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A Comment on the Value of Time and the Demand for Money

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In two recent papers Edi Karni attempts to incorporate the value of time into the theory of money demand. While we consider this a fruitful approach we believe that his papers contain errors which stem mainly from his failure to begin with a utility maximizing model.

Karni begins by assuming a Baumol money demand function which he modifies by assuming that the value of total transactions is proportional to the sum of wage and non-wage income and that the cost of a financial transaction includes a money component and a time component. Then, assuming that the value of time equals the wage rate, Karni obtains his money demand function. He then proceeds to derive partial elasticities of money demand with respect to the variables in his function in order to obtain testable hypotheses.

The difficulties with the Karni model stem from his treatment of time allocated to work and of the marginal value of time. He assumes that work time is exogenous for the consumer and also that the marginal value of time equals the wage. However when work time is exogenous, the marginal value of time is not necessarily equal to the wage and there is no a priori reason to believe that such a relationship will hold.[1]

Section I is devoted to an analysis of Karni's model in a utility maximizing framework in which work time is endogenous. Given Karni's definition of transactions we show that the value of time is not equal to the wage and that the money demand function which emerges is not the same as the function he uses. His solution will drop out as a special case when work time is exogenous and the value of time is assumed to equal the wage. Unless this assumption is made, Karni's money demand function will not be obtained whether or not work time is exogenous. Therefore his tests of the value of time and inventory hypotheses are not valid except in this case when the value of time is assumed to equal the wage.

In section II we derive a money demand function incorporating the value of time hypothesis from a utility maximizing model. Instead of Karni's definition of transactions we assume that the value of transactions is equal to consumption expenditures and obtain a money demand function which looks very similar to his. However, it is not the same and Karni's empirical results cannot be used to test the value of time and inventory hypotheses which derive from our formulation.

I. The Karni Model

In this section we analyze Karni's model in a utility maximization framework. The individual maximizes utility subject to a time constraint and a budget constraint and chooses the optimal amounts of consumption, leisure and cash balances. The transaction of buying or selling a bond involves a time cost and a money cost.

The individual's utility is a function of consumption (x) and leisure (ℓ):

(1) $U = U(x, \ell)$.

The individual has a given amount of time (T) available to spend on work (t_w), leisure (ℓ) and bond transactions (nt_e) where n is the number of bond transactions made and t_e is the time requirement per transaction:

(2) $T = t_w + nt_e + \ell$

The budget constraint is written as follows:

(3) $y + wt_w - px + \pi = 0$

where, y = non-wage income

w = wage

p = price of commodities

π = net profit from cash management.

The net profit from cash management is the return on bonds minus the cost of money-bond transactions:

(4) $\pi = iB - nb$

where, i = interest rate on bonds

B = average bond holdings

b = the money cost of a bond market transaction.

Karni assumes that total transactions are proportional to income, i.e., transactions = $a(y + wt_w)$ where a is some constant. The transactions are spread evenly over the period. The consumer makes n evenly spaced bond market transactions. The first transaction involves buying bonds of $a(y + wt_w)(\frac{n-1}{n})$ while the remaining (n-1) transactions involve cashing in $[a(y+wt_w)/n]$ of bonds. Thus, average money (M) and bond holdings throughout the period are:

(5) $$M = \frac{a(y + wt_w)}{2n}$$

(6) $$B = \frac{a(y + wt_w)(n - 1)}{2n} = a(y + wt_w)(\frac{1}{2} - \frac{1}{2n})$$

Substituting equations (4) and (6) into (3) we have,

(3') $$y + wt_w - px + ia(y + wt_w)(\frac{1}{2} - \frac{1}{2n}) - nb = 0$$

The individual's problem is to maximize (1) subject to (2) and (3') or to maximize the following Lagrangian:

(7) $$L = U(x, \ell) + \lambda_1(T - t_w - nt_e - \ell) + \lambda_2[y + wt_w - px + ia(y + wt_w)(\frac{1}{2} - \frac{1}{2n}) - nb]$$

Differentiating the above, the first order conditions are:

$$(8)\quad \begin{aligned} &(a)\ U_x - \lambda_2 p = 0 \\ &(b)\ U_\ell - \lambda_1 = 0 \\ &(c)\ -\lambda_1 + \lambda_2 w[1 + ia(\tfrac{1}{2} - \tfrac{1}{2n})] = 0 \\ &(d)\ -\lambda_1 t_e + \lambda_2[\frac{ia(y + wt_w)}{2n^2} - b] = 0 \\ &(e)\ T - t_w - nt_e - \ell = 0 \\ &(f)\ y + wt_w - px + ia(y + wt_w)(\tfrac{1}{2} - \tfrac{1}{2n}) - nb = 0 \end{aligned}$$

Since the marginal utility of time is λ_1 and the marginal utility of income is λ_2 the money marginal value of time is λ_1/λ_2. Note that at the optimum $dU/dT = \lambda_1$ and $dU/dy = \lambda_2$. From (8c) the marginal value of time is:

$$(9)\quad -dy/dT = \lambda_1/\lambda_2 = w[1 + ia\ (\tfrac{1}{2} - \tfrac{1}{2n})]$$

Thus, the value of time which emerges when Karni's definition of transactions is used is not equal to the wage. To get his demand function out of the above formulation requires assuming that work time is exogenous (which would eliminate (8c)) and then making the ad hoc assumption that the value of time (λ_1/λ_2) is equal to the wage. There is no reason to believe that this assumption would be generally valid.[2]

If we solve (8c-d) for the optimal number of transactions and (using (5)) the optimal money holdings we obtain:

$$(10)\qquad n = \frac{iawt_e + \{(iawt_e)^2 + 4ia(y + wt_w)[2(wt_e + b) + iawt_e]\}^{1/2}}{2[2(wt_e + b) + iawt_e]}$$

$$(11)\qquad M = \frac{a(y + wt_w)[2(wt_e + b) + iawt_e]}{iawt_e + \{(iawt_e)^2 + 4ia(y + wt_w)[2(wt_e + b) + iawt_e]\}^{1/2}}$$

This is clearly not the demand function which Karni presents. Note that this function may not have the desired properties. For example, the relationship between money demand and the transactions time requirement is ambiguous. Karni's function, using our notation, would be

$$M = \left(\frac{(wt_e + b)(y + wt_w)a}{2i} \right)^{1/2}$$

In this formulation (to be distinguished from the formulation which we present below) the value of transactions, $a(y + wt_w)$, and, therefore, the profit from cash management, depend directly on the allocation of time to work. This can be seen more clearly by rewriting (8c):

$$\lambda_2 w[1 + ia(\frac{1}{2} - \frac{1}{2n})] = \lambda_1$$

For a maximum time must be allocated to work so that the marginal benefit equals the marginal cost. The marginal cost is λ_1 which equals the marginal utility of leisure foregone (see (8b)). The marginal benefit equals the marginal utility of the wage earned plus the marginal utility of the marginal profits earned from cash management. These marginal profits depend directly on the allocation of time to work.

This points to a special case in which the value of time will equal the wage. If $n = 1$, i.e., if no bonds are held (see equation (6)) then $\lambda_1/\lambda_2 = w$. In this case the marginal profits from cash management are zero.

II. An Alternative Formulation

If Karni's definition of transactions is dropped and instead we assume that total transactions are equal to expenditures on consumption, we can derive a money demand function which looks similar to the one which Karni assumes. In a model in which all income is consumed it seems reasonable to assume that the value of transactions is equal to the value of consumption rather than proportional to income.[3]

Specifically we are assuming the value of transactions = px. Then maximizing utility subject to the time and budget constraints the following Lagrangian is formed:

$$(12)\quad L = U(x, \ell) + \lambda_1(T - t_w - nt_e - \ell) + \lambda_2[y + wt_w - px + i(\frac{px}{2} - \frac{px}{2n}) - nb]$$

where profit from cash management = $iB - nb = i\ (\frac{px}{2} - \frac{px}{2n}) - nb$.

Note that in this formulation work time only indirectly affects the value of transactions and, therefore, the profit from cash management.

Maximizing (12) we have

$$(13)\quad \begin{aligned} &(a)\ U_x + \lambda_2 p(\frac{i}{2} - \frac{i}{2n} - 1) = 0 \\ &(b)\ U_\ell - \lambda_1 = 0 \\ &(c)\ - \lambda_1 + \lambda_2 w = 0 \\ &(d)\ - \lambda_1 t_e + \lambda_2(\frac{pxi}{2n^2} - b) = 0 \\ &(e)\ T - t_w - nt_e - \ell = 0 \\ &(f)\ y + wt_w - px + i(\frac{px}{2} - \frac{px}{2n}) - nb = 0 \end{aligned}$$

Clearly, from equation (13c) the value of time is equal to the wage:

$$\lambda_1/\lambda_2 = w$$

From (13c-d) the optimal number of bond market transactions is:

$$(14)\quad n = (\frac{pxi}{2(wt_e+b)})^{1/2}$$

Using this expression the money demand function is obtained:[4]

$$(15)\quad M = \frac{px}{2n} = (\frac{px(wt_e+b)}{2i})^{1/2}$$

Deflating by p, real money demand is

$$(15')\quad \frac{M}{p} = \left(\frac{x(\frac{w}{p}t_e + \frac{b}{p})}{2i}\right)^{1/2}$$

Using our notation, Karni's demand for real balances is

$$\frac{M}{p} = \left[\frac{(\frac{w}{p}t_e + \frac{b}{p})(\frac{y}{p} + \frac{w}{p}t_w)a}{2i}\right]^{1/2}$$

Karni's function, and his empirical work, treats work time as being exogenously determined. Our function, coming from a full utility maximizing model, treats prices, the interest rate, and the money and time cost of a bond market transaction as being exogenously determined. Consumption, work time and money demand are endogenous.

Consequently, Karni's empirical results are not applicable to our model. Proceeding as Karni does we might fit the following equation (assuming b and t_e constant):

$$(16)\quad d\log\frac{M}{P} = \alpha + \eta_{\frac{M}{P},\frac{w}{p}}\, d\log\frac{w}{p} + \eta_{\frac{M}{P},\frac{y}{p}}\, d\log\frac{y}{p} + \eta_{\frac{M}{P},i}\, d\log i$$

where,

$$(a)\quad \eta_{\frac{M}{P},\frac{w}{p}} = \frac{1}{2}\left[\eta_{x,\frac{w}{p}} + \frac{\frac{w}{p}t_e}{\frac{w}{p}t_e + \frac{b}{p}}\right]$$

$$(17)\quad (b)\quad \eta_{\frac{M}{P},\frac{y}{p}} = \frac{1}{2}\eta_{x,\frac{y}{p}}$$

$$(c)\quad \eta_{\frac{M}{P},i} = \frac{1}{2}(\eta_{x,i} - 1)$$

(where η refers to the partial elasticity).

The real wage effects money demand both through its effect on consumption and through its effect on the cost of a bond market transaction. The value of time hypothesis specifically refers to the latter effect. To test this hypothesis we would fit equation (16) and then compare the estimated real wage elasticity of money demand with an independent estimate of the wage elasticity of consumption demand (see equation (17a)).[5]

III. Conclusion

In this paper we have analyzed Edi Karni's value of time model and have shown that his specification of money demand is inconsistent with a utility maximizing approach except under very special conditions. Given his definition of transactions, the money demand function which comes out of a utility maximizing model is not the same as the function which is estimated in his paper. We show that, using an alternative definition of transactions, we can obtain a money demand function which reflects the value of time and inventory hypotheses from a full utility maximizing model.

Footnotes

1. Karni points out that money is held by firms as well as consumers. However his discussion concentrates entirely on the consumer.

2. See Moses and Williamson

3. Feige and Parkin make the same assumption as we do here in a more general inventory model.

4. The analogous development for the firm is straightforward. Assume the firm uses labor (L_p) and capital (K) to produce output, $q = q\ (L_p, K)$, but uses only labor (L_F) for financial transactions, $L_F = nt_e$, where $L = L_p + L_F$. Given prices, p, w, and r, the value of the firm's transactions is $wL + rK$ and profits may be written,

$$\pi = pq(L_p, K) - wL - rK + [i(wL + rK)(\frac{1}{2} - \frac{1}{2n}) - nb]$$

Maximizing profits will yield

$$(14') \quad n = [\frac{i(wL_p + rK)}{2(b+wt_e) - iwt_e}]^{1/2}$$

$$(15') \quad M = [\frac{(wL_p+rK)(b + wt_e - \frac{iwt_e}{2}}{2i}]^{1/2} + \frac{wt_e}{2}$$

The difference between (14) and (14'), and (15) and (15') is the presence of the terms iwt_e, and $(iwt_e)/2$ and $wt_e/2$, respectively. These occur because, for the firm, labor costs of transactions are actually part of the money value of transactions. This is not the case for the consumer who allocates part of this time endowment which has an imputed value only.

5. We did estimate equation (16) and got results which are similar to Karni's given the difference in the variables we used. We do not present these results because we do not have independent estimates of the consumption elasticity and, therefore, cannot test the value of time hypothesis.

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