that year for each iteration. In addition the annuities for five other arbitrarily chosen years are printed together with the randomly selected initial intercept of the yield function. Finally, this output is fed through the histogram subroutines of the *Statistical Package for the Social Sciences* (SPSS).

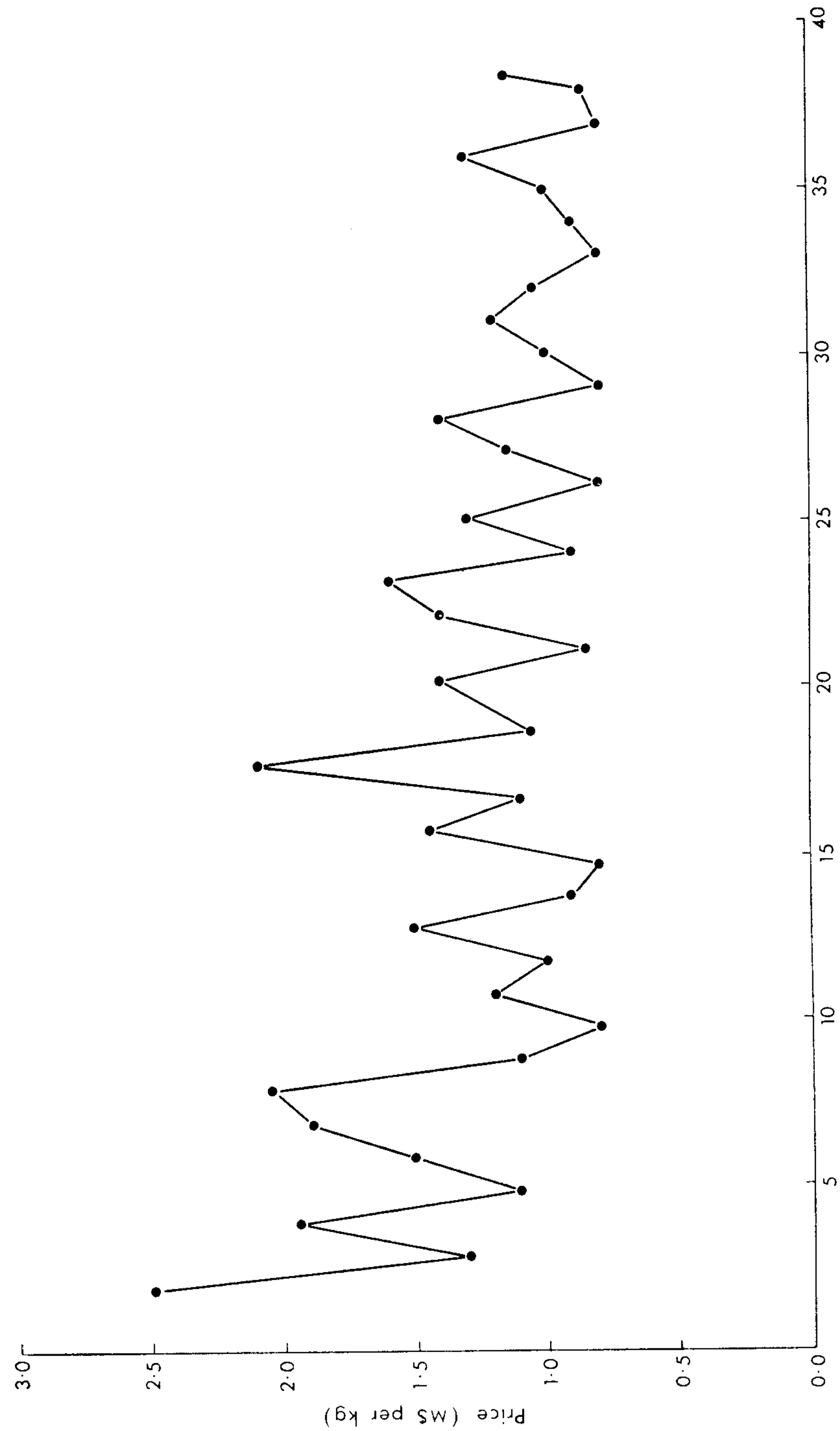


FIGURE 5.
Random Rubber Prices.

reproduced in Figure 6. Optimal replacement is in year 32 with a maximum annity of M$153.55 per year.[14]

From the iterations made with this model mean optimum replacement dates (T_0) and mean maximum annuity values (A_0) are obtained with their respective standard deviations (s_t and s_a). Because of the flatness anticipated in the annuity function, five alternative arbitrary dates were also selected and the means and standard deviations of the annuities on these dates were recorded.

The first test of the model was to check on the sensitivity of the optimal date as annual yields became more variable. Recent studies at the RRIM on three different clones have indicated a coefficient of variation of between 12.5 and 17.6 per cent.[15] Runs were made with V^1_y = 10, 15 and 20 per cent and with β set at a decline of 100 kg per annum. The results are reported in Table 2. There is very little variation about the mean optimal year (32) even when the variability in yields is increased substantially. The coefficient of variation of the optimal year (14) only increases from 4·59 per cent to 5 per cent while the mean optimal year itself remains at 32 years.

TABLE 2

Optimal Dates and Annuities for Different Coefficients of Variation in Yield (V^1_y)

Item	Coefficient of Variation in Yield %		
	10	15	20
Mean Optimal Year (T_0)	32·05	31·95	31·95
Standard Deviation (s_t)	1·47	1·47	1·60
$V_T\% = s_t/T_0$	4·59	4·60	5·01
Mean Optimal Annuity (A_0) $	157·18	156·81	156·10
Standard Deviation(s_a) $	53·48	56·78	61·14
$V_a\% = s_a/A_0$	34·03	36·20	39·17
Mean Annuities and Standard Deviations for Year:			
30	154·94 (53·80)	154·52 (57·09)	153·71 (61·45)
31	156·05 (53·60)	155·60 (56·88)	154·77 (61·22)
32	156·49 (53·41)	156·05 (56·71)	155·23 (61·07)
33	156·41 (53·25)	155·95 (56·52)	155·11 (60·83)
34	155·94 (52·94)	155·47 (56·19)	154·62 (60·49)

While the robustness of the optimal date to wide variations in yields (and prices) is an interesting and useful result for certain extension purposes, the extreme variability in optimum annuity values highlights the risks associated with rubber (coefficient of variation of maximum annuities, $V_a > 33$ per cent). Although, by definition, the optimal replacement date dominates the alternative arbitrary dates in the sense of first-degree stochastic dominance (FSD) [2], these annuity values are not significantly different from the mean optimal annuity.

[14] The next iteration for this same clone resulted in a maximum annuity of M$204.59 obtained in year 30.

[15] Personal communication, Rubber Research Institute, Malaysia.

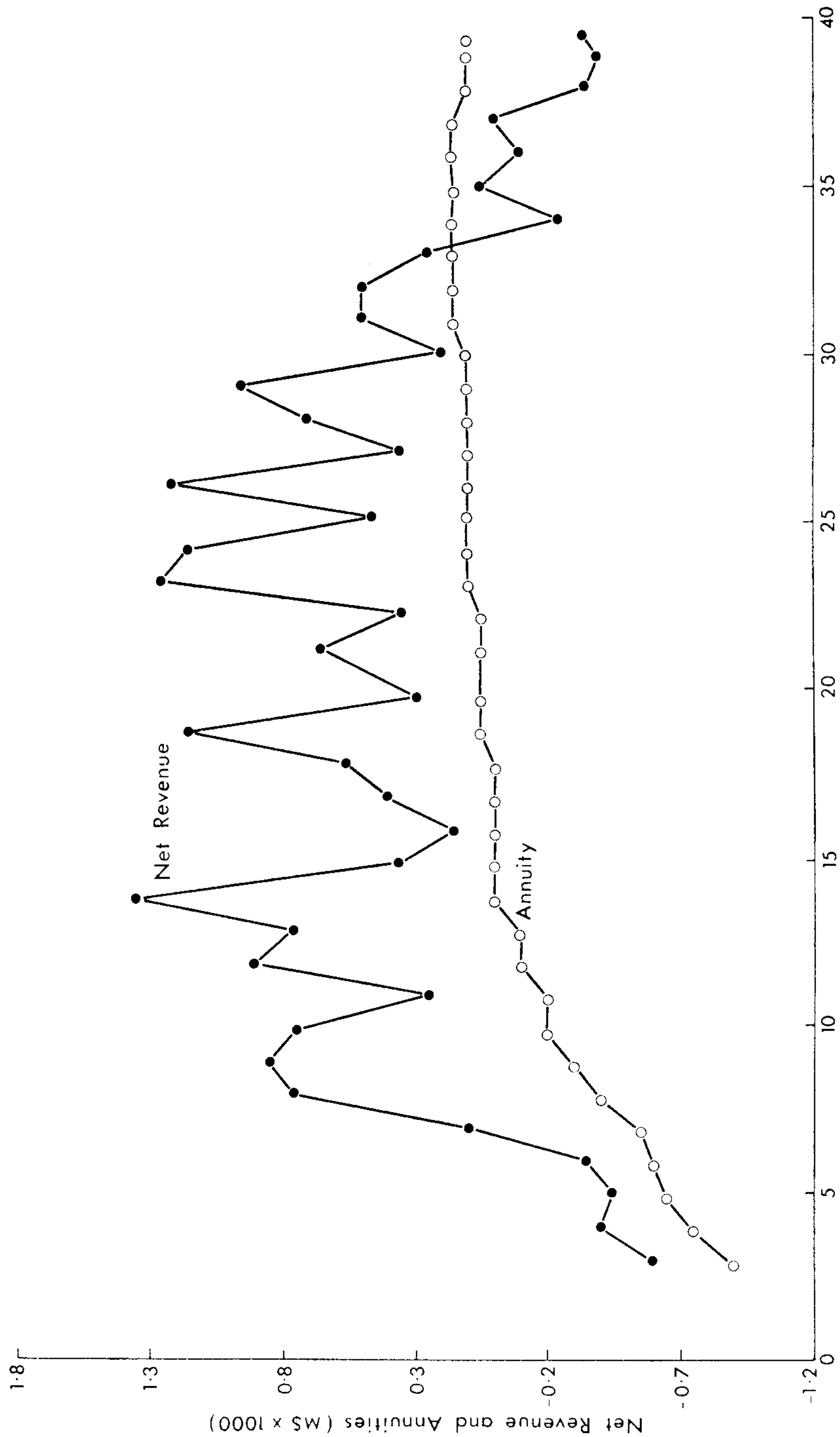


FIGURE 6.
Net Revenue and Annuities

It may be thought that extension advice can be confidently given regarding the optimal replacement date for this type of clone and that there is likely to be negligible cost to making a wrong decision. That is, that there would be little loss if the present stand of trees were replaced with a similar clone within a couple of years either side of the optimum date. However, these optimistic results are only true if, as the model assumes, the present defender, with all its fluctuations, is followed by a sequence of identical challengers. That is, if a high [low] cycle is followed by identical high [low] cycles of net revenues. The real problem arises when the defender is followed by any of the possible alternative challengers suggested by this simulation procedure.

Each manager knows the present position of his defending stand of trees but is faced with a distribution of challengers. Since no assumptions have yet been made about different levels of management skills, the distribution of maximum annuities is simply the result of random influences on yields and prices. If the challenger in fact turns out to be different [worse or better] from the defender then, obviously, the wrong decision is made by assuming a perpetual sequence of identical net revenue cycles. This suggests that, given neutrality towards risk and the fact that the distributions in Tables 2 and 3 showed little evidence of skewdness, a high [low] yielding defender should delay [bring forward] replanting in the expectation of a challenger represented by the mean maximum rather than the annuity derived from a perpetuation of the particular experience of the present defender.

The logic of this argument is given in Figure 7 which shows low and high revenue streams ($R(t)_L$ and $R(t)_H$) and their associated annuities ($A(t)_L$ and $A(t)_H$) together with a frequency distribution of maximum annuities. The small divergence of optimal dates for perpetual sequences of identical net revenue cycles is shown (T_L and T_H) and the quite different optimal dates (T_1 and T_2) that should be adopted on the basis of the mean maximum annuity.

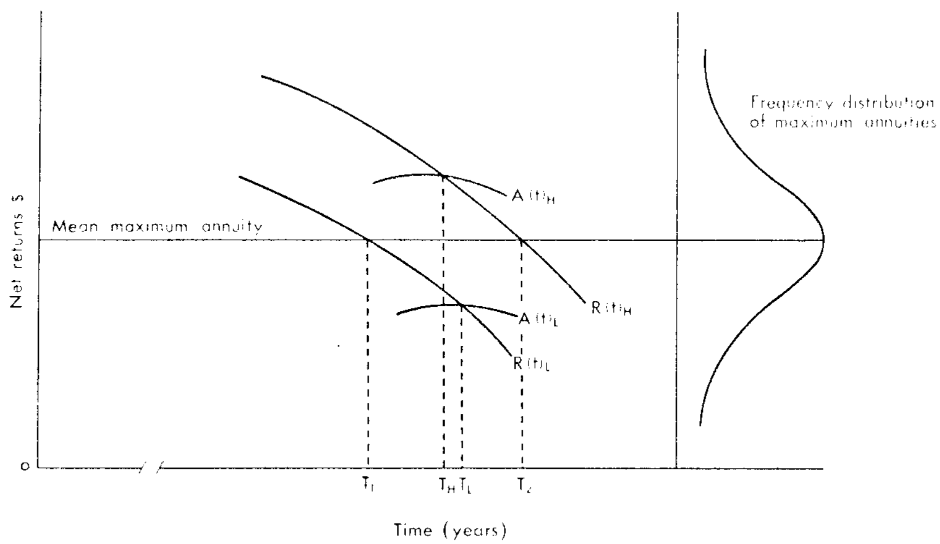


FIGURE 7.
Alternative Annual Net Revenues and Annuities and Frequency Distribution of Maximum Annuities.

These results imply a positive supply response policy for the Malaysian Rubber Research and Development Board (MRRDB): that during 'good times' estates should be encouraged to delay replanting (things won't be so good in future) while during 'bad times' earlier replanting should be encouraged. The RRIM has the necessary detailed data from which to derive yield curves by clone and ecological zone. It also has the necessary cost and price data to derive 'benchmark' mean maximum annuities by clone and region. Such benchmark annuities could be used to guide estate managers to replacement policies having more economic logic to them than the current fixed or rule of thumb replacement dates.

Considerable efforts are made by the extension services of the Malaysian Rubber Industry Smallholders Development Authority (RISDA) to persuade smallholders to replant their rubber at similar cycle lengths to those adopted by the estate sector. It is an observation common to Sri Lanka, Indonesia and Malaysia that smallholders generally have lower yields and adopt longer cycle lengths than the estates [4]. That these longer cycles may be rational is suggested by the result of ranking the optimal annuities against the year in which they occur. This is done in Table 3. It will be noted that there is a reasonably consistent pattern where the optimal cycle length increases as average maximum annuities decline. Thus obeying the optimal replacement rule and assuming perpetual sequences of identical cycles means that, in general, low yield/price sequences lead to later replacement dates than higher yield/price sequences. The implication is that extension efforts should be directed to raising yields as a means of reducing cycle lengths.

TABLE 3

Mean Annuities at Optimal Replacement Dates with Changes in Yield Variability

	Coefficient of Variation (V^1_y)								
	10%			15%			20%		
Optimal Year	Mean	Std Dev	N	Mean	Std Dev	N	Mean	Std Dev	N
	157·2*	53·5*	200*	156·8*	56·8*	200*	156·1*	61·1*	200*
28	207·6	0·0	1	215·3	0·0	1	179·0	50·8	3
29	222·7	49·7	8	228·5	52·6	8	230·1	53·9	9
30	182·3	48·4	22	179·0	52·4	24	177·4	58·3	24
31	157·1	48·6	37	152·2	54·7	45	155·2	60·2	42
32	160·4	58·7	53	167·3	61·5	47	167·3	64·6	48
33	149·5	46·4	47	143·4	47·4	45	136·3	50·7	39
34	133·8	44·3	26	133·1	47·4	25	131·0	49·2	27
35	102·0	30·8	4	107·4	27·1	3	137·6	57·2	5
36	103·9	37·8	2	104·6	41·4	2	88·0	43·2	3

* Values for entire population.

In the light of the international buffer stock agreement signed by the Association of Natural Rubber Producing Countries (ANRPC) in Jakarta in late 1976, it is interesting to note the effects which such an agreement could have if it does indeed stabilize prices.[16] To allow direct comparison with previous results the secular decline in prices assumed in equation (14) is maintained. Such a decline in real prices would be viewed as being unduly pessimistic by the natural rubber industry in view of its improved competitive strength in the light of the increase in the prices of the petroleum stock for synthetic rubber [see 17 and 26]. The maintenance of a stable (but declining) price reduced the coefficient of variation of the optimal annuity from 34 to 11 per cent when V^1_y is set at 10 per cent. This result, together with the very slight increase in V_a (Table 2) as V^1_y is doubled, indicates the extent to which incomes might be stabilized by appropriate price/buffer stock policies. The assumption of a stable but declining price had little effect on the optimal date (32·05 (0·69)) but naturally reduced the expected annuity to (136·39 (15·24)) since the random prices fluctuated in a log-normal manner and given the heavy investment costs any upward fluctuation in prices will be reflected in high net revenues. If the international agreement on price stabilization not only stabilizes short term price fluctuations but removes the declining trend then the expected annuities would be substantially increased.

Table 4 presents the results for change in β with V^1_y set at 15 per cent. In reality the rate of decline in yields in the final years can be controlled by the tapping system used but it would not be entirely independent of the tapping system used in Phase 2. For this reason little cognizance should be taken of the increase in mean optimal annuity values in the Table.

TABLE 4

Changes in Optimal Date and Annuity with Changes in the Rate of Decline in Yields while Tapping Second Renewal Bark

Item	Value of β		
	—140	—100	—50
Mean optimal year	30·28	31·95	37·03
Standard deviation	1·16	1·47	2·08
Mean optimal annuity	153·00	156·81	165·57
Standard deviation	57·05	56·78	55·80

In addition to extending the optimal cycle length it was anticipated that there would be an increase in the variability of the optimal date as β was reduced. The full extent of this increase in variability is not shown in the table because the optimal date (37) for $\beta = -50$ is only two years removed from the limit of the data. The result was to produce a highly skewed distribution with 66 per cent of the maximum annuities actually taking place in year 39. In practice it is highly unlikely that

[16] The first issue of the new journal of the Malaysian Rubber Research and Development Board (MRRDB), the *Malaysian Rubber Review*, carries an analysis of the Malaysian Crash Programme to stabilise rubber prices initiated in December 1974 [Lim 17]. The success of the scheme is an indication of the likely success of the international agreement on price stabilization.

such high yield levels could be maintained for such an extended period since little tappable bark would remain. Longer, low yield, cycles can only be sustained by less intensive tapping.

The results reported in this paper were based on a Normal distribution of yields in each year (equation 11) and a log-normal distribution of prices (equation 14). In an effort to examine the sensitivity of the results to alternative assumptions regarding the probability distribution, additional runs were done with simple rectangular and log rectangular distributions.[17] Yields and prices naturally fluctuated more widely but the overall conclusions reported here changed very little. The robustness of the results to these changes in assumptions regarding the nature of the probability distributions strengthens the conclusions.

Conclusions

The paper has shown that there is little economic justification for spending a great deal of extension effort in ensuring that estate managers make the complex calculations necessary to arrive at the optimum cycle length. However, the current practice of adopting an administratively convenient replacement date should be modified to allow for the difference between the present situation ('good' or 'bad') and long run expectations. To this end the RRIM could promote the use of area and clone specific benchmark or normal annuities.

Thus rather than a single optimal replacement date, the efficient set of replacement dates is very wide for three quite different reasons: *first* because the flat topped annuity curve shown in Figure 6 is indicative of the range over which replacement can take place with negligible loss if perpetual sequences of identical cycles are assumed. *Secondly,* because lower yielding estates will, by obeying the same optimal rule, generally replant later than higher yielding estates.

Finally, because if replacement is done on the basis of mean maximum annuities optimal replacement dates will be spread further apart than if based on the perpetuation of present cycles.[18] The policy conclusions follow that *across* management groups (e.g. efficient versus inefficient estates), extension efforts should be directed towards raising annual net returns rather than emphasising earlier replanting *per se.* Emphasis on earlier replanting *within* management groups will be warranted on commercial grounds where current net revenues are low compared to long run expected annuities from the same clone or where technological change substantially raises yields and, consequently, future annuities. The manner in which these conclusions must be modified by different types of technological change, by different clones, by high salvage values and by social as opposed to private costs and returns must be the subject of future research with this stochastic model.

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