The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

Australian Journal of Agricultural Economics, Vol. 25, No. 2 (August 1981), pp. 149-159.

PRODUCTION RISK AND INPUT USE: PASTORAL ZONE OF EASTERN AUSTRALIA

JOCK R. ANDERSON and WILLIAM E. GRIFFITHS

University of New England, Armidale, N.S.W. 2351

Few attempts have been made to estimate production functions for the Australian grazing industries. The question of the nature of the effect of input levels on production risk has been broached even more rarely. Previous investigators had to employ models and methods of estimation which embody highly restrictive implicit assumptions about the nature of risk effects. A typical restrictive feature has been the implication that increasing input intensity leads to increasing risk. In this paper, a much less restrictive model and corresponding estimation techniques are brought to bear on individual farm data for 38 properties with 10 continuous years of production records. Perhaps not surprisingly, it is found that some inputs (especially those capital inputs which might normally be thought of as increasing the safety of production) tend to reduce risk.

Economists' ignorance of the effect of various categories of inputs on aspects of production, such as mean output and measures of variability of output, in the Australian grazing industries is profound—at least as judged by the paucity of successful attempts to model such phenomena adequately. A cross-sectional study by Anderson (1972) of properties in the Eastern Pastoral Zone involved descriptions of output and its variability but, because no attempt was made to disaggregate the effects of individual groups of inputs via the estimation of production coefficients, only the influence of overall size of enterprise was examined. The only investigation of relationships represented in the paradigm of the production function for this region has seemingly been that of Duloy (1959), who did not concern himself with aspects of risk and variability.

Speculations as to the reasons for neglect of empirical work on these phenomena could follow several lines. One could be the inherent limitations of cross-sectional production functions for grappling with questions of resource use efficiency (Anderson and Jodha 1973). Another could be the evident irrelevance of farm production models which abstract from the existence of production uncertainty in an environment where uncertainty is so pervasive.

Related to the latter possibility is the fact that appropriate methods and models for recognising and quantifying such impacts of uncertainty have appeared only recently. One approach, perhaps best described as a crude method, is represented by the work of Anderson (1973) in connection with crop-fertiliser response processes. Observations over time, at controlled levels of factors, were used to estimate moments of probability distributions of performance and, in turn, these were related to the factors of production using ordinary least squares (OLS) regressions. The moments estimated included those beyond the mean and variance and there was no presumption of normality. However, the cost of such generality was the lack of efficiency and consistency in the *ad hoc* estimators.

A more recent and more formal approach has been proposed by Pope and Just (1977) and Just and Pope (1978, 1979). They suggest econometrically efficient estimators which are necessary if one is to model consistently the random effects in production functions, particularly their unrestrictive error specification. To date, attention has not transcended the influence of factors on mean and variance of production. The formal models feature explicit modelling of heteroscedastic (especially relative to input levels) error structures and, almost invariably, a necessity for multi-stage nonlinear estimation.

The Stochastic Specification

Using, for illustrative purposes, the Cobb-Douglas or power function form of a whole-farm production function, the traditional stochastic specification can be written as:

$$(1) \qquad y = \gamma \Pi_k X_k^{\alpha_k} e^v$$

where y is the level of output;
X_k is the level of the kth input;
γ and the α_k, $k=1,2,\ldots,K$, are parameters; and
v is an 'error' term with zero mean and constant variance σ^2.

Apart from the theoretical work of Pope and Just (1977) and Just and Pope (1978, 1979), recent developments in the production function literature (e.g. McKay, Lawrence and Vlastuin 1980, 1981; in the tradition of Diewert 1974) seem to have concentrated on indirect estimation of production function parameters, rather than on a stochastic specification more realistic than that in equation (1). Typically, such studies involve estimation of functions such as cost or profit functions because, under the assumption of profit maximisation, these functions can be linked directly with production functions, and they avoid the problem of the inputs being jointly endogenous. However, they ignore the fact that output, and hence profit, might be stochastic and, therefore, that it might be more realistic to assume that the producer is maximising *expected profit*, or *expected utility*, which could, for example, be a function of the mean and variance of profit.[1]

If one recognises the risky nature of output, then one of the first steps towards providing a framework for expected utility maximisation is the specification and estimation of certain aspects of the probability distribution of output. It is this question with which we are concerned. In particular, an undesirable restrictive feature of the stochastic structure in equation (1) is that the signs of the marginal products and the respective 'marginal risks' (derivatives of variance of output with respect to input, $\partial V[y]/\partial X_k$, according to the terminology of Magnússon (1966) and Anderson, Dillon and Hardaker (1977, Ch. 6), are constrained to be the same—presumably positive!

This has led Just and Pope to suggest a more flexible stochastic specification which can be indicated by the form:

$$(2) \qquad y = \gamma \Pi_k X_k^{\alpha_k} + v \Pi_k X_k^{\beta_k}.$$

[1] It is well known that, under the traditional stochastic specification in equation (1), the problem of simultaneity of inputs and output does not arise if the producer is maximising expected profits (Zellner, Kmenta and Drèze 1966) or expected utility (Blair and Lusky 1975).

This involves the additional parameters β_k, $k=1,2,\ldots,K$, and the sign of a marginal risk here depends on the sign of the respective 'risk elasticity', $2\beta_k$. Of course, if all the β_ks were zero, equation (2) would be an 'additive risk' specification of a production function that could be estimated nonlinearly and which would feature zero marginal risk effects.

Data

To estimate a production function such as equation (2), it seems essential to have data with two broad characteristics. First, to have data with sufficient variation in input levels and combinations, it is necessary to resort to a cross-sectional approach. Without such variation, a production relationship would be unlikely to be 'identified'. Second, to observe adequately risk effects, it seems desirable to have data on year-to-year farm performance, that is, a time series. Combined cross-sectional and time-series data on farm performance are not widely available and we were fortunate in being given access to BAE data which have these characteristics. The BAE kindly made available suitably camouflaged individual-farm data for 38 properties with continuous annual records from 1964-65 to 1973-74. These farms are all grazing properties in the Pastoral Zone of Queensland and New South Wales, and are sample members of the Australian Agricultural and Grazing Industries Survey. We formulated the following variables from the available data.

Output

We sought to have basically wool growing properties. While the major enterprise on all 38 properties was the sheep enterprise, there were significant numbers of beef cattle and, rather less frequently, wheat was produced on some properties. It was not possible to allocate input use amongst these alternative enterprises and so we were forced to convert the different enterprise outputs into a common measure. The output measure chosen, Y, is 'wool equivalent'. This was defined as actual wool produced plus output other than wool expressed in wool units by dividing the total non-wool revenue by the individual farm's unit value of wool in the respective year.[2] This procedure can result in a negative output for a particular year (for instance, if there are significant trading losses in either the sheep or beef enterprise).

Inputs

Where possible, inputs were expressed in physical terms or their equivalents. Where it was necessary to express the inputs as financial

[2] This procedure seems reasonable. It is equivalent to weighting the physical outputs of the sheep, cattle and grain enterprises by their respective product prices in each year, and then standardising these annual revenues over different price regimes by deflating by wool price. This produces a series which is in intertemporally comparable physical units of wool.

Although prices of the different commodities have varied over the period of observation and between farms, there is no statistically significant or persistent change in product mix over the period of observation. The average proportions of revenue derived from wool, sheep, cattle and grain are 0.73, 0.84, 0.12, and 0.03, respectively. The authors believe that, although there have been slight variations in output mix over time, the variation among the data is insufficient to permit estimation of a consistent multi-product production model (Mundlak 1964, El-Issawy 1970, McKay, Lawrence and Vlastuin 1981). Indeed, the extension of the present methods of accounting for differential marginal risk effects in such a multi-product context is a challenge for further investigators.

measures, these were deflated by the BAE Index of Prices Paid (with resulting data expressed in 1960-61 dollars).

Labour. The labour input, L (man-weeks), was measured on a yearly basis by dividing the total (including imputed operator and family contributions) expenditure on labour by the operator's allowance (which in turn is based on prevailing award rates for stockhands) and multiplying by 52.

Biological capital. Capital invested in livestock, S (sheep equivalents), was measured in physical terms as the average number of sheep carried (the average of opening and closing numbers) plus the average number of cattle carried times eight.

Other capital. An attempt was made to measure non-biological capital as flows of resources for mutually exclusive and fairly exhaustive categories. These were watering services (W), fencing services (F), plant and machinery services (P), and buildings and land services (B). The first three of these, and the building and improvement components of the final variable, were defined as follows: depreciation plus expenditure on associated non-factor inputs and an arbitrary share of the relevant expenditure on maintenance (where this was not formally allocated in the survey data file) plus an interest charge on the stock of the resource (10 per cent of the closing value). The closing value includes an allowance for depreciation, sales of assets and purchases of new assets. Treatment of the land component of the final variable differed from that in the first three variables in that there was no depreciation allowance and, included in the land services, was expenditure on agistment and droving, and rates and taxes on the land resource.

Estimation

Two approaches to estimation are presented here. The first is that of Anderson (1973) applied to equation (1) and an analogous variance function, which is categorised as a 'crude' approach and model. The second is an error components model applied to equation (2).

The crude model

The crude model consisted of independent power function specifications for mean of production, $\bar{y}$, and for variance of production, $V[y]$, namely:

$$\log\bar{y}_i = \alpha_0 + \Sigma_k\alpha_k\log\bar{X}_{ki} + v_i, \tag{3}$$

$$\log V[y]_i = \beta_0 + \Sigma_k\beta_k\log\bar{X}_{ki} + w_i, \tag{4}$$

where the parameters are analogous to but different from those of equation (2);

$\bar{X}$ is a vector of mean inputs; and

$\bar{y}$ is mean output, averaged in each case over time for each of the $N = 38$ farms indexed by i.

The variation of production for each firm was computed as the average over time of the squared deviations between observed output and the transformed (antilog) predictions of mean output from equation (3).

As noted, the crudity of these specifications has several dimensions beyond merely the limitations of working only with power function forms. First, when elasticities are positive, the estimation via the log-linear equations with additive disturbances implies a restrictive multiplicative (positive marginal risk) specification of the mean production function. It was largely to avoid such an unfortunate specification that Just and Pope proposed their alternative error structure. Second, in equation (4) for variance, risk effects are specified explicitly and more flexibly (i.e. whether risk is 'increasing' or 'decreasing' depends on the signs of the β_k coefficients) but this is inconsistent with the assumption of a well behaved disturbance in equation (3).

Third, there is a logical difficulty of ascribing any feature of production (e.g. mean and variance) to mean levels of inputs. That is, aggregation over time to assess the effects of inputs sacrifices information on the productive effects of different levels of inputs in particular years on subsequent production.[3]

With such intrinsic limitations, what virtue is there in describing the crude model and associated results? Three reasons are offered: (a) some basis is required for contrasting a superior approach; (b) a link is forged with past related work; and (c), whilst it is inadequate on *a priori* grounds, perhaps the crude approach is deserving of some attention because of its overwhelming (comparatively speaking) ease and economy of estimation.

Taking the results reported in Table 1 at face value, the 'mean' production function seems plausible with positive elasticities of production (which sum to 1.17) and diminishing marginal returns, although the elasticity for sheep is remarkably small (especially relative to its standard error). The variance of production function has two remarkable features: (a) only two effects are (statistically) significantly non-zero (biological capital and buildings and land services) and these illustrate 'increasing risk' (positive marginal variance effects) and (b), of the effects not so clearly significant, two (labour and watering services) have negative marginal risk effects.

Whilst not denying any of the expressed reservations about these results, they carry an intuitive validity. Most inputs, and especially livestock carrying intensity, tend to increase production, and risk as measured by variance. However, some inputs (here, notably labour and watering facilities) serve to reduce variance and thus presumably lead to safer production. One might well quibble with measuring risk by variance. An alternative measure of some appeal is the coefficient of variation. The elasticity of this measure with respect to input k is $(0.5\beta_k - \alpha_k)$ which for inputs 1,2, . . ., 6 is -0.50, 0.72, -0.20, 0.05, -0.34 and 0.13, respectively. Viewed this way, the interpretation of marginal risk effects basically still stands (is rather reinforced for labour, livestock and water), although it does change for plant and machinery services.[4] This factor is now seen as a risk-reducing factor, and thus ex-

[3] Also, it is possible that the firms with a high variance of output also had a high variability in input levels. This will not necessarily be captured by the average input level in equation (4), but it will be captured by the more complicated model outlined later.

[4] The change in sign implies that, as plant and machinery services increase, the standard deviation of output will increase, but at a rate lower than that of mean output. No single measure provides an ideal accounting of risk, and the varying interpretation noted here can be resolved unambiguously only by complete specification of the relevant probability distribution and utility or preference functions.

D

TABLE 1

Summary of Regression Statistics for the Crude Model

k	Variable	Factor of production	Mean production	Variance of production
0	–	Level (log)	0.81 (0.56)[a]	4.45 (2.39)
1	L	Labour	0.19 (0.16)	−0.82 (0.67)
2	S	Biological capital	0.01 (0.16)	1.47 (0.67)
3	W	Watering services	0.05 (0.06)	−0.29 (0.24)
4	F	Fencing services	0.28 (0.10)	0.66 (0.40)
5	P	Plant and machinery services	0.39 (0.12)	0.09 (0.49)
6	B	Building and land services	0.25 (0.07)	0.77 (0.29)
		R^2	0.96	0.84
		SEE(log)	0.025	0.45
Sample size 38				

[a] Numbers in parentheses are respective standard errors.

penditures on plant can complement labour and watering expenditures in adding to the relative safety of production. Mechanised drought feeding is one exemplification of such an effect.

The heteroscedastic error-components model

An alternative model that exploits the data structure via an error-components model of the type discussed by Fuller and Battese (1974) and the Just-Pope heteroscedastic structure of equation (2) is briefly sketched. It is described more comprehensively by Griffiths and Anderson (1982).

Assume that there are T time series observations and N firms. Then write:

(5) $$y_{it} = \gamma \Pi_k X_{kit}^{\alpha_k} + \epsilon_{it},$$

where[5]

(6) $$\epsilon_{it} = (\mu_i + \lambda_t + \nu_{it}) \Pi_k X_{kit}^{\beta_k},$$

(7) $$E(\mu_i) = E(\lambda_t) = E(\nu_{it}) = 0,$$

and

(8) $$E(\mu_i^2) = \sigma_\mu^2,\ E(\lambda_t^2) = \sigma_\lambda{}^2,\ E(\nu_{it}^2) = \sigma_\nu^2,$$

where $i = 1,2, \ldots, N$, and $t = 1,2, \ldots, T$.

[5] As an alternative to equation (6) the terms μ_i and λ_t could appear additively rather than multiplied by $\Pi X_{kit}^{\beta k}$. However, this specification seems less realistic than the one employed because it assumes that the contribution of the time and firm effects to the variance of output does not depend on the level of input use.

In addition, we assume that the μ_i, λ_t and v_{it} are mutually uncorrelated for all i and t. Letting $\sigma^2 = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_v^2$ and $z_{it} = \Pi_k X_{kit}^{\beta_k}$, the above assumptions imply that the ϵ_{it} have the following properties:

$$E(\epsilon_{it}) = 0; \tag{9}$$

$$E(\epsilon_{it}^2) = \sigma^2 z_{it}^2; \tag{10}$$

$$E(\epsilon_{it}\epsilon_{is}) = \sigma_\mu^2 z_{it} z_{is} \text{ for } t \neq s; \tag{11}$$

$$E(\epsilon_{it}\epsilon_{jt}) = \sigma_\lambda^2 z_{it} z_{jt} \text{ for } i \neq j; \text{ and} \tag{12}$$

$$E\epsilon_{it}\epsilon_{js}) = 0 \text{ for } i \neq j \text{ and } t \neq s. \tag{13}$$

Equations (10) to (13) also represent the variance and covariance properties of y_{it}. In (10), the variance of output is a function of input levels, while equations (11) and (12) allow for the existence of non-zero correlation between outputs from the same firm in different time periods, or between outputs from different firms in the same time period. Outputs from different firms in different time periods are assumed to be uncorrelated.

The suggested six-step estimation procedure is outlined in the Appendix and described fully in Griffiths and Anderson (1982). The first step is to obtain the nonlinear least squares estimator $(\hat{\alpha}', \hat{\gamma})'$ for equation (5) parameters, $\alpha' = (\alpha_1, \alpha_2, \ldots, \alpha_K)$ and γ by minimising the sum of squares:

$$\epsilon'\epsilon = \Sigma_i \Sigma_t (y_{it} - \gamma \Pi_k X_{kit}^{\alpha_k})^2, \tag{14}$$

where $\underset{1xT}{\epsilon_i'} = (\epsilon_{i1}, \epsilon_{i2}, \ldots, \epsilon_{iT})$; and

$\underset{1xNT}{\epsilon'} = (\epsilon_1', \epsilon_2', \ldots, \epsilon_N')$.

To estimate the other parameters in the model and eventually to obtain a more efficient estimator for α, we follow Just and Pope (1978) and consider the logarithm of the square of equation (6):

$$\log(\epsilon_{it}^2) = \beta_0 + 2\Sigma_k \beta_k \log X_{kit} + w_{it}, \tag{15}$$

where $w_{it} = \log\{(\mu_i + \lambda_t + v_{it})^2\} - \beta_0$; and
$\beta_0 = E[\log\{(\mu_i + \lambda_t + v_{it})^2\}]$.

We shall denote ordinary least squares estimates of the parameters in equation (15) by $\hat{\beta}_0, 2\hat{\beta}_1, \ldots, 2\hat{\beta}_K$, and these are obtained by replacing the ϵ_{it}s with the nonlinear least squares residuals.

By adding to equations (7) and (8) the assumption that μ_i,λ_t and v_{it} are normal, an intermediate chi-square variable with one degree of freedom can be derived. Knowledge of the mean and variance of this variable can then be used to derive the stochastic structure of the disturbances in equation (15), namely zero mean, constant known variance ($E[w_{it}^2] =$ 4.9348) and covariances:

$$E[w_{it}w_{is}] = C_\mu^2 \text{ for } t \neq s; \tag{16}$$

$$E(w_{it}w_{jt}] = C_\lambda^2 \text{ for } i \neq j; \text{ and} \tag{17}$$

$$E[w_{it}w_{js}] = 0 \text{ for } t \neq s \text{ and } i \neq j. \tag{18}$$

TABLE 2

Summary of First-Stage and Final-Stage Statistics for the Heteroscedastic Model

k	Variable	Mean production $(\hat{\alpha}', \hat{\gamma})$	Mean production $(\tilde{\alpha}', \tilde{\gamma})$		Variance of production $(\hat{\beta}_0, 2\hat{\beta})$	Variance of production $(\tilde{\beta}_0, 2\tilde{\beta})$
0	–	2.81 (0.75)	3.34 (1.10)		1.97 (1.61)	1.92 (1.66)
1	L	0.25 (0.08)	0.16 (0.08)		−0.28 (0.44)	−0.29 (0.45)
2	S	0.59 (0.07)	0.79 (0.08)		1.13 (0.39)	1.02 (0.41)
3	W	−0.02 (0.03)	−0.04 (0.04)		−0.10 (0.17)	−0.12 (0.18)
4	F	0.11 (0.05)	0.12 (0.06)		−0.24 (0.28)	−0.18 (0.28)
5	P	−0.21 (0.07)	−0.07 (0.07)		0.02 (0.36)	0.15 (0.37)
6	B	0.20 (0.04)	0.07 (0.05)		0.69 (0.24)	0.68 (0.25)
	$\hat{\sigma}_v^2$	420	15.4	R^2	0.20	0.18
	$\hat{\sigma}_\mu^2$	–	5.4	$\hat{\sigma}_w^2$	4.987	4.875
	$\hat{\sigma}_\lambda^2$	–	4.7	F	15.36	13.72
				$\hat{C}_\mu^2$	–	0.09
				$\hat{C}_\lambda^2$	–	0.07
				$\hat{C}_v^2$	–	4.77

Sample size 380

The covariances C_μ^2 and C_λ^2 depend on σ_μ^2/σ^2 and $\sigma_\lambda^2/\sigma^2$, respectively. These covariances can be estimated from estimates of σ^2, σ_μ^2 and σ_λ^2 (see the Appendix) and, in turn, used in a generalised least squares regression of log $\hat{\epsilon}_{it}^2$ on the logX_{kit} variables to yield estimates $\tilde{\beta}_0, 2\tilde{\beta}_1, 2\tilde{\beta}_2, \ldots, 2\tilde{\beta}_K$ which exploit the times-series/cross-section structure of the data. Finally, these estimates of β can be used to obtain revised, now asymptotically efficient, estimates $(\tilde{\alpha}', \tilde{\gamma})$, via a further nonlinear least squares estimator. Naturally, this cycle of refinement could be continued, but further iterations would not lead to a gain in asymptotic efficiency.

The results of applying the procedure to the data for individual farms and years are reported in Table 2, along with the estimates from the first stage (Appendix Steps 1 and 2).[6]

[6] The standard errors of the nonlinear least squares estimates are those provided by the Gauss-Newton algorithm. Specifically, for the first stage estimates, they are given by the square roots of the diagonal elements of $\hat{\sigma}_\epsilon^2 \left[\frac{\partial\epsilon}{\partial\theta'}\right]'\left[\frac{\partial\epsilon}{\partial\theta'}\right]$, where $\hat{\sigma}_\epsilon^2 = \hat{\epsilon}'\hat{\epsilon}/NT$, $\theta' = (\gamma, \alpha')$ and the matrix of partial derivatives is evaluated at $(\hat{\gamma}, \hat{\alpha}')$. For the final stage estimates the relevant matrix is $\hat{\sigma}_\epsilon^{2**} \left[\frac{\partial\epsilon^{**}}{\partial\theta'}\right]'\left[\frac{\partial\epsilon^{**}}{\partial\theta'}\right]$ where the elements of the vector ϵ^{**} are defined in equation (A9). Under certain conditions (Judge et al., 1980, Chapter 17), and our model specification, the standard errors of the final stage are asymptotically justified. However, those of the first stage are inappropriate because they incorrectly assume $E[\epsilon\epsilon'] = \sigma_\epsilon^2 I$.

The mean production results are generally more plausible than those from Table 1, especially in acknowledging the central importance of biological capital, but with the difficulty that production elasticities for watering services and plant and machinery services are negative. Happily, in the final stage these latter two estimates are not significantly different from zero, indicating that their population counterparts could be positive. The impact of accounting for the data and error structures is also apparent through the differences between the respective coefficients, although standard errors are little changed.

The results for the variance of production functions are, again, perhaps of greater intrinsic interest. These are somewhat disappointing in terms of explanatory power and the extent of statistical significance in individual effects. The 'risk elasticities' reported here ($2\hat{\beta}_k$ and $2\bar{\beta}_k$) are to be interpreted in the same way as those of Table 1 (earlier designated in equation (4) as β_k).

There are some broad similarities between the marginal risk effects implicit in Tables 1 and 2. Of particular note are the strongly positive risk-inducing effects of biological capital or sheep and land and building services, and the understandable but not so clear-cut risk-reducing effects of labour and watering services. The most striking difference between the two models is for fencing services which, in Table 2, is now revealed as having a variance-reducing effect, albeit not strongly statistically significant. The close similarities of the final two columns of Table 2 mean that, at least for this set of data, OLS estimation of β is sufficient, and the gain in efficiency presumably slight.

The alternative measure of a marginal risk derived also for the crude model can again be computed. Those reported here are, for the elasticity of input k on the coefficient of variation, found as $\tilde{\beta}_k - \tilde{\alpha}_k$. They are respectively, -0.45, 0.23, -0.08, -0.30, 0.22 and 0.61.

Conclusion

Our results are rather more diverse and less satisfactory than we would have wished—a situation which seems to reflect the state of the art of estimation of firm-level production functions.

The low R^2s and high standard errors in the estimation of the variance function make it impossible to conclude that our heteroscedastic error components model is necessarily a good specification. Nevertheless, the signs of the coefficients are in sufficient agreement with our *a priori* expectations to suggest that we are moving in the right direction. Despite our inability to estimate accurately the separate effects, it seems likely that the various inputs do indeed influence the riskiness of production and that their relative contributions differ. It appears that animal stocking and land improvements greatly enhance variability of output and, somewhat equivocally, labour, stock watering facilities and possibly fencing facilities mitigate such variability.

As to estimation method, our multi-stage procedure has been argued to be superior to the crude approach and to possess desirable statistical properties. However, it is far from being straightforward, and a minimal requirement for employing it is access to a fairly flexible generalised non-linear least squares computer program.

APPENDIX

The Multi-Stage Estimation Procedure

Step 1: Use nonlinear least squares to find $(\hat{\alpha}', \hat{\gamma})$ which minimises $\epsilon'\epsilon$, and obtain the corresponding residuals $\hat{\epsilon}$.

Step 2: Regress $\log\hat{\epsilon}_{it}^2$ on the $\log X_{kit}$s to obtain estimates $\hat{\beta}_0$, $2\hat{\beta}_1$, $2\hat{\beta}_2$, . . ., $2\hat{\beta}_K$, and corresponding $\hat{z}_{it}$s.

Step 3: Estimate the required variances as:

(A1) $$\hat{\sigma}^2 = \exp(\hat{\beta}_0 + 1.2704),$$

(A2) $$\hat{\sigma}_\mu^2 = \frac{1}{TN(T-1)/2} \sum_{i=1}^{N} \sum_{t=1}^{T-1} \sum_{s=t+1}^{T} \frac{\hat{\epsilon}_{it}\,\hat{\epsilon}_{is}}{\hat{z}_{it}\,\hat{z}_{is}},$$

(A3) $$\hat{\sigma}_\lambda{}^2 = \frac{1}{TN(N-1)/2} \sum_{t=1}^{T} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\hat{\epsilon}_{it}\,\hat{\epsilon}_{jt}}{\hat{z}_{it}\,\hat{z}_{jt}},$$

(A4) $$\hat{\sigma}_v^2 = \hat{\sigma}^2 - \hat{\sigma}_\mu^2 - \hat{\sigma}_\lambda^2.$$

Step 4: Use these variance estimates to estimate the covariances C_μ^2 and C_λ^2 (Johnson and Kotz 1972, p. 226), albeit at the cost of evaluating infinite sums involving gamma functions, as:

(A5) $$\hat{C}_\mu{}^2 = \sum_{h=1}^{\infty} \left(\frac{\hat{\sigma}_\mu^2}{\hat{\sigma}^2}\right)^{2h} \frac{h!\ \Gamma(½)}{h^2\ \Gamma(h+½)},$$

and analogously for $\hat{C}_\lambda^2$. The following related items can then be computed:

(A6) $$\begin{aligned} &\hat{C}_v^2 = 4.9348 - \hat{C}_\mu^2 - \hat{C}_\lambda^2,\\ &\hat{C}_1^2 = T\hat{C}_\mu^2 + \hat{C}_v^2,\\ &\hat{C}_2^2 = N\hat{C}_\lambda^2 + \hat{C}_v^2,\\ &\hat{C}_3^2 = T\hat{C}_\mu^2 + N\hat{C}_\lambda^2 + \hat{C}_v^2,\\ &b_1 = 1 - \hat{C}_v/\hat{C}_1,\\ &b_2 = 1 - \hat{C}_v/\hat{C}_2, \text{ and}\\ &b_3 = b_1 + b_2 - 1 + \hat{C}_v/\hat{C}_3. \end{aligned}$$

Step 5: Use generalised least squares to regress $u_{it} = \log\hat{\epsilon}_{it}^2$ on the $x_{kit} = \log X_{kit}$ variables to obtain estimates $\tilde{\beta}_0$, $2\tilde{\beta}_1$, $2\tilde{\beta}_2$, . . ., $2\tilde{\beta}_K$. This is achieved (Judge et al. 1980, p. 343) by applying OLS to the transformed equation

(A7) $$u_{it}^* = \beta_0(1 - b_1 - b_2 + b_3) + 2 \sum_{k=1}^{K} \beta_k x_{kit}^* + \hat{w}_{it}^*,$$

where, using u_{it}^* as an example, a transformed observation is given by:

(A8) $$u_{it}^* = u_{it} - b_1\bar{u}_{i\cdot} - b_2\bar{u}_{\cdot t} + b_3\bar{u}_{\cdot\cdot},$$

with $\bar{u}_{i\cdot} = \Sigma_t u_{it}/T$, $\bar{u}_{\cdot t} = \Sigma_i u_{it}/N$, and $\bar{u}_{\cdot\cdot} = \Sigma_t\Sigma_i u_{it}/NT$. Let this estimator be denoted by $\tilde{\beta}$.

Step 6: Use nonlinear least squares to find $(\tilde{\alpha}', \tilde{\gamma})$ which minimises $\Sigma_i \Sigma_t \epsilon_{it}^{**2}$ where, the ϵ_{it}^{**} are obtained from:

$$\epsilon_{it}^{**} = \epsilon_{it}^{*} - a_1 \bar{\epsilon}_{i.}^{*} - a_2 \bar{\epsilon}_{.t}^{*} + a_3 \bar{\epsilon}_{..} \tag{A9}$$

where $\epsilon_{it}^{*} = \epsilon_{it}/\tilde{z}_{it}$, $\bar{\epsilon}_{i.}^{*} = \Sigma_t \epsilon_{it}^{*}/T$, $\bar{\epsilon}_{.t}^{*} = \Sigma_i \epsilon_{it}^{*}/N$,

$\bar{\epsilon}_{..}^{*} = \Sigma_t \Sigma_i \epsilon_{it}^{*}/NT$,

$a_1 = 1 - \hat{\sigma}_v/\hat{\sigma}_1$, $a_2 = 1 - \hat{\sigma}_v/\hat{\sigma}_2$, $a_3 = a_1 + a_2 - 1 + \hat{\sigma}_v/\hat{\sigma}_3$,

$\hat{\sigma}_1^2 = T\hat{\sigma}_\mu^2 + \hat{\sigma}_v^2$, $\hat{\sigma}_2^2 = N\hat{\sigma}_\lambda^2 + \hat{\sigma}_v^2$, and $\hat{\sigma}_3^2 = T\hat{\sigma}_\mu^2 + N\hat{\sigma}_\lambda^2 + \hat{\sigma}_v^2$.

References

Anderson, J. R. (1972), 'Risk and farm size in the pastoral zone', *Australian Journal of Agricultural Economics* 16(1), 1-10.

——— (1973), 'Sparse data, climatic variability, and yield uncertainty in response analysis', *American Journal of Agricultural Economics* 55(1), 77-82.

———, Dillon, J. L. and Hardaker, J. B. (1977), *Agricultural Decision Analysis*, Iowa State University Press, Ames.

——— and Jodha, N. S. (1973), 'On Cobb-Douglas and related myths', *Economic and Political Weekly* 8(26), A65-6.

Blair, R. D. and Lusky, R. (1975), 'A note on the influence of uncertainty on estimation of production function models', *Journal of Econometrics* 3(4), 391-4.

Diewert, W. E. (1974), 'Applications of duality theory', in M.D. Intriligator and D. A. Kendrick (eds), *Frontiers of Quantitative Economics, Volume II*, North Holland, Amsterdam, 106-71.

Duloy, J. H. (1959), 'Resource allocation and the fitted production function', *Australian Journal of Agricultural Economics* 3(2), 75-85.

El-Issawy, I. H. (1970), 'Towards a multiproduct production function', *Farm Economist* 11(2), 515-25.

Fuller, W. A. and Battese, G. E. (1974), 'Estimation of linear models with crossed-error structure', *Journal of Econometrics* 2(1), 67-78.

Griffiths, W. E. and Anderson, J. R. (1982), 'Using time-series and cross-section data to estimate a production function with positive and negative marginal risks', *Journal of the American Statistical Association* (forthcoming).

Heady, E. O. and Dillon, J. L. (1961), *Agricultural Production Functions*, Iowa State University Press, Ames.

Johnson, N. L. and Kotz, S. (1972), *Continuous Multivariate Distributions*, Wiley, New York.

Judge, G. G., Griffiths, W. E., Hill, R. C. and Lee, T-C. (1980), *The Theory and Practice of Econometrics*, Wiley, New York.

Just, R. E. and Pope, R. D. (1978), 'Stochastic specification of production functions and economic implications', *Journal of Econometrics* 7(1), 67-86.

——— and ——— (1979), 'On the relationship of input decision and risk', in J. A. Roumasset, J-M Boussard and I. Singh (eds), *Risk, Uncertainty and Agricultural Development*, SARCGSRA, College, Laguna, Philippines, 177-9.

McKay, L., Lawrence, D. and Vlastuin, D. (1980), 'Input demand and substitution in the Australian sheep industry', *Review of Marketing and Agricultural Economics* 48(2), 57-70.

———, ——— and ——— (1981), Production flexibility and technical change in Australia's major mixed farming region, the wheat-sheep zone. Paper presented to the 25th Annual Conference of the Australian Agricultural Economics Society, Christchurch, February.

Magnússon, G. (1969), *Production under Risk*, Almqvist and Wiksells, Uppsala.

Mundlak, Y. (1964), 'Transcendental multiproduct production functions', *International Economic Review* 5(3), 273-83.

Pope, R. D. and Just, R. E. (1977), 'On the competitive firm under production uncertainty', *Australian Journal of Agricultural Economics* 21(2), 111-8.

Zellner, A., Kmenta, J. and Drèze, J. (1966), 'Specification and estimation of Cobb-Douglas production function models', *Econometrica* 34(4), 784-95.