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Production economics in the presence of risk*

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This paper provides an overview of the literature on production under the influence of risk. Various specifications of stochastic production function such as models with additive and multiplicative uncertainty, Just and Pope model, output-cubical, state-allocable and state-general models are discussed. Further, criteria determining optimal producer behaviour are derived for deterministic production technology and for various kinds of state-contingent technologies such as output-cubical, state-specific, state-allocable and state-general technologies. Finally, a brief discussion is presented about the drawbacks of each of these specifications of technology.

Key words: output-cubical, risk-neutral, state-allocable.

1. Introduction

The problem of production under uncertainty has been analysed using two different approaches. The first of these has been state-contingent approach in general equilibrium framework developed by Debreu (1952) and Arrow (1953). The other approach has been based on stochastic production functions: Sandmo (1971) and Just and Pope (1978).

Duality approach to producer theory that originated with Shephard (1953,1970) argues that under standard regularity conditions,[1] any production technology can be conveniently represented by either production possibility set or by cost (or profit) functions and these two representations are equivalent. As state-contingent production under uncertainty is a special case of multi-input, multi-output technology, duality tools can be readily applied to state-contingent production technology. Furthermore, Chambers and Quiggin (1998, 2003) show that when the input sets are closed and nonempty, a well-behaved cost function can be derived from any stochastic production or revenue function.

In the stochastic production approach introduced by Sandmo (1971) and Just and Pope (1978), the main idea was to derive the first-order conditions for optimisation and use the implicit function theorem to describe comparative static responses to changes in parameters of technology.

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1 That is, if the production or cost functions are continuous and twice differentiable in their respective arguments.

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In most production processes, risk plays an important part in the choice of inputs and supply of output. Just and Pope (1978) quantify production risk in terms of output variance by specifying the risk function in such a way that the inputs increase or decrease production risk. However, an important limitation of Just and Pope framework is that it does not model decision-makers' behaviour towards risk. Love and Buccola (1991, 1999) address this limitation by explicitly taking into account producer attitude towards risk by including a utility function in their model. The main shortcoming of Love and Buccola's approach is that they assume specific functional form for the utility function that describes producer attitude towards risk and a restrictive probability distribution for modelling the error term, which represents producer risk.

Kumbhakar (2002) extends Love and Buccola's model by incorporating an efficiency term in addition to the risk term in their model. The limitation of Kumbhakar's model is that it does not account for the uncertain environment that the producers find themselves operating in. Therefore, it does not allow substitutability between state-contingent outputs (and inputs). These shortcomings of existing models can be resolved if the modelling is carried out in a state-contingent framework.

For the sake of simplicity and for conserving space, we restrict our analysis to single input and single output technology. The analysis of more general multi-input and multi-output technology is not conceptually harder and only notationally more involved.

This paper is organised as follows. Section 2 discusses conventional deterministic production technology, highlighting producers' optimising behaviour and limitations of the corresponding technology. Section 3 presents the state-contingent approach using concepts and terminology of Hirshleifer and Riley (1992), and Chambers and Quiggin (2000). In Sections 4, 5 and 6, respectively, we describe producers' optimising behaviour with examples for output-cubical, state-allocable and state-general technology. Further, in these three sections, limitations of the respective technologies are highlighted. Finally, some concluding comments are offered in Section 7.

2. Conventional technology

Section 2.1 defines the conventional technology, Section 2.2 describes producers optimising behaviour for this technology and Section 2.3 discusses the limitations of conventional technology.

2.1. Representation

For the conventional production technology, output can be written as a function of input as

$$z = f(x). \tag{2.1}$$

There are many alternative representations of a production technology, including cost functions, transformation functions, distance function, production possibilities sets and input correspondences. For example, the input correspondence associated with Equation (2.1) is:

$$X(z) = \{x : x \text{ can produce } z\}. \tag{2.2}$$

The input set contains all inputs x that can produce a given output z. Given that the production function is continuous and twice differentiable, the properties of the input sets can be summarised as follows:

$$X(z) \text{ is closed for all } z. \tag{2.3}$$

$$X(z) \text{ is convex for all } z. \tag{2.4}$$

Inputs are said to be weakly disposable if

$$x \in X(z), \Rightarrow \forall \lambda \geq 1,\ \lambda x \in X(z); \text{ and} \tag{2.5}$$

Inputs are said to be strongly disposable if

$$x \in X(z) \text{ and } x^* > x \Rightarrow x^* \in X(z). \tag{2.6}$$

2.2. Optimising behaviour

Given cost of input w and price of output p, producer maximises profit $\Pi = pz - wx = pf(x) - wx$. The first-order condition for profit maximisation is given by

$$\frac{d\Pi}{dx} = pf'(x) - w = 0, \tag{2.7}$$

or

$$f'(x) = \frac{w}{p}, \tag{2.8}$$

where $f'(x)$ represents first derivative of the production function $f(x)$ with respect to x. Hence, the optimal production choice (z^*,x^*) is a point on the production function where the isoprofit line with a slope w/p is tangent to the production function. In Figure 1, the optimal production bundle is represented by (z^*,x^*) located on the production function.

2.3. Limitation

The conventional production technology is deterministic and does not account for production risk. For example, in agriculture, a farmer's *ex ante*

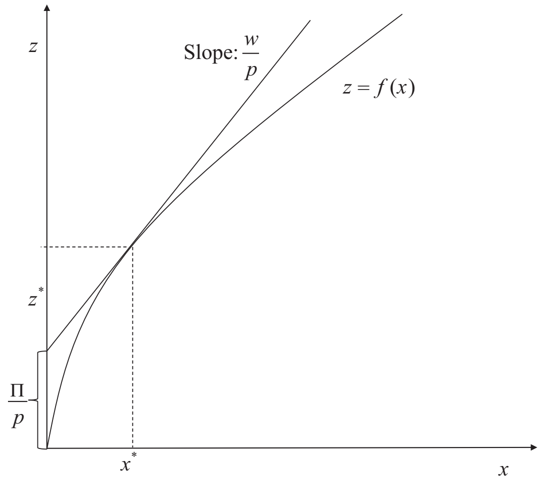

Figure 1 Optimal production choice for conventional technology.

production decisions are based on her expectation about the future weather conditions. Hence, a deterministic production technology is unrealistic in modelling a farmer's decision in an inherently uncertain production environment.

3. Producer behaviour under uncertainty in state-contingent framework

Uncertainty in production environment is described by a set $\Omega = \{1,\ldots,S\}$ consisting of all possible future states of nature, where nature selects one of the states from this set and this choice by nature is independent of production choices made by the producer. Hence, production can be thought as two-period game between producer and nature. In period 0, the producer commits input x costing w, to the production process, and in period 1, nature resolves the uncertainty by picking a state of nature from the Ω. Decision-makers belief about future states of nature is described by subjective probability vector $\boldsymbol{\pi} = (\pi_1,\ldots,\pi_S)$. Based on the state of nature $\{s\}$ realised in period 1, the output z_s in the corresponding state of nature is unearthed by the transformation function $t(x, \mathbf{z})$, where $\mathbf{z} = (z_1,\ldots,z_S)$ represents the state-contingent output vector. In other words, if state of nature $\{s\}$ is chosen by nature and the producer had chosen *ex ante* input–output combination $(x, \mathbf{z})$, then the *ex post* output is z_s, which is the sth element of $\mathbf{z}$. The price and net return vectors associated with state-contingent output vector are given by $\mathbf{p} = (p_1,\ldots,p_S)$ and $\mathbf{y} = (y_1,\ldots,y_S) = (p_1 z_1 - wx,\ldots,p_s z_S - wx)$, respectively. The producer chooses input bundle x to maximise her utility W, which is assumed to be nondecreasing in state-contingent net returns $\mathbf{y} = (y_1,\ldots,y_S)$.

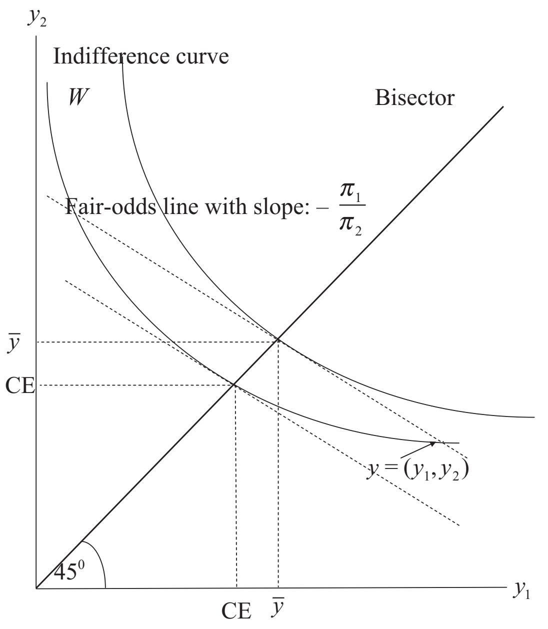

Figure 2 Illustration of the main ideas behind state-contingent production.

Under relatively weak assumptions of monotonicity, continuity and differentiability on the utility W, Rasmussen (2003) defines a risk-averse decision-maker to satisfy the condition that:

$$W(\bar{y}, \ldots, \bar{y}) \geq W(y_1, \ldots, y_S), \tag{3.1}$$

where $\bar{y}$ is the expected net return given by

$$\bar{y} = \pi_1 y_1 + \pi_2 y_2 + \ldots + \pi_S y_S. \tag{3.2}$$

Figure 2 depicts the preferences of a risk averse producer. From the figure, it is evident that the indifference curves for the risk averse decision-makers are convex. The figure also shows producers output choices $\mathbf{y} = (y_1, y_2)$, net return $\bar{y} = \pi_1 y_1 + \pi_2 y_2$ and certainty equivalent CE.[2] The bisector (see Hirshleifer and Riley 1992, for further discussion) indicates certainty because along this line the net return is the same no matter what state of nature materialises *ex post*. In Figure 2, the line with slope $-(\pi_1/\pi_2)$ that is tangent to the indifference curve which also intersects the bisector is referred to as fair-odds[3] line. At the point of tangency, we have:

[2] The certain return that provides the same utility as the uncertain net return vector **y**.
[3] See p.89–90 Chambers and Quiggin (2000) for a more formal explanation.

$$\frac{\pi_1}{\pi_2} = \frac{W_1(y,y)}{W_2(y,y)}. \quad (3.3)$$

Furthermore, Figure 2 shows that at **y** the absolute slope of the indifference curve W is less than the slope of the fair-odds line, which means:

$$\frac{\pi_1}{\pi_2} > \frac{W_1(y_1,y_2)}{W_2(y_1,y_2)}. \quad (3.4)$$

Equation (3.1) determines whether the decision-maker is risk-neutral or risk averse. When Equation (3.1) holds with equality the producer is said to be risk-neutral and when Equation (3.1) hold with inequality the producer is said to be exhibiting risk-averse behaviour. Hence, in Figure 2 the utility of risk-neutral decision-maker coincides with the fair-odds line and is given by

$$W^{\mathrm{RN}}(\mathbf{y}) = \pi_1 y_1 + \pi_2 y_2 + \cdots + \pi_S y_S. \quad (3.5)$$

In general (see Rasmussen 2003), it is not possible to derive a criteria for optimum input usage for a risk averse (or a risk loving) producer but one may be able to compare a risk averse (or risk loving) decision-maker with a risk-neutral decision-maker. In other words, it is possible to conclude whether a risk averse (or risk loving) producer will employ more or less input when compared with a risk-neutral producer. For making such comparisons, it is important to define 'good' and 'bad' states of nature, both of which are subjective ideas.

A risk-neutral producer (Eqn 3.5) would choose an input x^* (optimal) to achieve an utility of $W^{\mathrm{RN}}(y_1(x^*),\ldots,y_S(x^*)) = \pi_1 y_1(x^*) + \pi_2 y_2(x^*) + \cdots + \pi_S y_S(x^*)$. As the utility is ordinal rather than cardinal in nature, it can be rescaled[4] in such a way that

$$\sum_{s=1}^{S} W_s(y_1(x^*),\ldots,y_S(x^*)) = 1, \quad (3.6)$$

which implies that the sum of the partial first derivative of utility function W with respect to y_s evaluated at net return vector $\mathbf{y}(x^*)$ is one.

A risk averse producer faces a 'good'[5] state of nature $\{s\}$ if

$$W_s(y_1(x^*),\ldots,y_S(x^*)) < \pi_s \quad (3.7)$$

and she faces a 'bad' state of nature $\{s\}$ if

[4] In fact, it can be scaled in any arbitrary way as long as the scale factor is a positive number.
[5] We follow Rasmussen's definition of 'good' and 'bad' states (see Rasmussen 2003).

$$W_s(y_1(x^*), \ldots, y_S(x^*)) > \pi_s. \tag{3.8}$$

And in state of nature $\{s\}$ when

$$W_s(y_1(x^*), \ldots, y_S(x^*)) = \pi_s \tag{3.9}$$

the producer is said to be facing a neutral state of nature.

Therefore, in a 'good' state of nature an additional dollar of state-contingent net income provides a lower marginal utility than the probability of that state. On the contrary, in a 'bad' state of nature an additional dollar of state-contingent net income provides a higher marginal utility than the probability of that state. It is important to note that marginal income is represented by the net return (income) evaluated at a risk-neutral producer's optimal input bundle (x*) and the marginal utility in a particular state is represented by the first derivative of a risk averse producers' utility with respect to the net return of the corresponding state of nature evaluated at the net income that a risk-neutral producer's optimal bundle would produce.

4. Output-cubical technology

Most conventional frontier models are output-cubical. An output-cubical technology is a special but restrictive type of state-contingent technology as it does not allow substitution between output realised in different states of nature. Section 4.1 defines an output-cubical technology and Section 4.2 describes producers' optimising behaviour for this technology. In Sections 4.3 and 4.4, respectively, we discuss some examples and limitations of an output-cubical technology.

4.1. Representation

For the output-cubical production technology, output can be written as a function of input as

$$z_s = f(x, \varepsilon_s) \quad s \in \Omega = \{1, \ldots, S\}, \tag{4.1}$$

where ε_s is a random term that producers cannot control.

Chambers and Quiggin (2000, p. 59) use input correspondence to represent stochastic technology. While analysing uncertain production processes, it is assumed that the state-contingent vector of outputs is produced by an input managed by the producer and a random vector over which the producer has no jurisdiction. If $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_S) \in \mathbb{R}_+^S$ is the random vector which is out of the producer's control, then the stochastic production function specification requires the following relationship between inputs and the stochastic output

$$z_s \le f(x, \epsilon_s), \text{ where } f : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+. \tag{4.2}$$

The output-cubical (state-contingent) input correspondence associated with Equation (4.2) is

$$\begin{aligned} X(\mathbf{z}) &= \{x : z_s \le f(x, \epsilon_s), s \in \Omega\} \\ &= \bigcap_{s\in\Omega} \{x : z_s \le f(x, \epsilon_s)\} \\ &= \bigcap_{s\in\Omega} \{\bar{X}(z_s, \epsilon_s)\}, \end{aligned} \tag{4.3}$$

where $\bar{X}(z_s, \epsilon_s)$ can be conceived as the *ex post* input set associated with the production function for a given realisation of the random variable.

4.2. Optimising behaviour

If w is cost of input and p_s is price of output in the state of nature $s \in \Omega = \{1,\ldots,S\}$, then producers profit is given by

$$\Pi = \sum_{j\in\Omega} p_j e_j f_j(x, \varepsilon_j) - wx \tag{4.4}$$

where $e_s = 1$, if sth state of nature is realised *ex post* and $e_j = 0$, $\forall j \ne s \in \Omega$.

The first-order condition for profit maximisation is given by

$$\frac{d\Pi}{dx} = \sum_{j\in\Omega} p_j e_j f_j'(x, \varepsilon_j) - w = 0 \tag{4.5}$$

or

$$f_s'(x, \varepsilon_s) = \frac{w}{p_s}, \tag{4.6}$$

where $f_s'(x, \varepsilon_s)$ represents the first derivative of the production function $f_s(x, \varepsilon_s)$ with respect to x when $\{s\}$ is the realised state of nature. Hence, the optimal production choice (z_s^*, x^*) is a point on the production function where the isoprofit line with a slope w/p_s is tangent to the production function in the realised state of nature $\{s\}$. In Figure 3,[6] the optimal production bundle is represented by $(z_s^*, x^*), s \in \{1, 2\}$ located on the respective production functions.

[6] OC is an abbreviation for Output-Cubical.

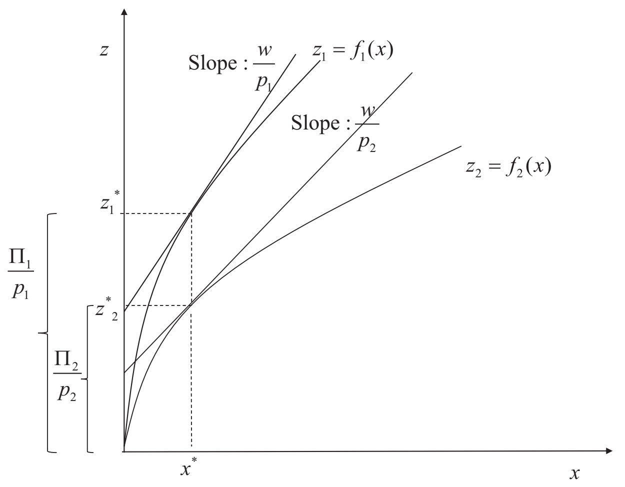

Figure 3 Optimal production choices for OC technology: $S = 2$.

4.3. Examples

4.3.1. Additive uncertainty

In this case, we have

$$f(x, \epsilon_s) = g(x) + \epsilon_s. \tag{4.7}$$

For this specification,

$$X(\mathbf{z}) = \bigcap_{s \in \Omega} \{x : z_s - \epsilon_s \leq g(x)\}. \tag{4.8}$$

If $g(x)$ is increasing in variable input x under producer's control

$$\begin{aligned} X(\mathbf{z}) &= \{x : \mathrm{Max}\{z_1 - \epsilon_1, \ldots, z_S - \epsilon_S\} \leq g(x)\} \\ &= X_g(\mathrm{Max}\{z_1 - \epsilon_1, \ldots, z_S - \epsilon_S\}), \end{aligned} \tag{4.9}$$

where $X_g(m) = \{x : \mathrm{Max}\{g(x) \geq m\}$ and the corresponding cost function is given by

$$c(w, \mathbf{z}) = c_g(w, \mathrm{Max}\{z_1 - \epsilon_1, \ldots, z_S - \epsilon_S\}), \tag{4.10}$$

where c_g is dual to X_g.

If $W : \mathbb{R}^S_+ \longrightarrow \mathbb{R}_+$ represents a continuous preference (Yaari 1969; Quiggin and Chambers 1998) structure that is strictly increasing in state-contingent net returns,[7] if there is no price uncertainty, and if the price of stochastic

[7] Net return of a producer is defined as the total revenue obtained by selling the products minus the total cost incurred in the production process.

output is normalised to one, then the producer's objective function (see Chambers and Quiggin 2002) is

$$\begin{aligned}\text{Max}\{\mathbf{W}(\{z_1 - c_g(w, \text{Max}\{z_1 - \epsilon_1, \ldots, z_S - \epsilon_S\}), \ldots,\\ \{z_S - c_g(w, \text{Max}\{z_1 - \epsilon_1, \ldots, z_S - \epsilon_S\}))\}.\end{aligned} \tag{4.11}$$

The producer with this objective function will choose the kinked point on her optimal isocost curve (see Figure 1 in Chambers and Quiggin 2002). So her choice is determined by

$$z_1 - \epsilon_1 = z_s - \epsilon_s, s \in \Omega. \tag{4.12}$$

This implies that all producers, irrespective of their risk attitudes, share a common expansion path that is parallel to nonstochastic production vector, that is,

$$z_s = z_1 + \epsilon_s - \epsilon_1, s \in \Omega. \tag{4.13}$$

So,[8] for stochastic production technology with additive uncertainty risk lovers, risk-neutral and risk averse, all choose the same state-contingent output and the corresponding input.

4.3.2. Multiplicative uncertainty

In this case, we have

$$f(x, \epsilon_s) = h(x)\epsilon_s, \tag{4.14}$$

where h is a nonstochastic production function.

The state-contingent correspondences are given by

$$\begin{aligned} X(\mathbf{z}) &= \bigcap_{s\in\Omega}\left\{x : \frac{z_s}{\epsilon_s} \leq h(x)\right\}\\ &= \left\{x : \text{Max}\left\{\frac{z_1}{\epsilon_1}, \ldots, \frac{z_S}{\epsilon_S}\right\} \leq h(x)\right\}\\ &= X_h\left(\text{Max}\left\{\frac{z_1}{\epsilon_1}, \ldots, \frac{z_S}{\epsilon_S}\right\}\right)\end{aligned} \tag{4.15}$$

and the corresponding cost function is given by

$$c_h\left(w, \text{Max}\left\{\frac{z_1}{\epsilon_1}, \ldots, \frac{z_S}{\epsilon_S}\right\}\right), \tag{4.16}$$

[8] For a detailed discussion, see Chambers and Quiggin (2002).

where c_h is the minimal cost function for stochastic production function h. Again, as in the case of additive uncertainty, a rational producer will disregard her risk preference and input price and choose expansion path given by

$$z_s = z_1 \frac{\epsilon_s}{\epsilon_1}, s \in \Omega. \tag{4.17}$$

Therefore, if the uncertainty is multiplicative, then the producer's risky returns from production in state of nature $\{s\}$ would be ϵ_s/ϵ_1 times her returns from production in a risk-less state of nature $\{1\}$.

4.3.3. Just and Pope model

Just and Pope (1978) observed the unrealistic restriction that the output expansion paths in state-contingent output space were linear and independent of input prices and producers' risk preferences. The Just–Pope technology can be written as

$$f(x, \epsilon_s) = g(x) + h(x)\epsilon_s, \tag{4.18}$$

where $g(x)$ and $h(x)$ are nonstochastic technologies and the term $h(x)\epsilon_s$ represents multiplicative uncertainty. It can be seen from Equation (4.18) that additive uncertainty and multiplicative uncertainty are special cases of Just–Pope technology. The state-contingent input correspondence for Just–Pope specification is given by

$$X(\mathbf{z}) = \bigcap_{s \in \Omega} \{x : g(x) + h(x)\epsilon_s \geq z_s\}. \tag{4.19}$$

This specification takes care of the problems related with producer's linear expansion path but it still does not allow for substitutability between state-contingent outputs.

4.3.4. Kumbhakar model

Kumbhakar (2002) extends Just and Pope technology by incorporating producer behaviour towards risk in his model. Kumbhakar's model is specified as

$$z = g(x) + h(x)\varepsilon, \tag{4.20}$$

where $\varepsilon \sim N(0,1)$ is the stochastic error term representing production uncertainty. The mean output and output risk function are defined as $E(z) = g(x)$ and $\mathrm{Var}(z) = h^2(x)$, respectively.

Furthermore, it is assumed that producers maximise their expected utility[9]$(E(U(\Pi)))$ of profit[10] $(\Pi = z - wx)$. The first-order condition for maximising producers' utility can be written as

[9] The utility function $U(.)$ is assumed to be continuous and differentiable function of profit Π.

[10] The actual profit is normalised by output price p to get the normalised profit Π.

$$g'(x) = w - \theta h'(x), \tag{4.21}$$

where $g'(x) = \partial g(x)/\partial x$, $h'(x) = \partial h(x)/\partial x$ and $\theta = E(\varepsilon U'(\Pi))/E(U'(\Pi))$.

The variable input x is risk-increasing when $h'(x)$ is positive[11] and it is risk-decreasing when $h'(x)$ is negative. There is no change in risk if $h'(x) = 0$. In Equation (4.21), θ captures producers' risk preferences. For a risk-averse producer θ is greater than zero and the converse is true for risk-loving producers. For a risk-neutral producer θ is equal to zero. Finally, Equation (4.21) clearly indicates that input allocated by producers depends on risk (ε).

4.4. Limitations

Chambers and Quiggin (2000) mention three important observations regarding output-cubical stochastic production function. First, the strong functional restriction on the interaction between the random factors and controllable inputs does not appear to have an empirical basis. The stochastic error term is supposed to capture the effect of random inputs, which imposes the restriction that stochastic inputs must be weakly separable from controllable inputs.

Second, irrespective of whether the stochastic production function exhibits continuity, the cost function dual to this technology is not everywhere differentiable in outputs. Hence, traditional methods that equate marginal cost to marginal benefit are no longer applicable for such technologies.

Third, the nondifferentiability of cost function with respect to state-contingent outputs implies that output sets of the stochastic production function are cubes in state-contingent output space. Hence, technologies linked to stochastic production function are referred as 'Leontief-in-output' or 'output-cubical'. Therefore, output-cubical technology does not allow producers to manage production risk by allocating inputs to different states of nature based on their expectations about these future states of nature.

5. State-specific, state-allocable technology

State-specific, state-allocable technology is a flexible and realistic kind of state-contingent technology that allows for substitution between outputs realised in different states of nature. Section 5.1 defines a state-specific, state-allocable technology and Section 5.2 describes producers optimising behaviour for this technology. In Sections 5.3–5.5, respectively, we discuss two examples and a limitation of a state-specific, state-allocable technology.

[11] As the marginal risk $\partial Var(z)/\partial x = 2\ h(x)h'(x) > 0$ when $h'(x) > 0$.

5.1. Representation

For the state-specific, state-allocable production technology, output can be written as a function of input as

$$z_s = f(x_s, \varepsilon_s) \quad s \in \Omega = \{1, \ldots, S\}. \tag{5.1}$$

The input correspondence associated with Equation (2.1) is:

$$X(z) = \sum_{s \in \Omega} X_s(z_s), \tag{5.2}$$

where each $X_s(z_s) \subseteq \mathbb{R}_+$.

Alternatively, state-allocable input technology can be represented by output correspondence given by

$$Z(x) = \left\{ \mathbf{z} = (z_1, \ldots, z_S) : \sum_{s \in \Omega} x_s \leq x \right\}, \tag{5.3}$$

where each $z_s \subseteq \mathbb{R}_+$.

Comparing Equations (4.1) and (5.1), we observe that for output-cubical specification of technology the state-contingent output z_s depends on the total input x allocated to the production process, but for state-specific, state-allocable specification of technology the state-contingent output z_s depends on the input allocated to the corresponding state of nature, that is, x_s.

5.2. Optimising behaviour

The optimisation problem for state-allocable input is given by

$$\underset{x_1, \ldots, x_S}{\text{Max}} \; W(y_1, \ldots, y_S), \tag{5.4}$$

where $y_j = p_s f_j(x_j) - w \sum_{s=1}^{S} x_s$, $j \in \Omega$.

The first-order condition for state-allocable input that is state-specific[12] is therefore given by

$$W_j(\mathbf{y}) \left(p_j \frac{\partial f_j(x_j)}{\partial x_j} \right) = w \sum_{s=1}^{S} W_s(\mathbf{y}), \quad j \in \Omega. \tag{5.5}$$

Since, $\sum_{s=1}^{S} W_s(\mathbf{y}) = 1$ (see Eqns 3.6,5.5) can be written as

$$W_j(\mathbf{y}) \left(p_j \frac{\partial f_j(x_j)}{\partial x_j} \right) = w, \quad j \in \Omega. \tag{5.6}$$

Similarly, the optimality condition for a risk-neutral decision-maker is given by

[12] In this paper, we only consider state-allocable input that is state-specific in nature.

$$\pi_j\left(p_j\frac{\partial f_j(x_j)}{\partial x_j}\right) = w, \quad j \in \Omega. \tag{5.7}$$

A risk-averse decision-maker would use more input than a risk-neutral decision-maker if

$$\left.\frac{\partial W(\mathbf{y})}{\partial x_j}\right|_{x=x^*} = \left.\left(W_j(\mathbf{y})p_j\frac{\partial f_j(x_s)}{\partial x_j} - w\sum_{s=1}^{S} W_j(\mathbf{y})\right)\right|_{x=x^*} > 0. \tag{5.8}$$

From Equation (5.7), it follows that when the risk-neutral producer uses an optimal amount of input, the marginal product times the price of input in a particular state of nature is equal to the ratio of cost of input and producer's risk-neutral probability in the corresponding state of nature, that is, $(p_j\partial f_j(x_j)/\partial x_j|_{x=x^*} = w/\pi_j$. Therefore, Equation (5.8) reduces to

$$W_j(\mathbf{y}) > \pi_j\sum_{s=1}^{S} W_s(\mathbf{y}). \tag{5.9}$$

Equations (5.8 and 5.9) imply (since $\sum_{s=1}^{S} W_s(\mathbf{y}) = 1$ from Eqn 3.6) that in 'bad' state of nature a risk-averse decision-maker will apply more input than a risk-neutral decision-maker. And using similar argument, in 'good' state of nature a risk-averse decision-maker will apply less input than a risk-neutral decision-maker.

Figure 4a depicts a two-state technology where the total amount of the input used in the production process has been fixed at x^*. This 'beaker' diagram shows that state-contingent outputs can be substituted for one another. The horizontal axis of this 'beaker' measures total input allocated to the production process. Input committed to state of nature {1} is measured from right to left and the input committed to state of nature {2} is measured from left to right.

The left vertical axis measures output produced in state $s = 1$ and the right vertical axis measures the output produced in state $s = 2$. Figure 4a also shows that state-contingent output increases at diminishing rate with an increase in the input allocated to the corresponding state of nature. Panel (b) shows state-contingent product transformation curve where output in state of nature {1} is traded off with output in state of nature {2}. The negative slope of the state-contingent product transformation curve indicates that an increase in output in a given state of nature can be achieved by decreasing output in another state of nature.

At A in Figure 4a x_1^A is allocated to state of nature {1} and $x_2^A = x^* - x_1^A$ is allocated to state of nature {2}, such that the state-contingent outputs are the same in every state of nature, that is, $z_1^A = z_2^A$. Therefore, by choosing A firms can eliminate risk. However, any other allocation of x^* involves risk.

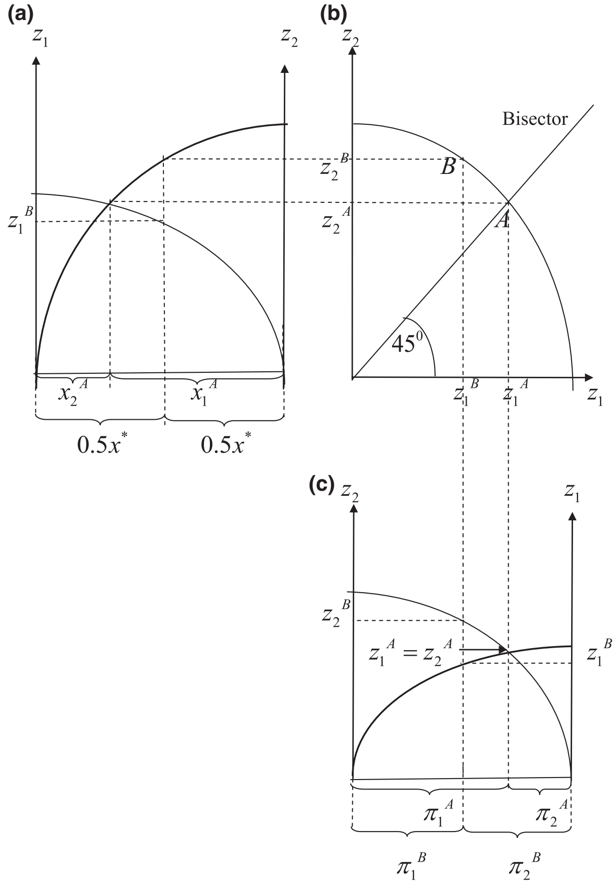

Figure 4 A state-allocable, state-contingent technology.

For example, even if the input is equally allocated between states of nature, firms will obtain a higher output in state of nature {2} than in state of nature {1} at point B ($z_1^B < z_2^B$)). The bisector (45° line) in panel (b) gives the locus of all riskless state-contingent output pairs.

Using Equation (5.7) and normalising output prices (assuming the prices are the same for every state of nature) to one, the state-contingent output as a function of the corresponding risk-neutral probability for the O'Donnell *et al*. (2010) model is given by

$$z_s = \left(\frac{\pi_s}{a_s bw}\right)^{\frac{1}{b-1}}. \tag{5.10}$$

Equation (5.10) indicates that for decreasing ($b > 1$) returns to scale, state-contingent output increases with risk-neutral probability in the corresponding state of nature.

Figure 4c shows that for given total input allocated to the production process, an increase in the risk-neutral probability π_1 in state of nature $\{1\}$ results in an increase in state-contingent output in that state of nature, that is, z_1 increases. Simultaneously, as the risk-neutral probability in state of nature $\{2\}$ is $\pi_2 = 1 - \pi_1$, the state-contingent output in state of nature $\{2\}$ decreases. Therefore, the output choices (see Figure 4b) of an efficient and rational firm are determined by the stochastic technology (a_1,a_2), the total input x^* allocated to the production process and the risk-neutral probabilities (π_1,π_2) associated with the various states of nature.

5.3. Example 1: O'Donnell, Chambers and Quiggin model

O'Donnell *et al*. (2010) model production technology using the following Cobb-Douglas function:

$$z_s = \left(\frac{x_s}{a_s}\right)^{1/b}, \quad s \in \Omega = \{1, 2\}, \tag{5.11}$$

where z_s is the amount of stochastic output produced in period 1 by employing x_s amount of nonstochastic input in period 0, $a_s > 0$ is the technology parameter related to production of output in state of nature $\{s\}$ and the parameter b represents the degree of substitutability between state-contingent outputs.

From Equation (5.11), the input allocated to a specific state of nature $\{s\}$ is given by

$$x_s = a_s z_s^b, \quad s \in \Omega = \{1, 2\}. \tag{5.12}$$

Assuming that the firms are rational and efficient, the total input used in the production process in period 0 is the sum of the inputs allocated to each state of nature, that is

$$x = x_1 + x_2 = a_1 z_1^b + a_2 z_2^b. \tag{5.13}$$

5.4. Example 2: Chavas model

Chavas (2008) provides a cost-based approach to represent state-contingent technologies. The specification of *ex ante* output in Chavas (2008) is as follows:

$$z_{st} = \mu_t e_s^{\sigma_t} s \in \Omega = \{1, \ldots, S\}, \quad t \in \{1, \ldots, T\}, \tag{5.14}$$

where $\{s\}$ represents state of nature and t is an index for time or technology. Here, e_s is a random variable that takes different values across different states of nature and σ_t is the spread parameter that varies across time or technology. It is important to observe that in Equation (5.14) the spread parameter σ_t is independent of the state of nature realised *ex post*.

Further it is assumed that there exists an auxiliary variable y_t such that

$$y_t = k_t e_s^{\sigma_t} s \in \Omega = \{1, \ldots, S\}, \quad t \in \{1, \ldots, T\}. \tag{5.15}$$

Here, y_t is a proxy variable that measures production under uncertainty. Taking log of both sides of Equation (5.15), $\ln y_t = \ln k_t + \sigma_t \ln e_s$ which basically is an ordinary least squares regression with heteroskedastic error having a variance of σ_t.

If for the tth observation, the state of nature $\{s\}$ is realised, from Equation (5.15) it follows that $e_s = (y_t/k_t)^{1/\sigma_t}$. Substituting this value of e_s in (5.14), the simulated state-contingent output can be written as

$$z_t^e = \{z_{rt} : z_{rt} = z_t (y_r/k_r)^{\sigma_t/\sigma_r} / y_t/k_t; r = 1, \ldots, T\}. \tag{5.16}$$

The exponential term σ_t/σ_r varies across states of nature and across observations, while μ_t merely represents[13] the underlying technology and does not effect the simulated output z_t^e.

To get a consistent estimate for cost function $C(\mathbf{w}_t, z_t^e, t)$ and consequently input demand function $x(\mathbf{w}_t, z_t^e, t)$, it is necessary first to get consistent estimates for k_t and σ_t. For tth observation, Chavas (2008) defines $K - 1$ values of b_{kt} such that $b_{1t} < b_{2t} < \cdots < b_{(k-1)t}$. He further defines K partitions, $V_{1t} = [-\infty, b_{1t}]$, $V_{kt} = [b_{(k-1)t}, b_{kt}]$, $k = 2, \ldots, K$ and $V_{Kt} = [b_{Kt}, \infty]$, $t = 1, \ldots, T$ so that at least one observation $z_{rt} \in V_{kt}\ \forall t$.

Finally, the following assumptions are made

$$z_{kt} = \left(\sum_{r=1}^{T} I_{krt} z_{rt} \right) \Big/ \left(\sum_{r=1}^{T} I_{krt} \right) \text{ and} \tag{5.17}$$

$$z_t^K = \{z_{kt} : k = 1, \ldots, K\}, \tag{5.18}$$

where the indicator variable I_{krt} is defined as

$$I_{krt} = 1 \text{ if } z_{krt} \in V_{Kt} \text{ else } I_{krt} = 0.$$

A generalised-Leontief functional form is assumed for the cost function, that is,

$$C(\mathbf{w}_t, z_t^K, t) = h(\mathbf{w}_t, z_t^K, t) \left[\sum_{i=1}^{M} \sum_{j=1}^{M} \alpha_{ij} {w_{it}}^{1/2} {w_{jt}}^{1/2} \right] + \sum_{j=1}^{M} w_{jt} g_j(z_t^K, t), \tag{5.19}$$

where $\alpha_{ij} = \alpha_{ji}\ \forall i \neq j$ and M is the number of inputs used in the production

[13] For this model to be valid, it is crucial that both Equations (5.14 and 5.15) hold good.

process. In Equation (5.19), $h(\mathbf{w}_t, z_t^K, t)$ and $g_j(z_t^K, t)$, respectively, have the following functional forms

$$h(.) = \sum_{k=1}^{K} \beta_k z_{kt} + \sum_{k \neq k'} \sum_{k'} \beta_{kk'} z_{kt} z_{k't} \text{ and} \tag{5.20}$$

$$g_j(.) = \gamma_{0j} + \gamma_{jt} t, \quad j = 1, \ldots, M, \tag{5.21}$$

where $\beta_{kk'}$ measures the substitution between state-contingent outputs.

Using Shephard's Lemma the input demand function can be written as

$$x_{it}(\mathbf{w}_t, z_t^K, t) = h(\mathbf{w}_t, z_t^K, t) \left[\sum_{j=1}^{M} \alpha_{ij} {w_{jt}}^{1/2} / {w_{it}}^{1/2} \right] + g_j(z_t^K, t), \\ i = 1, \ldots, M \text{ and } t = 1, \ldots, T. \tag{5.22}$$

Chavas (2008) applies his model to US agricultural data and finds that the substitution of outputs across the states of nature is negligible.

5.5. Limitation

If the technology is state-specific state-allocable, then the output in any state of nature is zero if no input is allocated to that state of nature, which is unrealistic. For example, in agricultural production, use of a particular nutrient in 'wet' state may be more effective than its use in a 'dry' state of nature, resulting in better yields in 'wet' state and significantly diminished, yet nonzero yields in 'dry' state of nature.

6. State-general technology

Section 6.1 defines a state-general technology and compares it with state-specific, state-allocable technology, and Section 6.2 describes producers' optimising behaviour for this technology. In Sections 6.3 and 6.4, respectively, we discuss an example and some limitations of a state-general technology.

6.1. Representation

For the state-general production technology, output can be written as a function of input as

$$z_s = f(x_1, x_2, \ldots, x_S, \varepsilon_s) \quad s \in \Omega = \{1, \ldots, S\}. \tag{6.1}$$

The output correspondence for state-general technology is given by

$$Z(x) = \left\{ z = (z_1, \ldots, z_S) : x = \sum_{s \in \Omega} x_s \right\}, \tag{6.2}$$

where each $z_s \subseteq \mathbb{R}_+$.

The difference between state-specific, state-allocable technology and state-general technology is that when the technology is state-specific state-allocable, input allocated to any given state of nature affects output only in that particular state of nature, whereas in the case of state-general technology input allocated to any particular state of nature affects output in more than one state of nature. Mathematically the following conditions apply for state-specific, state-allocable and state-general technology, respectively,

$$\frac{\partial f_s(x)}{\partial x_s} > 0 \text{ and } \frac{\partial f_j(x)}{\partial x_s} = 0,\ j \neq s \text{ and } j, s \in \Omega \tag{6.3}$$

and

$$\frac{\partial f_s(x)}{\partial x_k} \neq 0\ \forall s \in \omega \subseteq \Omega, \omega \neq \{\phi\} \text{ and } k \in \Omega. \tag{6.4}$$

It can be seen from Equations (6.3 and 6.4) that state-specific, state-allocable technology is a special case of state-general technology.

6.2. Optimising behaviour

For state-general inputs, the optimisation problem is defined as

$$\operatorname*{Max}_{x} W(y_1, \ldots, y_S), \tag{6.5}$$

where

$$y_s = p_s f_s(x) - wx, \quad s \in \Omega. \tag{6.6}$$

Optimal input usage can be arrived at by setting the first derivative of Equation (6.5) to zero, that is

$$\frac{\partial W(\mathbf{y})}{\partial x} = \sum_{s=1}^{S} W_s(\mathbf{y}) \left(p_s \frac{\partial f_s(x)}{\partial x} - w \right) = 0. \tag{6.7}$$

For a risk-neutral decision-maker (having linear utility), Equation (6.7) is replaced by

$$\frac{\partial W(\mathbf{y})}{\partial x} = \sum_{s=1}^{S} \pi_s \left(p_s \frac{\partial f_s(x)}{\partial x} - w \right) = 0 \tag{6.8}$$

which can be further rewritten as

$$\sum_{s=1}^{S}\pi_s p_s \frac{\partial f_s(x)}{\partial x} = E\left(p_s \frac{\partial f_s(x)}{\partial x}\right) = w \tag{6.9}$$

which implies that a risk-neutral producer increases her input until the expected marginal product of the particular input exceeds its cost.

Furthermore, it is obvious from Equations (6.7 and 6.8) that the marginal net return will be greater than zero in some states of nature and less than zero in the remaining states of nature. This has an important implication, which is that both risk-neutral and risk-averse producers would apply higher input in some states of nature and lower input in the remaining states of nature relative to the input that they would have applied to the production process had they been fully aware about the state that would prevail in the future. In mathematical terms there exist at least two states, say $s, j \in \Omega$ such that

$$p_s \frac{\partial f_s(x)}{\partial x} - w \neq p_j \frac{\partial f_j(x)}{\partial x} - w. \tag{6.10}$$

A risk-averse decision-maker would apply more input relative to a risk-neutral decision-maker, if

$$\left.\frac{\partial W(y_1(x),\ldots,y_S(x))}{\partial x}\right|_{x=x^*} > 0, \tag{6.11}$$

where x^* is the optimal input applied by a risk-neutral producer to the production process. Using the fact that $y_s(x) = p_s f_s(x) - wx$, Equation (6.11) can be written as

$$\left.\frac{\partial W(y_1(x),\ldots,y_S(x))}{\partial x}\right|_{x=x^*} = \sum_{s=1}^{S} W_s(\mathbf{y})\left.\left(p_s \frac{\partial f_s(x)}{\partial x} - w\right)\right|_{x=x^*} > 0. \tag{6.12}$$

As at x^* Equation (6.8) holds, Equation (6.11) can be rewritten as

$$\left.\frac{\partial W(y_1(x),\ldots,y_S(x))}{\partial x}\right|_{x=x^*} = \sum_{s=1}^{S} (W_s(\mathbf{y}) - \pi_s)\left.\left(p_s \frac{\partial f_s(x)}{\partial x} - w\right)\right|_{x=x^*} > 0. \tag{6.13}$$

Partitioning the set containing all possible states of nature ($\omega \cup \omega' = \Omega$ and $\omega \cap \omega' = \phi$) into subsets containing 'good' and 'bad' states of nature represented by ω and ω', respectively, Equation (6.13) can be written as

$$\left.\frac{\partial W(y_1(x),\ldots,y_S(x))}{\partial x}\right|_{x=x^*} = \sum_{i\in\omega}(W_i(\mathbf{y})-\pi_i)\left(p_i\frac{\partial f_i(x)}{\partial x}-w\right)\Bigg|_{\mathbf{x}=\mathbf{x}^*} + \sum_{j\in\omega'}(W_j(\mathbf{y})-\pi_j)\left(p_j\frac{\partial f_j(x)}{\partial x}-w\right)\Bigg|_{x=x^*} > 0. \tag{6.14}$$

Further, if at the optimal input used by a risk-neutral producer, the marginal net returns in the 'good' states of nature are negative and positive in the 'bad' states of nature, then a risk-averse producer will apply more input than a risk-neutral producer.

Thus, a risk-averse decision-maker will apply more input to the production process than a risk-neutral decision-maker (whose optimal input usage is x^{RN}), if the marginal return in 'bad' state of nature is positive and the marginal net return in 'good' state of nature is negative. On the contrary, a risk-averse decision-maker will use less input than a risk-neutral decision-maker, if the marginal return in 'bad' state of nature is negative and the marginal net return in 'good' state of nature is positive.

6.3. Example: Nauges, O'Donnell and Quiggin model

Taking logarithm on both sides of Equation (5.11), the production technology in O'Donnell *et al.* (2010) can be rewritten as

$$\ln z_s = \frac{1}{b}(\ln x_s - \ln a_s), s \in \Omega = \{1,\ldots,S\}. \tag{6.15}$$

Nauges *et al.* (2009) extend production technology represented by Equation (6.15) into a more flexible specification of technology given by

$$\ln z_s = \frac{1}{b}[\ln(\theta x - \theta x_s + x_s) - \ln a_s], \quad s \in \Omega = \{1,\ldots,S\}, \tag{6.16}$$

where $0 \leq \theta \leq 1$ is the parameter that determines whether an input allocated to a particular state is state-specific or state-general and $x = \sum_{s=1}^{S} x_s$ is the total input allocated across all states of nature.

Nauges *et al.* (2009) apply their model to a panel data of Finnish farms. The data contain yearly farm-level observations on land (in acres) allocated to each crop, crop output and expenditures on labour, fertilisers and pesticides. The major source of production risk arises owing to the variable nature of weather conditions. The output in the model is an output index which is derived by dividing the sum of the value of wheat, barley and oats by an output price index. Land is the only state-allocable input. In their application, the state-specific, state-allocable input allocations to different states of nature are observed.

The three cases of interest for this technology are

(i) $\theta = 0$.
In this case, this technology boils down to O'Donnell *et al.* (2010) technology given by Equation (6.15).

(ii) $\theta = 1$.
In this case, technology takes the form of output-cubical technology and can be written as

$$\ln z_s = \frac{1}{b}[\ln x - \ln a_s], s \in \Omega = \{1, \ldots, S\}. \quad (6.17)$$

(iii) $0 < \theta < 1$.
Here, the technology becomes state-general because a nonzero output is produced in any given state of nature even if no input is allocated to that particular state of nature.

An example from agriculture can best describe state-general input. In wheat production, any farmer may experience either a 'wet' season or a 'dry' season. By using x amount of fertiliser she can produce z_1 in 'wet' season and z_2 in 'dry' season. The yields in 'wet' and 'dry' seasons are mutually exclusive and they only depend on the total input applied in the production process. The farmer thus produces nonzero output no matter what state of nature she experiences by applying a given amount of fertiliser to the production process.

Jaramillo *et al.* (2010) using a methodology similar to Nauges *et al.* (2009) argue that state-contingent approach can be used for studying dynamics of seed trait adoption. In their application, the authors find that the traits which protect yield against production uncertainty are state-specific and the traits which supplement production factors that are more stable are state-general.

6.4. Limitation

Whether the technology is state-allocable or state-general, when the total input used in the production process is fixed, the substitution between state-contingent outputs is brought about by re-allocating input among the various states of nature. In the case of state-specific, state-allocable technology, the substitution between state-contingent outputs is exclusively accomplished by substituting inputs between various states of nature. But this may not be true in the case of state-general technology because if the input is state-general, then it is possible to produce output in a given state of nature even if no input is allocated to the corresponding state of nature. Therefore, the 'beaker' diagram in Figure 4 also holds for state-general technology, but unlike state-specific, state-allocable technology, it is not possible to identify input allocations to individual states of nature.

Again, the empirical implementation of Nauges *et al.* (2009) model requires knowledge of the input allocations to different states of nature. In most practical applications, input allocated to different states of nature is not observed.

7. Conclusion

A major shortcoming of conventional productivity analysis is that it does not explicitly take into account the substitutability of inputs (and hence the outputs) between the potential states of nature. These potential states of nature arise owing to the inherently stochastic nature of the operating environment faced by the decision-makers. To account for the stochastic nature of the production process, O'Donnell and Griffiths (2006) estimate state-contingent production frontiers. A limitation of their approach is that they assume the technology to be output-cubical, thereby ruling out substitution of inputs (and hence the outputs) between the various states of nature.

O'Donnell *et al.* (2010) by way of simulation show that in an uncertain decision environment, rational decision-makers, using the same stochastic technology and operating in the same markets, often make different production choices. However, O'Donnell *et al.* (2010) specify a state-allocable technology that is state-specific. In most empirical applications, the production technology is usually state-allocable but not state-specific.

This limitation is overcome by Nauges *et al.* (2009) as they estimate a technology that is state-general in nature. However, in their application, input allocations to various states of nature are observed. But, in many real-world applications, the input allocated to different states of nature is unobserved and only the total input used in the production process is observed. Therefore, future research needs to be focused on devising techniques to estimate state-general production technology, when only the total input applied to production process is observed.

The state-contingent theory holds good even when there is continuum (see Chambers and Quiggin 1998; Briec and Cavaignac 2009, for details) of states of nature. However, Just (2003) and Rasmussen (2004) mention that the main limit to the empirical application of the state-contingent approach is the theoretically large number of states that need to be defined to realistically represent agricultural environments. Rasmussen (2004) shows that even considering a limited number of state variables results in an impossibly large number of states. Given these comments, in future research it is important to explore the factors determining the appropriate number of states.

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