EXTRA TAX RESULTING FROM INCOME VARIATION WITH PARTICULAR REFERENCE TO NEW ZEALAND

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The extent by which expected tax payments are increased by income variance is discussed in the New Zealand setting. The cases of both normal and rectangular distributions of income are examined. The absolute magnitude of the expected extra tax for income fluctuation is lower than was expected. In Australia income variance may be higher but the more rapidly progressive tax rates of New Zealand mean that for a given mean income and standard deviation the extra tax in that country would be higher.

Introduction

The proportion of income demanded by the New Zealand Government increases as income increases. Because of this a tax-payer with an income which does not vary pays less tax than a tax-payer with the same average income but whose income fluctuates. The larger the income variance the higher the additional tax payments.

Farmers as a group have incomes which vary because of the vicissitudes of their environment. They attempt to save tax by adjusting their farm expenditure around annual balance day. In New Zealand, since 1965, there has been an income equalization scheme by which farmers can deposit surplus income with the Government in 'good years' and withdraw it within five years, adding to the taxable income of 'bad years'.

The purpose of the project reported here was to determine the extent by which expected tax payments are increased by income variance. The results show the extent to which farmers on fluctuating incomes are penalized by tax laws which treat those on varying and constant incomes alike.

Table 1 shows the New Zealand personal tax function for 1968. \( X \) is taxable income which is total income with exemptions subtracted. The tax function can be divided into five phases. Some of these phases are linear, where income variation within the phase will not result in extra tax payments because the proportion of income paid as tax is constant within the range. Other phases are quadratic. Hence the proportion of income taken as tax increases as income rises, and income variation results in extra tax payments.

In Phase 1 where taxable income is zero or less, tax is zero.

Over the next $1000 the income tax function is linear. Over the next $800, Phase III, the tax rate rises steeply by 6d every £100 increase in taxable income.\(^1\) A quadratic function approximates this discontinuous function with only very small errors. Over the fourth phase the tax rate

\(^1\) The most recent rates of income tax are levied under the New Zealand Land and Income Tax Annual Act of 1968 but the form of the tax function was specified by Part C of the First Schedule of the Land and Income Tax Act of 1954 and hence the rates of increase are in terms of £s.d.
TABLE 1

The New Zealand Personal Income Tax Function for 1967–68
(X is Taxable Income and T is tax)

<table>
<thead>
<tr>
<th>Phase</th>
<th>Range</th>
<th>Class of Function</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$0 \leq X \leq 0$</td>
<td>Linear</td>
<td>$T = 0$</td>
</tr>
<tr>
<td>II</td>
<td>$0 &lt; X \leq 8000$</td>
<td>Linear</td>
<td>$T = 0.135X$</td>
</tr>
<tr>
<td>III</td>
<td>$8100 &lt; X \leq 81800$</td>
<td>Quadratic</td>
<td>$T = (0.562 \times 10^{-4})X^2 + 0.034X + 45.0$</td>
</tr>
<tr>
<td>IV</td>
<td>$8180 &lt; X \leq 86470$</td>
<td>Quadratic</td>
<td>$T = (0.281 \times 10^{-4})X^2 + 0.129X - 36.0$</td>
</tr>
<tr>
<td>V</td>
<td>$86470 &lt; X \leq 7200$</td>
<td>Quadratic</td>
<td>$T = (0.312 \times 10^{-4})X^2 + 0.143X - 237$</td>
</tr>
<tr>
<td>VI</td>
<td>$7200 &lt; X$</td>
<td>Linear</td>
<td>$T = 0.6(X - 7200) + 2415$</td>
</tr>
</tbody>
</table>

rises less steeply (by 3d per £100). Up until taxable income reaches $6470 there is a tax rebate of 10%. This rebate no longer applies after this point, accounting for the slightly different constants in Phase V. The last phase is linear, 60% of income above $7200 being taken as tax.

Three factors affect the magnitude of the extra tax payments caused by taxable income varying.

1. The magnitude of the variance of taxable income.
2. Mean taxable income.
3. The form of the frequency distribution of taxable income.

The first factor needs no further explanation. The second factor, mean taxable income, determines (with the other two factors) the phases of the tax function in which incomes fall. Thus with a very high income all incomes may fall in Phase VI and hence no extra tax will be paid. The form of the frequency distribution also affects the distribution of incomes in the phases of the tax function.

*An Analytical Method*

If the tax function of Phase IV is assumed to apply over the entire domain of the tax function, then both the mean taxable income and the form of the distribution can be ignored. Extra taxation is a function of variance in taxable income only.

Let $\overline{T}_v$ be the mean tax payable over $N$ years on a non-varying income and $T_v$ be the mean tax payable over $N$ years with a variable income. We wish to determine the extra mean tax payable due to a variable income, $\overline{T}_v - \overline{T}_c$. Also, let $X_i$ be the taxable income in the $i^{th}$ year and $a, b$ and $c$ be the coefficients of the tax function.

Tax in the $i^{th}$ year ($T_i$) is

$$T_i = aX_i^2 + bX_i + c$$

and

$$\overline{T}_v = \frac{\sum_{i=1}^{N} T_i}{N}$$

$$= \frac{a\sum X_i^2 + b\sum X_i + Nc}{N}$$

$$= \frac{a(\sum X_i^2)}{N} + \frac{b(\sum X_i)}{N} + c$$

(3)
If taxable income is non-varying over $N$ years,
\[ \bar{T}_e = a[(\Sigma X_i)/N]^2 + b[(\Sigma X_i)/N] + c \]  \hspace{1cm} (4)

and subtracting equation (4) from equation (3) we have
\[ \bar{T}_o - \bar{T}_e = a[(\Sigma X_i^2)/N - (\Sigma X_i)/N]^2 \]
\[ = a\sigma^2_x \]  \hspace{1cm} (5)

Thus all that is necessary to calculate mean extra tax payments is to multiply the variance of income by the constant $a$ of the simplified income tax function. Because New Zealand farm incomes tend to fall in Phase IV of the tax function, I have used $a = 0.281 \times 10^{-4}$ in applying this analytical method.

The Enumeration Method

The extra tax payments resulting from income variance can also be calculated by enumeration but a mean and parameters for the income distribution are needed.

\[ E(T_e) - E(T_o) = \sum_{j=a}^{j=b} f(X_j)P(X_j) - f(\mu_X) \]

where $f$ is the tax function and $X_j$ is the mid point of $\$100$ deviation intervals from the mean, $\mu_X$, and $P(X_j)$ is the probability of the $j^{th}$ deviation occurring. The summation starts at the $a^{th}$ deviation and finishes at the $b^{th}$.

Calculations were carried out using a normal and a rectangular distribution. The limited relevant historical data of New Zealand farmers' incomes available to the author for constructing a frequency distribution suggests a uni-modal probability distribution. The normal distribution was selected on the ground that farmers' incomes are a function of a number of variables and hence can be expected to be normally distributed. The rectangular distribution was selected as an extreme case because it is the flattest 'uni-modal' distribution.

In the case of the normal distribution the probability of the $j^{th}$ $\$100$ deviation interval occurring was determined with the aid of Fisher's table for the Normal Probability Integral. The values of $a$ and $b$ were adjusted for each standard deviation of income considered ($\sigma$) so that the summation of $\$100$ deviation intervals commenced $3\sigma$ below $\mu_X$ and finished $3\sigma$ above.

In the case of the rectangular distribution, the range of income for a particular standard deviation being considered ($\sigma_X$) equals $\sigma_X \sqrt{12}$. This range was divided into $\$100$ intervals and $b$ and $a$ were set so that half the intervals were above and half were below the mean. As all the $P(X_j)$'s are equal in the rectangular distribution, $P(X_j)$ was the inverse of the mean number of $\$100$ intervals.

The tax function used included the exact set of rules for calculating tax, both income and social security tax. A tax exemption of $\$2000$ for income tax was assumed. (In practice this exemption depends on family size and insurance premiums.)

Results

Table 2 and Figure 1 present the results. Figure 1 shows part of the data from the table and demonstrates the effect that increasing the
<table>
<thead>
<tr>
<th>Standard Deviation of Income</th>
<th>Analytical Result</th>
<th>Expected Income</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>N*</td>
</tr>
<tr>
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<td>7</td>
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<td>5000</td>
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</table>

* N = Normal  ** R = Rectangular
Fig. 1—The effect of increasing variation in income on the extra tax paid at expected levels of income of $5,000 and $10,000 distributed normally and rectangularly.

standard deviation of income has on the expected extra tax at mean incomes of $5000 and $10,000 distributed normally and rectangularly. The estimate of expected extra tax using the analytical method is also shown in the figure. The conclusions drawn from the figure are supported in more detail by the results in Table 2.

1. The analytical method gives a reliable approximation of the expected extra tax in the 'middle incomes' where most farmers incomes fall (in the $1800-$6470 range). This is where \( a = 0.281 \times 10^{-4} \) is the constant for the quadratic function.

2. Those taxpayers with high incomes are penalized least by having incomes that fluctuate. This is because many taxable incomes fall beyond $7200 where tax is a linear function of income. Because the tax rate remains constant here, the additional tax paid where incomes are high is compensated for where incomes are low.

3. Generalizing from the two distributions, it appears that the form of the distribution is a minor factor in determining the additional tax for a given income variance.
Discussion

The small absolute magnitude of the expected extra tax for income fluctuation was less than that which the author had been led to believe from discussions with farm accountants and farm management experts. An expected farm income of $5000 is unlikely to have a standard deviation of greater than $2500. Yet this only results in a $200 expected additional tax payment—an increase in tax of 20%. Proportionally those on low expected incomes are most likely to be penalized. A farmer with an expected income of $2000 with a standard deviation of $1000 (the same coefficient of variation as the example just given), pays $60 more tax which is 44% more than if his income was constant. However, provisions already exist for New Zealand farmers to carry losses forward for tax purposes. In practice the calculations over-emphasize the disadvantage of undue fluctuation to him.

The small magnitude of the expected extra tax payments means that schemes which smooth income declared for tax are neither likely to be a great boon to farmers nor are they likely to cost the New Zealand exchequer much.

In the Australian situation, Hinkley and Taplin [1] used a case study from a property covering a sequence of 8 years income with the variance reduced by adjusting incomes for trends over time using linear regression. They observed a coefficient of variation of about 50%. This degree of variation is higher than that faced by the New Zealand farmer.

On the other hand the New Zealand tax rates increase more rapidly than in Australia. The marginal rate of income tax reaches 60 cents in the dollar when taxable income is only $7200 compared with $16,000 in Australia. At an average taxable income of $6000 and with a standard deviation of $3000, Hinkley and Taplin, using the Australian tax rates, calculated that additional payments were $159 annually. Table 2 shows that an equivalent New Zealand income of $8000 (adding $2000 for exemptions) with the same standard deviation, paid an extra $266 due to the sharply rising rate of taxation in New Zealand.

Reference