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SUPPLY RESPONSES, RISK AVERSION AND COVARIANCES IN AGRICULTURE

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In recent research (Fraser 1984; Grant 1985), the problem of a risk averse producer facing uncertainty of both price and output where these two uncertainties are correlated has been analysed. These papers represent extensions of the work of Anderson, Dillon and Hardaker (1977) and Newbery and Stiglitz (1981) to the examination of the impact of such a covariance on the optimal hedging decision (Grant 1985) and on a producer's expected utility of income (Fraser 1984). The objective in this paper is to extend the latter analysis by examining the subsequent supply response of a producer to the effect of a covariance on the expected utility of income. Using the model of a risk averse producer of Newbery and Stiglitz (1981, pp. 80–5) to examine this problem, it is possible to show the circumstances in which the influence of this covariance will lead to an increase or a decrease in the producer's optimal level of effort (and consequently, planned supply).[1]

Apart from its contribution to the understanding of the behaviour of risk averse producers, this paper represents a useful theoretical contribution to the empirical evaluation of price stabilisation schemes. Existing empirical evaluations (for example, Newbery and Stiglitz 1979, 1981; Hinchy and Fisher 1985) have been conducted on the assumption of no supply response by risk averse producers. However, in the context of a simple model with supply variability only, Newbery and Stiglitz (1981, Ch. 22) show that allowing for supply responses can, under certain conditions, qualitatively reverse the welfare impact on producers of a price stabilisation scheme. This important result shows that price stabilisation schemes should not be evaluated on the basis of their short-run (no supply response) welfare impact alone. Moreover, it would seem worthwhile to devote additional research to extending the simple model of Newbery and Stiglitz (1981, Ch. 22) to clarify further the conditions under which these welfare reversals can occur. One such extension involves allowing for both supply and demand variability in the producer's decision environment as is done in this paper. More specifically, the following analysis could be incorporated into the Newbery and Stiglitz model to allow for supply responses to price stabilisation in a more general production environment. A simple example is used later in the paper to show the particular importance of allowing for both supply and demand variability in determining such supply responses.

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[1] Note that such an analysis has previously been attempted by Anderson, Dillon and Hardaker (1977, p. 172).

The Analysis

The model of a risk averse producer used by Newbery and Stiglitz (1981, pp. 80–5) assumes that the only input to production is the producer's own labour, l, and that a single crop is produced which is subject to multiplicative risk:

(1) $\tilde{x} = \tilde{\theta} f(l)$

where $f(l)$ = planned output $[f'(l) > 0, f''(l) < 0]$;
$\tilde{\theta}$ = multiplicative risk term $[E(\tilde{\theta}) = 1]$; and
$\tilde{x}$ = uncertain actual output $[E(\tilde{x}) = \bar{x} = f(l)]$

With price also uncertain, the producer's random income ($\tilde{y}$) is thus given by:

(2) $\tilde{y} = \tilde{p}\,\tilde{x}$

where $\tilde{p}$ = uncertain price $[E(\tilde{p}) = \bar{p}]$.

Newbery and Stiglitz also assume the producer's utility is (additively) separable in income and leisure so that the objective is to maximise by choice of labour input:

(3) $E[U(\tilde{p}\,\tilde{x})] - wl$

where w = marginal disutility of labour; and
$U(\tilde{p}\,\tilde{x})$ = utility of random income $(U' > 0,\ U'' < 0)$.

As mentioned previously, Fraser (1984) examines the effect on equation (3) of the covariance between uncertain price and output. To see the effect of such a covariance on optimal labour input, differentiate equation (3) with respect to l to give the producer's first-order condition:[2]

(4) $E[U'(\tilde{p}\,\tilde{\theta}\,f(l))\,\tilde{p}\,\tilde{\theta}]f'(l) = w$

Next, define:

(5) $g(\tilde{p}, \tilde{\theta}) = U'[\tilde{p}\,\tilde{\theta}\,f(l)]\,\tilde{p}\,\tilde{\theta}$

and use the result of Mood, Graybill and Boes (1974, p. 181) to observe that:

(6) $$E[g(\bar{p}, \tilde{\theta})] \approx g(\bar{p}, 1) + 0.5(\sigma_p^2/\bar{p} + \sigma_\theta^2 \bar{p})[2U''(\bar{p}\,\bar{x})\bar{p}\,\bar{x} + U'''(\bar{p}\,\bar{x})\bar{p}^2\,\bar{x}^2] + \sigma_{p\theta}[U'(\bar{p}\,\bar{x}) + 3U''(\bar{p}\,\bar{x})\bar{p}\,\bar{x} + U'''(\bar{p}\,\bar{x})\bar{p}^2\bar{x}^2]$$

where σ_p^2 = variance of $\tilde{p}$;
σ_θ^2 = variance of $\tilde{\theta}$; and
$\sigma_{p\theta}$ = covariance of $\tilde{p}$, $\tilde{\theta}$.

[2] It is assumed in what follows that the second-order condition is satisfied.

Then define:

(7) $R = -[U''(\bar{p}\,\bar{x})/U'(\bar{p}\,\bar{x})]\bar{p}\,\bar{x}$
= the producer's coefficient of relative risk aversion evaluated at $\bar{p}\,\bar{x}$

and note that:[3]

(8) $U'''(\bar{p}\,\bar{x})\bar{p}^2\bar{x}^2/U'(\bar{p}\,\bar{x}) = R^2 + R - \bar{p}\,\bar{x}\,R'$

Finally, substitute the approximation of $E[g(\tilde{p},\ \tilde{\theta})]$ in (6) into (4) and rearrange to give:

(9) $U'(\bar{p}\,\bar{x})\{\bar{p} + 0.5(\sigma_p^2/\bar{p} + \sigma_\theta^2\bar{p})[R(R-1) - \bar{p}\,\bar{x}\,R'] + \sigma_{p\theta}[(R-1)^2 - \bar{p}\,\bar{x}\,R']\}f'(l) = w$

Equation (9) shows how the parameters of the uncertain environment combine with the characteristics of the producer's risk aversion to determine the optimal labour input. Of particular significance here is that the qualitative effect of a covariance between price and output on the optimal labour input may actually be independent of the size of the producer's coefficient of relative risk aversion. Specifically, if R is constant or decreasing, then labour input is increased by a positive covariance and decreased by a negative covariance between $\tilde{p}$ and $\tilde{\theta}$.[4] Only if R is increasing does its size play a role in determining the direction of the effect of the covariance. In this case, for values of R sufficiently close to unity, $R' > 0$ means that labour input will increase or decrease as $\sigma_{p\theta}$ is negative or positive. For other values of R, the effect of $\sigma_{p\theta}$ on labour input is ambiguous and depends both on the sign of $\sigma_{p\theta}$ and on whether:

$$\bar{p}\,\bar{x}\,R' \gtrless (R-1)^2$$

For Newbery and Stiglitz (1981, p. 82), decreasing R 'seems likely', while the typical covariance between price and output in agriculture could be expected to be negative. In this case, labour input would be lower than that predicted in circumstances which omitted the interdependence of price and output, regardless of the actual magnitude of R. Moreover, if the magnitude of this negative covariance were to increase then, *ceteris paribus*, labour input would be lowered. Examples of developments which might bring this about are a decrease in the elasticity of the product demand curve and an increase in the correlation between the producer's output and the total output of the product.

The significance of allowing for both supply and demand variability in determining supply responses and consequently for evaluating the long-run effects of price stabilisation schemes can be shown by reference to the following simple example. Specifically, assume the relationship between price and output exhibits a constant negative correlation:

[3] I am particularly grateful to one anonymous referee for pointing this out to me.

[4] This is an unusual result concerning the effects of a change in risk, which typically depend both on the size of R and on whether it is increasing or decreasing. See, for example, Newbery and Stiglitz (1981, pp. 81–2).

(10) $\sigma_{p\theta} = \rho\sigma_p\sigma_\theta \qquad (\rho < 0)$

and that all producers are risk neutral. With these assumptions equation (9) simplifies to:

(11) $U'(\bar{p}\,\bar{x})(\bar{p} + \rho\sigma_p\sigma_\theta)f'(l) = w$

Ignoring the relationship between $\tilde{p}$ and $\tilde{\theta}$ is equivalent to setting $\rho = 0$ and means that labour input (and, therefore, planned output) is predicted to be unresponsive to a reduction in σ_p (which leaves $\bar{p}$ unchanged). It follows that the short-run and long-run effects of a price stabilisation scheme of this sort would be identical. However, recognising $\rho < 0$ means that a reduction in σ_p is predicted to increase labour input. Allowing for the relationship between $\tilde{p}$ and $\tilde{\theta}$ must therefore alter the evaluation of the long-run effects of the scheme.

Conclusion

The focus in this note has been on the role of a covariance between a producer's uncertain price and output in determining the optimal level of planned production. Using the Newbery and Stiglitz (1981, pp. 80–5) model of a risk averse producer, it was shown that, in the case where the producer's coefficient of relative risk aversion is constant or decreasing, whether this covariance increases or decreases, the optimal level of effort depends simply on the sign of the covariance and is independent of the degree of risk aversion. If, however, the producer's relative risk aversion is increasing, then the effect of the covariance depends not just on its sign, but also on the degree of this risk aversion.

For the purposes of the empirical evaluation of the effects of price stabilisation schemes, this analysis has shown that the supply response of a risk averse producer who faces correlated price and output uncertainties will at least be of a different magnitude to that predicted when omitting this correlation, and may even be of the opposite sign. Given the demonstrated significance of supply responses in determining the long-run welfare impact of such schemes, incorporating this more general production environment into their evaluation would seem an important item for further research.

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