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# A NOTE ON OPTIMAL RULES FOR STOCHASTIC EFFICIENCY ANALYSIS

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The concepts of stochastic efficiency have been formalised only in the past 20 years. The literature expanded rapidly throughout the 1970s (see, for example, Whitmore and Findlay 1978), with a significant input from agricultural economists (for example, Anderson 1974). Subsequent theoretical developments have been relatively few. Publications on stochastic efficiency in the agricultural economics literature have continued to appear, principally applications papers in the *American Journal of Agricultural Economics* (for example, Lee, Brown and Lovejoy 1985) and the regional US journals (for example, Lemieux, Richardson and Nixon 1982; Rister, Skees and Black 1984).

This note has been prompted by the recent paper by Buccola and Subaei (1984) in this *Journal*, and aims to draw attention to some long established, but little used, results from the stochastic efficiency literature.[1] These results on mixed, or $k$-way, dominance (Drynan 1977; Fishburn 1978; Meyer 1979) stem from Fishburn's (1974) work on convex stochastic dominance. In essence, the results generalise standard stochastic dominance rules. More significantly, they mean that application of the standard rules to a set of prospects may not be sufficient to identify all prospects that are inefficient. In addition to drawing attention to these older results, the paper reports $k$-way results for a wider class of utility functions than previously considered. Means of implementation are outlined.

The paper is organised as follows. In the next section relevant concepts are defined. Then follow the major mixed dominance results and means of implementation by linear programming. Results for the wider class of utility functions are then presented. The paper is concluded with a discussion of the significance of both the particular mixed dominance results and stochastic efficiency more generally. Some readers may find it helpful to read this final section before reading the main results section.

## *Preliminaries*

Assume that a choice is to be made between $n$ feasible prospects, $a_i$, $i = 1, 2, \ldots, n$, each with uncertain outcome and monetary consequence $w$, $w \varepsilon [a, b]$. Assume each prospect can be described in terms of its density function on monetary consequences, $F_{0,i}(w)$. Additionally, define for each prospect the cumulative functions:

* The material in this paper, with the exception of the penultimate section, is drawn from the author's Ph.D. thesis at the University of New England. The assistance of Jock Anderson, supervisor, is gratefully acknowledged.

[1] One of the few publications in the agricultural economics literature to note these results is Anderson, Dillon and Hardaker (1977).

$$(1)\quad F_{j,i}(w)=\int_a^w F_{j-1,i}(z)\,dz, \qquad j=1,\,2,\,3$$

For $j=1$, the cumulative function for prospect $i$ is the distribution function of monetary consequences for that prospect.

Assume that choice between prospects is based on a von Neumann-Morgenstern utility function for monetary consequences, $u(w)$, with $u'(w)>0$, $w\varepsilon[a,b]$. The preferred prospect is that for which expected utility is a maximum, where the expected utility of the $i$th prospect is defined by:

$$(2)\quad EU_i=\int_a^b u(w)\,F_{0,i}(w)\,dw$$

Assume that $u(w)$ is known only to the extent that $u(w)\varepsilon U$, where the class of utility functions $U$ is a subset of the set of von Neumann-Morgenstern utility functions. Then, if for each member of $U$, a prospect other than $a_j$ is preferred to $a_j$, the prospect $a_j$ is said to be *dominated.* Dominance can occur in several ways. In the special case where prospect $a_i$ is preferred to $a_j$ by all members of $U$, then the prospect $a_j$ is said to be *pairwise dominated by* $a_i$, to be *one-way dominated*, or to be *dominated in the pure sense.* When a union of $k$ prospects exists such that some member of the union, perhaps different for all $u(w)$, is always preferred to prospect $a_j$ by all members of $U$, then prospect $a_j$ is *k-way dominated*, or *dominated in the mixed sense.*

The subset of prospects formed of all prospects which are preferred to all other prospects by at least one member of $U$ is called the *efficient set.* The efficient set consists only of prospects which are not dominated. Those prospects not in the efficient set comprise the *dominated set.* The objective of efficiency analysis is to divide the set of prospects into the efficient set and the dominated set.

Any rule used to decide some form of dominance is *optimal* if it is necessary and sufficient for the given form of dominance, *valid* if it is sufficient, and *invalid* otherwise.[2] Only an optimal rule will identify the efficient set.

## *Efficiency Analysis*

In this section, necessary and sufficient conditions for a prospect to be dominated are determined for various classes of utility functions. Three classes are of particular interest. They are:

$U_1=\{u(w)$: $u(w)$ and $u'(w)$ exist and are continuous with $u'(w)>0$ for $w\varepsilon[a,b]\}$;

[2] Most stochastic efficiency analyses have relied on pure dominance rules. When valid, such rules are too strong for the concept of dominance defined above and as used in these same analyses. A prospect which is inefficient because, for every utility function, there is another prospect, not always the same, which is preferred to it, will not be detected. The mixed dominance rules defined in later sections are optimal. One common pair-wise efficiency rule which is invalid when applied to the class of risk averse utility functions is the 'E-V' rule. It may indicate that an efficient prospect is inefficient, and that an inefficient prospect is efficient (Porter 1973).

$U_2 = \{u(w)$: $u(w)$, $u'(w)$, and $u''(w)$ exist and are continuous with $u'(w) > 0$, $u''(w) < 0$ for $w \varepsilon [a, b]\}$; and
$U_3 = \{u(w)$: $u(w)$, $u'(w)$, $u''(w)$, and $u'''(w)$ exist and are continuous with $u'(w) > 0$, $u''(w) < 0$, $u'''(w) > 0$ for all $w \varepsilon [a, b]\}$.

The following necessary and sufficient conditions for dominance of one prospect by another for these classes are well known (see, for example, Anderson 1974 and references therein). For the class of utility functions $U_r$, $r = 1, 2, 3$, prospect $a_j$ is dominated by prospect $a_i$, if, and only if, $F_{1,i}(w)$ stochastically dominates $F_{1,j}(w)$ in the $r$th degree, where stochastic dominance relationships between distributions are defined as follows. The distribution function $F_{1,i}(w)$ stochastically dominates the distribution function $F_{1,j}(w)$ by $r$th degree stochastic dominance if, and only if, (a) $F_{r,i}(w) \leq F_{r,j}(w)$ for all $w \varepsilon [a, b]$ with strict inequality for at least one value of $w$; and (b) $F_{2,i}(b) \leq F_{2,j}(b)$.

The stochastic dominance ordering rules are optimal for one-way dominance for the appropriate utility classes. By applying optimal one-way ordering rules to all pairs of the basic prospects, all prospects which are dominated by another prospect can be identified and discarded, leaving a reduced set. This set has often been referred to mistakenly as the efficient set. That this procedure, based on pair-wise comparisons of prospects using an optimal one-way ordering rule, does not necessarily yield the efficient set can be shown by example.

Consider the class of utility functions containing only two utility functions $u_1(w)$ and $u_2(w)$. Suppose there are three prospects, $a_1$, $a_2$, and $a_3$. Further suppose that the ordering of the prospects in terms of $u_1(w)$ expected utility is $EU(a_1) > EU(a_2) > EU(a_3)$; and for $u_2(w)$, $EU(a_3) > EU(a_2) > EU(a_1)$. The efficient set then consists of $a_1$ and $a_3$, and prospect $a_2$ is dominated. However, from a pair-wise comparisons analysis, $a_2$ is not dominated by any other prospect, and the reduced set from a pair-wise comparisons analysis contains all three prospects.

To identify the efficient set, it is necessary to compare potentially dominated prospects with all the remaining prospects simultaneously. Theorem 1 provides a sufficient condition for dominance of a prospect for any class of von Neumann–Morgenstern utility functions.

*Theorem 1*

For any class of von Neumann–Morgenstern utility function, $U$, and a set of prospects $a_i$, $i = 1, 2, \ldots, n$, prospect $a_j$ is dominated if there exists an $n$-vector, $\lambda$, $\lambda_i \geq 0$, $i = 1, 2, \ldots, n$, $\lambda_j = 0$, $\sum_{i=1}^{n} \lambda_i = 1$, such that the random strategy prospect, $\sum_{i=1}^{n} \lambda_i F_{1,i}$, formed by choosing $a_i$ with probability $\lambda_i$, dominates prospect $a_j$.

*Proof:* Consider the prospect $\sum_{i=1}^{n} \lambda_i F_{1,i}$. The expected utility of this prospect is:

$$(3) \quad EU\left(\sum_{i=1}^{n} \lambda_i F_{1,i}\right) = \sum_{i=1}^{n} \lambda_i EU(a_i)$$

If this prospect dominates $a_j$, then $\sum_{i=1}^{n} \lambda_i EU(a_i) > EU(a_j)$ for all $u(w) \varepsilon U$, and hence for each $u(w)$, at least one $EU(a_i) > EU(a_j)$, $i \neq j$. Hence, $a_j$ is dominated.

It is important to appreciate that the random strategy prospect does not correspond to a sum or linear combination of random variables as one encounters in portfolio construction or whole-farm planning. Instead, the random strategy prospect has a distribution formed as the sum or linear combination of other distributions. As Theorem 1 says, the prospect is formed in a probabilistic fashion by randomly drawing one of the $n$ actual prospects which are available for choice.

Theorem 2 establishes that the condition is necessary for some classes of utility functions.

*Theorem 2*

For a class of von Neumann–Morgenstern utility functions, $U$, such that any positive linear combination of members of $U$ also lies in $U$, and a set of prospects $a_i$, $i = 1, 2, \ldots, n$, prospect $a_j$ is dominated if, and only if, there exists an $n$-vector $\lambda$, $\lambda_i \geq 0$, $i = 1, \ldots, n$, $\lambda_j = 0$, $\sum_{i=1}^{n} \lambda_i = 1$, such that the random strategy prospect, $\sum_{i=1}^{n} \lambda_i F_{1,i}$, formed by choosing $a_i$ with probability $\lambda_i$, dominates $a_j$.

*Proof:* Sufficiency follows from Theorem 1. For the best proof of necessity, see Fishburn (1978).

Many interesting classes of utility functions will satisfy the requirements for Theorem 2. The theorem holds, for example, for $U_1$, $U_2$, $U_3$. It also holds for the class of quadratic utility functions, and for the risk aversion constrained classes considered by Meyer (1977*a*), namely, classes such that each member utility function has a local risk aversion function that never lies above an upper bound function $r_U(w)$ and never lies below a lower bound function $r_L(w)$.

To apply the theorem to $a_j$ for any of these classes, it is necessary to search for a constrained convex combination of the prospects which one-way dominates $a_j$. Making use of the existing theorems giving necessary and sufficient conditions for one-way dominance, the following theorem can be established.

*Theorem 3*

For the class of utility functions $U_r$, $r = 1, 2, 3$, and a set of prospects $a_i$, $i = 1, 2, \ldots, n$, prospect $a_j$ is dominated if, and only if, there exists an $n$-vector $\lambda$, $\lambda_i \geq 0$, $i = 1, 2, \ldots, n$, $\lambda_j = 0$, $\sum_{i=1}^{n} \lambda_i = 1$, such that the distribution function $\sum_{i=1}^{n} \lambda_i F_{1,i}(w)$ stochastically dominates the distribution $F_{1,j}(w)$ in the $r$th degree.

*Proof:* Proof follows from Theorem 2 and the one-way dominance results.

Theorem 3 can be seen as a specialisation of the convex stochastic dominance results obtained by Fishburn (1974). He considers two sets of $n$ prospects $\{a_i\}$ and $\{b_i\}$ with distribution functions $F_{1,i}(w)$ and $G_{1,i}(w)$, $i=1, 2, \ldots, n$, respectively, and establishes the following result (among others).

For $U_r$, $r=1, 2$, if, and only if, there exists an $n$-vector $\boldsymbol{\lambda}$, $\lambda_i \geq 0$, $\sum_{i=1}^{n} \lambda_i = 1$, such that the distribution function $\sum_{i=1}^{n} \lambda_i F_{1,i}(w)$ stochastically dominates the distribution function $\sum_{i=1}^{n} \lambda_i G_{1,i}(w)$ in the $r$th degree, then for each utility function in $U_r$, there will be an $i$ such that $a_i$ is preferred to $b_i$. By setting all $G_{1,i}(w)$, $i=1, 2, \ldots, n$, equal, the necessary and sufficient conditions for dominance of one prospect by others are obtained.

By applying Theorem 3 successively to the prospects, all dominated prospects will be identified, and the efficient set obtained for the particular class of utility functions.

### *Implementing Dominance Analysis for $U_r$ Classes*

At the practical level, the optimal one-way dominance ordering rules have often been abandoned in favour of less theoretically attractive, but more easily implemented, rules such as the E–V criterion, the mean lower partial variance rule (Bawa 1975), the mean lower partial average negative deviation rule (Tauer 1983), or the mean-Gini rule examined by Buccola and Subaei (1984). A $k$-way dominance analysis is even more difficult to implement without resorting to some pragmatic approximation. For Theorem 3 cases, the search for suitable $\lambda_i$ values can be performed efficiently by linear programming.

For an $r$th degree one-way stochastic dominance analysis of $\sum_{i=1}^{n} \lambda_i F_{1,i}(w)$ and $F_{1,j}(w)$, the two distributions must be compared for all $w \varepsilon [a,b]$. This comparison can be effected, arbitrarily well, by comparing the two $r$th cumulative functions at a set of $m$ points, $w_s$, $s=1, 2, \ldots, m$, in $[a,b]$. Consider the following linear programming problem.

(4) $$\max_{\lambda_i, d_s} \sum_{s=1}^{m} c_s d_s$$

subject to:

(5) $$\sum_{i=1}^{n} \lambda_i F_{r,i}(w_s) + d_s = F_{r,j}(w_s) \qquad s=1, 2, \ldots, m$$

(6) $$\sum_{i=1}^{n} \lambda_i F_{2,i}(b) \leq F_{2,j}(b)$$

(7) $$\sum_{i=1}^{n} \lambda_i = 1$$

(8) $$\lambda_j = 0$$

(9) $$\lambda_i \geq 0, \; d_s \geq 0$$

The activities $d_s$, $s=1, 2, \ldots, m$, are slack or disposal activities with arbitrary positive coefficients $c_s$, $s=1, 2, \ldots, m$. The first $m$ constraints require that the $r$th cumulative function of the random strategy prospect be no greater than that of the prospect $a_j$ at the $m$ points at which comparisons are made, since each $d_s$ is required to be non-negative. The next constraint requires that the random strategy prospect have at least as large a mean as prospect $a_j$. The remaining constraints restrict the $\lambda_i$ values to form a convex combination, with no contribution from prospect $a_j$.

The solution of this problem by the simplex method can be divided into two phases (Hadley 1962): (a) locating a feasible solution; and (b) locating that feasible solution which maximises the objective function. If no feasible solution exists, $F_{1,j}(w)$ is not stochastically dominated in the $r$th degree by any allowable distribution $\sum_{i=1}^{n} \lambda_i F_{1,i}(w)$, and hence, prospect $a_j$ is not dominated. If a feasible solution exists with one or more of the $d_s$, $s=1, 2, \ldots, m$, positive then $a_j$ is dominated. Once a feasible solution with one $d_s>0$ is found, further optimisation can be abandoned.

When analysing a set of prospects to determine the efficient set, some modifications can be made to the linear programming formulation to improve computational efficiency. First, variable $\lambda_j$ and the constraint $\lambda_j=0$, can be omitted. Second, any prospect which is found to be dominated can be discarded and not used in subsequent analysis of dominance of other prospects. Third, performing a one-way dominance analysis of all prospects initially may help to reduce the size of the set of prospects to be considered.

Finally, since $U_{r+1}$ is a subset of $U_r$, the efficient set for $U_{r+1}$ is a subset of that for $U_r$. Hence, if an efficiency analysis has been performed for $U_r$, one for $U_{r+1}$ need only consider the prospects in the efficient set for $U_r$.

In summary, the proposed algorithm to locate the efficient set for $U_r$ is as follows.

[1] Compute the $F_{r,i}(w)$ functions, $i=1, 2, \ldots, n$, using some form of integration.
[2] Perform the appropriate one-way dominance analysis on the feasible prospects, producing a reduced set of prospects.
[3] Set $j=1$.
[4] For the reduced set, determine if the $j$th prospect is dominated by attempting to solve the appropriate linear programming problem. If a feasible solution with a positive objective function value is found, discard prospect $a_j$ from the reduced set.
[5] If $j=n$, all prospects have been examined, and the current reduced set is the efficient set for $U_r$. Go to 7.
[6] Increment $j$. Go to 4.
[7] If an analysis for $U_s$, $s>r$ is required, go to 2. Otherwise, terminate the analysis.

### *Mixed Dominance for Risk Constrained Classes*

As noted earlier, Theorem 2 holds for many classes of utility function. But practical implementation of the $k$-way dominance concept depends on access to an easy means of detecting one-way dominance and to a search procedure over alternative convex combinations of distributions to find a dominating random strategy prospect. While Theorem 2 holds for the risk constrained classes defined by Meyer (1977*a*), the one-way optimal conditions for these classes are not available in closed form. Useful mixed dominance results are not apparent. But in a subsequent paper Meyer (1977*b*) considered those risk aversion constrained classes which are constrained on only one side. For these classes, he showed that the optimal pair-wise dominance rules are closely related to the rule for $U_2$. Define second degree stochastic dominance with respect to a function of one distribution by another as follows. The distribution function $F_{1,i}(w)$ stochastically dominates the distribution function $F_{1,j}(w)$ in the second degree with respect to function $h(x)$ if, and only if,[3]

$$(10) \quad \int_a^y F_{1,j}(w)h(w)dw - \int_a^y F_{1,i}(w)h(w)dw \geq 0 \qquad \text{for all } y \varepsilon [a,b]$$

with strict inequality holding for at least one value of $y$. Meyer showed that prospect $a_j$ is dominated by prospect $a_i$ if, and only if, $F_{1,i}(w)$ stochastically dominates the distribution function $F_{1,j}(w)$ in the second degree with respect to function $h(w)$.

This result generalises that for $U_2$ since the latter relates to the special case $h(w)=1$. The practical cost of the generality is negligible since the only difference in the procedure for detecting one-way dominance is the inclusion of the extra factor in the integrations. Moreover, the detection of mixed dominance also parallels that for $U_2$, except that one must now search for a suitable linear combination of the integrated weighted distribution functions. The result can be stated as follows.

*Theorem 4*

For the class of utility functions such that each member has a risk aversion function no less than $r_L(w)$ and a set of prospects $a_i$, $i=1, 2, \ldots, n$, prospect $a_j$ is dominated if, and only if, there exists an $n$-vector $\lambda$, $\lambda_i \geq 0$, $i=1, 2, \ldots, n$, $\lambda_j=0$, $\sum_{i=1}^{n} \lambda_i = 1$, such that the distribution function $\sum_{i=1}^{n} \lambda_i F_{1,i}(w)$ stochastically dominates the distribution $F_{1,j}(w)$ in the second degree with respect to $h(w)$ where $r_L(w) = -h'(w)/h(w)$.[4]

*Proof:* Proof follows from Theorem 2 (which applies for this class of utility functions) and from Meyer's one-way dominance results.

## *Discussion*

Stochastic efficiency analysis may be used in two situations. First, if a number of decision makers with differing utility functions confront the

[3] Meyer's (1977*b*) terminology is different. His 'function' is actually the integral of the $h(w)$ called the 'function' here.

[4] A similar theorem can be proved for utility classes with risk aversion constrained only on the upper side. Meyer (1977*b*) gives the necessary one-way rules.

same decision problem, efficiency analysis identifies which of the prospects would not be selected by any of the decision makers. Second, in the case where a particular decision maker faces a decision problem, but the utility function is not known, at least not fully known, efficiency analysis helps to identify which of the prospects might be selected and which would definitely not be selected. In both cases, the more information about the possible utility functions, the greater the potential to identify inefficient prospects. In the limiting case of only one known utility function, all optimal and non-optimal prospects are identified.

From a theoretical perspective, efficiency analysis is appealing. One attempts to draw the strongest possible conclusions given the information that is available about the utility functions of the decision makers. It is possible to derive general statements about behaviour, for example, that decision makers with concave utility functions will prefer the sure actuarial value of a gamble to the gamble. But from a practical perspective, stochastic efficiency analysis, at least as currently conceived, would seem to have a more limited role. Do decision makers ever confront identical decision problems? Are there not usually differences in monetary and non-monetary payoffs and in beliefs about events from one decision maker to another? To be more useful, stochastic efficiency analysis will have to be extended to allow for variation in these aspects of the decision problem. On the other hand, this variation does not arise when the analysis relates to a particular decision maker with a particular decision problem. In this case, however, it would seem to be possible, if not ultimately necessary, to say more about the utility function than implied by the $U_r$ classes commonly used in stochastic efficiency analysis. Meyer's risk constrained classes would seem more appropriate.

One of the practical difficulties with stochastic efficiency analyses has been their frequent failure to reduce the set of feasible prospects sufficiently. If the efficient set has been determined, and is considered large, the analyst has three options: (a) accept the situation; (b) inject further information into the analysis (for example, by placing more restrictions on the utility function); or (c) abandon high principles and opt for a workable, non-optimal and possibly invalid rule. The point made here is that the problems which have been encountered with the size of reduced sets lie in part in failure to find the efficient set through failure to use optimal ordering rules. In order to judge the efficacy of stochastic efficiency analysis, agricultural economists, and others, need first to develop experience in the application of optimal rules.

Given that many decision analysts have found stochastic efficiency analysis an appropriate form of analysis, the question of why there has not been greater interest in the notion of mixed dominance remains. The easy answer is that diffusion of the knowledge has been slow, but this still leaves the question of why. A more fundamental reason is that explicit notions of mixed dominance are not required under the assumptions made in many efficiency analyses.

A major area of application of efficiency analysis has been in portfolio selection problems in which the set of feasible prospects is formed from linear combinations of a set of available investment activities. E–V analysis has been the most common form of portfolio efficiency analysis, the 'efficient' prospects forming an E-V frontier which is convex to the E axis. The E-V rule is justified on the basis of either quadratic utility functions or normal distributions along with diminishing marginal utility. The rule is a valid but not an optimal rule for efficiency analysis when quadratic utility is assumed. It can be demonstrated that some prospects which appear efficient on the basis of a pair-wise E-V analysis are actually inefficient on a $k$-way basis.[5] But in a portfolio analysis, every prospect within the E-V frontier is one-way dominated by many individual prospects on the frontier.[6] Only one-way efficient prospects on the frontier need to be examined for possible mixed dominance. But as Baron (1977) has shown, the variance of a random strategy prospect is greater than the same linear combination of the variances of the component prospects. Because the frontier is convex to the E axis, there can be no random strategy prospect formed from one-way efficient prospects on the E-V frontier which can dominate other one-way efficient prospects on the frontier. Thus an analysis for mixed dominance is not needed in a quadratic utility portfolio analysis.

Under the alternative justification for E-V analysis, namely normality and risk aversion, the explicit notion of mixed dominance is again unnecessary. Despite Meyer's (1979) claim to the contrary, if all distributions are normal, the reduced set from a pair-wise analysis is the efficient set.[7] This applies whether or not one has a portfolio type problem or a problem with only a discrete set of feasible prospects.

Because of the predominance of portfolio type problems in financial decision analysis, from where much of the stochastic efficiency analysis has emanated, it is not so surprising that $k$-way dominance has not commanded greater interest. But in many applications in agricultural economics and elsewhere, decision problems do entail only a finite set of prospects. The $k$-way dominance concepts should then be considered explicitly.

[5] Consider the three prospects with means 118, 120 and 122, and variances of 4, 18 and 20. All prospects are efficient under the E-V rule. Consider the random strategy prospect formed by selecting each of the first and third prospects with probability 0.5. Following Baron (1977), this prospect has mean 120 $(=0.5*118+0.5*122)$ and variance 16 $(=0.5*4+0.5*20+0.5*0.5*(122-118)^2)$. This prospect one-way dominates the second by the E-V rule, and since the class of quadratic utility functions satisfies the conditions of Theorem 2, it follows that one or other of the first or third prospects will always be preferred to the second.

[6] This does not mean that all points on the E-V frontier are one-way efficient. Hanoch and Levy (1970) show that the E-V rule is not the optimal pair-wise rule. It fails to detect some prospects on the frontier which are one-way dominated for quadratic utility functions by other prospects on the frontier.

[7] In this case, members of the reduced set from a pair-wise analysis necessarily intersect, one having the greater variance and the greater mean. Any random strategy prospect that might dominate one prospect necessarily includes at least one prospect with a larger mean, and hence with larger variance. The density function (and cumulatives) of this particular component prospect, irrespective of the positive weight assigned to it, will exceed that of the potentially dominated prospect for sufficiently negative monetary outcomes. Hence no member of the reduced set can be dominated. Meyer's error would appear to lie in incorrectly assuming that the distribution formed as a convex combination or mixture of normal distributions is also normal. Johnson and Kotz (1970) give details of the form of such distributions.

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