ON THE DERIVATION OF INDIFFERENCE CURVES FOR ESTIMATING CONSUMER SURPLUS

J. O. S. KENNEDY
La Trobe University, Bundoora, Vic. 3083

There is a growing interest in methods of valuing experiences which benefit or harm us, but for which there is no money payment or compensation. Common examples of non-market goods we may wish to value are recreation sites and pollution. Alternative measurement approaches, such as the travel cost method, hedonic prices, and multi-attribute utility theory are well established in the research literature. Their application to agricultural and environmental projects is discussed by authors such as Anderson et al. (1977), Freeman (1979), and Sinden and Worrell (1979). An interesting, relatively recent approach based on consumption characteristics is proposed by Greig (1978).

Sinden (1973 and 1974), Sinden and Wyckoff (1976) and Sinden and Worrell (1979) have advocated a method of deriving indifference curves for different combinations of use of two alternative recreation sites. The indifference curves are derived from total utility schedules which in turn are derived from an interview procedure. The theoretical justification of the procedure rests on the postulates and implications of the expected utility theorem originally propounded by von Neumann and Morgenstern (1947). The advantage of an approach in which indifference curves are obtained is that measures of changes in consumer surplus associated with proposed projects may be derived – measures which are theoretically respectable in conventional cost-benefit analysis. However, it is contended that the indifference curves derived by Sinden rely on an assumption which is unrecognised and is unnecessarily restrictive. A simpler and more direct approach to deriving indifference curves would suffice. Another point made is that the discussion by Sinden (1978) of the different types of consumer surplus and their relevance is at odds with the established literature he purports to summarise.

The Method Suggested by Sinden

Let us first examine Sinden's method of obtaining indifference curves. The description and argument are similar in Sinden (1973 and 1974) and Sinden and Wyckoff (1976). Discussion here is based on the article by Sinden (1974).

The Ramsey method of requesting the subject to compare his utility for two lotteries each with two mutually exclusive and equally likely outcomes is followed. The subject has to adjust one of the outcomes of one of the lotteries until he is indifferent between the two lotteries. In Sinden's application, the outcomes consist of the number of days of recreation at a state park in Oregon (*CSP*), at a city pool (*PL*) and at Crater Lake (*CL*).

Sinden (p. 69) obtained the following equations for one subject from the Ramsey procedure:

(1) $$U(1\ PL) - U(0\ CL) = U(0.5\ CSP) - U(0\ CSP)$$

(2) $$U(1\ PL) - U(0\ CL) = U(1.25\ CSP) - U(0.5\ CSP)$$

where $U(1\ PL)$ is the utility derived from spending one day at the pool and so on. It should be noted that really the utility functions for different recreation sites should be distinguished but, for ease of comparison, Sinden's notation is still followed here.

The utility functions are unique up to a positive linear transformation, so the origin and scale of the utility measure for *CSP* are arbitrary. They are set so that $U(0\ CSP) = 0$ and $U(1\ PL) - U(0\ CL) = 5$ utiles. From (1) and (2) we obtain $U(0.5\ CSP) = 5$ utiles and $U(1.25\ CSP) = 10$ utiles. A further iteration of the Ramsey method determines n_3 such that

(3) $$U(1\ PL) - U(0\ CL) = U(n_3\ CSP) - U(1.25\ CSP)$$

and $U(n_3\ CSP) = 15$ utiles. In this way a total utility schedule for *CSP* is derived.

The problem with Sinden's method appears when Sinden attempts to interpret the single attribute utility function for *CSP* as giving information on utility derived from recreation at *CSP* and *PL*. In one paragraph on p. 69 Sinden states $U(1.25\ CSP) = 10$ utiles, but in the following paragraph he states: 'For example, at a level of 10 utiles on the original total utility curve, the combination was 1.25 *CSP* and 1 *PL*'. By 'combination' Sinden clearly means combined consumption because 1.25 *CSP* and 1 *PL* are shown on an indifference map giving 10 utiles. Alternative consumption seems to be confused with joint consumption.

In deriving a second total utility schedule for *CSP* with a new scale, Sinden obtains the following:

(4) $$U(1.25\ CSP) + U(1\ PL) = U(1\ CSP) + U(2\ PL).$$

From this it is inferred that

(5) $$U(1.25\ CSP, 1\ PL) = U(1\ CSP, 2\ PL),$$

which gives a second point, 1 *CSP*, 2 *PL* on Sinden's 10 utile indifference curve. But (5) does not necessarily follow from (4). If (5) does not follow from (4), the indifference mapping procedure suggested by Sinden fails.

Only if utility for *CSP* and *PL* is additive in the utility for *CSP* and the utility for *PL* can (4) be said to imply (5). But if utility is additive, there are two consequences which are not borne out in Sinden's results. First, presumably both 0 *CL* and 0 *CSP* are equivalent to 'staying at home' (p. 68), and so $U(0\ CSP) = U(0\ CL) = 0$. From (1) and the scaling system, $U(1\ PL) = 5$ utiles. This gives a utility of 15 and not 10 utiles for both sides of (4). Second, the marginal utility for days at *CSP* would be independent of days spent at *PL*. In other words, the total utility schedules for *CSP* drawn for levels of *PL* varying from 0 to 4 should be vertically parallel.

Alternative Methods

As Sinden points out, the idea of deriving indifference curves is attractive because they can be used to estimate consumer surplus. There are some conceptual problems which make any results only approximate, but this is true of all attempts to value nonmarket flows. One problem is the settling on the position of the budget line on the recreation indifference map. Is there a fixed recreation budget for each individual, unaffected by changing costs of access to recreation sites? But one may also ask if the budget line on conventional indifference maps for two goods does not depend on labour supplied, which in turn depends on the prices of the goods. Empirical analysis in terms of utility functions for all goods consumed and all reservation uses of resources is desirable but impracticable.

Accepting the desirability of trying to derive indifference curves, an alternative method of derivation could be based on multi-attribute utility theory as pioneered by Keeney and Raiffa (1976). It is shown how, if certain assumptions hold and with some additional information, single attribute utility functions, say $U_a(a)$ and $U_b(b)$, may be combined to give $U(a, b)$. Keeney and Raiffa are interested in using such multi-attribute utility functions for ranking projects with different combinations of levels of attributes. A drawback with this use of the functions is that a ranking of projects is all that is obtained. It gives no indication of whether any of the projects are desirable on economic grounds in the first place. However, it would be quite possible to use multi-attribute utility functions to calculate consumer surpluses based on derived indifference curves and assumed budgets.

The single attribute utility functions would still be obtained by questioning procedures based on the expected utility theorem – the von Neumann-Morgenstern method or the Ramsey method preferred by Sinden. But the obtaining of such utility functions is subject to a possible theoretical objection. As has been pointed out, for example by Alchian (1953) and Handa (1971), one of the axioms or postulates on which the expected utility theorem is based, that of strong independence, is inconsistent with gambling behaviour or the love for gambling *per se*. It has been claimed by Officer and Halter (1968, p. 260) that the Ramsey method of deriving total utility schedules overcomes this problem, because 'the subject has to choose between two gambles so that there is no bias if a subject has a utility (disutility) for gambling'. This claim appears to be accepted by Dillon (1971) and Sinden (1974). However, if a necessary postulate of expected utility theory fails, the expected utility theorem fails. It is pointless to persevere with further analysis based on the Ramsey method which relies on the expected utility theorem if it is thought love of gambling *per se* does exist.

However, if the aim is to derive indifference curves merely as an intermediate stage in estimating consumer surplus, interview procedures which lead to cardinal utility indices for recreation extract more information than is necessary. Indifference curves based on ordinal utility are sufficient for proceeding to estimate consumer surplus. There have been experimental attempts to directly derive unscaled indifference curves for ordinary market goods (see, e.g. MacCrimmon and Toda 1969). Such methods could be used to derive indifference curves for recreation use, with fewer conceptual and practical problems than plague the derivation of scaled indifference curves. Direct elicitation would obviate the need to pose choices between lotteries which are highly artificial in a recreation context.

Appropriate Measures of Consumer Surplus

Relevant measures of consumer surplus may be deduced from indifference curves. Sinden (1978) refers to four types of consumer surplus discussed by Currie et al. (1971), which are the surpluses adopted by Hicks (1956). However, Sinden unintentionally gives quite different definitions of the four surpluses from those definitions accepted in the literature which he surveys. A table (Sinden 1978, Table 1) shows the appropriate measure of consumer surplus (compensating (CV) or equivalent (EV) variations) depending on whether price falls or rises. In a footnote (p. 182), he states, 'Mishan (1971) promoted the price compensating variation when price falls and the price equivalent variation when price rises'. However, reference to the sources quoted shows these claims to be incorrect. CV or EV can apply in situations in which price either falls or rises. The distinction between the two relies on the level of welfare after a price change. To quote Mishan (1975, p. 419):

> More generally, the definition of CV is the sum of money—to be paid by the consumer when the price falls; to be received by him when the price rises—which, following a change in the price, leaves him at his initial level of welfare. The EV, on the other hand, is that sum of money—to be received by the consumer when the price falls; to be paid by him when it rises—which, if he were exempted from the change in price, would yet provide him with the same welfare change.

Mishan argues that, to be consistent with the spirit of cost-benefit analysis, and the principle of accepting potential Pareto improvements, CV rather than EV measures should be employed. However, this argument is not universally accepted (see Freeman 1979, Ch. 3). But in any case, as Willig (1976) has argued, the difference between the two measures in practice is almost certain to be so small as to be swamped by measurement error.

The two variations are relevant to situations in which changes in levels of consumption are adaptations to price changes. If consumption of a good is not a function of price, but, as in the case of a public good, is a function of availability, then similar concepts to CV and EV are applicable. Sinden refers to these as quantity compensating and equivalent variations. Hicks (1956) ultimately uses the terminology compensating (CS) and equivalent (ES) surpluses. If changes in the provision of a public good such as a national park are being considered, then CS and ES measures can be read off indifference-curve maps without reference to changing positions of budget lines. In such situations, Freeman (1979, Ch. 3) is happy to define CV so that it includes CS, and EV so that it includes ES. The possible confusion about the relevance of variation or surplus measures which concerns Sinden should not arise in practice.

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