Australian Journal of Agricultural Economics, Vol. 23, No. 3 (December 1979), pp. 231-232.

AUSTRALIAN WHEAT STORAGE: A DYNAMIC PROGRAMMING APPROACH — A CORRECTION

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G. Hird of the Department of Mathematics, Monash University, has drawn to our attention some technical errors in the text of our article in an earlier issue of the *Journal* (Alaouze et al. 1978). The purpose of this note is to correct these errors and to clarify the exposition of material in our paper.

Despite these corrections, the conclusions appearing on pp. 168-9 of the paper are correct. In making the corrections, the notation and equation numbers appearing in the original paper are retained. For simplicity we redefine some of the variables and functions appearing in the original paper.

P_t	is the wheat price in the period with t stages of the process remaining,
C_t	is the level of carryover,
S_t	is the level of supply,
$K(C_t)$	is the cost of storage function,
α	is the discount factor,
S^*_t	is the maximum permissible level of carryover associated with the supply level S_t, and
$g_t(C_t)$	is the maximised discounted expected net revenue associated with carrying over C_t.

The aim is to find the values of C_{jt} which correspond to the l row maxima of the matrix:

$$(13) \qquad \{P_t\,.\,(S_{it} - C_{jt}) - K(C_{jt}) + \alpha g(C_{jt})\},$$

subject to $0 \leqslant C_{jt} \leqslant S^*_{it}$ for $i = 1, \ldots, l$ and $j = 1, \ldots, m$.

We then claimed to show that the row maximum of (13) is unique and that it is independent of the row. The elements of row i of (13) are partitioned into:

$$(14a) \qquad T_{1ij} = P_t(S_{it} - C_{jt}) - K(C_{jt}),$$

$$(14b) \qquad T_{2ij} = \alpha g(C_{jt}).$$

We then argued that, since T_{1ij} decreases monotonically with C_{jt} and T_{2ij} increases monotonically with C_{jt}, each element of a row of (13) is unique. This argument is false. The sum of two functions, one of which is monotonically decreasing in a variable, and the other monotonically increasing in the same variable, is not necessarily monotonic.

Fortunately, we do not require the row maximum (13) to be unique for the remaining results of the paper to hold. The optimal policy can still be readily computed if more than one element of the set C_{jt} yields the row maximum (Bellman and Kalaba 1965, p. 59). The computer program used to compute the results presented in the paper is general enough to yield correct answers if more than one element of the set

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C_{jt} yields the row maximum. Furthermore, the arguments leading to the graph of the storage rule (Figure 1, p. 169) still hold if more than one element of the set C_{jt} yield the maximum. The only change is that there is a storage rule (and a graph similar to Figure 1) for each element of C_{jt} that yields the row maximum. Operationally, any of these storage rules can be used, since they all yield the same value of the return function. The storage rules presented in Tables 1 and 2 (pp. 170-1) show the smallest element of the set C_{jt} that corresponds to the row maximum of (13) if more than one element of the set C_{jt} yielded the row maximum.

We also stated that the row maximum of (13) is independent of the row. This was poorly worded. What was intended (and in fact proved on p. 168) is that the *column* in which the row maximum occurs is invariant with the row.

Finally, it was stated that, when the row maximum of (13) corresponds to an interior maximum, the following marginal condition holds in the continuous case:

$$- \partial T_{1i}/\partial C_t = \partial T_{2i}/\partial C_t. \tag{14c}$$

This is the basis of our economic interpretation of Bellman's principle of optimality for inventory problems. An additional requirement for (14c) to be correct is that the function:

$$f_i(C_t) = T_{1i} + T_{2i} \tag{14d}$$

is differentiable at the maximum (Avriel 1976, pp. 10-11).

References

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