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NOTES ON THE REGIONAL PRODUCT FUNCTION

by

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Notes on the Regional Product Function

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In this paper we show the existence of a regional product function RPF under assumptions weaker than have been made before. This RPF has the usual properties of a standard concave function when the law of non-increasing returns holds for every subsector of the economy.

Under constant returns to scale we show how the isoquant of the RPF can be constructed from the isoquants of the sectoral production functions and we relate many of the comparative statics propositions in development and in international trade theory to this construction.

Associated with the RPF is a variable production state of the economy i.e. a description of the output and employment levels in each sector, and the rewards for the resources. We point out in the concluding section that regional projects that imply changes in regional technology and in resource endowments should be evaluated on the basis of the RPF and of the production state of the economy that is implied by the RPF.

1. The Regional Product Function. Definition

Most introductory textbooks of economics describe the relationship between regional resources v and output combinations x that can be produced from these through the construction of the so-called production possibility or transformation

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* The author is Professor of Economics at the University of California, Davis. The paper is a revision of a set of notes delivered to the International Economics Seminar at the K.U.L., in the fall of 1972, while the author was visiting at the Center for Operations Research and Econometrics. I wish to thank the students of this seminar and Professor Theo Peeters for their many valuable observations.

curve. All output points along this curve as well as all points of the non-negative quadrant bounded by this curve are points of the production possibility set $X(v)$ and could be produced if society so desired. Next, given the prices of output p, we could find the output combination $\hat{x}$ in the production possibility set $X(v)$ at which the value of output $p\hat{x} = p_1\hat{x}_1 + p_2\hat{x}_2 + \ldots + p_n\hat{x}_n$ is larger than the value of output px of any other output combination x in $X(v)$. This value $p\hat{x}$ is the regional product, say F.

Clearly the regional product F depends on the production possibility set $X(v)$ and on the prices p, so that we may represent the regional product as a function $F(p, v)$. The purpose of this paper is to characterize certain properties of this function, i.e. how does F change if prices p change and what can be said if the regional resources v change.

So far we have been rather vague about the meaning of region, regional resources and outputs. Many definitions are possible, see Dubey [4] for a survey. For our purpose a region may be any area however defined but we do need two precise data, the set of regional technologies and the classification of the commodities into mobile and immobile commodities. The census of technologies existing in the region should determine the production sets Y_j for each industry j, i.e. what outputs (positive numbers) are possible and what inputs (negative numbers) are possible (see [3]). We assume that if an immobile commodity is an input in some production set, then it is not an output in some other or the same production set. We call such an immobile input a regional resource, such as types of land, labor, natural resources. All other commodities will be called goods and these therefore include traded goods, domestic goods as well as traded inputs. Denoting a goods vector by x and resource vector by v as before, we can define a production possibility set as the set of all goods that can be obtained from a given vector v of resources. Given a list of prices for goods we then determine the largest possible value of goods that can be obtained from the set of regional technologies for the given vector v of regional resources. This is the regional product, or the regional value added remembering that the negative components in the goods vector x are the imported inputs.

To summarize: There are two types of commodities, regional immobile resources and goods. There exist J regional production sets Y_j, $j = 1, \ldots, J$. $(x^j, v^j) \in Y_j$ means that the production plan (x^j, v^j) is possible for the j-th firm, where x^j is a bundle of goods and v^j is a bundle of resources. The regional total production set $Y \equiv \sum_j Y_j$ is the set of all possible production plans $(x, v) \equiv (\Sigma x^j, \Sigma v^j)$, $(x^j, v^j) \in Y_j$. Given Y, we have defined the production possibility set $X(v) \equiv \{x | (x, v) \in Y\}$ and the regional product function $F(p, v) \equiv \text{Max}\{px | x \in X(v)\}$, where p are the prices for goods in the region under consideration. For the sake of the argument we assume that X(v) is bounded.

2. The Relevance of the RPF F(p, v)

What we want to show in this section is that under profit maximizing perfect competitive behavior the regional product will exactly equal F(p, v) so that the construction above is meaningful in that sense.

Suppose the rentals for resources are equal to w. Profit maximizing behavior on the part of the j-th producer implies that the profit associated with the chosen production plan $(\hat{x}^j, \hat{v}^j)$ is not less than the profit associated with any alternative production plan, say (x^j, v^j) in the production set Y_j. Algebraically $p\hat{x}^j + w\hat{v}^j \geqq px^j + wv^j$, for all $(x^j, v^j) \in Y_j$. Adding over all producers in the region we find $p\hat{x} \geqq px + w(v - \hat{v})$, for all $(x, v) \in Y$. If

$\hat{v}$ is a resource market clearing quantity in the sense that $v \leqq \hat{v}$ and $w(v-\hat{v}) = 0$, we have $p\hat{x} \geqq px$ for all $x \in X(v)$.

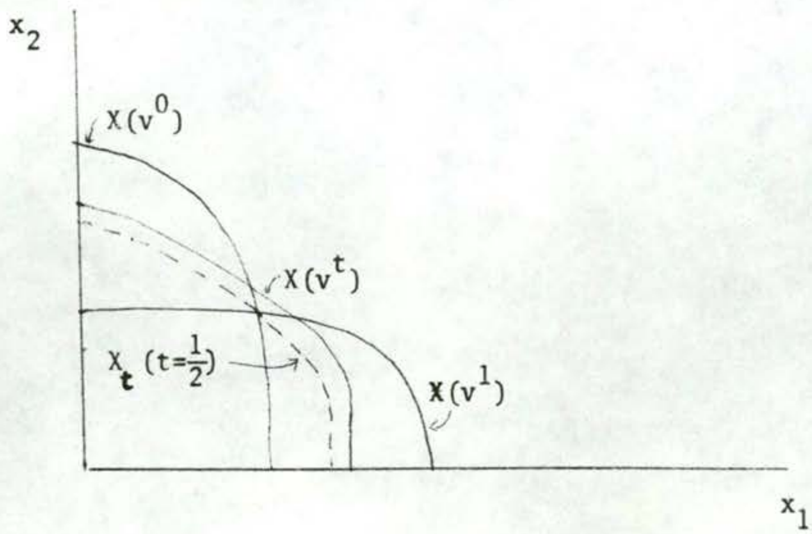
Therefore regional product is maximized under profit maximizing, resource market clearing, perfectly competitive behavior and F(p, v) does represent observable regional product under those assumptions. To the extent that the behavior of the economic agents differs from the stated behavior, the RPF will not correspond to the regional product.

3. Concavity of F(p, v) under Convexity of the Total Production Set Y

In this section and the next we deal with properties of the RPF in a more general framework than has been done before, see e.g. [8]. Geometrically speaking, a set is convex if the points along the line segment connecting any two points of the set are all in the set. Economically speaking, convexity of the total production set is the law of non-increasing returns, couched in modern terms. This is most clearly seen if we consider the subset of all production plans having the same components for all but two of the commodities. This subset must be convex too. In particular if one of the two commodities is an output and the other an input, convexity implies that we must have a set of points bounded above by a curve, a so-called production function, with the property that the increments in output associated with equal increments in the input are not increasing. Alternatively, convexity implies that if we take two points along that production function curve, the line segment connecting the two points does not lie above the curve. The last geometric property implies that the production function is concave where by definition a function f(z) is concave if $z^t \equiv (1 - t)z^0 + tz^1$ implies $f(z^t) \geqq (1-t)f(z^0) + tf(z^1)$, $0 \leqq t \leqq 1$.

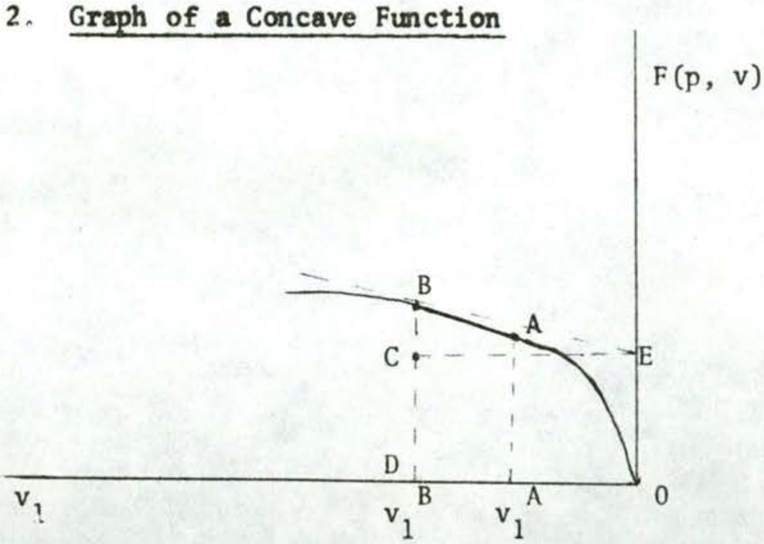
We now show (see [9] for a comparable proof) that the regional product function $F(p, v)$ is concave, and in particular satisfies the law of non-increasing returns with respect to each component of the regional resources vector v. From the statements made before this is equivalent to showing that $F(p, v^t) \geq (1 - t)F(p, v^0) + tF(p, v^1)$, when $v^t = (1 - t)v^0 + tv^1$, $0 \leq t \leq 1$. By definition $F(p, v^t)$ is the largest value added among all possible choices of goods vectors in the production possibility set $X(v^t)$ and $(1 - t)F(p, v^0) + tF(p, v^1)$ is the largest value added among all choices in the set $X_t \equiv (1 - t)X(v^0) + tX(v^1)$. The concavity of $F(p, v)$ in v would follow immediately if it can be shown that X_t is a subset of $X(v^t)$, since the maximum over a set $X(v^t)$ cannot be smaller than the maximum over one of its subsets X_t. To see that X_t is in fact a subset of $X(v^t)$, let us choose $x^0 \in X(v^0)$, $x^1 \in X(v^1)$ so that $x^t \equiv (1 - t)x^0 + tx^1 \in X_t$, by construction. By the choice just made we have $(x^0, v^0) \in Y$ and $(x^1, v^1) \in Y$, and since Y is convex we have $(x^t, v^t) \in Y$ i.e. $x^t \in X(v^t)$. By repeating the same argument for all possible points $x^t \in X_t$, we find that each $x^t \in X(v^t)$, so that X_t is indeed a subset of $X(v^t)$. The geometric interpretation of this property of production possibility sets is as in Figure 1.

Figure 1. Production Possibility Sets.



In economic terms the above property of the production possibility sets implies that $X(v^0) + X(v^1)$ i.e. the total production possibility set associated with v^0 and v^1, keeping both resources separated, is a subset of $X(v^t) + X(v^{1-t})$ which is the total production possibility set obtained after resources have been repooled in batches v^t and v^{1-t} through migration say. In particular for $t = 1/2$ and under constant returns to scale, we have $X(v^0) + X(v^1) \subset 2X(v^{(1/2)}) = X(v^0 + v^1)$. This means that under constant returns to scale and assuming that all regional resources can migrate, the reason for their initial immobility being say political, we find that for two regions with identical technologies integration of their marketing channels through a customs union is never more efficient in production than a complete integration that includes in addition free migration of all their resources through a common market agreement. Pooling of resources is a source of gain above and beyond the gains from trade in the traditional sense.

Figure 2. Graph of a Concave Function



Having shown the concavity of the regional product function, see Figure 2, it is important to realize that this function measures the increase in value added associated with an increment in a regional resource. It is not a production function in the traditional sense, it does not measure how much more output of a certain commodity will be produced. What it measures is the increment in value added for a given increment in some resource when this additional resource is employed somewhere in the region in the most optimal way. If this function were known it would tell us what an additional laborer coming into the region would contribute to the regional value added. It is not surprising to find that the law of non-increasing returns continues to apply in the aggregate, but in general there will be intervals of constant returns to a factor despite the fact that in each industry returns to a single factor are strictly falling. We will discuss this in more detail later. Another peculiarity of the function is that it is not differentiable even when all industry production functions are differentiable. The answers given by the regional product function are very important from a regional policy point of view. In particular along all straight line segments of the function, e.g. AB of Figure 2, there is no reason for a labor union to be concerned about the impact of new migration into the region on the wage levels of the existing labor force. This impact would be zero as long as the labor force stays below v_1^B. But, eventually the law of diminishing returns sets in beyond v_1^B.

To summarize this section: The regional product function $F(p, v)$ is concave in v i.e. $F(p, v^t) \geqq (1 - t)F(p, v^0) + tF(p, v^1)$ if the regional total production set Y is convex. This property follows trivially from the definition of $F(p, v)$ and the property that the production possibility sets satisfy $(1 - t)X(v^0) + tX(v^1) \subset X(v^t)$.

4. Slopes of $F(p, v)$ are Equal to Resource Rental Prices

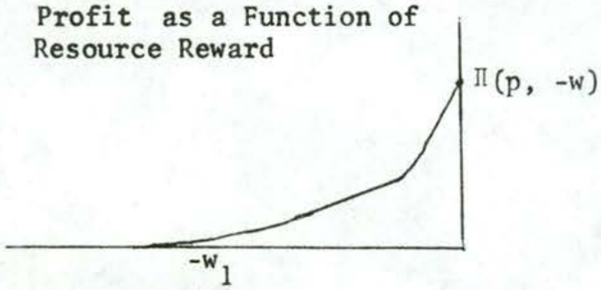
In the last paragraph of the preceeding section we have compared the slopes of the RPF to the rewards of the regional resources. In fact under profit-maximizing perfect competitive behavior and clearance of the resource markets the slopes of $F(p, v)$ are exactly equal to the resource rewards.

First we have to be clear about the meaning of the slopes of $F(p, v)$. We follow [7] for definitions in this area. As indicated above $F(p, v)$ is not differentiable everywhere. In Figure 2 there are kinks in the curve and we can draw many lines through the point B and that do not lie below $F(p, v)$. The slopes of all the lines through B and not lying below $F(p, v)$ are called supergradients at v^B, and these slopes are the quantities we want to interpret.

There is an interesting interpretation to the construction of lines that go through one point of the curve such as OAB of Figure 2, but otherwise do not fall below it. If we are given a vector of slopes, say $-w$, we could from above drop a line with the given slopes $-w$ (negative numbers) onto the graph of $F(p, v)$. From the moment the line comes to rest on a point of the curve, we will have determined its intercept on the vertical axis. This intercept is equal to the maximum profit $\Pi(p, -w) = \sup_v \{F(p, v) + wv\}$ that can be obtained when resource rewards are equal to w, remembering that resources are measured as negative quantities. This is clear since in Figure 2 BC is equal to variable cost $-wv$ and BD is total net revenue. For a different w we will obtain a different intercept, always equal to the profit $\Pi(p, -w)$, and the line will come to rest on a different point of the graph. The v-coordinate of this point is the level of employment at which a maximum profit is obtainable and the slope $-w$ of the line will be a supergradient of the function $F(p, v)$.

The profit function measured by the intercept is known in the technical literature as the conjugate of $F(p, v)$, more correctly, $-\Pi(p, -w)$ is the v-conjugate of the regional product function $F(p, v)$. From [7, p. 308] the relationship between the regional product function $F(p, v)$ and the profit function $\Pi(p, -w)$ is such that if a vector $-w$ is a supergradient of $F(p, v)$ at v then $-v$ is a subgradient of the profit function at $-w$, and conversely. Further the profit function is convex in $-w$, as shown in Figure 3.

Figure 3. Profit as a Function of Resource Reward



This can be verified from Figure 2 by plotting the intercepts such as OE when we vary the slopes $-w$. It is also clear that both profit and employment levels v, are not increasing with increasing w.

We summarize: *Under perfectly competitive profit maximizing behavior the supergradients (slopes) of $F(p, v)$ are equal to the resource rental prices. This follows directly from the v-conjugacy relation between $F(p, v)$ and (minus) the profit function $\Pi(p, -w)$.*

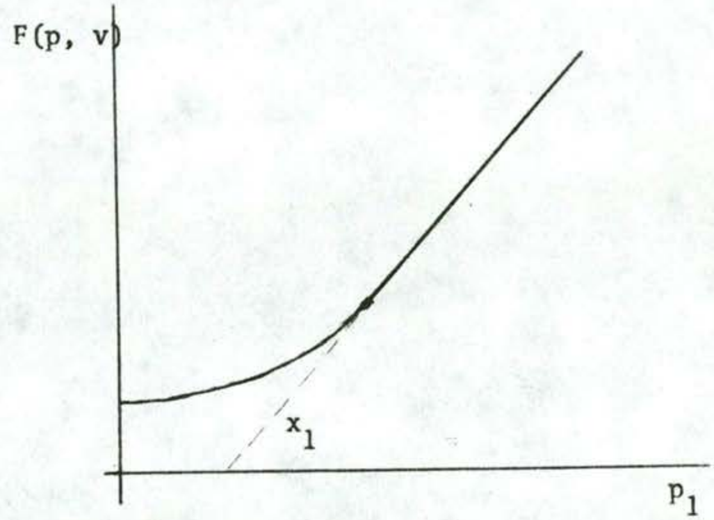
5. The RPF F(p, v) as a Function of Prices

So far we have concentrated on the regional product function $F(p, v)$ as a function of the regional resources, but it is also a function of the prices p. As in Figure 1, if we double the prices p, the optimal goods vector in the production possibility set will remain the same, so that $F(2p, v)$ will be exactly equal to 2F(p, v). In general F(p, v) is homogeneous of degree one in p i.e. $F(tp, v) = tF(p, v)$, $t \geqq 0$.

It is also well known that F(p, v) is convex in p i.e. $F(p^t, v) \leqq (1-t)F(p^0, v) + tF(p^1, v)$, where $p^t = (1-t)p^0 + tp^1$. This is easily seen as follows. By definition $F(p^t, v) = \text{Max } \{px | x \in X(v)\}$, so that $p^0x^0 \geqq p^0x^t$, and $p^1x^1 \geqq p^1x^t$, where x^t is the optimal choice when prices are p^t. Adding the last two inequalities after multiplication by 1-t and t respectively, the convexity follows.

Figure 4.

The RPF as a Function of Price



Similar to the v-conjugate above we could look into the p-conjugate function of F(p, v), say I(x, v), but this is simply the indicator function of the production possibility set i.e. $I(x, v) = 0$ for $x \in X(v)$, $I(x, v) = \infty$ for $x \notin X(v)$, and one has x is a subgradient of F(p, v) at p if and only if p is a subgradient of the indicator function I(x, v) at x. Thus in Figure 4 we assumed that x_1 is an output and p_1 is nonnegative. We have that the subgradients (slopes) of F(p, v) measure the amount of x_1 produced if and only if p_1 is a subgradient of the indicator function I(x, v). In particular an interior point of the production possibility could be selected if and only if p is zero.

Another well known way to look at the relation between prices p and levels of output is to observe that by definition $p^0x^0 \geqq p^0x^1$ and $p^1x^1 \geqq p^1x^0$, so that after subtracting we obtain $(p^1 - p^0)(x^1 - x^0) \geqq 0$ or quantities produced tend to follow prices.

Summary: The regional product function is convex and homogeneous of degree one in prices p.

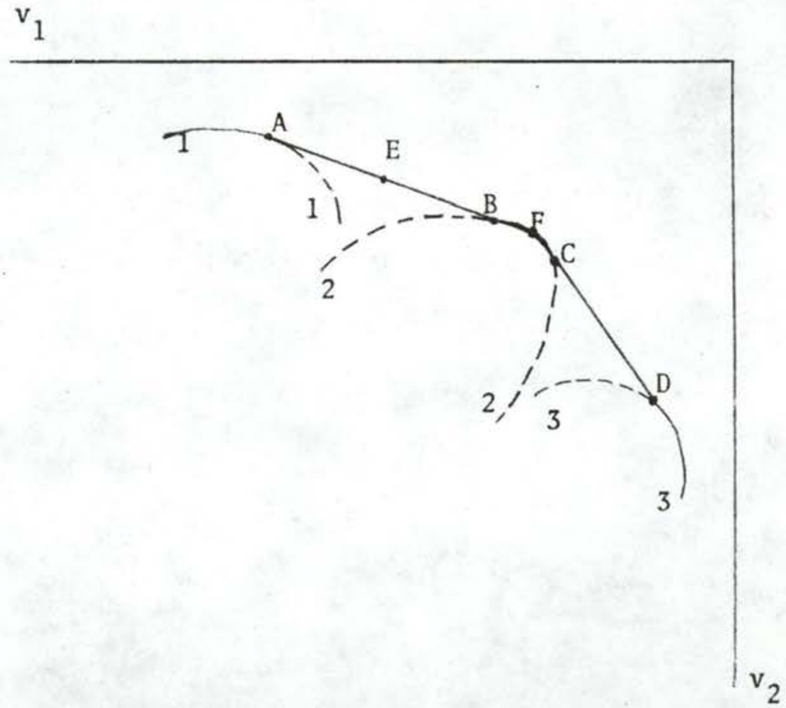
6. The Isoquant Map of the RPF F(p, v) under Constant Returns to Scale

So far we have concentrated on the regional product as the single number that summarizes the region's productive activity. In addition to that we also showed that the slopes of the RPF are equal to the resource rentals under competitive conditions. But more than that, once we have determined the regional product we will also have found the optimal bundle of goods $\hat{x}$ in the production possibility set and hence the sectoral levels of output and of employment. We will refer to the latter as the production state of the economy. Given the technologies and the resources v, the wages and the state of the economy can be calculated, as part of the same parametric programming problem that defines the RPF. Policies directed towards changing v or p can all be analyzed along these lines. Evidently numbers will be needed to describe the technologies and the resource levels.

Beyond the properties of F(p, v) stated above not much can be said further without numbers. There is however one more geometric construction that can enhance our intuitive understanding of certain relationships between prices p and outputs x, or resource remunerations w and resources v. This construction is the well known device of looking at isoquants thereby allowing us to deal with two prices or two resources. Under constant returns to scale the regional product function is homogeneous of degree one in the resources v i.e. $F(p, tv) = tF(p, v)$, $t \geq 0$, and the unit isoquant $\{v | F(p, v) = 1\}$ incorporates all the relevant information in the space of resources.

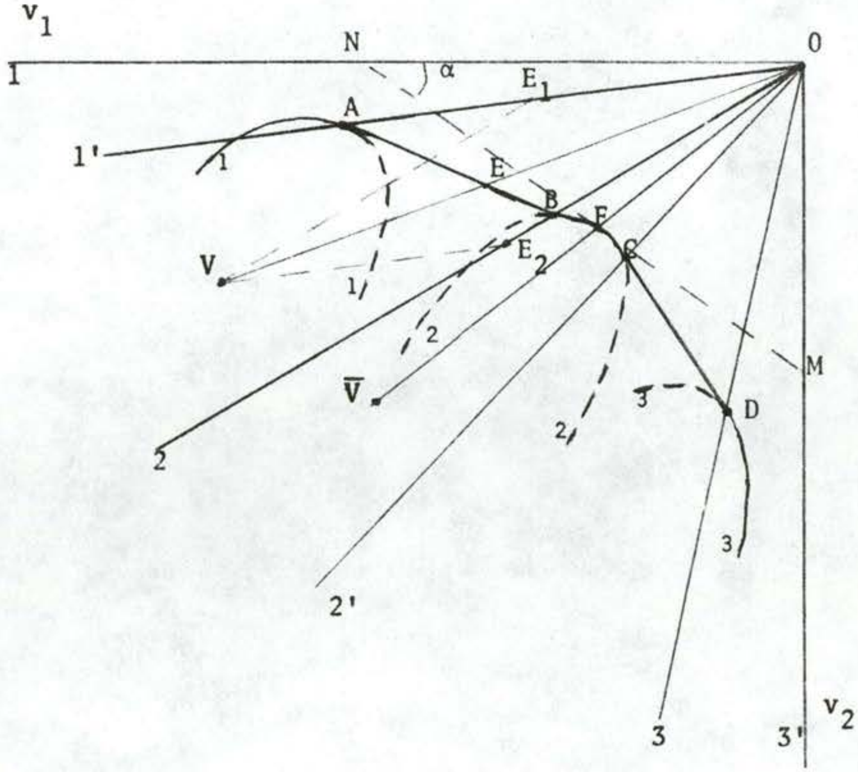
A unit isoquant in this case means the set of resources v that produce exactly one monetary unit, say 1 \$. We begin by finding the 1 \$ isoquant for each industry i.e. the set $\{v^j | px^j = 1, (v^j, x^j) \in Y_j\}$, such as the curves (1, 1), (2, 2), (3, 3) in Figure 5. We could call them micro-isoquants. Any input combination along the micro-isoquants yields 1 \$ of output if that input combination is employed in the corresponding industry. But this is not all, there

Figure 5. Regional Product Function Isoquant



are many more pairs of resources that produce 1 \$ of output. Let us consider the point E, on the segment between A and B, i.e. $E = (1 - t)A + tB$, $0 \leq t \leq 1$. We could split the resource point E into its two parts, employ $(1 - t)A$ in the first industry and tB in the second industry. The sum total so produced would be (1 - t)\$ in the first industry and t \$ in the second industry. Hence the point E is an element of the 1 \$ regional product isoquant. By a similar argument one shows that all points along AB and CD produce a 1 \$ of output. Technically speaking given the micro-isoquants (1, 1) (2, 2) and (3, 3), the regional product function isoquant consists of all boundary points of the convex hull of the micro-isoquants, that is the smallest convex set that contains the sets $\{v^j | px_j^j \geqq 1, (v^j, x^j) \in Y_j\}$, j = 1, ..., J. Thus in Figure 5, the curve ABCD is the regional product function isoquant. Any point of this isoquant contains the information what industries are engaged in producing something and which are producing nothing. At E, industries 1 and 2 produce something, but not industry 3, whereas at F only industry 2 is engaged in production, not industries 1 and 3.

Figure 6. RPF Isoquant and the Production State of the Economy



Given a resource vector v we can determine graphically the level of the regional product, the \$ levels produced in each industry as well as the ratio of resource rewards w_1/w_2. Thus if the resource vector is at V, regional product is \$ VO/VE, the \$ level of output is E_1O/AO in sector 1 and E_2O/BO in sector 2, where $VO = E_1O + E_2O$ is the decomposition of VO according to the parallelogram rule. Also the slope of the isoquant at E is equal to w_1/w_2 and the output of sector 3 is zero. In contrast if the resource vector is at $\overline{V}$, regional product is \$ $\overline{V}O/FO$, the regional output being produced entirely in sector 2 and nothing anywhere else. The region is completely specialized in sector 2. Hence given any resource vector one can find the rentals as well as the complete production state of the economy in Figure 6.

The resource space is subdivided in cones, so-called cones of diversification [1], such as the cones 101', 1'02, 202', 2'03, 303'. If the resource vector v falls in say the 101', 202', or 303' cone, we have complete specialization in the sectors 1, 2 or 3 respectively. If v falls in the cone 1'02 or 2'03 the two sectors (1, 2) or (2, 3) respectively are engaged in producing something. Given v, all the rest is determined by the shape of the isoquant. In particular all sectors are in equilibrium, in the sense that all producing sectors break even, no profits, and for the non-producing sectors revenue falls short of cost if they were to produce. This is seen as follows. Suppose the w_1/w_2 ratio is equal to α of Figure 6 and suppose the line NM through F is the locus of resource inputs costing 1 \$. Then all resource pairs below NM are more expensive than 1 \$. The pair represented by F costs exactly 1 \$ and hence breaks even whereas all other pairs producing 1 \$ are more expensive and do not break even. Hence when w_1/w_2 is equal to α, the economy specializes in sector 2 and in nothing else.

The slope of the RPF measures the marginal rate of substitution. As in Figure 6, the rate of substitutability between resources depends on the substitutability in a sector only in the case of complete specialization. In general, the rate of substitutability depends much more on the differences in factor proportions between the different sectors. Moving from A to B the rate of factor substitutability is a constant and the \$ output of sector 1 declines relatively to the \$ output of sector 2. Therefore substitutability between the resources is accomplished through changes in the output levels of each sector and not through substitution of one resource for another in the same sector. As a matter of fact along AB the resource proportions in each sector remain fixed, only output levels change. Through these changes in output levels the region can fully employ any resource endowment in the cone 1'02 without any change in w_1/w_2. All this presupposes of course that the immediate regional resources are perfectly mobile between industrial sectors that are located in the region.

7. Rentals and the Production State under Incomplete Specialization

From Figures 5 and 6 we observed that the isoquant has straight line segments such as AB, CD, or alternatively the regional product function is linear in the resources if v falls in the cones of diversification 1'02 or 2'03. In general if the number of resources is m, F(p, v) is linear in v if there are at least m sectors that produce a positive \$ level of value added. If this is the case one says that the economy is incompletely specialized.

It should be noticed that there may be many cones of incomplete specialization, each one with its own w's and these cones may be separated by other areas of specialization such as 202' in Figure 6. Whether or not a region is specialized in production or incompletely specialized is not something that can be assumed independently of anything else. It is already determined by the maximization of the regional product given the set of technologies and the vectors p and v. In this section we assume that it has already been determined that the region's maximum value added is obtainable through incomplete specialization.

The following propositions have been advanced in the international trade literature:

a) Factor-Price Equalization: (Samuelson [8])

<u>If two regional economies are incompletely specialized and the technologies and prices p are the same, resource rewards w are identical in the two regions if the resource endowments fall in the same cone of diversification.</u>

The proof is trivial if it is recognized that under incomplete specialization the regional product function is linear in the resources, such as for all resources in the cones 1'02 or 2'03 of Figure 6. Hence rewards w are constant under incomplete specialization in the same sectors.

The geometry of Figure 6 suggests that comparisons can be made between micro-sectors according to the intensity of resource uses per unit of value added. The lines OA, OB, OC, OD can all be ranked according to the ratio of v_1/v_2. Thus at A sector 1 is relatively more intensive in the use of resource 1 than is sector 2 at B. The problem is how to extend that concept to three or more resources. To do that we need some symbols. Let $a(w) \equiv (a_{ij}(w))$ be the matrix of resource input coefficients, where $a_{ij}(w)$ is the amount of resource i per \$ of value added in sector j, for all m sectors j that are producing a positive value added under incomplete specialization. Thus a(w) is a square matrix.

Definitions:

Under incomplete specialization sector j is locally relatively intensive in the use of resource j for all j, if the inverse of a(w) has the sign pattern

$$\begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix}$$

. If a(w) has this property for all w compatible with incomplete specialization in the same sectors, we say that sector j is globally relatively intensive in the use of resource j, for all j. (No factor intensity reversals).

This definition is a generalization of the ranking based on resource proportions in the two resources model. In particular the condition implies that $a_{ii}/a_{ji} > a_{ij}/a_{jj}$ i.e. resource i is used relatively more intensively in sector i than in any other sector. For a more detailed treatment see [2], [5], [10].

We now write two more propositions:

b) Generalized Strong Samuelson-Rybczynski Proposition:

<u>Under incomplete specialization and sector j locally relatively intensive in the use of resource j all j, if the region's i-th resource increases, the endowments remaining in the same cone of diversification, value added will increase in the i-th sector and fall in all other sectors.</u>

Proof:

By definition the value added levels y are determined by the full employment conditions $-v = a(w)y$. Hence $y = -[a(w)]^{-1}v$ and the sign pattern of the inverse of a(w) implies y_i increases, and y_j falls for all $j \neq i$. The proof for $m = 2$ is in Figure 7a with $OE_1 < OE_3$ and $OE_2 > OE_4$ when the two resource points fall in the same cone.

c) Generalized Strong Stolper-Samuelson Proposition:

<u>In the case of single output technologies with sector j producing good j, under incomplete specialization and with sector j globally relatively intensive in the use of resource j all j, if the i-th price p_i increases by a small amount, the economy remaining incompletely specialized in the same sectors, the i-th resource reward w_i will increase and w_j, $j \neq i$ will fall.</u>

The proof is almost identical as before under 2 with the price equals marginal cost i.e. the $p = a'(w)w$ conditions, instead of the full employment equations.

Figure 7a.

Samuelson-Rybczynski Theorem

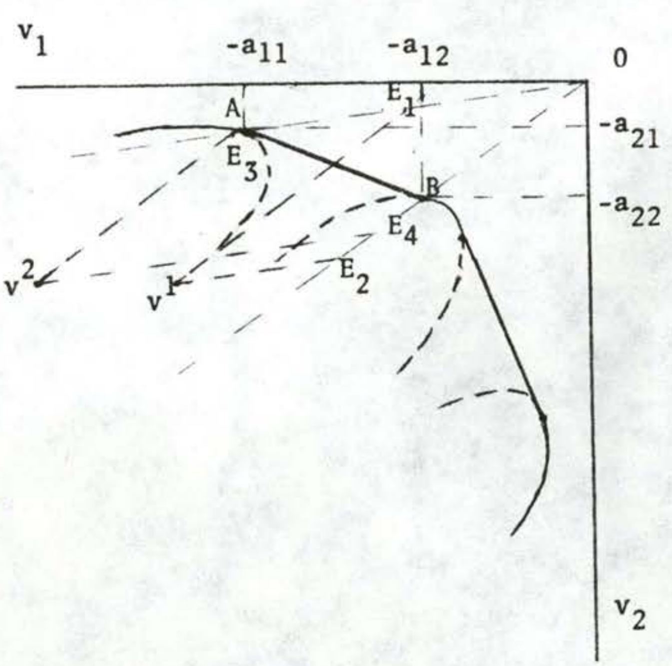
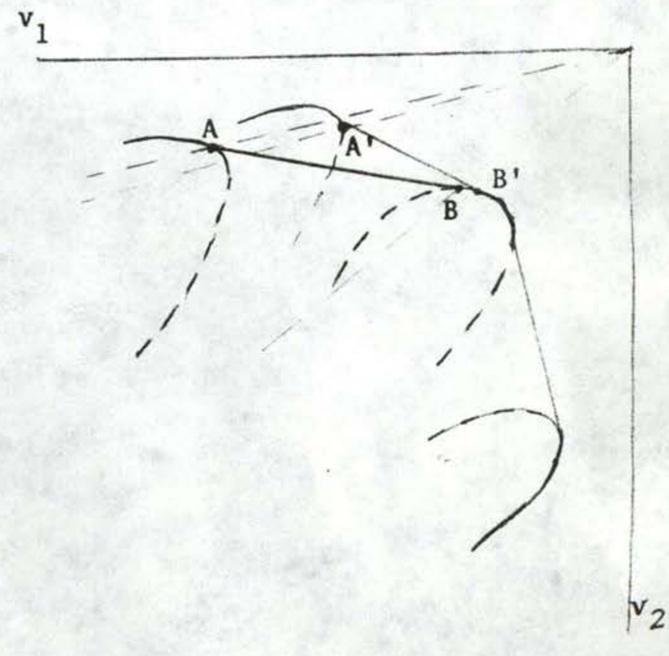


Figure 7b.

Stolper-Samuelson Theorem



In Figure 7b an increase in p_1 shifts the first value isoquant towards the origin and the slope of A'B' is steeper than the slope of AB, implying that w_1/w_2 went up. Notice that the cones of diversification change when p_1 changes. Proposition c) lists the conditions under which an increase in the price of wheat implies an increase in the wheat-growing land rent. In a two resource, two good model the Stolper-Samuelson proposition implies that p_1/p_2 and w_1/w_2 are changing in the same direction if resource 1 is used relatively intensively in the first sector.

The propositions above are subject to the same crucial qualification that the economy be incompletely specialized in the same industries before and after the change in resources and prices. Again this can only be determined through a calculation of the regional product function F(p, v), and after a check on the relative intensity of the sectors.

Corollary:

If sector j produces good j with no joint outputs and is globally relatively intensive in the use of resource j for all j, a small regional economy trading in all goods except the first one at international prices p, will not decrease its production of good 1 after opening of trade in the first market if before trade the resource reward w_1^0 is smaller than in the rest of the world.

Proof:

From Proposition c), p_1 and w_1 both increase together or fall together. Hence w_1^0 smaller before trade implies the pre-trade domestic price p_1^0 is smaller than the international price p_1 and from the property of the regional product function $\Delta p \Delta x \geq 0$ we have from Section 5 $(p_1 - p_1^0)(x_1 - x_1^0) \geq 0$ i.e. the post-trade level of output is not less than the pre-trade level x_1^0.

This corollary is the Heckscher-Ohlin theorem in the 2 resources, 2 goods case. Thus if we say that a country is abundant in resource 1 if the pre-trade reward w_1^0 is lower than in the rest of the world and if we ignore perverse demand conditions, the Corollary says that a country does not import that good in which its abundant resource is used relatively intensively. It is, however, not an interesting generalization of the Heckscher-Ohlin theorem because of the information being asked for in the pre-trade situation. The conceptual and measurement problems in the Heckscher-Ohlin model with more than two factors are well known from [6].

8. Concluding Remarks

a) In this paper we have concentrated on the relationship between the regional technologies and resources and what the region can produce in monetary value added. This relation is the RPF and it is a function with all the characteristics of a concave production function, obeying the law of non-increasing returns if the micro-sectors satisfy this law.

b) Associated with the RPF is a complete production state of the regional economy i.e. a distribution of output levels and employment levels for each sector, and the regional rentals that are paid to the resources under competitive conditions.

c) The results that can be obtained through a calculation of the RPF and its associated production state are fully general equilibrium results if the prices p and the resources v are the perfectly competitive equilibrium quantities. In particular this implies that we should be dealing with a small region whose production state should have a negligible affect on the prices.

d) The RPF and its associated production state are the ideal data to have in the evaluation of regional projects. Thus if every project is translated in terms of what it adds to the resource endowments or how it changes the set of regional technologies, it can be socially ranked according to the contribution to the regional product and the changes in the production and employment levels. Except for the proviso expressed under c), the general equilibrium ramifications of the project are taken into consideration. Similarly trade and taxation policies that imply a change in the prices p could be evaluated along similar lines. In contrast the traditional cost and benefit analysis either ignores these linkages or proceeds on the basis of unjustified guess work as to the general equilibrium implications.

e) The RPF should not be identified with the aggregate production function. In the latter all resources are pooled together in say a total of man-hours and a capital aggregate, whereas in the former all resources v are kept separate from each other. In econometric studies the hypothesis made is that a production function in aggregate labor and capital exists, whereas here in the paper the recommended computational technique is parametric programming. For programming models one needs much more information than for econometric studies of the aggregate production function. However it should be realized that an econometrically estimated aggregate production function is too crude to give an answer to the question of how to evaluate a specific regional policy project, since it does not tell us anything about the details of the production state or of the resource rewards in the economy.

f) I should not try to minimize the data problem inherent in the RPF approach suggested here. Suffice it to say that the main purpose here was to put forward that the regional product function and its associated state are useful raw data in the evaluation of regional policies. From the theory point of view they are the ideal pieces of information against which other simpler methods should be compared.

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