AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

Australian Journal of Agricultural Economics, Vol. 38, No. 2 (August 1994), pp 143-170

PRICE-LINKED FARM AND SPATIAL EQUILIBRIUM MODELS

R. L. BATTERHAM & T. G. MACAULAY*
Department of Agricultural Economics,
University of Sydney, NSW, 2006

The integration of detailed farm supply models with the basic spatial equilibrium model, is outlined. The direct linking of farm linear programming models with the spatial equilibrium model is achieved so that both prices and quantities are endogenous. Both the farm model and the spatial equilibrium model must be specified in primal-dual form to make the linkages. Limited details of the use of such a model in a study of a segment of the grain handling system in New South Wales are presented along with conclusions relating to the pricing of grain handling services.

For some time it has been recognised that a simple input-output structure could be used to generate a supply relationship in a spatial trading system (Takayama and Judge 1971). A method of linking spatial equilibrium models and linear programming representations of farm models is outlined in this paper. The links are made through endogenous price and quantity variables. The models used to illustrate the approach were developed as part of research conducted for the Royal Commission into Grain Storage, Handling and Transport (MacAulay, Batterham and Fisher 1988a and 1988c). The detailed policy analysis resulting from this work is presented in the Royal Commission reports.

The approach used is a modified application of the activity analysis models of Takayama and Judge (1964b and 1971, ch. 14). Farm linear programming models are embedded in the spatial equilibrium model and replace the estimated farm or regional supply functions of the standard spatial equilibrium model. The contribution of this work is to illustrate that such farm models can be linked both in terms of prices and quantities when a primal-dual form of the spatial equilibrium model is used.

With price and quantity linked farm and spatial equilibrium models, assumed changes in any part of the production and marketing system can

* Revised version of a Contributed Paper to the 34th Annual Conference of the Australian Agricultural Economics Society, University of Queensland, St Lucia, 13-15th February, 1990. The authors acknowledge the support of Brian Fisher in the early development work for this paper and also the Royal Commission into Grain Storage, Handling and Transportation. The assistance of Stephen Whelan in the final stages of the preparation of the paper, and helpful comments from two anonymous referees is also acknowledged.

© Copyright 1994 The Australian Agricultural Economics Society

be analysed in terms of the consequences on any other parts of that system. For example, the effects of a policy change at some point in the marketing chain can be analysed in terms of its possible effects at the farm level and at the market level. Similarly, any change in technology or farmer behaviour that may affect farm or regional supply can be considered in terms of its effects further up the marketing chain.

In addition, incorporating farm models into spatial equilibrium models provides a means of generating an endogenous estimate of supply when econometric estimates are difficult to make, such as in the case of very limited time series data. The technique used in the linking of farm models within a spatial equilibrium model is to specify both as primal-dual models.

Spatial Equilibrium Model

A primary purpose for developing a spatial equilibrium model is to determine equilibrium values for prices, quantities and trade flows between spatially (and/or temporally) separated regions or markets. In the simplest form of the model the assumption of perfect competition between regions is adopted and supply, demand and transport costs between each of the regions are assumed to be known. A two-region, single-commodity model can be solved graphically (see Bressler and King 1970 or Tomek and Robinson 1981). Slightly more complex models can be solved algebraically using the concepts of consumer and producer surplus (Samuelson 1952).

The formulation of the problem as a quadratic programming model by Takayama and Judge (1964a) allowed direct numerical solution of reasonably large models. More importantly, their formulations permitted incorporation of policy interventions of various types such as tariffs and quotas between regions and non-competitive market structures. Since the development by Takayama and Judge, spatial equilibrium models have been used to analyse many applied economic problems in agriculture and other sectors of the economy (see, for example, Judge and Takayama 1973, Takayama and Labys 1986, Harker 1985).

Takayama and Judge (1971) showed that the original form of model based on a net social welfare objective function could also be solved using what they termed 'net social monetary gain' as an objective function (referred to in this paper as 'net revenue'). This form of the objective function required a self-dual form of the spatial model. This more general form of the model could incorporate non-symmetric supply and demand coefficient matrices and still provide a solution which satisfied the requirements for a competitive market solution. In addition, it was recognised that it could be solved as a complementarity problem (Takayama and Judge 1971, p. 255).

In this paper the price form of the spatial equilibrium model as outlined by Takayama and Judge (1971) will be used (a simpler presentation is given in Martin 1981). With a net revenue objective function the model is self-dual model[1] and thus has both price and quantity variables included as active variables (MacAulay and Casey 1987). Inclusion of price and quantity variables provides additional scope in the specification of policy intervention mechanisms that can be included in the model. It also has the interesting consequence that the optimum value of the objective function is zero since the dual problem is subtracted from the primal and if the mathematical problem is a concave programming problem then at the optimum the primal value is equal to the dual. This provides a helpful check on the logic of model specification and on the accuracy of data entry.

It is also the primal-dual character of the model that permits the connection of farm models with the spatial model by linking the output of the commodity modelled at the farm level to both price and quantity variables for the same commodity in the spatial equilibrium model. Thus, there is a simultaneous determination of equilibrium prices and quantities in the farm and spatial models. The result of this simultaneous solution is that if a higher price is generated for the commodity modelled in the spatial equilibrium model this price is transmitted to the farm model. The farm model solution will simultaneously generate a larger amount of the commodity since relative prices in the farm component of the model will have changed in favour of the higher priced commodity. However, at the same time, the farm solution will be subject to the input-output coefficients and resource constraints so that with a price rise for the commodity, the imputed shadow values on the effective farm resource constraints will also rise.

A detailed matrix representation of the model, along with a simple numerical example are presented in the Appendix.

Farm Linear Programming Models

Linear Programming Models and Estimation of Farm Level Supply Functions

Three methods have been commonly used to estimate supply functions at the farm level. They are econometric methods, producer panels and linear programming models. Each method has well known advantages and limitations when applied to particular situations (see, for example, Cowling and Gardner 1963 and Shumway and Chang 1977). The major difficulties with the use of econometric methods are the lack of historical farm level data, and the difficulty of specifying the complexity of supply response functions. Producer panels rely on the correspondence between interview responses and actual farm behaviour in the supply of the commodity in question.

[1] A special purpose Fortran program was written to convert a primal spatial equilibrium model into a primal-dual model. The program incorporates several error checking routines, and is available from the authors.

Major limitations to using linear programming in supply response analysis are the costs of data collection, model construction and solution interpretation. The data required are technical relationships drawn from the biological sciences such as agronomy and animal science, and from engineering. Model construction and the interpretation of solutions require a skilful analyst and are therefore costly. The principal advantage of linear programming is the flexibility of the behavioural assumptions that can be accommodated in such models. It is easy to model alternative farmer objectives, or the impact of alternative government policies on farm decision making.

In the late 1920s, Black and Johnson pioneered the use of budgeting methods to estimate farm supply functions. The use of linear programming models to estimate farm supply response functions is based on the same theoretical concepts, that is, the generally accepted neoclassical theory of the firm (Wu and Kwang 1960). Many studies have made use of linear programming in supply estimation (for example, Ladd and Easley 1959). Similar early Australian work includes Knight and Taplin (1971), and the Aggregate Programming Model of Australian Agriculture constructed at the University of New England (see Kennedy 1972).

Linear programming models for supply estimation are generally built using a series of alternative farm activities (production, marketing, financing, and so on) which are constrained by the resources (physical, financial and human), assumed to be available to a farm entrepreneur. The entrepreneur is assumed to maximise profit or a similar quantifiable objective function. The supply functions are then estimated using parametric programming methods. The parametric programming technique systematically changes the objective function coefficient that represents the price of the commodity in question. This then permits a stepped marginal cost or supply function to be traced out. The economic meaning of the resultant supply function was examined in detail by Kottke (1967). It is relatively easy to use the technique to trace out cross-commodity supply effects by observing the change in the supply of one commodity in the model as a result of the change in the price of another commodity.

Representative Farm Models and Aggregation of Farm Level Supply Functions

The theory of aggregating farm level supply functions to regional or industry supply functions is derived from the theory of the firm. Industry (or regional) supply functions are the horizontal summation of the individual farm supply functions. Obviously, it is generally impossible to model all farms in an industry (or region). The problem therefore becomes one of selecting representative farms to model and aggregating the representative supply functions to the industry or regional level.

The concept of the representative firm and the horizontal summation of the firms supply functions to an industry supply function originally came from Marshall (1959). However, problems of 'aggregation bias' arise when supply functions that would be estimated using models of all farms are different from those that would be estimated using representative farm models (Frick and Andrews 1965). Two solutions to the problems of aggregation and aggregation bias have been developed. The first is a pragmatic one with farm types within regions being stratified into farm enterprises having similar yields, prices and costs. Representative farm models are constructed for each strata in a region using secondary data. Sufficient conditions for exact aggregation using this approach were developed by Day (1963). The conditions are that each representative farm model should exhibit proportional variation in the constraints and net incomes, and that similar technology be employed by the farms modelled. The second approach stratifies farms based on the most limiting resource with different representative farms being constructed depending on which resource is limited (Barker and Stanton 1965). In more recent work Onal and McCarl (1989 and 1991) argue that firms are essentially heterogeneous and that, under these circumstances, aggregation can be achieved using decomposition techniques in linear programming.

Primal Model Links to the Spatial Equilibrium Model

For the models considered in this paper the representative farm linear programming models can be initially specified in a primal form in a similar way to that outlined in any textbook on the use of linear programming in farm management (see, for example, Dent, Harrison and Woodford 1987). There are, however, some minor differences to the specification of the farm models when they are linked to spatial equilibrium models.

The first of these modifications is in the objective function. In 'stand alone' farm models the objective function is usually one of maximising profit. When combined with the spatial model this becomes part of the overall objective function of net social revenue in the spatial equilibrium model. The objective of maximising farm profit means that farm fixed costs have to be estimated and deducted from the total gross margin which is the usual objective of linear programming farm planning models. This is required since in a regional context individual farm fixed costs will be variable and is accomplished in the farm models by inserting a fixed cost activity constrained to equal one.

In addition, the production, marketing and financing activities for each of the agricultural activities considered to be feasible in the region or strata represented were included in the model. Although often done in conventional farm planning models, it is essential to separate the production and marketing activities in the combined models. The marketing activity forms the connection between the farm and spatial equilibrium models for the commodity in question. It is the transfer vector from the farm model into the regional (or strata) supply row of the spatial equilibrium model. The objective function coefficient for this marketing activity for the farm model is zero, as it is specified in the spatial equilibrium part of the model. The objective function value for the farm models thus represents the net variable supply cost of the commodity to the spatial equilibrium model.

The resource constraints for the farm models can either be defined in terms of the total resources represented by the representative farm model or they can be for one representative farm and the production and objective function coefficients then scaled to represent all farms.

The use of an advanced non-linear programming code, such as MINOS (Murtagh and Saunders 1987), makes possible more realistic farm models (Fox 1986). Nonlinear cost functions or production functions and nonlinear functions for cash flows and credit constraints are among the possibilities. The introduction of integer programming into MINOS further extends the possibilities.[2]

Primal-dual Farm Models and Links to the Spatial Equilibrium Model

To incorporate the farm models into a primal-dual spatial equilibrium model so as to replace the supply functions in the standard spatial equilibrium model it is necessary that the farm models be in a primal-dual form also. The farm models can be transformed to dual models in the standard way using the method described in Baumol (1977, p. 107). The rows become the columns and the right hand side the objective function, and the dual objective function is subtracted from the primal objective function. If the primal-dual form of the farm models are solved as models in their own right then, as is the case with the spatial equilibrium model, the optimum value of the objective function is zero.

The dual variables of the farm models are the shadow prices on the resource constraints given the prices determined in the spatial equilibrium part of the system. In the dual part of the spatial equilibrium model the regional (or strata) supply row of the primal model becomes a regional supply price in the dual part of the model. This regional supply price is the price used in the dual part of the farm model. Thus the farm model is linked to the spatial equilibrium model in two ways, by quantity through the primal part of the farm and spatial equilibrium models, and by price through the dual part of these models. The tableau for a simple farm and spatial equilibrium model is given in Table A.1 in the Appendix.

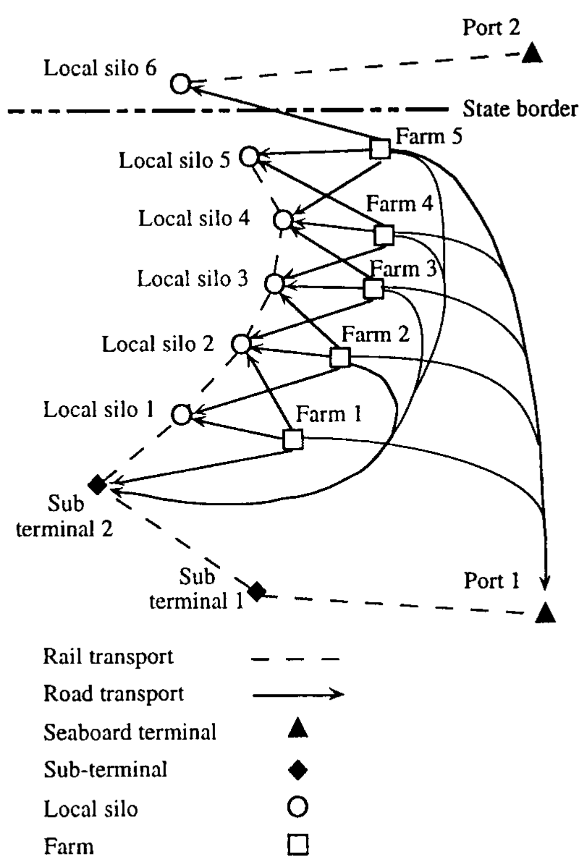
The solution of standard spatial equilibrium models is normally carried out using quadratic programming methods (Takayama and Judge 1971) or general non-linear programming methods such as included in MINOS (Murtagh and Saunders 1987). A listing of alternative solution methods is given in MacAulay (1992). Recent methods have involved fixed-point algorithms (Mackinnon 1975) and variational inequality approaches (Nagurney 1992).

[2] Personal communication, B.A. Murtagh, University of New South Wales, 1989.

An Example of Linked Farm and Spatial Equilibrium Models

To illustrate the type of model discussed above a 'farm-to-ship' model developed as part of research conducted for the Royal Commission into Grain Storage, Handling and Transport (MacAulay, Batterham and Fisher 1988a and 1988c) will be outlined. This model was designed to permit an examination of the effects of different wheat payment arrangements, of changes in transport costs and of changes in on- and off-farm grain storage charges on the shipment of wheat from farms through to ports. The model was constructed so as to represent the shipping of grain from a series of farms at varying distances from a number of country receival points and sub-terminals through to two ports. Transport by road and rail was permitted where appropriate and the storage and handling of grain at each of the sites was included. The possibility of shipment by road to a port terminal was also incorporated into the model. The transport possibilities are illustrated in Figure 1.

FIGURE 1
Grain Transport Schematic



The model consisted of two parts: the first was a non-linear spatial model of the grain transport and handling system and the second a representation of farms producing grain. These models were tested separately and then combined with the spatial system. The result was a non-linear model of grain production, transport, storage and shipping. Quadratic average cost functions, reflecting the cost of handling grain at receival sites, were used for some experiments so that the total cost functions were cubic functions and the whole problem solved as a concave cubic programming problem (MacAulay, Batterham and Fisher 1989). The model was designed to be half-yearly in character.

Network Structure

The spatial equilibrium model was designed to cater for eight grain handling sites and two ports. The port-level demands were represented by linear functions. Associated with the local delivery sites were five representative farms. Each farm was assumed to have six or seven destinations to which wheat could be sent by road; the three nearest local receival sites, two sub-terminals and the two ports. A rail network also transported grain from the local receival sites and sub-terminals to the ports. To take into account the supply of grain from outside the area represented by the five representative farms, additional supplies were allowed for the sub-terminals and the ports. These additional supplies were assumed to be sensitive to price and by consideration of the data on the historical flows through the system, supply functions were derived. Demand functions representing wheat for human consumption were also specified for some of the sites. Thus the model included both standard linear supply and fixed supply quantities and representative farm models.

Objective Function

As with the simplified representation discussed above, the complete model was based on a net revenue objective function with which the farm models were linked. This type of objective function is suitable for linking directly to farm models which have net profit as their objective function.

Demand functions

Little, if any, information was available on the demand for wheat faced by exporters from ports around Australia. Hence, it was necessary to make a judgement as to the likely elasticities of demand at the ports and to use these to derive linear demand functions. Myers, Piggott and MacAulay (1985) estimated an elasticity of export demand of –6.17 based on a price flexibility of –0.162. Given the elasticity value, the volumes of grain outloaded from the ports, together with an export price for Australian Standard White (ASW) wheat of $175 per tonne, demand function coefficients could then be calculated. To derive the demand functions for each of the ports it was assumed that they had the same slope as the national demand function with the implication that the elasticities were much larger in absolute value than the national export demand elasticity. It was assumed that the export of grain from the ports was evenly spread between the two six-month periods. The six-monthly functions were calculated by dividing the intercept and slope coefficients of the quantity-dependent demand functions by two.

The demand for wheat for human consumption was specified in a similar way. Again, very little information could be obtained on the elasticities of demand at individual terminals. For purposes of the study an elasticity of demand of –0.3 was assumed. This is consistent with the range of elasticities cited in Myers (1982, p. 46). The price used to derive the demand functions was $213.89 (free on rail). To keep the size of the model as small as possible, it was assumed that the margin between the domestic consumption price of $213.89 and the port price less transport to the port was constant so that the demand by mills could be represented in terms of a 'silo-door' price for ASW wheat. The demand functions used in the model are given in Table 1.

TABLE 1
Estimated Demand Functions for Selected Locations

Location	Price ($/tonne)	Elasticity	Intercept (000 tonnes)	Slope
Port 1	173.22	–48.9	99478.0	–562.78
Port 2	173.00	–71.4	98726.0	–562.78
Local silo 3	199.75	–0.3	9.246	–0.0107
Local silo 4	199.64	–0.3	4.249	–0.0049
Sub terminal 1	198.49	–0.3	62.283	–0.0724

Source: MacAulay, Batterham and Fisher (1988c), p. 114.

Farm Supplies

The supply of grain to the local receival sites in the base year was derived from inloadings data. These supplies were then apportioned to the individual farm models as discussed below. The supplies from the farm models were endogenous. However, the model farms represented an area from which grain could be shipped to various specific receival points. The relative contributions of each of these areas was set according to their observed shares in a base year. The farm supplies were based on an allocation of the deliveries to the nearest local delivery site. The local supplies at each of the country receival points were assumed to be only from the five representative farms. The individual farm supplies were weighted by appropriate aggregation factors so that the total supply equalled the total amount delivered by farms to the five receival points in the base year.

The supplies to each of the sub-terminals consisted of deliveries by rail and by road from farms other than those located in areas represented by the farm models. The supply at the ports consisted of deliveries from points other than the area studied. In this way the grain movements through each of the receival points and the sub-terminals could be approximated. To provide for price sensitive supply response at the various delivery sites, estimates of supply functions were prepared for the two ports, the sub-terminals, and one local delivery point. The addition of these supply functions completed the supply side of the model. These functions are shown in Table 2.

The linear programming models generated the necessary supplies for the local receival points and the additional supply functions provided for the interaction with the rest of the regions' wheat production. To obtain the functions, a supply elasticity of 0.8 was assumed. This estimate is consistent with those for the wheat-sheep zone made by Vincent, Powell and Dixon (1982) and Wall (1987).

TABLE 2
Estimated Supply Functions

Location	Price ($/tonne)	Elasticity	Intercept (000 tonnes)	Slope
Port 1	156.07	0.8	276.6	7.089
Port 2	155.85	0.8	249.7	6.409
Local silo 1	138.69	0.8	20.9	0.602
Sub terminal 1	132.58	0.8	37.4	1.128
Sub terminal 2	138.34	0.8	40.12	1.160

Source: MacAulay, Batterham and Fisher (1988c), p. 117.

Carryover Stocks

The levels of opening and closing stocks in the system were specified according to the levels reported at the beginning and end of the base year. In some instances these levels were zero or negative because of drying and other losses. In these circumstances, a very small amount (5 tonnes) was used so that suitable prices resulted from the model solutions. The cost of storing grain was assumed to be largely the interest cost of the funds tied up in the grain valued at the price generated by the model. The carryover charge was thus endogenous to the model. A real interest rate of 5 per cent per annum was used. The carryover charge was applied to end-of-period stocks.

Transport Costs

Two main sets of transport rates were required for the model. These were the rates from the farm gate to local receival points and, from receival points to another receival point, sub-terminal or port. The farm to receival point rates were derived by considering a representative location for each of the five farms, calculating the distance and applying a trucking rate obtained after discussion with local farmers and transporters. In locating each representative farm it was necessary to limit the number of possible delivery routes available to each farm so as to keep the dimensions of the model to a reasonable size. In calculating the truck rate for farm deliveries it was assumed that the contract rate and the implicit rate charged by farmers for their own trucks were equivalent.

Attempts were also made to collect information on road freight rates for grain transported direct from the farm to seaboard terminal. The information collected was for small volumes of grain carted to cities where the seaboard terminals are located and were responses from a very small and non-random sample of carriers. They do not include any estimate of the social costs of road damage or accident that may be involved in such long-distance haulage.

Various rail freight rates were used in the model based on a number of alternative policy assumptions. The rates used reflected the nature of the research questions being investigated.

Storage Capacities

The setting of storage capacities for the model proved to be difficult because of the absolute nature of programming model restrictions. For the purposes of carrying out experiments with the model, it seemed that the sub-terminals, at least, should not cause restrictions on the flow through the system because of their relatively large capacity in relation to the volume of grain delivered from the representative farms. Hence, very large capacities were specified for these sites.

In the case of the local receival points the actual physical capacity of vertical, horizontal and bunker storage was specified. The possibility of a turnover rate greater than 1.0 within a six month period was not allowed. This assumption in the model could be changed easily. The effective annual turnover rate was specified at 2.0 but the rate allowed for in the model was less because the major deliveries take place within a short part of the summer period.

Components of the Farm Models

The representative farm models were based on the physical characteristics of actual farms delivering wheat in the area studied. Farms delivering wheat to the local receival points were identified from delivery records for the base season. Each farm was given a location on maps of the local government areas. Information on the areas of farms, distance from receival point and the apparent area of wheat grown on each farm was estimated using average shire yields (Australian Bureau of Statistics 1985) and these were used to calibrate the programming models.

The activities included in the representative farm models were two winter cereal crops (wheat and barley); two summer crops (sorghum and sunflower); a sheep activity (wethers); a cattle activity (breeding weaners); two pasture activities (on arable and non-arable land); and fodder sorghum and oat activities. The data for these activities were based on O'Sullivan (1985) and Rickards and Passmore (1977). Some of the farm activities spanned two periods. The possibility of storing wheat on farms after harvest for subsequent delivery in the following period was also included in the model. Storage costs were based on Benson *et al.* (1987).

The wheat selling activities form the major link between the representative farm models and the spatial equilibrium model. Wheat prices were determined in the dual part of the spatial equilibrium model and then formed the basis for supply decisions in the farm models.

A common rotation in the area is four years of wheat (or wheat and barley) followed by two years of other crops (often sorghum). To reproduce the pattern, a rotational constraint was introduced which required that for every two hectares of wheat or barley grown there must be at least one hectare of sorghum, sunflower, fodder oats or sorghum or native pasture on arable land. However, it was also assumed that this constraint could be relaxed by undertaking an additional weed spraying activity.

It was assumed that owner-operator labour was available on the representative farms. No cost allowance was made for the operator labour, nor was labour hiring allowed in the model.

Each of the modelled farms represents a number of similar farms stratified by size. The aggregation factors were calculated using delivery records for each receival point for the base season and represent the factor by which the output from each representative farm is multiplied to give sub-regional output. The farm models were constructed by assuming that land and labour were the most limiting resources. Following the approach by Day (1963) outlined above, farms within a strata were assumed to have proportional variation between constraints, and to use similar technology. These assumptions seemed realistic given the nature of the agriculture of the region being modelled and the dominance of wheat growing in the region.

Results

The policy questions investigated with the aid of the model included: 1) the effects on the grain handling system of pooled charging for handling and transport compared with charges disaggregated for the various services provided; 2) the effect of the introduction of competition (or the lack of it) between the ports; 3) the effects of applying various rail transport charges on the optimal transport pattern of grain; and 4) the effects of more efficient handling and storage systems. This latter question was examined by varying the cost function used in the model, but this set of experiments is not considered further in the paper.

The results were reported in terms of changes to the handling and storage system, including the volume of grain transported by road and rail, and the effects of various options on the modelled farms. A large volume of information on the system was obtained for any given model run (MacAulay, Batterham and Fisher 1988b, 1988d). The price information included the end of year price of stored grain, supply and demand prices at the silo door (equal for any given receival point), offer prices at the silo door, and at the farm gate. Shadow values were generated where storage or transport capacity was limiting. Similarly, opportunity costs on the potential use of non-optimal transport routes were generated. Quantity information included grain throughput at any site, grain transported by individual transport methods and routes and the production of various commodities in the farm models.

An initial run was used to characterise the then existing grain production, handling and storage system. Average costs of running the grain handling and storage system were pooled, together with a capital cost recovery charge. Rail freight rates were set at actual charges (without a 'drought' subsidy that was widely used in New South Wales at the time). Equilibrium prices were determined for the system given the export and local demand functions, the charges by the handling authority and transport charges, the price sensitive supply functions representing regions outside the area of the farm models and the endogenous supply from the modelled farms. The equilibrium prices generated were consistent with the price regime in operation throughout the actual handling and storage system, thus providing reasonable validation of the model.

By disaggregating the costs of handling and storage charges in the model it was found that the flow of exports would be more evenly distributed through the year and that most local silo offer prices and hence farm gate prices would increase. With such a change more grain would be stored on farms, incomes would increase, and hence returns to land, labour and management would increase on most farms. However, one of the modelled local silos and attached farms showed price decreases. A consequence was decreased income and returns to resources on that modelled farm. This illustrates the possibility of estimating the redistributive effects of a policy change by using such price-linked farm and spatial equilibrium models. Clearly, these results are dependant on the nature of the cost functions at each of the local silos, the throughput of the silos and the location of the farms in relation to the silos. With the closure of one of the silos grain had to be transported over longer distances, again implying a distributional effect.

An experiment comparing competition between the ports versus 'no competition' was modelled by assuming f.o.b. prices which were the same for the ports in the 'no competition case', and differed by the transport and wharfage charges in the 'competition' case. The port demand functions were also recalculated for the competition case. The obvious result of the experiment was a decrease in the exports from the higher cost port under the competitive assumption. The consequences in terms of changes in prices, and quantities shipped, throughout the system were observed. There were losses for modelled farms that shipped through the higher cost port.

An analysis of the effects of applying various rail transport charges on the optimal transport pattern of grain was accomplished relatively easily by varying the rail transport charges. Road transport charges were also varied with the changes based on various estimates of road transport charges. With rail freight rates only a little above the then current rates considerable quantities of grain would be trucked to the nearest port. Obviously, this result was interpreted cautiously, as no constraints were imposed to limit the trucking of large volumes of grain to the port.

Summary and Suggestions for Further Research

In this paper a method of linking farm models to a primal-dual spatial equilibrium model through price and quantity variables has been demonstrated. The linked models can be used to determine the consequences of change in any part of a production and marketing system on all other parts of that system. An example of the use of a large-scale model was given in the 'farm-to-ship' model employed to examine of the effects of different transport, storage and payment arrangements in the wheat industry.

There are many possible extensions and uses of linked farm and spatial equilibrium models. Linked models could be used in situations where the farm (or firm) level consequences of government interventions in a marketing system are being examined. Thus, both price and quantity variables are endogenous within the models. It is worth noting also, that this concept could be extended to model the behaviour of individual consumers or households and so replace the regional demand functions. Such modelling could also prove useful for studies of farm-household interactions in developing countries.

An obvious extension of linked farm and spatial equilibrium models is to multi-commodity situations. In the example outlined above, three grades of wheat were represented in the farm models, and had data been available on the demand for each grade, each of the grades could have been considered in the spatial equilibrium model. Representation of different grades, and different commodities is simple conceptually. A similar extension from two time periods, as represented in the above model, to many time periods is also straightforward. The limitation to these extensions is model size, in terms of computer capacity and the cost of building and debugging large-scale models. These limitations are becoming less restrictive with continuing developments in computer hardware and software.

Another possible extension is to develop 'rest of the world' components in the spatial equilibrium model. The influence of changes in the international trade regime (for example, increasing demand for wheat in another part of the world, or the freeing of trade) could be traced through the processing, storage and transport sector, back to the farm level.

A further extension is to explicitly model behaviour through time. Recursive models could be constructed to analyse marketing or trade policy issues by simulating such systems through time (MacAulay 1976). Alternatively, they could be used to consider farm adjustment problems

(Kingma and Kerridge 1977). Recursive models provide a means of considering multiple time period problems in a relatively compact form. Linked recursive models can be used to simultaneously solve the farm and spatial equilibrium models. The models simplify the process of considering the dynamic effects of policy changes on farm production patterns, and hence farm income. The modelled effect of the policy change on other parts of the marketing system is resolved simultaneously.

References

Australian Bureau of Statistics (1985), *Crops and Pastures, N.S.W.* Cat. No. 7321.1, Canberra.

Barker, R. and Stanton, B. F (1965), 'Estimation and aggregation of firm supply functions', *Journal of Farm Economics* 47(3), 701-712.

Baumol, W. J. (1977), *Economic Theory and Operations Analysis*, 4th ed. Prentice Hall, London.

Benson, D., Espinas, V., Andrews, A., Johnston, J., Price, D., and Small, A. (1987), 'On farm storage from a policy perspective–models, costs and utilisation', Paper presented to the 31st Agricultural Economics Society Conference, Adelaide, February 1987.

Blyth, M. J. (1982), A Spatial Equilibrium Linear Activity Analysis of the New South Wales Dairy Industry, unpublished PhD thesis, Department of Agricultural Economics, University of Sydney, Sydney.

Bressler, R. G. and King, R. A. (1970), *Markets, Prices, and Interregional Trade*, Wiley, New York.

Cowling, K. and Gardner, T. W. (1963), 'Analytical models for estimating supply relations in the agricultural sector: a survey and critique', *Journal of Agricultural Economics* 15(3), 439-450.

Cutler, L. and Pass, D. S. (1971), *A Computer Program for Quadratic Mathematical Models to be Used for Aircraft Design and Other Applications Involving Linear Constraints,* Rand Corporation, Santa Monica.

Day, R. H. (1963), 'On aggregating linear programming models of production', *Journal of Farm Economics* 45(4), 797-813.

Dent, J. B., Harrison, S. R. and Woodford, K. B. (1986), *Farm Planning with Linear Programming*, Butterworths, Sydney.

Fox, G. (1986), Mathematical Programming with MINOS: A Handbook for Economists. AEB/86/9, Department of Agricultural Economics and Business, University of Guelph, Guelph.

Frick, G. E. and Andrews, R. A. (1965), 'Aggregation bias and four methods of summing farm supply functions', *Journal of Farm Economics* 47(3), 696-700.

Harker, P. T. (1985), *Spatial Price Equilibrium: Advances in Theory, Computation and Application*, Proceedings, Denver, Colorado, 1984, Springer-Verlag, Berlin.

Judge, G. G. and Takayama, T. (1973), *Studies in Economic Planning over Time and Space*, North-Holland Publishing Company, Amsterdam.

Kennedy, J. O. S. (1972), On using aggregate programming to model agriculture in New South Wales, Aggregate Programming Model for Australian Agriculture Report 1, University of New England, Armidale.

Kingma, O. T. and Kerridge, K. W. (1977), 'Towards an analytical base for studying farm adjustment problems — a recursive stochastic model of the farm-firm', *Quarterly Review of Agricultural Economics* 30(2), 91-116.

Knight, R. C. and Taplin, J. H. E. (1971), 'Supply response in the peanut industry: a linear programming approach', *Quarterly Review of Agricultural Economics* 24(2), 112-122.

Kottke, M. (1967), 'The anatomy of a step supply function', *Journal of Farm Economics* 49(2), 107-118.

Ladd, G. W. and Easey, E. V. (1959), An Application of Linear Programming to the Study of Supply Responses in Dairying, Research Bulletin 467, Agricultural and Home Economics Experiment Station, Iowa State University, Ames.

MacAulay, T. G. (1976), A Recursive Spatial Equilibrium Model of the North American Beef Industry for Policy Analysis, unpublished PhD thesis, Department of Agricultural Economics and Extension Education, University of Guelph, Guelph.

MacAulay, T. G. (1992), 'Alternative spatial equilibrium formulations: a synthesis', in Griffiths, W.E., Lütkepohl, H. and Bock, M.E. (eds) *Readings in Econometric Theory and Practice*, Ch. 13, North-Holland, Amsterdam, 297-326.

MacAulay, T. G. and Casey, H. C. (1987), 'An exact linear programming solution for spatial equilibrium models', Paper presented to the Australian Agricultural Economics Society Conference, Adelaide, February, 1987.

MacAulay, T. G., Batterham, R. L. and Fisher, B. S. (1988a), 'The grain production, transportation, storage and shipping system: model construction', Ch 9 in Fisher, B.S. and Quiggan, J.C. (eds), *An Analysis of the Characteristics of the Australian Grain Handling System*, Research Report No. 13, Department of Agricultural Economics, University of Sydney, October 1988, pp. 64-79.

MacAulay, T. G., Batterham, R. L. and Fisher, B. S. (1988b), 'The grain production, transportation, storage and shipping system: the harvester-to-ship model results', Ch 10 in Fisher, B.S. and Quiggan, J.C. (eds), *An Analysis of the Characteristics of the Australian Grain Handling System*, Research Report No. 13, Department of Agricultural Economics, University of Sydney, October 1988, pp. 80-112.

MacAulay, T. G., Batterham, R. L. and Fisher, B. S. (1988c), 'A spatial equilibrium model of regional grain flows: details of methodology and data used for Northern New South Wales Programming Model', Appendix C in *Royal Commission into Grain Storage, Handling and Transportation, Evaluation of Alternatives Systems,* Volume 3, Supporting Paper 8, pp. 109-125.

MacAulay, T. G., Batterham, R. L. and Fisher, B. S. (1988d), 'Northern New South Wales Programming Model Results', Appendix D in *Royal Commission into Grain Storage, Handling and Transportation, Evaluation of Alternatives Systems*, Volume 3, Supporting Paper 8, pp 126-186.

MacAulay, T. G., Batterham, R. L. and Fisher, B. S. (1989), 'Solution of spatial trading system with concave cubic programming', *Australian Journal of Agricultural Economics* 33(3), 170-186.

Mackinnon, J. G. (1976), 'A technique for the solution of spatial equilibrium models', *Journal of Regional Science* 16 (3), 293-207.

Marshall, A. (1959), *Principles of Economics*, 8th ed, Macmillan, London.

Martin, L. J. (1981), 'Quadratic single and multi-commodity models of spatial equilibrium: a simplified exposition', *Canadian Journal of Agricultural Economics* 29(1), 21-48.

Murtagh, B. A. and Saunders, M. A. (1987), MINOS 5.1 User's Guide, Technical Report SOL 83-20R, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford.

Myers, R. J. (1982), *An Econometric Evaluation of Australian Wheat Pricing Policies,* Agricultural Economics Bulletin No. 27, Department of Agricultural Economics and Business Management, University of New England, Armidale.

Myers, R. J., Piggott, R. R. and MacAulay, T. G. (1985), 'Effects of past Australian wheat price policies on key industry variables', *Australian Journal of Agricultural Economics* 29(1), 1-15.

Nagurney, A. (1992), 'The application of variational inequality theory to the study of spatial equilibrium and disequilibrium', in Griffiths, W.E., Lütkepohl, H. and Bock, M.E. (eds) *Readings in Econometric Theory and Practice*, Ch. 13, North-Holland, Amsterdam, 327-355.

Nerlove, M. and Backman, K. L. (1960), 'The analysis of changes in agricultural supply: problems and approaches', *Journal of Farm Economics* 42(3), 531-554.

Onal, H. and McCarl, B. A. (1989) 'Aggregation of heterogeneous firms in mathematical programming models', *European Review of Agricultural Economics* 16(4), 499-513

Onal, H. and McCarl, B. A. (1991), 'Exact aggregation in mathematical programming sector models', *Canadian Journal of Agricultural Economics* 39(2), 319-334.

O'Sullivan, S. (1985), *Enterprise Budgets for the North West of N.S.W.,* Complan Handbook Number 6, University of New England, Armidale.

Rickards, P. A., and Passmore, A. L. (1977), *Planning for Profit in Livestock Grazing Systems,* Professional Farm Management Guidebook Number 7, University of New England, Armidale.

Samuelson, P. A. (1952), 'Spatial price equilibrium and linear programming', *American Economic Review* 42(3), 283-303.

Shumway, C. R and Chang, A. A. (1977), 'Linear programming versus positively estimated supply functions: an empirical and methodological critique', *American Journal of Agricultural Economics* 59(2), 344-357.

Takayama, T. and Judge, G. G. (1964a), 'Spatial equilibrium and quadratic programming', *Journal of Farm Economics* 46(1), 67-93.

Takayama, T. and Judge, G. G. (1964b), 'An interregional activity analysis model for the agricultural sector', *Journal of Farm Economics* 46(3), 349-365.

Takayama, T. and Judge, G. G. (1971), *Spatial and Temporal Price and Allocation Models*, North-Holland Publishing Company, Amsterdam.

Takayama, T. and Labys, W. (1986), 'Spatial equilibrium analysis', Ch 5. in P. Nijkamp (ed.), *Handbook of Regional and Urban Economics,* North-Holland Publishing Company, Amsterdam.

Takayama, T. and MacAulay, T. G. (1991), 'Recent developments in Spatial (Temporal) Equilibrium Models: Non-Linearity and Existence', In Lesourd, J.-B. (ed), *International Commodity Market Modelling: Advances in Methodology and Applications*, Chapman and Hall, London, pp. 71-89.

Tomek, W. G. and Robinson, K. L. (1981), *Agricultural Product Prices*, Cornell University Press, Ithaca.

Vincent, D. P., Powell, A. A. and Dixon, P. B. (1982), 'Changes in supply of agricultural products', in Williams, D. B. (ed.), *Agriculture in the Australian Economy,* Sydney University Press, Sydney.

Wall, C. A. (1987), Modelling a Multiple Output Production System: Australian Sheep Industry, unpublished PhD thesis, Department of Agricultural Economics, University of Sydney, Sydney.

Wu, Y. and Kwang, C. (1960), 'An analytical and graphical comparison of marginal analysis and mathematical programming in the theory of the firm', in Boulding, K. and Spivey, W. (eds), *Linear Programming and the Theory of the Firm*, Macmillian, New York.

APPENDIX A
Mathematical Formulation of the Price-Linked Models: An Illustrative Example

The Standard Spatial Model

For a set of n regions, the inverse supply and demand functions may be defined in linear terms as follows:

(A.1) $p_y = \lambda - \Omega y$

(A.2) $p_x = \upsilon + Hx$

where p_y and p_x are (n x 1) vectors of unrestricted demand and supply prices for n regions; y and x are (n x 1) vectors of demand and supply quantities; λ and υ are (n x 1) vectors of the intercepts of the demand and supply functions respectively; Ω and H are (n x n) matrices of slope coefficients for the demand and supply functions respectively. Both matrices may include off-diagonal elements and be non-symmetric.

The standard spatial equilibrium model is a quadratic programming problem. The set of constraints ensure that the characteristics of a competitive spatial equilibrium are defined such that the supply and demand functions must hold, that the supply and demand quantities and the quantities traded must balance and that the spatially competitive price arbitrage conditions must hold (Takayama and Judge 1971; Martin 1981). The primal–dual form of the model in the quantity domain can be defined as follows:

(A.3) Maximise
$G(y, x, X, \rho_y, \rho_x) = (\lambda - \Omega y)'y - (\upsilon + Hx)'x - T'X - 0'\rho_y - 0'\rho_x$

subject to

(A.4)	$-G_yX + y \leq 0$	$(-G_yX + y)'y = 0$
(A.5)	$-G_xX - x \leq 0$	$(-G_xX - x)'x = 0$
(A.6)	$-\rho_y + \lambda - \Omega y \leq 0$	$(-\rho_y + \lambda - \Omega y)'\rho_y = 0$
(A.7)	$\rho_x - \upsilon - Hx \leq 0$	$(\rho_x - \upsilon - Hx)'\rho_x = 0$
(A.8)	$-T + G'_y\rho_y + G'_x\rho_x \leq 0$	$(-T + G'_y\rho_y + G'_x\rho_x)'X = 0$

and

(A.9) $y, x, X, \rho_y, \rho_x \geq 0$,

where for all n regions ρy and ρx are non-negative Lagrangian multipliers which at the optimal solution represent the market prices. $T'X$ is the total transportation cost where T is a (n^2 x 1) vector of unit transport costs (t_{ij} between region i and j) and X a (n^2 x 1) vector of of the quantities transported (x_{ij} from region i to region j) between each of the n regions.

The matrices G_x and G_y are (n x n^2) and designed to ensure that the sum of the quantities transported into or out of a region can be equated to the supply and demand quantities and they are of the following form:

$$G_y = \begin{bmatrix} 1 & & & 1 & & & & 1 & & \\ & 1 & & & 1 & & & & 1 & \\ & & \ddots & & & \ddots & \cdots & & & \ddots \\ & & & 1 & & & 1 & & & 1 \end{bmatrix}_{(n \times n^2)},$$

$$G_x = \begin{bmatrix} -1\ -1 \ldots -1 & & & \\ & -1\ -1 \ldots -1 & & \\ & & \ddots & \\ & & & -1\ -1 \ldots -1 \end{bmatrix}_{(n \times n^2)}.$$

In the constraints (A.4) to (A.8) the second set of conditions are 'complementary slackness' conditions and require that if the variable concerned is not zero then the marginal condition contained in the brackets will be zero (Lee, Moore and Taylor 1981, p. 130 and pp. 696-698 and Takayama and Judge 1964). These conditions are automatically incorporated into the solution algorithm of quadratic programming routines.

The objective function of the problem (A.3) to (A.9) can be interpreted as a measure of the social monetary gain ($p_y y$) less the total social production cost ($p_x x$) less the total transport cost $T'X$ for the regional trading system. The net social monetary gain objective may be conveniently referred to as a net revenue objective function. Thus:

(A.10) $\quad$ Net revenue $= p_y\, y - p_x\, x - T'X$

For the quantity form of the spatial equilibrium model (as indicated above) the supply and demand prices are replaced by the indirect supply and demand functions (A.1) and (A.2) while for the price form of the model the quantities are replaced by the supply and demand functions. The net revenue form of the objective function is convenient for the purpose of linking farm models to the spatial equilibrium model.

Representative Farm Models

Farm models can be represented as linear programming problems in which net profit or the total gross margin is the objective function. This function is then subject to a set of resource constraints (A.13) and constraints which translate per unit production activities to supply by multiplying by yield. To incorporate a farm model into the supply side of a spatial equilibrium model it is appropriate to envisage the farm as producing a given level of the output indicated by $\bar{x}$. This given supply is then made endogenous to the complete model and valued at the market price determined in the spatial equilibrium part of the model. Thus:

Find $z \geq 0$ that maximises the primal problem Z_p

(A.11) $Z_p = c'z$

subject to

(A.12) $-Yz \leq \bar{x}$

(A.13) $Az \leq b$

and

(A.14) $z \geq 0$

where z is an (r x 1) vector of r farm-level activities, some of which contribute to the product supply and can be considered as marketing activities related to the products in the spatial equilibrium model; c is an (r x 1) vector of costs and returns associated with the farm activities; A is a (k x r) matrix of input-output coefficients reflecting the production activities and the constraints under which the farm operates; Y is an (n x r) matrix of yield coefficients reflecting the yield of the products generated by the farm activity assumed to be on a per hectare basis (the matrix may be structured to allow single activities to generate multiple products and to include the possibility that a number of activities may have no product output); b is a (k x 1) vector of right-hand-side values reflecting the resources available to the farm; and $\bar{x}$ represents the fixed supply of the products (or product) required to be generated from the resources used.

The dual form of this sub-model is

Find ρ_z, $\rho_x \geq 0$ that minimises the dual objective value Z_d

(A.15) $Z_d = \bar{x}\,\rho_x - b'\rho_z$

subject to

(A.16) $Y'\rho_x - A'\rho_z \leq c$

(A.17) $\rho_z \geq 0$

where ρ_z is a (k x 1) vector of shadow prices for the resources used and ρ_x is a (k x 1)vector of implicit product prices which link to the spatial equilibrium part of the model for the combined model.

Combined Price-linked Farm and Spatial Models-quantity Form

The linear programming primal-dual form of the model (A.11) to (A.17), when combined with the spatial equilibrium model, becomes:

(A.18) Maximise $G(y, x, X, z, \rho_y, \rho_x, \rho_z) = (\lambda - \Omega y)'y - c'z - T'X - 0'\rho_y - 0'\rho_x - b'\rho_z$

(A.19)	$-G_y X + y \leq 0$	$(-G_y X + y)'y = 0$
(A.20)	$-G_x X - Yz \leq 0$	$(-G_x X - Yz)'z = 0$
(A.21)	$Az - b \leq 0$	$(Az - b)'z = 0$
(A.22)	$-\rho_y + \lambda - \Omega y \leq 0$	$(-\rho_y + \lambda - \Omega y)'\rho_y = 0$
(A.23)	$Y'\rho_x - A'\rho_z \leq c$	$(Y'\rho_x - A'\rho_z)'z = 0$
(A.24)	$-T + G_y \rho_y + G_x \rho_x \leq 0$	$(-T + G_y \rho_y + G_x \rho_x)'X = 0$

and

(A.25) $y, z, X, \rho_y, \rho_x, \rho_z \geq 0$.

The primal constraints (A.19) and (A.20) reflect the requirement that supply and demand quantities must balance with the trade flows and, constraint (A.21) that the resource availability (vector b) cannot be exceeded in the production of the products in the system. The dual constraint (A.22) implies that the demand relationship must hold, and (A.23) reflects the condition that the yield-weighted supply prices less the imputed cost of producing the products from the available resources valued at their shadow prices must be less than or equal to the gross margins involved in producing the products. The constraint (A.25) is the equivalent of the spatial arbitrage condition for a single region and requires that demand prices be less than or equal to the supply prices. In a more compact matrix form the problem can be written in the quantity form as follows.

(A.26) Find $(\bar{y}'\bar{z}'\bar{X}'\bar{\rho}_y'\bar{\rho}_x'\bar{\rho}_z') \geq 0'$ that maximizes

$$(A.27)\quad Z_q = \left\{ \begin{bmatrix} \lambda \\ -c \\ -T \\ 0 \\ 0 \\ -b \end{bmatrix} + \begin{bmatrix} -\Omega & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & Y' & -A' \\ 0 & 0 & 0 & -G_y' & -G_x' & 0 \\ I & 0 & G_y & 0 & 0 & 0 \\ 0 & -Y & G_x & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ X \\ \rho_y \\ \rho_x \\ \rho_z \end{bmatrix} \right\}' \begin{bmatrix} y \\ z \\ X \\ \rho_y \\ \rho_x \\ \rho_z \end{bmatrix}$$

subject to

$$(A.28)\quad \begin{bmatrix} \lambda \\ -c \\ -T \\ 0 \\ 0 \\ -b \end{bmatrix} + \begin{bmatrix} -\Omega & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & Y' & -A' \\ 0 & 0 & 0 & -G_y' & -G_x' & 0 \\ I & 0 & G_y & 0 & 0 & 0 \\ 0 & -Y & G_x & 0 & 0 & 0 \\ 0 & A & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ X \\ \rho_y \\ \rho_x \\ \rho_z \end{bmatrix} \leq 0$$

$$(A.29)\quad \text{and } (y'\, z'\, X'\, \rho_y{}'\, \rho_x{}'\, \rho_z{}') \geq 0',$$

where the Marshallian (indirect) market demand and supply functions are used, and T is a transport cost vector (n^2 x 1) and

$$(A.30)\quad \rho_y = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_n \end{bmatrix} \geq 0, \quad \rho_x = \begin{bmatrix} \rho^1 \\ \rho^2 \\ \cdot \\ \cdot \\ \cdot \\ \rho^n \end{bmatrix} \geq 0, \text{ and } \rho_z = \begin{bmatrix} \rho_{z1} \\ \rho_{z2} \\ \cdot \\ \cdot \\ \cdot \\ \rho_{zk} \end{bmatrix} \geq 0$$

are non-negative demand, ρ_y, and supply, ρ_x, price vectors each (n x 1) and resource shadow prices, ρ_z (k x 1).

Price Formulation

The alternative formulation of the model in the price domain is often convenient as supply and demand functions are frequently estimated with quantity as the dependent variable (Takayama and Judge 1971, ch. 8). In this case the supply and demand functions are defined as

$$(A.31)\quad y = \alpha - Bp_y$$
$$= \alpha - B(\rho_y - w)$$

$$(A.32)\quad x = \theta + \Gamma p_x$$
$$x = \theta + \Gamma(\rho_x + v)$$

where for all n regions $p_y = \rho_y - w$ is a (n x 1) unrestricted vector of demand prices and w is a (n x 1) non-negative vector of slack variables; $p_x = \rho_x + v$ is a (n x 1) vector of unrestricted supply prices and v is an (n x 1) non-negative vector of slack variables; α is an (n x 1) vector of demand intercept terms; B is an (n x n) matrix of demand slope coefficients; θ is an (n x 1) vector of supply intercept terms; and Γ is an (n x n) matrix of supply slope coefficents. The slack variables are used to ensure that

ρ_y and ρ_x are non-negative. For most practical purposes w and v can be ignored provided that any calculated solutions are not likely to include negative prices or quantities.

By substituting for the quantities y and x instead of the prices in the net revenue objective function (A.10) and choosing the appropriate constraints the price formulation of the combined model can be specified as follows:

(A.33) Find $(\overline{\rho}_y{}' \,\overline{\rho}_x{}' \,\overline{\rho}_z{}' \;\overline{X}' \;\overline{z}') \geq 0'$ that maximizes

$$\text{(A.34)}\quad Z_P = \left\{ \begin{bmatrix} \alpha \\ 0 \\ -b \\ -T \\ -c \end{bmatrix} + \begin{bmatrix} -B & 0 & 0 & -G_y & 0 \\ 0 & 0 & 0 & -G_x & -Y \\ 0 & 0 & 0 & 0 & A \\ G_y{}' & G_x{}' & 0 & 0 & 0 \\ 0 & Y' & -A' & 0 & 0 \end{bmatrix} \begin{bmatrix} \rho_y \\ \rho_x \\ \rho_z \\ X \\ z \end{bmatrix} \right\}' \begin{bmatrix} \rho_y \\ \rho_x \\ \rho_z \\ X \\ z \end{bmatrix}$$

subject to

$$\text{(A.35)}\quad \begin{bmatrix} \alpha \\ 0 \\ -b \\ -T \\ -c \end{bmatrix} + \begin{bmatrix} -B & 0 & 0 & -G_y & 0 \\ 0 & 0 & 0 & -G_x & -Y \\ 0 & 0 & 0 & 0 & A \\ G_y{}' & G_x{}' & 0 & 0 & 0 \\ 0 & Y' & -A' & 0 & 0 \end{bmatrix} \begin{bmatrix} \rho_y \\ \rho_x \\ \rho_z \\ X \\ z \end{bmatrix} \leq 0$$

(A.36) $(\rho'_y \,\rho'_x \,\rho'_z X' \,z') \geq 0'.$

Model Constraint Types

The constraints in the model can be classified into two types:

- Those relating to the supply and demand balances of the primal model and those relating to supply and demand prices in the dual part of the model; and
- Constraints relating to the farm model, which include limits on land areas, pasture production and utilisation, family and hired labour, machinery availability, cash constraints and the related resource valuation constraints in the dual part of the model.

A general representation of the matrix used in the model in its primal-dual form and in the quantity domain is given in Table A.1.

TABLE A.1
Quadratic Programming Tableau for the Price-linked Farm and Spatial Equilibrium Model in the Quantity Domain

Maximise	y	z	X	ρ_y	ρ_z	ρ_x		*RHS*
Quadratic	$-\Omega$							
Linear	λ	$-c$	0		$-b$			
Constraints	I		$-I_y$				$\leq$	0
		A					$\leq$	b
		$-Y$	I_x				$\leq$	0
	$-\Omega$			$-I$			$\leq$	$-\lambda$
					$-A'$	Y'	$\leq$	c
				I'_y		$-I'_x$	$\leq$	0

An Illustrative Example

To illustrate the basic structure of the price-linked farm and spatial equilibrium models an artificial example was constructed. The data for the basic spatial equilibrium model were those of Takayama and Judge (1971, p. 169). The model was constructed so as to include one spatially traded commodity 'wheat' and 'barley' and 'oats' as non-traded goods but sold from the regional farms (it was assumed that the trade in these two commodities was of no interest). It would be possible to include such trade in a much expanded multi-commodity model.

The indirect demand functions for 'wheat' were assumed to be as follows:

$$\rho_1 = 200 - 10\,y_1 \qquad \rho_1 = 100 - 5\,y_1 \qquad \rho_1 = 160 - 8y_1\,.$$

Since the supply of 'wheat' was generated in the farm models, supply was endogenous to the model and so no supply functions were specified. The transfer costs for 'wheat' were as follows:

$t_{11} = 0$	$t_{12} = 2$	$t_{13} = 2$
$t_{21} = 2$	$t_{22} = 0$	$t_{23} = 1$
$t_{31} = 2$	$t_{32} = 1$	$t_{33} = 0.$

The input-output coefficients for the farm models in each of the regions can be seen in the full tableaus in Table A.3 (quantity form) and A.4 (price form), but the detailed values for region 1 are given in Table A.2. The values for the farms in regions 2 and 3 are slightly changed from those in region 1. Note that the full tableau is specified as a minimisation problem so that there is a change in sign of the objective function values. The problem was solved using RAND QP (Cutler and Pass 1971).

TABLE A.2
Illustrative Farm Model for Region 1

Item	Wheat	Oats	Barley	Available Supplies
Gross margin ($)	–3.0[a]	8.0	10.0	–
Land (ha)	1.0	1.0	1.0	100
Cash ($)	3.0	5.0	4.0	280
Labour (hours)	0.5	0.5	0.5	50

a The gross margin for wheat is the variable costs of production only since the return from the sale of wheat is included in the spatial part of the model.

The solution to the complete model is given in Table A.5. For a primal-dual model it is necessary that the primal solution be duplicated in the dual variables. In addition, the objective function value must be zero at the optimum since the dual problem is subtracted from the primal and both must have the same objective function value at the optimum.

TABLE A.3
Illustrative Tableau for a Price-Linked Farm and Spatial Equilibrium Model-Quantity Formulation[A]

Rows	Y1	Y2	Y3	W1	B1	O1	W2	B2	O2	W3	B3	O3	X11	X12	X13	X21	X22	X23	X31	X32	X33	DP1	DP2	DP3	SP1	SP2	SP3	PLND1	PCSH1	PLAB1	PLND2	PCSH2	PLAB2	PLND3	PCSH3	PLAB3		Right-hand side
Y1	0.1																																					
Y2		0.2																																				
Y3			0.125																																			
OBJ	-20	-20	-20	3	-8	-10	4	-9	-10	4	-7	-9	0	2	2	2	0	1	2	1	0	0	0	0	0	0	0	100	280	50	100	300	50	100	300	50		
RY1	-0.1																					-1																≤	-20
RY2		-0.2																					-1															≤	-20
RY3			-0.125																					-1														≤	-20
RW1																									1			-1	-3	-0.5							≤	3
RB1																												-1	-5	-0.5							≤	-8
RO1																												-1	-4	-0.5							≤	-10
RW2																										1					-1	-3.1	-0.6				≤	4
RB2																															-1	-5.2	-0.6				≤	-9
RO3																															-1	-4.1	-0.7				≤	-10
RW3																											1							-1	-3.2	-0.8	≤	4
RB3																																		-1	-5.1	-0.7	≤	-7
RO3																																		-1	-4.2	-0.6	≤	-9
RX11																						1			-1												≤	0
RX12																							1		-1												≤	2
RX13																								1	-1												≤	2
RX21																						1				-1											≤	2
RX22																							1			-1											≤	0
RX23																								1		-1											≤	1
RX31																						1					-1										≤	2
RX32																							1				-1										≤	1
RX33																								1			-1										≤	0
RDP1	1												-1			-1			-1																		≤	0
RDP2		1												-1			-1			-1																	≤	0
RDP3			1												-1			-1			-1																≤	0
RSP1				-1									1	1	1																						≤	0
RSP2							-1									1	1	1																			≤	0
RSP3										-1									1	1	1																≤	0
RPLND1				1	1	1																															≤	100
RPCSH1				3	5	4																															≤	280
RPLAB1				0.5	0.5	0.5																															≤	50
RPLND2							1	1	1																												≤	100
RPCSH2							3.1	5.2	4.1																												≤	300
RPLAB2							0.6	0.6	0.7																												≤	50
RPLND3										1	1	1																									≤	100
RPCSH3										3.2	5.1	4.2																									≤	300
RPLAB3										0.8	0.7	0.6																									≤	50

[a]The quadratic objective function rows are given in the top part of the tableau above the linear part. The variable names represent the following: Y is the demand quantity, DP is demand price, SP is supply price, PLND is the shadow value of land, PCSH is the shadow value of working capital, PLAB is the shadow value of labour, X_{ij} indicates the shipments of the commodity from region i to region j, W is the production of the commodity that is traded indicated as 'wheat', B stands for 'barley' and O stands for 'oats' both non-traded commodities. The quadratic objective function rows have the same name as the columns, the linear part of the objective function is indicated as OBJ and the remaining row names are the same as the column names except that an R has been added as the first letter of the row name. Three regions are represented with numeric values 1 to 3. The model is a primal-dual model with the spatial equilibrium part of the model represented in the quantity domain rather the price domain. The signs of the objective function are set for minimisation rather than maximisation.

TABLE A.3

Illustrative Tableau for a Price-Linked Farm and Spatial Equilibrium Model-Quantity Formulation[A]

Rows	Y1	Y2	Y3	W1	B1	O1	W2	B2	O2	W3	B3	O3	X11	X12	X13	X21	X22	X23	X31	X32	X33	DP1	DP2	DP3	SP1	SP2	SP3	PLND1	PCSH1	PLAB1	PLND2	PCSH2	PLAB2	PLND3	PCSH3	PLAB3	Right-hand side
Y1	0.1																																				
Y2		0.2																																			
Y3			0.125																																		
OBJ	−20	−20	−20	3	−8	−10	4	−9	−10	4	−7	−9	0	2	2	2	0	1	2	1	0	0	0	0	0	0	0	100	280	50	100	300	50	100	300	50	
RY1	−0.1																					−1															≤ −20
RY2		−0.2																					−1														≤ −20
RY3			−0.125																					−1													≤ −20
RW1																									1			−1	−3	−0.5							≤ 3
RB1																												−1	−5	−0.5							≤ −8
RO1																												−1	−4	−0.5							≤ −10
RW2																										1					−1	−3.1	−0.6				≤ 4
RB2																															−1	−5.2	−0.6				≤ −9
RO2																															−1	−4.1	−0.7				≤ −10
RW3																											1							−1	−3.2	−0.8	≤ 4
RB3																																		−1	−5.1	−0.7	≤ −7
RO3																																		−1	−4.2	−0.6	≤ −9
RX11																						1			−1												≤ 0
RX12																							1		−1												≤ 2
RX13																								1	−1												≤ 2
RX21																						1				−1											≤ 2
RX22																							1			−1											≤ 0
RX23																								1		−1											≤ 1
RX31																						1					−1										≤ 2
RX32																							1				−1										≤ 1
RX33																								1			−1										≤ 0
RDP1	1												−1			−1			−1																		≤ 0
RDP2		1												−1			−1			−1																	≤ 0
RDP3			1												−1			−1			−1																≤ 0
RSP1				−1									1	1	1																						≤ 0
RSP2							−1									1	1	1																			≤ 0
RSP3										−1									1	1	1																≤ 0
RPLND1				1	1	1																															≤ 100
RPCSH1				3	5	4																															≤ 280
RPLAB1				0.5	0.5	0.5																															≤ 50
RPLND2							1	1	1																												≤ 100
RPCSH2							3.1	5.2	4.1																												≤ 300
RPLAB2							0.6	0.6	0.7																												≤ 50
RPLND3										1	1	1																									≤ 100
RPCSH3										3.2	5.1	4.2																									≤ 300
RPLAB3										0.8	0.7	0.6																									≤ 50

[A]The quadratic objective function rows are given in the top part of the tableau above the linear part. The variable names represent the following: Y is the demand quantity, DP is demand price, SP is supply price, PLND is the shadow value of land, PCSH is the shadow value of working capital, PLAB is the shadow value of labour, X_{ij} indicates the shipments of the commodity from region i to region j, W is the production of the commodity that is traded indicated as 'wheat', B stands for 'barley' and O stands for 'oats' both non-traded commodities. The quadratic objective function rows have the same name as the columns, the linear part of the objective function is indicated as OBJ and the remaining row names are the same as the column names except that an R has been added as the first letter of the row name. Three regions are represented with numeric values 1 to 3. The model is a primal-dual model with the spatial equilibrium part of the model represented in the quantity domain rather the price domain. The signs of the objective function are set for minimisation rather than maximisation.

TABLE A.5
Solution for the Illustrative Primal–Dual Model[a]

Primal variables		Dual variables	
Y1	85.34	RY1	85.34
Y2	37.67	RY2	37.67
Y3	52.27	RY3	52.27
W1	93.33	RW1	93.33
B1	6.11	RB1	6.11
O1	1.29	RO1	1.29
W2	61.11	RW2	61.11
B2	19.26	RB2	19.26
O2	2.54	RO2	2.54
W3	20.83	RW3	20.83
B3	3.71	RB3	3.71
O3	55.56	RO3	55.56
X11	85.34	RX11	85.34
X12	1.00	RX12	1.00
X13	7.99	RX13	7.99
X21	3.00	RX21	3.00
X22	37.67	RX22	37.67
X23	23.44	RX23	23.44
X31	4.00	RX31	4.00
X32	2.00	RX32	2.00
X33	20.83	RX33	20.83
DP1	11.47	RDP1	11.47
DP2	12.47	RDP2	12.47
DP3	13.47	RDP3	13.47
SP1	11.47	RSP1	11.47
SP2	12.47	RSP2	12.47
SP3	13.47	RSP3	13.47
PLND1	6.67	RPLND1	6.67
PCSH1	2.82	RPCSH1	2.82
RPLAB1	3.33	PLAB1	3.33
PLND2	17.09	RPLND2	17.09
PCSH2	0.25	RPCSH2	0.25
PLAB2	12.80	RPLAB2	12.80
PLND3	23.61	RPLND3	23.61
PCSH3	1.06	RPCSH3	1.06
PLAB3	7.61	RPLAB3	7.61

[a] The reported solution is for the quantity form of the model but both forms give the same set of solutions and have an objective function value of zero. Variables are as defined in Table A.3. The dual variables are the shadow values on the constraints of the full problem and in a primal-dual model represent a duplicate solution pair.