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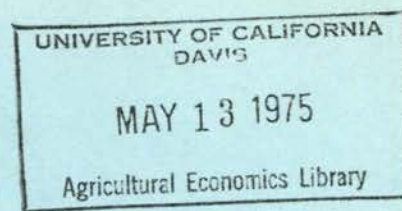
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IN GENERAL EQUILIBRIUM

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# Marketing Costs and Imperfect Competition in General Equilibrium\*

R. R. Cornwall

## 1. Introduction

General equilibrium theory has made substantial progress in untangling the logic of how a market can coordinate individual actions. This progress has been based, however, on a pristinely abstract concept of a "market." This "ethereal construct" [1, p. 3] was some sort of wondrous void (or black box) from which sprang prices to which all agents had equal access. This paper makes a tentative effort to enlarge the notion of a market in this theory. It extends earlier efforts (e.g. [6] and [8]) to make transactions and merchandising costs explicit in general equilibrium models. More basically, it attempts to allow explicitly for the fact that markets, too, are controlled by optimizing agents. In so doing, it is impossible to avoid the problem of how to describe imperfectly competitive solutions. Thus these agents may maintain more than one market in which to trade some commodity and the same "good" may have different prices in different markets in equilibrium -- both because of price discrimination and because of differing merchandising costs for different types of traders. In summary, this paper starts work on a model which permits the simultaneous determination of the nature of trading arrangements and of prices.

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\* This paper was prepared for discussion at the Conference on Equilibrium and Disequilibrium in Economic Theory at the Institute for Advanced Studies in Vienna, July 1974. I appreciate helpful comments of several of the participants at the Conference and particularly of Hans Keiding.

The next section of this paper describes the types of markets allowed for in this model. Section 3 presents the formal definition of this paper's model of markets. Section 4 makes a translation of this model into the standard Arrow-Debreu model of equilibrium. This is useful technically and it makes apparent the way in which this model extends the Arrow-Debreu analysis. Finally, Section 5 discusses a new solution concept which is suggested by this model.

## 2. Representation of markets

There are  $L$  markets, indexed  $\ell = 1, \dots, L$ . Each market consists of a list  $S_\ell$  of participants from among the  $m$  consumers (indexed  $i = 1, \dots, m$ ) and  $n$  firms (indexed  $j = m+1, \dots, m+n$ ). Among these participants is the "market-maker," denoted  $s_\ell$ . The other participants in  $S_\ell$  will be referred to as "market-players." In each market, the market-maker chooses the level of financial charges in that market. Of course, the observed (or, in this model, the equilibrium) charges may result from bargaining between the market-maker and the market-players. Furthermore, it is expected that the list of markets may contain two markets  $k$  and  $\ell$  with the same set of participants ( $S_k = S_\ell$ ) but with different market-makers. Finally, it is convenient in the representation of trading described below to allow for the possibility that all commodities are traded on each market. Of course, we would expect to observe in equilibrium that only certain commodities are traded on each market. In summary, markets are distinguished according to who the participants are and who the market-maker is and, in equilibrium, will also be distinguished by which markets are "open" and which commodities are traded on each market.

In each market  $\ell$ , there are two types of financial charge corresponding to a "two-part tariff" system<sup>1</sup>

- 1) marginal charge or price vector  $p_\ell \in \mathbb{R}^G$  (where  $G$  is the number of commodities);
- 2) fixed charge or rent scalar  $r_\ell$  which each market-player pays the market-maker if she participates in market  $\ell$ .

Thus if market-player  $i \leq m$  buys the vector  $t_{i\ell}$  on market  $\ell$ , then she pays  $s_\ell$  the sum  $p_\ell \cdot t_{i\ell} + r_\ell$ .

This description of markets allows for a wide variety of types of imperfect competition:

1) First-degree discrimination by agent  $j$ : Agent  $j$  makes a separate "market" with each trading partner. In this market,  $j$  charges a price equal to the resulting marginal cost of production and a rent equal to the buyers' total surplus (= compensating variation for a consumer).

2) Second-degree discrimination by agent  $j$ : Here all potential traders face the same price schedule but the price any one trader pays depends on the quantity she trades. This can be represented in the above framework in a number of ways; for example, trading partners of  $j$  may be grouped into separate "markets" according to which quantity they buy. The price in this market is then the price paid for the last unit traded in this market and the rent is the excess of the purchaser's total payment over the product of this price times the quantity traded. Of course this case could also arise in the way described in 1) above since it is an approximation to First-degree discrimination.

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1. This representation follows a suggestion made by my colleague and running partner, Wm. G. Moss.

3) Third-degree discrimination by agent  $j$ : In this case different trading partners are grouped together into several "markets" where different prices are charged and no rents are charged.

The choice among these various possibilities for a maker of markets will presumably depend both on the costs of carrying out the various kinds of segmenting or discrimination among trading partners (to be described in the next section) as well as on the extra revenue gained by so discriminating.

Before plunging into the formal analysis of trading on such markets, it is interesting to consider briefly how well observable trading procedures can be represented by the preceding description of markets. The specification of one market-maker dealing with a number of market-players obviously fits a number of familiar situations ranging from farmer  $j$  operating a roadside vegetable stand for consumers to Sears-Roebuck acting as a market-maker in a great variety of consumer goods to a large number of consumers. It also obviously covers cases where firms (e.g. petroleum companies) sell some output retail directly to consumers, some output to other agents called franchised dealers and some output to wholesalers.

This description of markets is not so immediately applicable in cases where (1) it is not clear who the market-maker is or where (2) there appear to be multi-agent market-makers. The first typically occurs where there is bilateral trading. For example, there may be just one buyer and one seller of a commodity. In this case, either agent could be designated as "market-maker" equally well without, I believe, affecting the set of solutions predicted by the model. Of course the list of markets might include both possibilities.

A rather different sort of bilateral trading is the trading of labor services conducted between a firm and a union. In this case one might treat

the firm as the market-maker and the workers as the other market participants. In this case, the union is not explicitly active in the market. Now it will be analytically convenient below to assume that all consumers (i.e. workers) are price-takers (= perfectly competitive). Thus this proposed representation of the union is unacceptable. A preferable way of representing the bilateral bargaining between union and firm is to treat the union as a "firm" (indexed by  $j > m$ ) which "buys" labor in a possibly discriminating way (fixed-charge  $r_l$  = union membership fee) and, according to its merchandising technology (operating a hiring hall, record keeping, pension-fund administration, etc.), supplies labor services to the firm through a market in which the union is the market-maker. The firm's personnel office becomes an internal administrative unit rather than a market-maker. In making this interpretation, it is important to note that it is not assumed below that "firms" maximize profits.

Other examples of multi-agent market-makers include (1) producer and consumer cooperatives, (2) nations setting tariffs and subsidies on international trade and (3) securities exchanges. In this model, each of these units is represented as a (non-profit-maximizing) firm. The choice of such an institution to be the "market-maker" rather than using some other procedure for trading a class of commodities or for a group of agents is reflected in the simultaneous determination by all agents of which market is "active." This choice will depend in part on transactions technologies and in part on which institution best reflects the economic strengths of the various groups. Thus an organized security exchange offers considerable economies in transaction costs (even in the static world of this model!) but also affords an opportunity for the brokers and dealers comprising the exchange to be non-perfectly competitive. Thus there may exist simultaneously over-the-counter markets in which some of the same securities are traded and in which the market-maker is a single dealer or broker.

### 3. Consistent outcomes on the markets

It seems convenient to assume that all consumers are perfectly competitive, or more exactly, that they can only act as non-price-taking market-makers when they form "firms." All firms then decide, for those markets in which they are the market-maker, whether or not to make that market open. For each open market  $\ell$ , the market-maker chooses the financial charges  $p_\ell$  and  $r_\ell$  and chooses her own commodity-vector  $t_{s_\ell \ell}$ . All market-players in each market then choose how much they will trade in each market. These choices are consistent if, on each market, supply equals demand and if each consumer's share of profits is just enough to permit him to carry out his proposed trades.

This scheme can be described formally as follows: A consistent outcome is a collection  $(L, (p_\ell, r_\ell, (t_{i\ell})_{i=1}^{m+n})_{\ell=1}^L)$  where  $L$  is the subset of  $\{1, \dots, L\}$  of open markets,  $p_\ell$  is the price vector on market  $\ell$ ,  $r_\ell$  is the fixed charge on market  $\ell$ , and  $t_{i\ell}$  is the G-vector of trades carried out by agent  $i$  on market  $\ell$  satisfying  $t_{i\ell} = 0$  if  $\ell \notin L$ . The usual sign convention is adopted where, for  $i \leq m$ ,  $t_{i\ell g} > 0$  means  $i$  is receiving  $t_{i\ell g}$  units of good  $g$  on market  $\ell$ ,  $t_{i\ell g} < 0$  means  $i$  is giving up good  $g$  and for  $j > m$  the opposite convention is used.

Part of the definition of a consistent outcome is that it must satisfy the following three conditions:

#### I. Supply equals demand on each market.

For each  $\ell$  in  $L$ ,

$$\sum_{i \leq m} t_{i\ell} = \sum_{j > m} t_{j\ell}$$

The term on the left is, of course, aggregate demand of all consumers on market  $\ell$  (which would be zero if this market only handled intermediate goods).

The term on the right is the supply on market  $l$  from production net of interindustry demand.

## II. Feasibility for each producer.

For each  $j > m$  there is a G-vector  $z_j$  of transactions costs (market-players as well as market-makers incur costs of participating in each market) satisfying

$$((t_{j\ell})_{\ell=1}^L, z_j) \in Z_j.$$

and

$$y_j = \sum_{\ell \in L} (t_{j\ell}) - z_j \in Y_j.$$

$Z_j$  is interpreted as the transactions technology set for firm  $j$ . This includes those costs of production which are peculiar to market  $l$  such as transportation, packaging, information gathering, computation, etc. It also includes the restriction that  $t_{i\ell} = 0$  if  $i \notin S_\ell$ . The set  $Y_j$  is the production possibilities set with the usual connotations.

## III. Optimality for each consumer

For each consumer  $i \leq m$ , there is a G-vector of transactions costs  $z_i$  satisfying

$$((t_{i\ell})_{\ell=1}^L, z_i) \in Z_i$$

and

$$x_i = \sum_{\ell \in L} (t_{i\ell}) - z_i \in X_i.$$

Here  $Z_i$  is the transactions technology set for consumer  $i$  and

$X_i$  is the consumption possibilities set. The consumption vector  $x_i$  is consumption net of the consumer's own endowment vector.

Consumer  $i$ 's vector of trades also must satisfy the budget constraint

$$\sum_{l \in L} p_l \cdot t_{il} + \sum_{l: t_{il} \neq 0} r_l \leq w_i$$

where  $i$ 's wealth  $w_i$  equals  $\sum_{j > m} \theta_{ij} \Pi_j$  where  $\theta_{ij}$  is  $i$ 's ownership share in firm  $j$  and where

$$\Pi_j = \sum_{l \in L} p_l \cdot t_{jl} + \sum_{l \in L} r_l N_{jl}$$

where

$$N_{jl} = \begin{cases} \text{number of market-players active in } l \text{ if } j = s_l \\ 0 & \text{if } j \neq s_l \text{ and } t_{jl} = 0. \\ -1 & \text{if } j \neq s_l \text{ and } t_{jl} \neq 0. \end{cases}$$

Finally,  $(t_{il})_{l \in L}$  must maximize utility for consumer  $i$  in the sense that for any alternative  $((t'_{il})_{l \in L}, z'_i)$  satisfying for  $l$  in  $L$ ,  $t'_{il} = 0$  for  $l \notin L$  and

$$((t'_{il})_{l=1}^L, z'_i) \in Z_i$$

$$x'_i = \sum_{l \in L} (t'_{il}) - z'_i \in X_i$$

$$x'_i \succ_i x_i$$

then this alternative must violate the budget constraint:

$$\sum_{l \in L} p_l \cdot t'_{il} + \sum_{l: t'_{il} \neq 0} r_l > w_i.$$

This completes the definition of a consistent outcome. It is useful for future reference to note that the utility maximization described in the preceding paragraph implicitly restricts how the vector  $z_i$  corresponding to  $(t_{il})_{l \in L}$  can be chosen; namely, for no other choice  $z'_i$  satisfying

$((t_{i\ell})_{\ell=1}^L, z_i) \in Z_i$  and  $\sum_{\ell \in L} (t_{i\ell}) - z_i \in X_i$  can it be true that

$$\sum_{\ell \in L} (t_{i\ell}) - z_i \succsim_i \sum_{\ell \in L} (t_{i\ell}) - z_i.$$

#### 4. Translation into an Arrow-Debreu model.

It is helpful for the analysis below and for a comparison with earlier general equilibrium models to show how this model can be viewed as an elaboration of the standard Arrow-Debreu model without the usual assumptions of lower semicontinuity of preferences, convexity or profit-maximization. The procedure is the familiar Arrowian trick of distinguishing commodities appropriately. In this case they will be distinguished according to the market on which they are traded. This new formulation will be called the trade-space model to distinguish it from the original formulation which will be called the activity-space model.

For any consumer  $i \leq m$ , define the trade possibilities set  $T_i$  by

$$T_i = \{t_i = (t_{i\ell})_{\ell=1}^L : t_{i\ell} = 0 \text{ if } i \text{ is not a potential participant in } \ell \text{ and there exists } z_i \text{ with } ((t_{i\ell}), z_i) \in Z_i \text{ and } \sum_{\ell=1}^L (t_{i\ell}) - z_i \in X_i\}.$$

For firm  $j > m$  define the trade possibilities set  $T_j$  analogously with  $X_i$  replaced by  $Y_j$ .

For each consumer  $i \leq m$  the preference relation  $\succsim_i$  on  $X_i$  induces a preference relation on  $T_i$  which is also denoted  $\succsim_i$ . To do so it is convenient to define the correspondence

$$Z_i(t_i) = \{z_i : (t_i, z_i) \in Z_i \text{ and } \sum_{\ell=1}^L (t_{i\ell}) - z_i \in X_i\}.$$

The preference relation  $\succsim_i$  on  $T_i$  can now be defined:

$$t_i \succsim_i t'_i \iff \text{for every } z'_i \text{ in } Z_i(t'_i) \text{ there exists } z_i \text{ in } Z_i(t_i)$$

$$\text{with } \sum_{\ell} (t_{i\ell}) - z_i \succsim_i \sum_{\ell} (t'_{i\ell}) - z'_i.$$

The notion of a consistent outcome is now more concisely restated in this trade space terminology: A consistent outcome is a collection  $(L, p, r, (t_i)_{i=1}^{m+n})$

where  $p \in \mathbb{R}^{LG}$ ,  $r \in \mathbb{R}^L$  and for every  $i$ ,  $\ell \notin L \Rightarrow t_{i\ell} = 0$  <sup>2</sup>

and

$$\text{I. } \sum_{i \leq m} t_i = \sum_{j > m} t_j$$

$$\text{II. for each } j > m, t_j \in T_j$$

$$\text{III. for each } i \leq m, t_i \in T_i$$

$$p \cdot t_i + \sum_{\ell: t_{i\ell} \neq 0} r_{\ell} \leq w_i$$

$$\text{where } w_i = \sum_{j > m} \theta_{ij} \Pi_j$$

$$\text{and } \Pi_j = p \cdot t_j + \sum_{\ell} r_{\ell} N_{j\ell}$$

$$\text{and if } t'_i \in T_i, t'_{i\ell} = 0 \text{ for } \ell \notin L, \text{ and } t'_i \succsim_i t_i,$$

$$\text{then } p \cdot t'_i + \sum_{\ell: t'_{i\ell} \neq 0} r_{\ell} > w_i.$$

The Arrow-Debreu model then gives us a particular consistent outcome which is rather well distinguished: A competitive outcome is a consistent outcome  $(L, p, r, (t_i)_{i=1}^{m+n})$  where  $L = \{1, \dots, L\}$  and which satisfies the additional profit maximization condition:

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2. For  $\ell \notin L$ ,  $p_{\ell}$  and  $r_{\ell}$  are arbitrary -- they are not determined.

IV. for each  $j > m$

$$p \cdot t_j = \sup\{p \cdot t_j^! : t_j^! \in T_j\}$$

The distinguishing features of a competitive outcome are that firms take prices as given and maximize profit and that all markets are open (though not necessarily active!). This definition of a competitive outcome does not rule out (indeed, it requires) that a firm offer each consumer a chance to deal with it in an "individualized" market as well as in a general market. However, the difference in prices on these two markets reflects only different merchandising costs of operating the different markets. This is ensured by the profit maximization condition:

A GL-vector  $\bar{t}_j$  solves

$$\text{maximize } p \cdot t_j \text{ subject to } t_j \text{ in } T_j$$

if and only if a  $G(L+1)$ -vector  $(\bar{t}_j, \bar{z}_j)$  solves

$$\text{maximize}_{(t_j, z_j)} \sum_{k=0}^2 f_k(t_j, z_j)$$

where  $f_{j0}(t_j, z_j) = p \cdot t_j$

$$f_{j1}(t_j, z_j) = \begin{cases} 0 & \text{if } (t_j, z_j) \in Z_j \\ -\infty & \text{otherwise} \end{cases}$$

$$f_{j2}(t_j, z_j) = \begin{cases} 0 & \text{if } \sum_{\ell=1}^L (t_{j\ell}) - z_j \in Y_j \\ -\infty & \text{otherwise} \end{cases}$$

Then appropriate assumptions of convexity, closedness and nonempty interiors (i.e. consistency of the dual and strong consistency of the primal in Rockafellar's terminology [10]) ensures that corresponding to  $(\bar{t}_j, \bar{z}_j)$  is a col-

lection of dual variables  $(\bar{x}_{jk}^*)_{k=0}^2$ , each of dimension  $G(L+1)$  where for each  $k$ ,  $\bar{x}_{jk}^*$  is in the subdifferential of  $f_k(\cdot)$  at  $(\bar{t}_j, \bar{z}_j)$ , and where  $\sum_{k=0}^2 \bar{x}_{jk}^* = 0$

(see [10, p. 323]). We get from the fact that  $\bar{x}_{jk}^*$  is in the subdifferential of  $f_{jk}(\cdot)$  that

$$\begin{aligned}\bar{x}_{j0}^* &= (-p_1, \dots, -p_L, 0) \\ \bar{x}_{j1}^* &= (\bar{t}_{j1}^*, \dots, \bar{t}_{jL}^*, \bar{z}_j^*) \\ \bar{x}_{j2}^* &= (\underbrace{\bar{q}_j, \bar{q}_j, \dots, \bar{q}_j}_L, -\bar{q}_j)\end{aligned}$$

where  $\bar{q}_j$  is a  $G$ -dimensional price vector. The condition  $\sum_{k=0}^2 \bar{x}_{jk}^* = 0$  means that for each  $\ell = 1, \dots, L$

$$p_\ell = \bar{t}_{j\ell}^* + \bar{q}_j \quad \text{and} \quad 0 = \bar{z}_j^* - \bar{q}_j$$

The second just states the obvious condition that the marginal cost to  $j$ ,  $\bar{z}_j^*$ , of the inputs  $\bar{z}_j$  to the transactions technology  $Z_j$  at the profit-maximizing output-input pair  $(\bar{t}_j, \bar{z}_j)$  is the same as  $\bar{q}_j$ , the marginal profitability of production for  $j$  at the net production vector  $\sum_{\ell=1}^L (\bar{t}_{j\ell}) - \bar{z}_j$  in the production set  $Y_j$ . The first condition states the intuitive conclusion that the marginal prices in market  $\ell$ ,  $p_\ell$ , equal marginal production costs  $\bar{q}_j$  plus marginal transactions costs  $\bar{t}_{j\ell}^*$  for market  $\ell$ . In particular, this justifies the preceding assertion that in competitive equilibrium, prices in different markets differ only by the different merchandising costs in the two markets:

$$p_k - p_\ell = \bar{t}_{jk}^* - \bar{t}_{j\ell}^*$$

This also makes clear the connection between this model of marketing costs and that of Foley [6].

In order to bring out the new features of the model of this paper as contrasted with standard general equilibrium models, we look for conditions which ensure the existence of a quasi-equilibrium. We look at quasi-equilibrium rather than competitive equilibrium because the former is enough to illustrate the essential features of the model of this paper. Conditions under which a quasi-equilibrium is in fact a competitive equilibrium are well known [5]. It is enough to note here that it would be unreasonable to assume zero is in the interior of any consumer's trade-possibilities set  $T_i$  since  $i$  is excluded from some markets.

Conditions on the trade-space model for a quasi-equilibrium to exist are standard [5]. For each  $i \leq m$  and each  $j > m$  the following conditions are imposed:

- T.1  $T_i$  is closed, bounded below and owns the zero vector.
- T.2  $\succsim_i$  is reflexive, transitive, complete and locally not satiated on  $T_i$ .
- T.3  $\succsim_i$  is upper semicontinuous on  $T_i$ ; i.e., for each  $t'_i$  in  $T_i$ ,  
 $\{t_i \in T_i : t_i \succsim_i t'_i\}$  is closed
- T.4  $\sum_{j>m} T_j$  is closed and there is a pointed cone  $C$  in  $\mathbb{R}^{GL}$  (i.e.  $C$  is a closed convex cone with  $C \cap -C = \{0\}$ ) such that  $0 \in T_j \subset C$ .  
 $(C \text{ is independent of } j.)$
- T.5  $-R_+^{GL} \subset T$
- T.6  $\succsim_i$  is lower semicontinuous on  $T_i$
- T.7  $T_i$  is convex and  $\succsim_i$  is weakly convex on  $T_i$ ; i.e.  $t_i \succsim_i t'_i$   
and  $0 < \lambda < 1$  imply  $\lambda t_i + (1-\lambda)t'_i \succsim_i t'_i$ .
- T.8  $\sum_{j>m} T_j$  is convex.

The basic question of this section is under what restrictions on the activity-space model are the above conditions on the trade-space model met.

For example, consider the following conditions on each  $i \leq m$  and each  $j > m$ :

- C.1  $X_i$  and  $Z_i$  are closed, bounded below and own the zero vector.
- C.2 a)  $\succsim_i$  is reflexive, transitive, complete and locally not satiated on  $X_i$ .  
 b) For every  $\epsilon > 0$  and every  $(t_i, z_i)$  with  $z_i \in Z_i(t_i)$ , there is  $\delta > 0$  such that for every  $x'$  in  $X_i$  within  $\delta$  of  $\sum_{\ell} (t_{i\ell}) - z_i$ , there exist  $(t'_i, z'_i)$  with  $z'_i \in Z_i(t'_i)$ , with  $t'_i$  within  $\epsilon$  of  $t_i$  and with  $\sum_{\ell} (t'_{i\ell}) - z'_i = x'$ .
- C.2.b is a type of continuity assumption which is discussed below.
- C.3  $\succsim_i$  is upper semicontinuous on  $X_i$ .
- C.4  $0 \in Y_j$  closed and  $0 \in Z_j$  closed and there is a pointed cone in  $\mathbb{R}^G$  (independent of  $j$ ) such that  $(t_j, z_j) \in Z_j$  implies  $t_{j\ell}$  is in this cone for each  $\ell$ .
- C.5 If, for each  $\ell$ ,  $t_{\ell} \in \mathbb{R}_+^G$ , then there is some  $z$  with  $((-t_{\ell})_1^L, z) \in Z_j$  and  $\sum_{\ell} (-t_{\ell}) - z \in Y_j$ .
- C.6  $Z_i(\cdot)$  is lower hemicontinuous and  $\succsim_i$  is lower semicontinuous.
- C.7  $X_i$  and  $Z_i$  are convex and  $\succsim_i$  is weakly convex on  $X_i$ .
- C.8  $Y_j$  and  $Z_j$  are convex.

In order to make the translation from the activity-space axioms to the trade-space axioms, it is essential to establish the continuity properties of the correspondence  $Z_i(\cdot)$  defined earlier. Clearly  $Z_i(\cdot)$  has a closed graph if  $X_i$  is closed and  $Z_i$  is closed. However, the latter assumption is not entirely innocuous as is illustrated by the following example of pure

set-up costs: Let  $G = 1$  and

$$Z_i = \{(t_i, z_i) \in \mathbb{R}_+^2 : t_i > 0 \Rightarrow z_i = c$$

$$\text{and } t_i = 0 \Rightarrow z_i = 0\}$$

where  $c$  is any positive number. Then  $Z_i$  is not closed. In this case, a type of free disposal in the transactions technology will restore closedness:

replace  $Z_i$  by the set

$$\{(t_i, z_i) \in \mathbb{R}_+^2 : t_i > 0 \Rightarrow z_i \geq c \text{ and } t_i = 0 \Rightarrow z_i \geq 0\}.$$

In what follows, we shall assume  $Z_i$  is closed without further ado.

Lemma If  $X_i$  and  $Z_i$  are closed and bounded below, then

$Z_i(\cdot)$  is upper hemicontinuous on  $T_i$ . In fact, the image

under  $Z_i(\cdot)$  of a compact set is compact.

Note:  $Z_i(\cdot)$  upper hemicontinuous means that if for some open set  $G$  and some  $t^0$  we have  $Z_i(t^0) \subset G$ , then there is an open neighborhood  $V$  of  $t^0$  such that  $t$  in  $V$  implies  $Z_i(t) \subset G$ .

Sketch of Proof:

By the preceding remarks, it is enough to show that if  $K$  is any bounded set, then  $\bigcup_{t \in K} Z_i(t)$  is bounded. But since  $Z_i$  is bounded below, so is  $\bigcup_{t \in T_i} Z_i(t)$ .

On the other hand, since  $X_i$  is bounded below then  $\sum_{\ell=1}^L (t_{i\ell}) - z_i$  is bounded below as  $z_i$  ranges over  $Z_i(t)$  and  $t$  ranges over bounded  $K$  so that  $z_i$  is bounded above as  $z_i$  ranges over  $\bigcup_{t \in K} Z_i(t)$ . Combining this with  $\bigcup_{t \in K} Z_i(t)$  being bounded below means it is also bounded above and hence is bounded as desired. |

A useful result of the compactness of  $Z_i(t)$  for each  $t$  in  $T_i$  is that if  $\gamma_i$  is upper semicontinuous on  $X_i$ , then for each  $t_i$  in  $T_i$  there exists some

$z_i(t_i)$  in  $Z_i(t_i)$  such that

$$\sum_{\ell} (t_{i\ell}) - z_i(t_i) \succsim_i \sum_{\ell} (t_{i\ell}) - z'_i$$

for all  $z'_i$  in  $Z_i(t_i)$ . This simplifies the definition of  $\succsim_i$  on  $T_i$  as well as the derivation of the properties of this preference relation from the relation on  $X_i$ . In particular, if  $\succsim_i$  on  $X_i$  is represented by a utility function  $u_i(\cdot)$  on  $X_i$ , then  $\succsim_i$  on  $T_i$  can be represented by  $u_i(\cdot)$  on  $T_i$  defined by

$$u_i(t_i) = u_i\left(\sum_{\ell} (t_{i\ell}) - z_i(t_i)\right).$$

We now list the conditions under which the activity-space axioms imply the trade-space axioms:

A. C.1 implies T.1

That  $T_i$  owns zero and is bounded below is clear from C.1. The closedness of  $T_i$  is based on the fact that  $Z_i(\cdot)$  maps compacts into compacts which follows from C.1 by the Lemma.

B. C.1 and C.2 imply T.2

That  $\succsim_i$  on  $T_i$  is reflexive and transitive is clear from these conditions on  $\succsim_i$  on  $X_i$ . It is also clear that local nonsatiation on  $X_i$  together with C.2.b give local nonsatiation on  $T_i$ . Condition C.2.b is not a very clever assumption. However, using it illustrates a difficulty of modeling consumer transactions costs. It implies that at least some market to which a consumer has access does not have pure set-up costs of trading there for the consumer; e.g. a corner market or a home delivery market. Of course there is no requirement that in equilibrium the consumer make use of such a market or that there cannot be other markets which do involve set-up costs for the consumer to participate. Finally, axiom C.1 and the Lemma give the existence of  $z_i(t_i)$

corresponding to each  $t_i$  in  $T_i$  which makes it easy to show that completeness on  $X_i$  means completeness on  $T_i$ .

C. a) C.1 and C. 3 imply T.3

b) C.1, C.3 and C.6 imply T.6

It is a well-known result (e.g. this is essentially the Maximum Theorem of Berge [2]) that the induced preference relation on  $T_i$  is continuous if the original preference relation is continuous and if  $Z_i(\cdot)$  is continuous. The upper hemicontinuity of  $Z_i(\cdot)$  is assured by C.1 and the Lemma. Unfortunately, the pure set-up costs example above illustrates that  $Z_i(\cdot)$  may not be lower hemicontinuous. Thus at times axiom C.6 may be invoked for lack of a better procedure. It does rule out set-up-cost transactions technologies for consumers but does not rule out the possibility that these technologies have increasing returns to scale of a sort which approximate set-up-cost technologies.

D. C.4 implies T.4

This is an easy result using the standard result that the sum of closed sets all contained in a pointed cone is closed. The pointed cone  $C$  in T.4 is the  $L$ -fold product of the cone in C.4.

E. C.5 implies T.5

F. a) C.7 implies T.7

b) C.8 implies T.8

The point of making this systematic comparison of conditions on the activity-space with conditions on the trade-space is to make it clear in what way axioms T.6 - T.8 are restrictive in a study of imperfect competition and marketing

costs. The restrictiveness of convexity assumptions in models of transactions costs is well known [7]. The restrictiveness of the assumption of lower semicontinuity of preferences is familiar in models of consumer behavior where consumers "produce" their own consumption characteristics from purchased inputs (e.g. see Rader [9]).

Thus far the activity-space model has been presented as the "true" underlying model whereas the trade-space model has been presented as a formulation which is analytically convenient. The trade-space model is also more general in that it could be gotten from a variety of different activity-space models. For example, a more general activity space model might permit consumer preferences to be defined on the graph of  $Z_i(\cdot)$ . This allows for the possibility of the "trading activity" being desirable as well as the "consumption activity." This generalization does not seem to ease any of the difficulties found above and so will not be explored here. However, any results that are formulated for the trade-space model can also be interpreted to apply to this more general activity-space model.

##### 5. The market core of an economy

In this section, we propose a solution concept which is based on the types of market defined earlier. A coalition  $S$  of consumers is a subset of  $\{1, \dots, m\}$ . This coalition "controls" the set  $J(S)$  of firms:

$$J(S) = \{j > m : \sum_{i \in S} \theta_{ij} > \mu_j\}$$

where  $\mu_j$  is the proportion of ownership shares of firm  $j$  which is needed to control firm  $j$ . A coalition  $S$  of consumers market-improves a consistent outcome  $(L, p, r, (t_i)_1^{m+n})$  if there exists an alternative consistent outcome

$(L^S, p^S, r^S, (t_i^S)_{i \in S \cup J(S)})$  such that  $L^S$  lists only markets all of whose participants are in  $S \cup J(S)$  and such that  $t_i^S \succ_i t_i$  all  $i$  in  $S$ . The market core of the economy is the collection of all consistent outcomes which cannot be market-improved on by any coalition.

The notion of improving described above has two essential features. First, it takes the institutions of markets and of "majority control" of firms as given and looks at the types of economic outcome which might occur through voluntary bargaining within that institutional context. Second, this notion seems to reproduce in a market setting the idea of the usual sort of improving that the improving coalition is self-sufficient. The outcome proposed by  $S$  for its members is independent of the choices of those not in  $S$  because the definition requires a kind of autarchy or physical self-sufficiency for any improving coalition. This autarchy is not complete, since some of the owners of a firm  $j$  in  $J(S)$  may not be in  $S$ . Thus they may have imposed on them profits or losses of  $j$  which result from the choices made by the majority of  $j$ 's owners who are in  $S$ . Nevertheless, this is a very conservative notion of improving.

The concept of a market core can be contrasted with the usual idea of the core: An outcome  $(t_i)_{i=1}^{m+n}$  is improved by a coalition  $S$  of consumers if there is an alternative outcome  $(t_i^S)_{i \in S \cup J(S)}$  with

$$a) \ t_i^S \text{ in } T_i \text{ for all } i \text{ in } S \cup J(S)$$

$$b) \ \sum_{i \in S} t_i^S = \sum_{j \in J(S)} t_j^S$$

$$c) \ t_i^S \succ_i t_i \text{ all } i \text{ in } S.$$

The core is the collection of all unimprovable, feasible outcomes. This solution concept does not take the institution of markets as given, but rather may be used to look at the development of trading institutions. However, as formulated above, it does follow Champsaur [3] in taking "majority-control" of firms as given.

It is of interest to compare the new concept of the market core with the standard concept of the core. It is possible to show under strong assumptions ensuring costless discrimination among consumers that the market core coincides with the core. It is also possible to give examples where because such discrimination is costly there exist in the market core Pareto inefficient outcomes with price greater than marginal cost. It is believed that this suggests that the market core is an appropriate concept for studying the development of market institutions which differ from the perfectly competitive paradigm. It is hoped that this solution concept will also serve as a useful tool for exploring the nature of social equilibrium in the presence of nonconvex marketing costs. This paper has set the stage for this work by concentrating its analysis on the relation between this model of marketing costs and the standard model of trade equilibrium.

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