EXTRA TAX RESULTING FROM INCOME VARIATION WITH PARTICULAR REFERENCE TO NEW ZEALAND

A. T. G. McARTHUR

Lincoln College

The extent by which expected tax payments are increased by income variance is discussed in the New Zealand setting. The cases of both normal and rectangular distributions of income are examined. The absolute magnitude of the expected extra tax for income fluctuation is lower than was expected. In Australia income variance may be higher but the more rapidly progressive tax rates of New Zealand mean that for a given mean income and standard deviation the extra tax in that country would be higher.

Introduction

The proportion of income demanded by the New Zealand Government increases as income increases. Because of this a tax-payer with an income which does not vary pays less tax than a tax-payer with the same average income but whose income fluctuates. The larger the income variance the higher the additional tax payments.

Farmers as a group have incomes which vary because of the vicissitudes of their environment. They attempt to save tax by adjusting their farm expenditure around annual balance day. In New Zealand, since 1965, there has been an income equalization scheme by which farmers can deposit surplus income with the Government in 'good years' and withdraw it within five years, adding to the taxable income of 'bad years'.

The purpose of the project reported here was to determine the extent by which expected tax payments are increased by income variance. The results show the extent to which farmers on fluctuating incomes are penalized by tax laws which treat those on varying and constant incomes alike.

Table 1 shows the New Zealand personal tax function for 1968. X is taxable income which is total income with exemptions subtracted. The tax function can be divided into five phases. Some of these phases are linear, where income variation within the phase will not result in extra tax payments because the proportion of income paid as tax is constant within the range. Other phases are quadratic. Hence the proportion of income taken as tax increases as income rises, and income variation results in extra tax payments.

In Phase 1 where taxable income is zero or less, tax is zero.

Over the next \$1000 the income tax function is linear. Over the next \$800, Phase III, the tax rate rises steeply by 6d every £100 increase in taxable income.[1] A quadratic function approximates this discontinuous function with only very small errors. Over the fourth phase the tax rate

[1] The most recent rates of income tax are levied under the New Zealand Land and Income Tax Annual Act of 1968 but the form of the tax function was specified by Part C of the First Schedule of the Land and Income Tax Act of 1954 and hence the rates of increase are in terms of £.s.d.

TABLE 1

The New Zealand Personal Income Tax Function for 1967–68
(*X is Taxable Income and T is tax*)

Phase	Range	Class of Function	
I	$X \leqslant 0$	Linear	$T = 0$
II	$0 < X \leqslant \$1000$	Linear	$T = 0.135X$
III	$\$1000 < X \leqslant \$1800$	Quadratic	$T \simeq (0.562 \times 10^{-4})X^2 + 0.034X + 45.0$
IV	$\$1800 < X \leqslant \$6470$	Quadratic	$T \simeq (0.281 \times 10^{-4})X^2 + 0.129X - 36.0$
V	$\$6470 < X \leqslant \$7200$	Quadratic	$T \simeq (0.312 \times 10^{-4})X^2 + 0.143X - 237$
VI	$\$7200 < X$	Linear	$T = 0.6(X - 7200) + 2415$

rises less steeply (by 3d per £100). Up until taxable income reaches \$6470 there is a tax rebate of 10%. This rebate no longer applies after this point, accounting for the slightly different constants in Phase V. The last phase is linear, 60% of income above \$7200 being taken as tax.

Three factors affect the magnitude of the extra tax payments caused by taxable income varying.

1. The magnitude of the variance of taxable income.
2. Mean taxable income.
3. The form of the frequency distribution of taxable income.

The first factor needs no further explanation. The second factor, mean taxable income, determines (with the other two factors) the phases of the tax function in which incomes fall. Thus with a very high income all incomes may fall in Phase VI and hence no extra tax will be paid. The form of the frequency distribution also affects the distribution of incomes in the phases of the tax function.

An Analytical Method

If the tax function of Phase IV is assumed to apply over the entire domain of the tax function, then both the mean taxable income and the form of the distribution can be ignored. Extra taxation is a function of variance in taxable income only.

Let $\overline{T}_c$ be the mean tax payable over N years on a non-varying income and T_v be the mean tax payable over N years with a variable income. We wish to determine the extra mean tax payable due to a variable income, $\overline{T}_v - \overline{T}_c$. Also, let X_i be the taxable income in the i^{th} year and a, b and c be the coefficients of the tax function.

Tax in the i^{th} year (T_i) is

$$T_i = aX_i^2 + bX_i + c \tag{1}$$

and

$$\overline{T}_v = (\sum_i^N T_i)/N \tag{2}$$

$$= (a\Sigma X^2_i + b\Sigma X_i + Nc)/N$$

$$= a(\Sigma X^2_i)/N + b(\Sigma X_i)/N + c \tag{3}$$

If taxable income is non-varying over N years,

$$\overline{T}_c = a[(\Sigma X_i)/N]^2 + b[(\Sigma X_i)/N] + c \qquad (4)$$

and subtracting equation (4) from equation (3) we have

$$\begin{aligned}\overline{T}_v - \overline{T}_c &= a(\Sigma X^2_i)/N - a[(\Sigma X_i)/N]^2 \\ &= a\sigma^2_X \end{aligned} \qquad (5)$$

Thus all that is necessary to calculate mean extra tax payments is to multiply the variance of income by the constant a of the simplified income tax function. Because New Zealand farm incomes tend to fall in Phase IV of the tax function, I have used $a = 0{\cdot}281 \times 10^{-4}$ in applying this analytical method.

The Enumeration Method

The extra tax payments resulting from income variance can also be calculated by enumeration but a mean and parameters for the income distribution are needed.

$$E(T_v) - E(T_c) = \sum_{j=a}^{j=b} f(X_j)P(X_j) - f(\mu_X)$$

where f is the tax function and X_j is the mid point of \$100 deviation intervals from the mean, μ_X, and $P(X_j)$ is the probability of the j^{th} deviation occurring. The summation starts at the a^{th} deviation and finishes at the b^{th}.

Calculations were carried out using a normal and a rectangular distribution. The limited relevant historical data of New Zealand farmers' incomes available to the author for constructing a frequency distribution suggests a uni-modal probability distribution. The normal distribution was selected on the ground that farmers' incomes are a function of a number of variables and hence can be expected to be normally distributed. The rectangular distribution was selected as an extreme case because it is the flattest 'uni-modal' distribution.

In the case of the normal distribution the probability of the j^{th} \$100 deviation interval occurring was determined with the aid of Fisher's table for the Normal Probability Integral. The values of a and b were adjusted for each standard deviation of income considered (σ) so that the summation of \$100 deviation intervals commenced 3σ below μ_X and finished 3σ above.

In the case of the rectangular distribution, the range of income for a particular standard deviation being considered (σ_X) equals $\sigma_X\sqrt{12}$. This range was divided into \$100 intervals and b and a were set so that half the intervals were above and half were below the mean. As all the $P(X_j)$'s are equal in the rectangular distribution, $P(X_j)$ was the inverse of the mean number of \$100 intervals.

The tax function used included the exact set of rules for calculating tax, both income and social security tax. A tax exemption of \$2000 for income tax was assumed. (In practice this exemption depends on family size and insurance premiums.)

Results

Table 2 and Figure 1 present the results. Figure 1 shows part of the data from the table and demonstrates the effect that increasing the

TABLE 2

The Effect of Standard Deviation of Income on Expected Extra Tax Payments in Dollars at Ten Expected Levels of Income Distributed Normally and Rectangularly

Standard Deviation of Income	Analytical Result	Expected Income 1000		2000		3000		4000		5000		6000		7000		8000		9000		10,0 00	
		N*	R**	N	R	N	R	N	R	N	R	N	R	N	R	N	R	N	R	N	R
		Extra Taxation Payments (Expected) in Dollars																			
500	7	1	0	27	25	10	10	10	7	9	7	10	7	11	7	21	17	15	6	9	0
1000	28	20	19	59	60	40	39	35	33	31	27	31	27	36	30	51	48	34	26	13	3
1500	63	55	56	104	103	87	88	78	74	69	64	69	62	77	73	93	92	64	55	27	14
2000	112	102	108	162	166	148	154	138	138	126	120	125	121	131	133	142	145	103	96	50	36
2500	176	158	165	228	239	222	230	213	215	200	198	196	194	198	200	201	204	149	146	83	72
3000	253	221	234	303	315	304	316	299	305	285	290	278	282	273	277	266	269	209	195	122	109
3500	344	293	313	386	407	396	420	395	418	382	403	370	388	357	371	340	347	263	259	170	159
4000	450	374	391	478	503	495	527	499	534	486	520	471	500	451	471	423	434	334	333	227	218
4500	569	460	486	575	608	599	644	607	655	594	641	576	615	549	576	511	524	411	404	291	276
5000	703	556	591	681	727	713	773	725	788	713	775	690	744	657	697	610	632	498	497	366	351

* N = Normal ** R = Rectangular

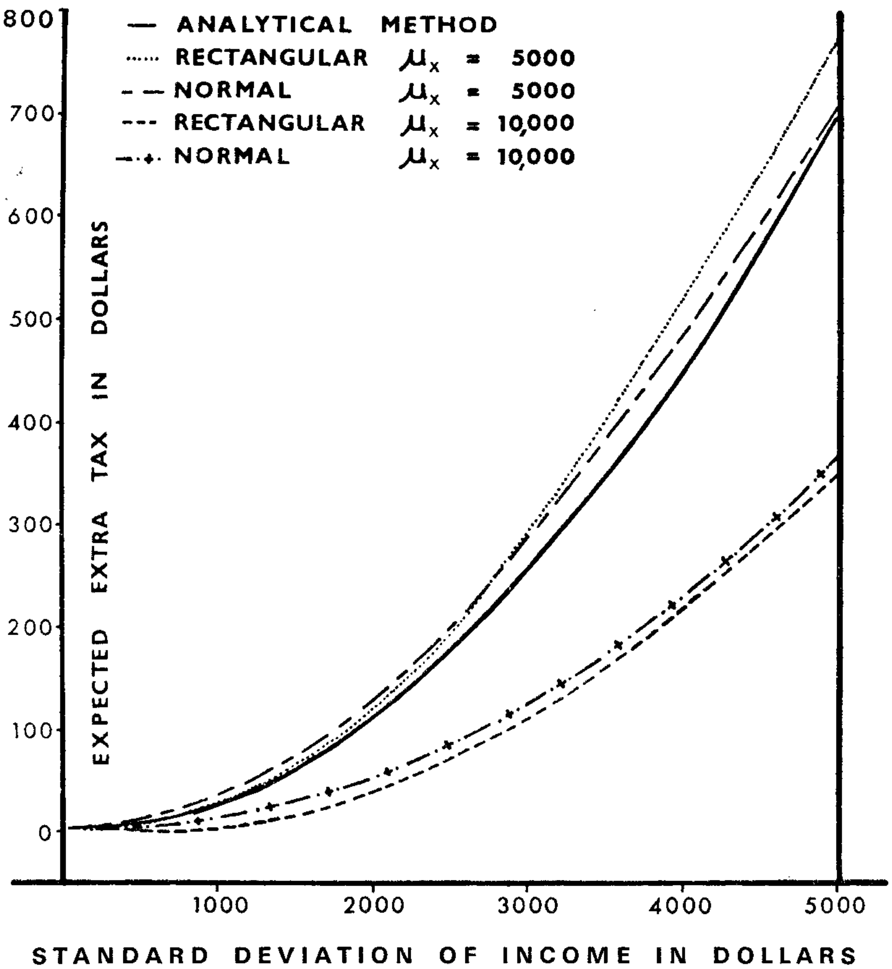

FIG. 1—The effect of increasing variation in income on the extra tax paid at expected levels of income of $5,000 and $10,000 distributed normally and rectangularly.

standard deviation of income has on the expected extra tax at mean incomes of $5000 and $10,000 distributed normally and rectangularly. The estimate of expected extra tax using the analytical method is also shown in the figure. The conclusions drawn from the figure are supported in more detail by the results in Table 2.

1. The analytical method gives a reliable approximation of the expected extra tax in the 'middle incomes' where most farmers incomes fall (in the $1800-$6470 range). This is where $a = 0{\cdot}281 \times 10^{-4}$ is the constant for the quadratic function.
2. Those taxpayers with high incomes are penalized least by having incomes that fluctuate. This is because many taxable incomes fall beyond $7200 where tax is a linear function of income. Because the tax rate remains constant here, the additional tax paid where incomes are high is compensated for where incomes are low.
3. Generalizing from the two distributions, it appears that the form of the distribution is a minor factor in determining the additional tax for a given income variance.

Discussion

The small absolute magnitude of the expected extra tax for income fluctuation was less than that which the author had been led to believe from discussions with farm accountants and farm management experts. An expected farm income of $5000 is unlikely to have a standard deviation of greater than $2500. Yet this only results in a $200 expected additional tax payment—an increase in tax of 20%. Proportionally those on low expected incomes are most likely to be penalized. A farmer with an expected income of $2000 with a standard deviation of $1000 (the same coefficient of variation as the example just given), pays $60 more tax which is 44% more than if his income was constant. However, provisions already exist for New Zealand farmers to carry losses forward for tax purposes. In practice the calculations over-emphasize the disadvantage of undue fluctuation to him.

The small magnitude of the expected extra tax payments means that schemes which smooth income declared for tax are neither likely to be a great boon to farmers nor are they likely to cost the New Zealand exchequer much.

In the Australian situation, Hinkley and Taplin [1] used a case study from a property covering a sequence of 8 years income with the variance reduced by adjusting incomes for trends over time using linear regression. They observed a coefficient of variation of about 50%. This degree of variation is higher than that faced by the New Zealand farmer.

On the other hand the New Zealand tax rates increase more rapidly than in Australia. The marginal rate of income tax reaches 60 cents in the dollar when taxable income is only $7200 compared with $16,000 in Australia. At an average taxable income of $6000 and with a standard deviation of $3000, Hinkley and Taplin, using the Australian tax rates, calculated that additional payments were $159 annually. Table 2 shows that an equivalent New Zealand income of $8000 (adding $2000 for exemptions) with the same standard deviation, paid an extra $266 due to the sharply rising rate of taxation in New Zealand.

Reference

[1] Hinkley, F. and Taplin, J., 'A Comparison of the Tax Saving Effects of Averaging and Income Equalization when Rural Incomes are Highly Variable', *Quarterly Review of Agricultural Economics*, 19 (4), 193, (1966).