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OPTIMAL BUFFER STOCK POLICIES FOR WHEAT AT THE WORLD LEVEL

JOHN O. S. KENNEDY*

La Trobe University

Alternative wheat storage policies which maximise the expected present value of returns for consumers, producers, a monopoly storage agency and society as a whole are derived using a dynamic programming model. Results are compared with those from an earlier simulation model, and are found to justify higher investment in storage capacity compared with that suggested by the simulation model. The model is extended to derive optimal storage policies if production follows a stochastic cobweb process.

Introduction

One of the roles of theoretical analysis of real-world problems is to highlight areas of uncertainty which require empirical investigation before solutions can be found. Recent literature on the theory of buffer stock policies has succeeded in doing this. Analysis of the simplest market model with linear supply and demand schedules subject to random parallel shifts and with costless storage leads to definite conclusions on the distribution of benefits between producers and consumers. It has been shown that it becomes increasingly difficult to draw conclusions about the outcomes of buffer stock policies as assumptions are modified to reflect reality more closely (Turnovsky 1974; Just et al. 1977). Hence, empirical models have an important function in complementing theoretical analysis.

A recent example of an empirical model of world wheat storage is Reutlinger's (1976) simulation model. Reutlinger admits that the model is crude and aggregative, but believes it is useful for obtaining rough estimates of the impact of alternative storage rules. The assumptions of the model are: world demand for wheat is a deterministic function of price, the demand schedule consisting of two linear segments; world wheat production is stochastic and can be represented by a triangular distribution; estimates can be made of storage costs and the relevant rate of interest for discounting; and results of alternative buffer stock policies can be evaluated in terms of consumer surplus, producer revenue and financial costs of storage.

The aim in this article is to argue the merits for an optimising rather than a simulation approach to this problem in particular, and more generally for research work on optimal buffer stock policies. In doing so, buffer stock policies alternative to those suggested by Reutlinger

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are evaluated, initially keeping as close as possible to Reutlinger's model structure. Subsequently policies are obtained for an extended model in which production in any year is a stochastic function of price realised in the previous year.

Optimising versus Simulation Models

An important conclusion which Reutlinger draws from the results of his model is that, on the economic criteria he initially set for evaluating his buffer stock policies, only relatively low investment in storage capacity would be worthwhile. Economic storage capacity would be about 5 to 10 Mt. However, taking wider considerations into account, such as increased price stability leading to better resource allocation decisions and lower rates of inflation, Reutlinger argues that a 20 Mt program could be justified.

However, only two storage policies, A and B, are tested by Reutlinger, for storage capacities ranging from 5 to 30 Mt. The policies consist of two decision rules: if production in any year is below a certain lower trigger level, release stocks to bring consumption up towards this level to the extent that stocks permit; if production in any year is above a certain higher trigger level, accumulate stocks to bring consumption down towards this level to the extent that vacant storage capacity permits. Policies A and B differ in the specification of the lower trigger level.

Whilst such policies are guaranteed to increase the stability of world consumption and price, they do not do so optimally with respect to any defined economic criterion. Reutlinger argues against the use of an optimising model because of the difficulty of obtaining a social welfare function in which agreed weights can be attached to the interests of producers, consumers and the storage agency. However, implicit weights are necessarily attached to the interests of different groups in selecting any policy as desirable from the results of a simulation model. The danger in not taking an optimising approach is that the existence of some desirable storage policies may not be recognised. In this article, optimal policies derived from a dynamic programming model are investigated using an objective function which allows for weights to be specified for different interest groups. The returns from these policies are compared with those from Reutlinger's two policies.

A further reason for taking an optimising approach is that the behaviour of perfectly competitive storage operators can be modelled. The type of storage policy with which Reutlinger experimented may be described as a price-band or production-band approach. Stein and Smith (1977) review models developed by other workers who have taken a similar approach. They are critical of this approach because no account is taken of the reaction of private storage operators to a price-band policy implemented by government. An exception is the study by Gardner (1977). However, Stein and Smith (1977) prefer what they term the economic approach, under which discounted expected social welfare is maximised. Optimal policies for the economic approach can be interpreted as either the optimal policy to be implemented by government alone or the policy which would be implemented automatically by a perfectly competitive storage industry. They point out that Gustafson (1958) originally showed this.

Although Stein and Smith (1977) argue that the grain storage market in the U.S.A. closely approximates to a perfectly competitive market, it is of interest, at least for comparative purposes, also to determine optimal policies for a monopoly storage agency. This was the approach taken by Alaouze et al. (1978*b*) in considering policy for Australian wheat storage.

The Dynamic Programming Model

There are many variables besides current world production which could be taken into account in framing an optimal stocking policy. For example, optimal policies should depend on expected changes in demand and supply functions. However, even if expectations are assumed stationary, one easily observed and relevant variable is current stock. A dynamic programming model is developed below for finding optimal policies in terms of maximising the present value of returns, in which the state variables are current production and current stock.

The model structure and data are kept as close as possible to those of Reutlinger's simulation model, so that the results of following the simulation policies can be compared with the dynamic programming policies. Financial results should therefore be interpreted in terms of 1975 prices, and any shifts in demand and supply since 1975 should be assumed to have been in step with each other. However, two important model modifications are made. Firstly, steady-state optimal policies are determined assuming an infinite time horizon. A 30-year time horizon was used in Reutlinger's simulation model. The derivation of steady-state policies means that optimal policies for each of 30 years do not have to be appraised, which in any case can be assumed to approximate closely the steady-state optimal policies towards the end of 30 years. A second modification is the use of a range of seven discrete values of production and stocks. This constraint on model structure is introduced because the dynamic programming problem is solved numerically. Seven values of production and stocks were defined taking account of the trade-off between accuracy and computing costs.

The following recurrence equation describes the steady-state optimal return function starting with stock level s and current production x:

$$f(s,x) = \max_{\triangle s} \{r(s,\triangle s,x) + \alpha \sum_{i=1}^{7} \theta_i\, f\,(s+\triangle s,x_i)\}$$

$$\text{subject to} - s \leqslant \triangle s \leqslant c-s$$

$$\sum_{i=1}^{7} \theta_i = 1,$$

where $\triangle s$ (if positive) is the addition to be made to stock; r is the return function for the current period; α is the period discount factor assuming a rate of interest of 8 per cent per annum; θ_i is the probability that production in the next period will be x_i, assuming a triangular probability distribution; and c is the storage capacity. The first constraint stipulates that stock depletion cannot be greater than current stock, whilst stock accumulation cannot exceed vacant storage capacity.

Optimal policies were obtained by solving the recurrence equation numerically using policy iteration, as described by Nemhauser (1966). Solutions were generally obtained after five or six iterations, starting with an initial policy 'guess' of zero change in stock for every stock/production state. Long-run or steady-state probabilities of the occurrence of each stock/production state were calculated for each policy. Any policy implies a consumption level for each state, $x - \Delta s$. It was therefore possible to calculate the long-run probability of consumption being in any of a number of consumption bands.

The seven possible production levels ranged from 314 to 386 Mt at intervals of 12 Mt. The seven stock levels ranged at equal intervals from zero to the stipulated maximum, c. The three demand schedules suggested by Reutlinger are used, plus a fourth one which is a straight line with an elasticity of —0.2 at its mid-point. The coding of the demand schedules for future reference is given in Table 1.

TABLE 1

Classification of Demand Schedules

Demand schedule	General form[a]: ($p = a + bq$) $q < 350$ a	b	$q > 350$ a	b
D1	1374.5	—3.57	541.5	—1.19
D2	1374.5	—3.57	238.34	—0.32
D3	3003.29	—8.22	541.5	—1.19
D4	750.0	—1.79	750.0	—1.79

[a] p is price of wheat per tonne and q is quantity (Mt)

Optimal policies were derived for five values of c (3, 5, 10, 20 and 30 Mt) and five present-value objective functions: maximising gross financial benefits, being the net gains to the storage agency of buying, selling and storing[1] wheat according to the optimal policy, before charging for storage capacity costs; maximising consumer benefits, being the increase in consumer surplus resulting from storage; maximising producer benefits, being the increase in producer revenue resulting from storage; maximising gross economic benefits, being the sum of the previous three benefits; and minimising the weighted combination of the probability of consumption falling below 332.5 Mt and the negative of gross economic benefits. Each objective function implies a different period return function, r.

A second model, essentially a component of the dynamic programming model, was used for evaluating any storage policy in terms of any of the above objective functions. In this way it was possible to find how the policy optimal for one interest group affected other interest groups. It was also possible to obtain comparable results for Reutlinger's A and B policies for the same discrete, steady-state (DS) framework assumed for the dynamic programming model. For ease in referring to the results obtained from different combinations of model and policy, the coding scheme shown in Table 2 is used.

[1] Variable cost of storage is assumed to be \$2/t.

TABLE 2

Classification of Models and Policies

	Models
R	Reutlinger simulation model
DS	Discrete, steady-state model
DSPR	Discrete, steady-state model with production responsive to price
	Policies
RA	Reutlinger's policy A — release/accumulate production triggers 345/355 Mt
RB	Reutlinger's policy B — release/accumulate production triggers 335/355 Mt
MXGEB	Maximise gross economic benefits
MXGFB	Maximise gross financial benefits
MXCB	Maximise consumer benefits
MXPB	Maximise producer benefits
MNCS	Minimise probability of consumption falling below 332.5 Mt

Results

Reutlinger's original present-value results for a 30-year time horizon have been converted to estimates of infinite time horizon present-value results and are shown in Table 3. They can be compared there with results from the discrete, steady-state (DS) model for the same policies, A and B, tested by Reutlinger, with certain provisos. The present values over an infinite time horizon for the Reutlinger results were calculated assuming the same value flows every 30 years to infinity. However, such a calculation underestimates the true present values because Reutlinger assumes an opening stock of zero. Stocks at the beginning of subsequent 30-year periods are likely to be positive and returns in future 30-year periods higher. Present values for the DS model are expected present values assuming zero stocks at the beginning of the first year of operations. Secondly, there must be some differences in results for the two models because of the enforced discrete values of the state variables in the DS model. A comparison shows DS values on the whole larger in absolute terms than values for the Reutlinger model, but there are some very substantial differences for storage capacities of 20 and 30 Mt. However, Reutlinger's figures are only estimates of expected values derived from a simulation model. Some of the standard deviations of the present values quoted by Reutlinger (his Table 3, p. 5) imply relatively large standard errors. A noticeable difference in results concerns the probabilities of consumption being below 332.5 Mt. The results for the Reutlinger model are much more sensitive to increasing available storage capacity than are the DS results. Again, this is presumably due to the different structures of the two models.

DS results for Reutlinger's policy A are compared with DS results for policies maximising gross economic benefits in Table 3 for demand schedule D1, and in Figures 1 and 2 for demand schedules D2 and D3.

TABLE 3

Present Values of Storage Benefits for Alternative Models and Policies for Demand Schedule D1[a]

Type of benefit	Storage rule (Reutlinger)						
	A				B		
	Storage capacity (Mt)				Storage capacity (Mt)		
	5	10	20	30	10	20	30
	$m	$m	$m	$m	$m	$m	$m
Gross economic benefits	320 (388)	435 (511)	373 (366)	199 (12)	—6 (—41)	—432 (—593)	—960 (—1160)
Gross financial benefits	215 (256)	138 (184)	—322 (—458)	—726 (—920)	—173 (—238)	—776 (—1020)	—1410 (—1670)
Consumer benefits	5450 (6990)	8960 (11200)	12500 (15600)	13700 (14100)	2640 (3810)	1860 (3870)	—61 (2090)
Producer benefits	—5350 (—6860)	—8660 (—10800)	—11800 (—14800)	—12700 (—13200)	—2470 (—3610)	—1520 (—3440)	512 (—1580)
Percentage probability of consumption below 332.5 Mt		8.5 (12.5)	6.9 (12.5)	5.5 (9.9)	7.2 (12.5)	3.6 (5.3)	2.1 (3.7)

[a] Obtained from the following models and policies:

Line 1: R-RA and R-RB (infinite time horizon estimates from the 30-year time horizon results (Reutlinger 1976, Table 3)),
Line 2: DS-RA and DS-RB (both in brackets).

All estimates are given correct to 3 significant figures.

Of particular note in these comparisons is the substantially higher present values of gross economic benefits for the optimal policies for storage capacity of 10 Mt and above. However, consumers do not fare quite so well, and producers do not fare quite so badly.

Net economic benefits are calculated by subtracting the present value of investment in storage capacity. Reutlinger assumes a low present value of storage silos of \$50 per tonne of storage capacity over a 30-year period because of the existence of large underutilised capacity. He estimates the present cost of building new storage facilities to be \$150 per tonne of capacity. For the DS model, the \$50 estimate is used for the first 30 years, and the \$150 estimate for every subsequent 30-year period, giving a present cost of \$67 per tonne of capacity. Calculations for net economic benefits from following a MXGEB policy assuming demand schedule D1 lead to the same policy conclusions as Reutlinger reached with his model, namely that only a relatively small storage scheme of 5 to 10 Mt capacity would be worthwhile on narrow economic criteria. Figure 1 indicates that storage is hardly economic at all if demand schedule D2 is assumed (with a more elastic second demand segment). However, if demand schedule D3 is assumed (with a more inelastic first demand segment), optimal storage capacity of 9 Mt giving a return of \$920m for the Reutlinger A policy compares with 12 Mt and \$1200m, respectively, for the MXGEB policy.

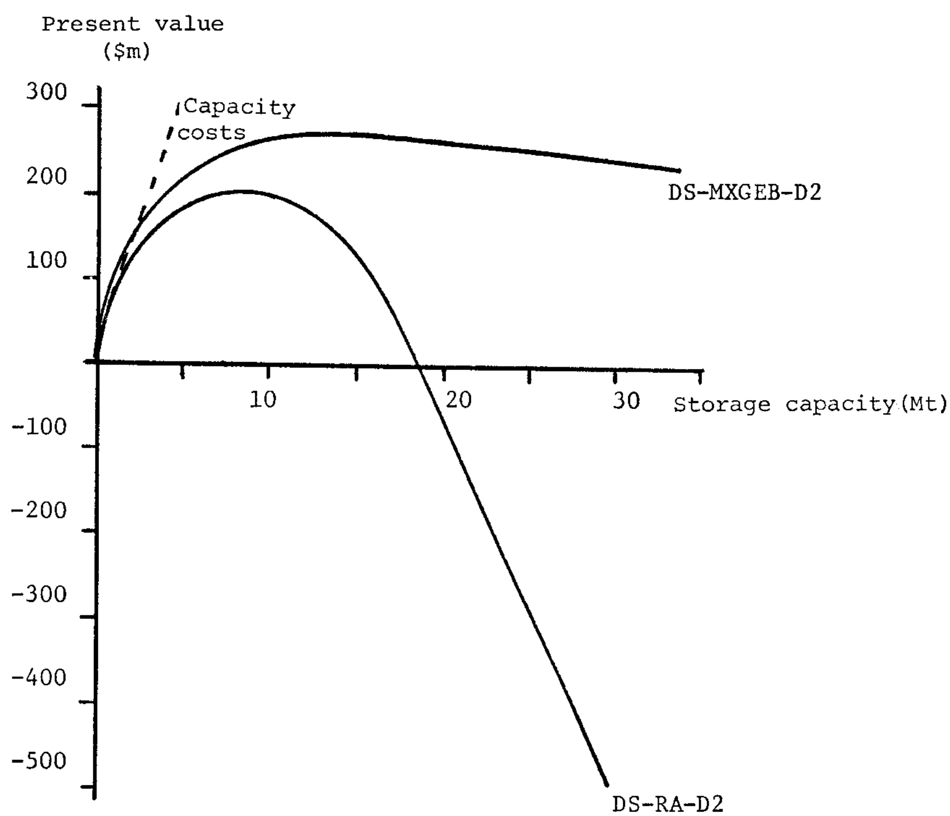

FIGURE 1—Present value of gross economic benefits for Reutlinger and optimal policies for demand schedule D2.

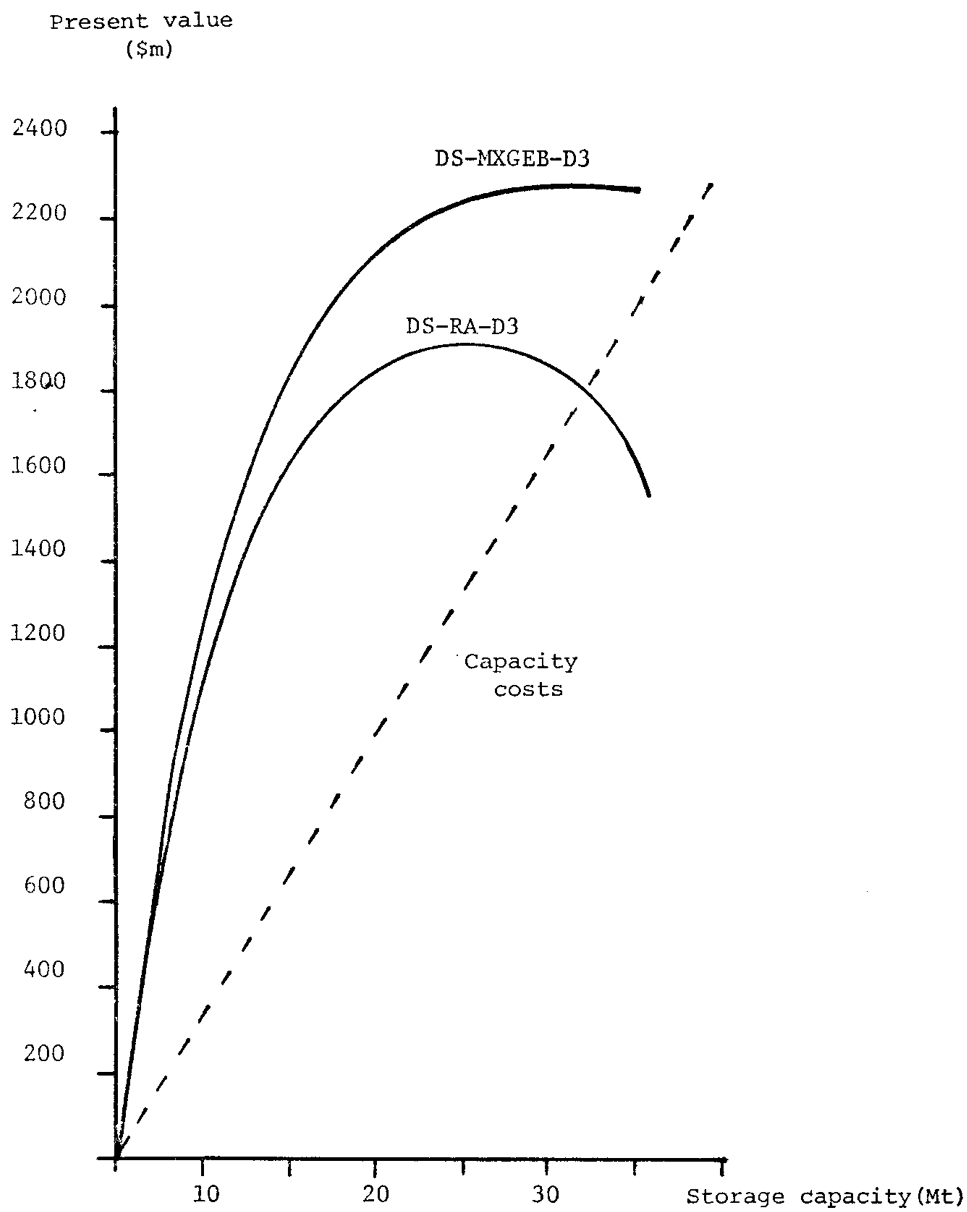

FIGURE 2—Present value of gross economic benefits for Reutlinger and optimal policies for demand schedule D3.

Returns from following policies optimal with respect to the different interest groups are compared in Table 4. Table 5 displays some of the storage policies which indicate optimal $\triangle s$ for any combination of production and opening stock levels.

These results give some quantitative estimates of the conflict in interests between consumers and producers in the operation of a buffer stock, and highlight the potential pressures which could be brought to bear on a storage agency. In particular, producers would gain very high returns if the MXPB policy were followed, which destabilises consumption and price.

TABLE 4

Evaluation of Alternative Storage Policies for Storage Capacity of 20 Mt

Model/policy/ demand schedule	Gross economic benefits	Gross financial benefits	Consumer benefits	Producer benefits	Probability of consumption shortfall[a]
	$m	$m	$m	$m	%
DS-RA-D1	366	—458	15592	—14768	12.5
DS-RB-D1	—593	—1023	3871	—3440	5.3
DS-MXGEB-D1	651*	45	13610	—13004	12.5
DS-MXGFB-D1	511	319*	7503	—7312	12.5
DS-MXCB-D1	443	—671	20219*	—19105	12.5
DS-MXPB-D1	—9418	—14169	—73291	78043*	30.0
DS-MNCS-D1	—532	—5254	6092	—1370	3.7*
DS-MXGEB-D4	536*	64	—3419	3890	12.5
DSPR-MXGEB-D1	1481*	—37	20588	—19071	14.4
DSPR-MXGEB-D4	283*	104	—1689	1867	4.0

[a] A consumption shortfall is defined as consumption below 332.5 Mt.
* Optimal value.

One of Reutlinger's policy conclusions was that, taking a broad view of the benefits of storage policies, policy B, operated with a storage capacity of 20 Mt, could be justified. A search was conducted with the optimising model for a policy which dominated policy B in terms of gross economic benefits and probability of a consumption shortfall. The result was policy MNCS, which can be seen to dominate the RB policy in Table 4.

Of interest in Table 5 is the reverse symmetry of the RA policy about the centre column. This does not appear in any of the optimal policies. Compared with the RA policy, the MXGEB and MXGFB policies tend to imply lower storage for any particular production and stock combination. Generally, for all policies, there is no simple linear relationship between optimal storage and stock level plus production.

The results clearly support Reutlinger's finding that the desirability of storage depends crucially on the assumed world demand schedule for wheat. For this reason sensitivity analysis has been conducted with alternative demand schedules. Indeed, it is because the three demand schedules, D1 to D3, are segmented that the theoretical result for a linear demand schedule (that producers gain at the expense of consumers with storage) is reversed. Optimal policies were also obtained for a linear demand schedule, D4, with a mid-point elasticity of —0.2, the estimate favoured by Reutlinger based on elasticities reported by Rojko et al. (1971). As can be seen in Table 4, the DS-MXGEB policy gave lower gross economic benefits for D4 than for D1.

Production a Function of Price

A desirable extension of Reutlinger's model is to allow production in any year to be a stochastic function of realised prices in previous

TABLE 5

Alternative Storage Policies for Storage Capacity of 20 Mt[a]

	Production							Production						
Stock	314	326	338	350	362	374	386	314	326	338	350	362	374	386
	DS-RA-D1							DS-RB-D1						
0	0	0	0	0	7	20	20	0	0	0	0	7	20	20
3	—3	—3	—3	0	7	17	17	—3	—3	0	0	7	17	17
7	—7	—7	—7	0	7	13	13	—7	—7	0	0	7	13	13
10	—10	—10	—7	0	7	10	10	—10	—10	0	0	7	10	10
13	—13	—13	—7	0	7	7	7	—13	—10	0	0	7	7	7
17	—17	—17	—7	0	3	3	3	—17	—10	0	0	3	3	3
20	—20	—20	—7	0	0	0	0	—20	—10	0	0	0	0	0
	DS-MXGEB-D1							DS-MXGFB-D1						
0	0	0	0	0	7	13	20	0	0	0	0	3	7	10
3	—3	—3	—3	0	7	13	17	—3	—3	—3	0	3	7	10
7	—7	—7	—7	0	3	10	13	—7	—7	—7	—3	0	3	7
10	—10	—10	—10	—3	3	10	10	—10	—10	—7	3	0	3	7
13	—13	—13	—10	—7	0	7	7	—13	—13	—7	—7	—3	0	7
17	—17	—17	—13	—7	0	3	3	—17	—13	—7	—7	—3	0	3
20	—20	—20	—13	—10	—3	0	0	—17	—13	—10	—10	—7	0	0
	DS-MXCB-D1							DS-MXPB-D1						
0	0	0	0	0	10	20	20	20	20	20	20	20	0	0
3	—3	—3	—3	0	10	17	17	17	17	17	17	—3	—3	—3
7	—7	—7	—7	0	10	13	13	13	13	13	13	—7	—7	—7
10	—10	—10	—10	0	10	10	10	10	10	10	10	—10	—10	—10
13	—13	—13	—13	0	7	7	7	7	7	7	—13	—13	—13	—13
17	—17	—17	—13	0	3	3	3	3	3	3	—17	—17	—17	—17
20	—20	—20	—13	0	0	0	0	0	0	0	—20	—20	—20	—20
	DSPR-MXGEB-D1[b]							DSPR-MXGEB-D4[c]						
0	0	0	0	7	10	17	20	0	0	0	0	3	10	20
3	—3	—3	—3	7	10	13	17	—3	—3	—3	—3	3	10	17
7	—7	—7	—7	0	7	13	13	—7	—7	—7	—7	3	10	13
10	—10	—10	—7	0	7	10	10	—10	—10	—10	—10	0	10	10
13	—13	—13	—7	0	3	7	7	—13	—13	—13	—13	—3	7	7
17	—17	—17	—7	—3	0	3	3	—17	—17	—17	—17	—3	3	3
20	—20	—17	—10	—7	0	0	0	—20	—20	—17	—10	—3	0	0

[a] All stock and production figures in integer megatonne units.
[b] Mid-point supply elasticity = 0.075.
[c] Mid-point supply elasticity = 0.01.

years. A simple extension along these lines, which does not entail any violation of the assumptions required for dynamic programming with two state variables, is to make expected production follow a cobweb process. For purposes of illustration, the dynamic programming model was modified to accommodate production responsiveness and is referred to as DSPR.

The recurrence equation remains the same, except that instead of one there are now n triangular production probability distributions, each with the same range (covering 8-n discrete production levels) but with different means. The relevant distribution for the following year depends on the current year's realised price, which in turn depends on current consumption or $x\text{-}\triangle s$. The recurrence equation can be rewritten as follows:

$$f(s,x) = \max_{\triangle s} \{r(s,\triangle s,x) + \alpha \sum_{i=1}^{8-n} \theta_i^j(x\text{-}\triangle s).f(s+\triangle s,x_i)\}$$

$$\text{subject to} - s \leqslant \triangle s \leqslant c\text{–}s$$

$$\sum_{i=1}^{8-n} \theta_i^j = 1; \quad j = 1, ..., n.$$

The discrete range of all production possibilities remains as before. The mean of the jth production probability distribution is $350 + 12(j\text{-}(n+1)/2)$, and the range of each distribution is $12(7\text{-}n)$. The index of the probability distribution for the following year is matched with current consumption $(x\text{-}\triangle s)$ by putting $j = n\text{-}k+1$ where k is an integer $(1 \leqslant k \leqslant n)$ and satisfies

$$p(386) + (k\text{-}1)a < p\,(x\text{-}\triangle s) \leqslant p(386) + ka$$
$$a = (p(314) - p(386))/n.$$

In this way, high current consumption, which implies a low realised current price, is matched with a production distribution for the following year having a low mean and vice versa. As a result of this change, the production distribution across all production possibilities is no longer symmetric in the absence of a storage policy. The distribution becomes positively skewed.

With expected production each year dependent on realised price in the previous year, account should be taken of corresponding changes in production costs. For simplicity, the marginal cost of production is assumed to be equal to the mid-point price, $p(350)$, and the total cost of production is calculated for the expected production level. Moreover, the present value of production costs and of benefits to infinity will be dependent on the storage policy, if any, that is adopted. This situation contrasts with that for the DS model, and means that the period return functions, r, can no longer be measures of changes in benefits resulting from following a storage policy compared with following no storage policy. In order to evaluate the return from a storage policy, total costs and returns from following no storage policy to infinity were subtracted from total costs and returns from following the storage policy.

Two DSPR-MXGEB policies for $n = 3$ for demand schedules D1 and D4 are shown in Table 5. Mid-point lagged supply elasticities are 0.075 and 0.1, respectively. Present value figures shown in Table 4 are expected values assuming zero initial stocks and long-run zero-storage production probabilities for production in the first year. The DSPR-MXGEB-D1 policy is similar to the DS-MXGEB-D1 policy. However, gross economic benefits are much higher, and are sufficient to justify a storage capacity of 20 Mt. For $n = 5$ and a supply elasticity of 0.125, gross economic benefits are even higher.

The DSPR-MXGEB policy for the linear demand schedule D4 (as opposed to that for D1) is one for which stocks, when available, are more readily released. Again, unlike the situation for the segmented demand schedules, consumers gain and producers lose. However, gross economic benefits are lower than those for the DS-MXGEB-D4 policy.

This result shows that, if production follows the particular stochastic cobweb process assumed, instead of being random, returns to society from storage are increased, provided the demand schedule is sufficiently convex. They are reduced if the demand schedule is linear.

Conclusion

Comparing the results for the dynamic programming model with those for Reutlinger's simulation model, it is concluded that there are many possibilities for making Pareto-optimal improvements on Reutlinger's A and B policies. Investments in storage capacity larger than those suggested by Reutlinger would be socially desirable.

The dynamic programming model was used to illustrate the potential conflicts of interests of consumers, producers and a monopoly storage agency in the operating of a buffer stock policy. These have implications for whether a storage agency should follow a profit maximising policy, and whether rules specified in terms of quantity or price triggers might not be more desirable than optimal policies if an objective function cannot be agreed upon and made public.

Finally, the author agrees with others (e.g. Turnovsky 1978; Blandford and Lee 1979; Taylor and Talpaz 1979) that there is scope for greater use of optimising approaches in theoretical and applied analyses of buffer stock policies. In the context of the present model, further experiments could usefully be conducted with alternative demand schedules and alternative supply response assumptions. Whether consumers or producers gain or lose from a storage policy which maximises economic benefits depends quite critically on the degree of convexity of the demand schedule. A variety of possible demand schedules facing the major exporters of wheat are discussed by Alaouze et al. (1978*a*). As regards supply, it would be possible to specify empirically derived supply schedules, and to experiment with alternative price expectation models such as mixes of rational and adaptive expectations as suggested by Turnovsky (1978).

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