A NOTE ON THE ECONOMICS OF GRAZING AND ITS EXPERIMENTAL INVESTIGATION*

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Some notes prove something ; others disprove something. This one does neither. It merely sketches a *simple* model of the grazing complex. Of itself, the model is no more than an attempt to specify the more important economic relationships of the grazing complex in an explicit, orderly fashion. Although of undoubted importance, these relationships so far appear to have received little attention.

Despite its naivete, the model establishes the virtual impossibility of estimating the parameters ideally needed to specify a profit maximizing system of grazing, even if we assume away climatic and price uncertainty, and the diversity of pasture types, history and location. On the positive side, the model suggests a framework for assessing grazing experiments in terms of their relevance for economic *analysis* (which, of course, is not to be confused with the simple cost accounting of most agricultural experimenters). Concomitantly, the model gives some clues to the experimental approach needed for the elucidation of the economics of grazing. In illustration, a few examples are presented of the types of experimental design that are perhaps best suited for investigation of the grazing complex.

Difficulties

Any respectable grazing model must take explicit account of three difficulties inherent in the nature of the grazing complex.

First, there is the role of time in the production process. Not only are there dynamic influences in the usual economic sense ; as well, the array of possible *time sequences* of input injections and output harvests implies innumerable *systems* of grazing, each system having its own production function. The economic problem, therefore, is not simply to decide on the level of input and output ; it is also necessary to choose between alternative grazing systems.

Second, livestock make decisions and have appetites ; they, not the grazier, will usually decide how much of what feed will be eaten. Even if livestock were consistent in their decisions, it would take much research to estimate the decision pattern for even a small array of feed situations.

Third, under any system of management, the grazing complex involves interdependent phenomena. Livestock influence pasture output ; pasture output influences livestock production. And when livestock have a

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choice of feeds, the choice of a consumption pattern for each feed is not made independently of the other feeds. Interdependence again exists, at least so long as the total feed offering is sufficient for survival. In consequence, any "realistic" grazing model must allow for the *simultaneous* determination of some variables.[1] Suffice to note that most agricultural researchers have given little *explicit* consideration to such problems, although they have noted that pasture and livestock production "form sensitively interacting phases that must be brought into delicately integrated adjustment if maximum production is to be secured".[2]

Given the difficulties listed above, no apology is offered for the simplicity of the model to be presented. The approach adopted is to balance scarce feed supplies and revenue-producing demands for feed within the restraints imposed by profit maximization and the cyclical nature of pasture growth and deterioration. Throughout, the relevant relations are expressed in functional form with an assumption that empirical investigation of such a model would utilize modern computing equipment on an iterative basis as need be. As becomes obvious, the analysis is on a quite different level from a mathematical programming attack on the grazing complex either in its inter- or intra-cyclical aspects. Rather, it draws attention to some of the structural relations and restrictions that should be allowed for, approximated, guessed, or ignored in a mathematical programme seeking guides for farmer decisions and hence justifiably able to ignore the intricacies of an ideal approach.

Assumptions

We distort the real world by assuming the following background.

(a) The firm operates on a fixed land area of uniform soil type devoted solely to established pasture of uniform character and history ;

(b) Newly produced pasture is of uniform nutritional quality regardless of its production date ;

(c) Perfect association, for a given type of season, between time of the pasture year and exogenous climatic influences ;

(d) A single class of livestock and a single fertilizer are the only variable endogenous inputs used in pasture production ; and that fertilizer, if used, is applied at the initial time-point of the production cycle ;

(e) Only a single type of conserved pasture is produced and only a single type of feed is purchased ;

(f) Sales of agistment or conserved fodder are infeasible ;

(g) Storage costs of conserved pasture and purchased feed are zero and there are no mechanical losses in conserving pasture or feeding out supplementary feed ;

(h) The rate of stocking (on a head basis) is constant over land and time ;

(i) Initially, only a single grazing system is to be considered. This

1. Reference to the potential usefulness of simultaneous equation models of biological processes is to be found in : Williams, E. J. "Simultaneous regression equations in experimentation", *Biometrika* 45 : 96-110, 1958 ; Lucas, H. L. "Experimental designs and analyses for feeding efficiency trials with dairy cattle". In Hoglund, C. R. *et al.*, eds. *Feed Utilization by Dairy Cows*, p. 188. Iowa State Univ. Press, Ames, 1959 ; and Heady, E. O. and Dillon, J. L., *Agricultural Production Functions*. Iowa State Univ. Press, Ames, 1960, pp. 198-202.

2. Wallace, L. R. "Animal production from grassland", *Aust. J. Sci.* 21 : 167, 1959. As is typical among agriculturalists, this quotation shows excessive preoccupation with maximum production.

system is specified by a given time sequence of purchased feed offerings and a given time sequence of minimal livestock liveweights (or some other criterion of the standard of livestock maintenance) ;

(j) The price and climatic regimes are known with certainty ;

(k) Profit is to be maximized merely over the twelve months of the pasture cycle, the current or initial time point being the beginning of the pasture flush ;

(l) A £ due in the future is worth a £ now.

The remaining assumptions or postulates about the workings of the grazing complex constitute, in large part, the model. They are most easily expressed symbolically.

Notation

Throughout, a dot over a variable is used to denote its time rate of change. Time subscripts are used in the form X_t to indicate the value of X at time t ; and in the form $X_{0,\,t}$ to indicate the value of $(X_t - X_0)$. Where necessary, an asterisk is used to distinguish levels of a variable decided by the grazier rather than by the livestock. In order of occurrence, the more important symbols used are as follows.

R^* : minimum allowable liveweight of livestock ;

t : time (in weeks, $0 \leqslant t \leqslant 52$) ;

B^* : quantity of purchased feed fed out ;

P : quantity of pasture produced ;

S : number of livestock grazed ;

F : input of fertilizer ;

H : quantity of pasture conserved ;

D : quantity of pasture deteriorated ;

C : quantity of pasture consumed ;

B : quantity of purchased feed consumed ;

E : quantity of conserved pasture consumed ;

Y : quantity of livestock product ;

R : liveweight of livestock ;

Z : net revenue ;

p_i : unit cost or price of the i-th good ;

E^* : quantity of conserved pasture fed out.

Grazing Model

For a given grazing system, specified by the preselected time sequences of minimal livestock liveweight (R^*_t) and purchased feed input ($B^*_{0,t}$), the model involves a system of eight equations, all except one of these being differential equations. As well, there are four restrictions (apart from the obvious non-negativity requirements) that ensure feasibility of the grazing system within the relevant range specified by the mechanics of grazing production. Infallibility is *not* claimed for the choice of variables entering the various relationships, even though errors due to unspecified variables have not been written into the equations.

The major relations or directions of influence allowed for by the model are depicted in Figure 1. The specific relations of the model are as follows.

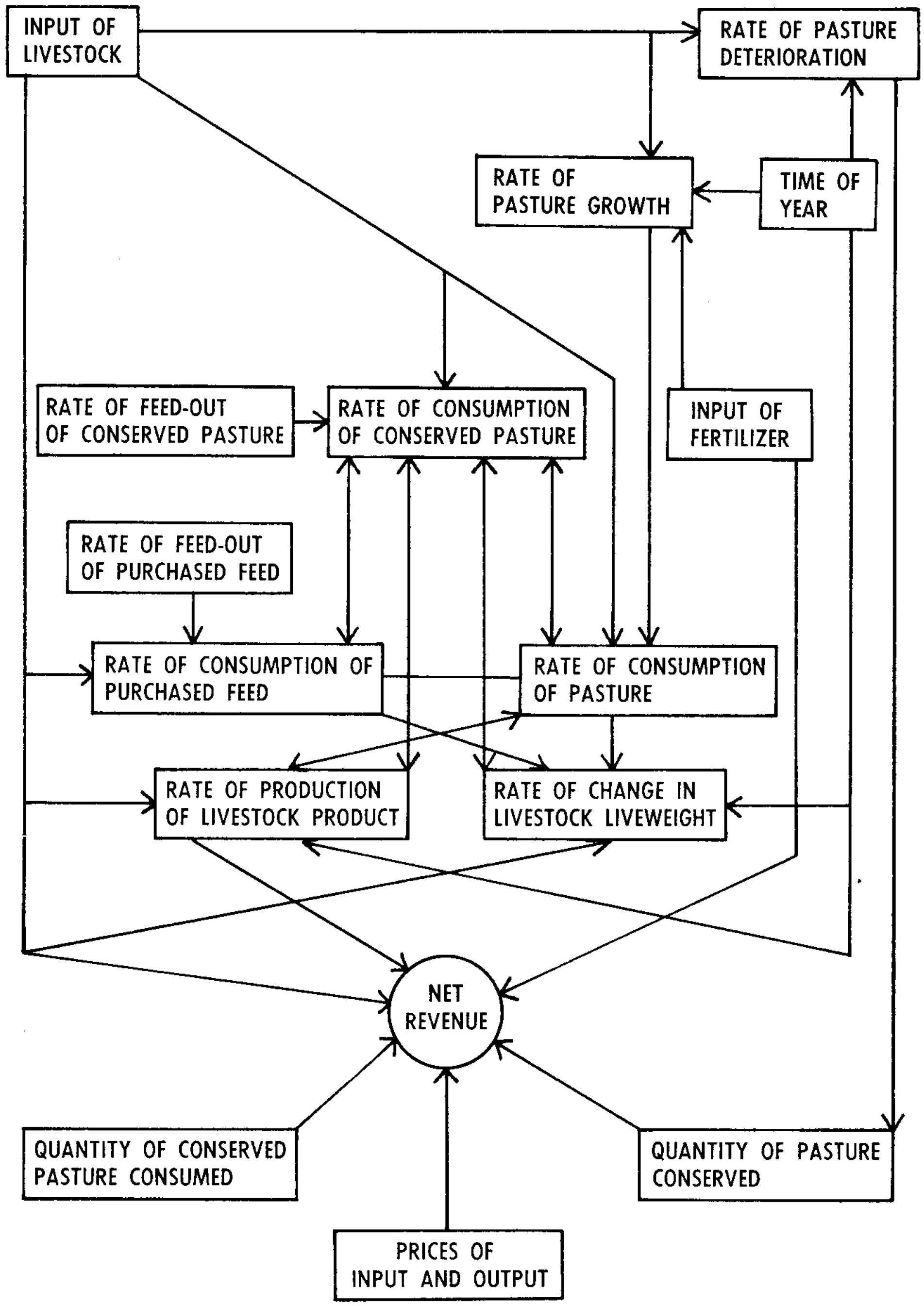

Figure 1.

Pasture Production :

$$\dot{P} = f_1(S,F,t) \tag{1}$$

Although the input of livestock and fertilizer in prior pasture cycles undoubtedly influence the rate of pasture production, these prior values are taken as given in the current context. Doubtless, a more realistic model would need to include the quantity of pasture conserved $(H_{0,t})$

in f_1. To make analysis feasible, this would necessitate further circumscribing the grazing system by a given $H_{0,t}$ sequence. For the purpose at hand the return from this additional complexity would not be worth its cost.

Pasture deterioration :

$$\dot{D} = f_2(S,t) \tag{2}$$

The rate of pasture deterioration is assumed to be measured in such a way that remaining pasture can be assessed in constant units (say on an absolute protein basis or some such). Some might argue that f_2 should include the stock of pasture on hand after livestock and mower have had their fill. For the real world they may be right. None the less, equation 2 suffices for present purposes.

Feed consumption :

$$\dot{C} = f_3(S,\dot{P},\dot{B},\dot{E},\dot{Y},\dot{R}) \tag{3}$$

$$\dot{B} = f_4(S,\dot{C},\dot{E},\dot{Y},\dot{R}) \tag{4}$$

$$\dot{E} = f_5(S,\dot{C},\dot{B},\dot{Y},\dot{R}) \tag{5}$$

These feed consumption relations reflect the decision making capabilities of livestock. Maybe the exclusion of the rate of pasture production ($\dot{P}$) from f_4 and f_5 implies non-negligible errors. Conversely, perhaps the rate of livestock production ($\dot{Y}$) should be excluded from f_3, f_4 and f_5. Who can say?

Livestock production and maintenance :

$$\dot{Y} = f_6(S,\dot{C},\dot{B},\dot{E},\dot{R},t) \tag{6}$$

$$\dot{R} = f_7(S,\dot{C},\dot{B},\dot{E},\dot{Y},t) \tag{7}$$

If animal product and liveweight are synonymous, equations 6 and 7 collapse to a single equation.

Profit :

Denoting fixed cost by K and the unit price or cost of the i-th good by p_i, net revenue is determined by :

$$Z = p_y Y_{0,52} - p_s S - p_h H_{0,52} - p_e E^*_{0,52} - K. \tag{8}$$

This profit function reflects two further assumptions. First, that there is no variable cost in the harvesting of livestock product, and second, that pasture conservation involves no set-up cost. The fixed cost K includes outlay on the purchase and distribution of purchased feed.

Restrictions :

Apart from obvious non-negativity requirements, there are four restrictions that must be met. The first of these relates to livestock maintenance. Using liveweight as a maintenance criterion, this restraint can be formulated as the necessity for the actual liveweight of livestock at any time-point (R_t) to equal or exceed the prespecified minimal allowable liveweight of livestock for the same time-point (R^*_t). This minimal liveweight may or may not vary over the production cycle since it depends on the grazier's discretion. Thus we require :

$$R_t \geqslant R^*_t. \tag{9}$$

The second restriction, specified below, accommodates the necessity for the quantity of purchased feed consumed ($B_{0,t}$) to equal or be less than the quantity of purchased feed fed out ($B^*_{0,t}$) over the period from 0 to t.

$$B^*_{0,t} \geqslant B_{0,t} \tag{10}$$

The remaining two restrictions, detailed in relation 11, ensure that only " excess " pasture is conserved and that the consumption of conserved pasture does not exceed its supply at any stage of the production cycle. Denoting the reciprocal of the transformation coefficient between pasture and conserved pasture by k, we require :

$$P_{0,t} - D_{0,t} - C_{0,t} \geqslant H_{0,t} \geqslant kE_{0,t}. \tag{11}$$

Except for the maintenance requirement, all the above restrictions would be automatically satisfied in the real world. This would not be the case, however, in predictive or normative manipulation of the model. Hence the necessity for the above precise specification of the restrictions.

Profit Maximization

Maximum profit under the given system of grazing (i.e. for given time sequences of minimal liveweights and purchased feed feed-outs) implies maximization of Z of equation 8 subject to the restrictions 9, 10 and 11. This necessitates some consideration of the econometric nature of the model, assuming (for the moment) that the requisite observations on the variables are available for estimation purposes.

Within the grazing system depicted by equations 1 to 7 there are three variables that are exogenous *relative to livestock behaviour.* These are S, F, and t, the first two being under the grazier's control. The remaining seven variables—$\dot{P}$, $\dot{D}$, $\dot{C}$, $\dot{B}$, $\dot{E}$, $\dot{Y}$ and $\dot{R}$—are determined within the system. Since $\dot{P}$ and $\dot{D}$ depend only on exogenous variables, satisfactory estimation of equations 1 and 2 would be no great problem. However, even under the usual simplifying econometric assumptions, equations 3 to 7 are underidentified so that the structural coefficients of these five relations could not be estimated.[3] But this underidentification does not matter for our purpose of profit maximization. We are merely interested in predicting the behaviour of the system for various inputs of livestock and fertilizer, the variables under the grazier's control. Accordingly, estimates of the reduced forms of equations 3 to 7 (i.e., their expression in terms of the endogenous variables) suffice. Making use of the fact that profit maximization implies all pasture conserved is consumed, the profit function could then be derived as a function of the inputs of livestock and fertilizer, together with the relevant price coefficients as detailed in equation 12.

$$Z = f_8(S,F\ ; p_y,p_s,p_f,p_h,p_e) \tag{12}$$

3. Thus should underidentification prove to be a general feature of biological systems, simultaneous equation models would be of little use to biologists as a fundamental research device.

Maximization of Z of equation 12 subject to the restrictions 9, 10 and 11 then yields the optimal level of livestock and fertilizer for the given grazing system. The optimal quantity of conserved pasture is given by the reduced form of equation 5 with livestock and fertilizer each at their optimal level. A harvest sequence to attain this quantity of conserved pasture could then be deduced from the amount of excess pasture available over the pasture cycle, as defined by restriction 11 with livestock and fertilizer each at their optimal level.

Still, there would be no guarantee that this grazing system would be efficient. Efficiency will only prevail if no purchased feed fed out is wasted, i.e. if $B^*_{0,t}$ equals $B_{0,t}$. If any purchased feed is not consumed, fixed costs could be reduced without decreasing production. However, once optimal livestock and fertilizer inputs have been determined for the initial grazing system, an efficient system could be deduced by nominating (on the basis of the reduced form of equation 4) a sequence of purchased feed offerings that allowed no waste.

So much for a single grazing system. In fact, the model is relevant to an infinity of efficient grazing systems. To specify the optimal system, it would be necessary to compare the profitability of each efficient system. Obviously this would be impractical ; it implies the preclusion of any ideal experimental approach to the determination of the necessary empirical data—even if the simple model used here did mirror the real world.

Experimental implications

Ideally, an experiment yielding data to estimate the reduced form of the profit function and enable profit maximization under the relevant restraints for a given grazing system would necessitate observations on $\dot{P}$, $\dot{D}$, $\dot{C}$, $\dot{B}$, $\dot{E}$, $\dot{Y}$ and $\dot{R}$ for an array of livestock and fertilizer combinations.

Assuming a linear profit function and availability of mechanical procedures for making the necessary observations, a minimum of four sets of observations would be required for least-squares estimation. However, for reasonably efficient estimation at least, say, 15 to 20 sets of observations would be needed. With a minimum of three animals per experimental unit, this implies a quite expensive experiment to investigate a single grazing system. Allowing for repetition of the experiment to encompass an array of grazing systems, and partial replication for the estimation of experimental error, the overall experiment would require a very large input of research resources quite apart from any expense involved in actually making the required observations. Moreover, some of the experimental grazing systems might be infeasible and hence void for estimational purposes. Since research resources are not free goods, idealistic estimation of our naive model (even if it were " true ") appears impractical. Obviously, experimental specification of a more realistic model involving a wider array of inputs, spillover effects between years, variable stocking rates over land and time, prior pasture history, probabilistic influences, etc. would be virtually impossible—and surely ridiculous in terms of the diversity of pasture types, history and location. Accordingly, experimental research into the economics of grazing must fall back to relatively simple experiments with primary emphasis on the relation between livestock product and the grazier's input of fertilizer, livestock and supplementary feed ; and, apart from the recognition of its existence, pay little account to the confounding influence of such factors as livestock behaviour, minimal livestock maintenance standards, and the time-pattern of input injections. Initially, the best way to handle the latter factors would be to set them at the levels in common use among graziers. This, in fact, is about what grazing researchers do—except for their usual cardinal error of failing to consider stocking rate, feed supply and fertilizer *concurrently* over a satisfactory number and range of combinations. For example, grazing trials investigating supplementary feeding have usually been conducted as single-variable experiments with stocking rate and fertilizer held fixed. As one economist has recently emphasized, this verges on the ludicrous since " within wide limits you can get any experimental response you want according to the stocking rate set ".[4] And yet extension inferences are often plucked from such single-variable trials with the implicit assumption that they are efficient sources of money-making advice for graziers. However, it could hardly be otherwise since most agriculturalists (and their statistical advisers) have failed to realize that the economist's position relative to experimentation is much the same as the statistician's : unless consulted in the planning stage of an experiment, it is most unlikely that he will be able to analyse the experimental results meaningfully.

Experimental Designs

Granted the wastefulness for economic analysis of the single-variable approach to grazing research, how might more reasonable experiments be designed ? The " physical " end point of the experimental observations should be an estimate of a response function along the lines of

$$Y_{0,t} = f_9(S,F,B^*_{0,t},H_{0,t}) \qquad (13)$$

for given t ; or some simpler function with one of the variables—preferably fertilizer or pasture conserved—held fixed.[5] To this end any number of experimental designs would be statistically feasible ;[6] for instance, complete or fractional factorials or composite designs might be used. However, given the relative scarcity of research resources and assuming the reasonableness of a (second order) polynomial approximation, the response function might best be estimated via a (second order) rotatable design.[7] [8] Such designs have the great attraction of being

4. Lloyd, A. G. " Experiments, the farmer and the economist ", *Agros*, 1961. Faculty of Agriculture, Univ. of Melbourne. (In press.)

5. Should the quantity of pasture conserved differ from the quantity of conserved pasture fed out, some adjustment of the variables entering equation 13 would be necessary. Moreover, whatever the variables entering the response function, it would be desirable to have the function estimated for an array (say " good, average and poor ") of climatic regimes.

6. See : Heady and Dillon, *op. cit.*, pp. 150-87 ; and Cochran, W. G. and Cox, G. M., *Experimental Designs*. Wiley, New York, 1957. Ch. 8A.

7. For discussion of rotatable and nearly-rotatable designs see : Heady and Dillon, *ibid.* ; Bose, R. C. and Draper, N. R. " Second order rotatable designs in three dimensions ", *Annals Math. Stat.* 30 : 1097-1112, 1959 ; Draper, N. R. " Second order rotatable designs in four or more dimensions ", *Annals Math. Stat.* 31 : 23-33, 1960 ; Box, G. E. P. and Behnken, D. W. *A class of three level second order designs for surface fitting*. Princeton Univ., Statistical Techniques Research Group Technical Report No. 26, Dec. 1958 ; De Baun, R. M. " Response surface designs for three factors at three levels ", *Technometrics* 1 : 1-8, 1959 ; and Dykstra, O. " Partial duplication of response surface designs ", *Technometrics* 2 : 185-95, 1960.

8. Designs oriented to the estimation of non-polynomial response functions are discussed in : Box, G. E. P. and Lucas, H. L. " Design of experiments in non-linear situations ", *Biometrika* 46 : 77-90, 1959 ; and Box, G. E. P. " Use of statistical methods in the elucidation of basic mechanisms ", *Bull. Inst. int. Statist.* 36 : 215-25, 1958.

specifically oriented to response surface estimation.[9] Moreover, these designs are relevant to more than just economic analysis ; they only assume the researcher's aim is the scientific one of purposive manipulation of the response complex to satisfy some criterion. This objective might just as easily (but less logically) be maximum physical product as maximum profit. In either case an estimate of the response function is needed ; mere estimation of mean response at a few arbitrary factor levels to test the significance of differences in response between factor levels is not enough. (Of course, like the dairy industry, such estimation of mean responses and significant differences has some role to play—but not to the extent that resources are wasted.)

An example of a suitable three factor second order rotatable design with, say, stocking rate, fertilizer and one type of supplementary feed each at five levels is the design in $(S,F,B^*_{0,t})$ or $(S,F,H_{0,t})$ space with experimental points located at $(0, \pm 1.902, \pm 1.176 ; \pm 1.176, 0, \pm 1.902 ; \pm 1.902, \pm 1.176, 0)$ together with eight replications of the center point (0, 0, 0).[10] This design involves only 20 treatments (compared with 125 for a single replicate of a complete factorial arrangement) and would allow seven degrees of freedom for experimental error. If the three variables were restricted to three levels each, a reasonable design would be one with the twelve treatments $(0, \pm 1, \pm 1 ; \pm 1, 0, \pm 1 ; \pm 1, \pm 1, 0)$ and four replications of the center point.[11] Again assuming a second order polynomial estimate of the response function, this 16 treatment nearly-rotatable orthogonal design would provide three degrees of freedom for experimental error. In terms of both statistical and economic efficiency, it compares most favourably with a complete factorial arrangement of three factors at three levels.

For grazing trials involving four variables, say livestock, fertilizer, supplementary feed and pasture conservation, the following arrangement with each factor at five levels would yield a satisfactory second order rotatable design so long as at least four replicates of the central treatment were used :[12]

$$
\begin{array}{lrrrr}
(& \pm a, & \pm a, & \pm a, & \pm .765a\ ), \\
(& \pm 1.682a, & 0, & 0, & \pm .765a\ ), \\
(& 0, & \pm 1.682a, & 0, & \pm .765a\ ), \\
(& 0, & 0, & \pm 1.682a, & \pm .765a\ ), \\
(& 0, & 0, & 0, & \pm 2.049a\ ), \\
(& 0, & 0, & 0, & \pm 1.122a\ ), \\
(& 0, & 0, & 0, & 0\ ),
\end{array}
$$

where $a = [.03661(32 + n)]^{\frac{1}{2}}$ and n is the number of replications of the central treatment. Thus for $n = 4$, a would be 1.148 ; and there would be 36 treatments yielding three degrees of freedom for experimental

9. See Box, G. E. P. and Draper, N. R. " A basis for the selection of a response surface design ", *J. Amer. Stat. Assoc.* 54 : 622-54, 1959 ; and Box, G. E. P. " Fitting empirical data ", *Annals New York Acad. Sci.* 86 : 792-816, 1960.

10. Adapted from Heady and Dillon, *op. cit.*, p. 179. To illustrate the notation, the treatments implied by $(0, \pm 1.902, \pm 1.176)$ are the following coded combinations of S, F, and B^* : (0, 1.902, 1.176), (0, −1.902, 1.176), (0, 1.902, −1.176), and (0, −1.902, −1.176).

11. From De Baun, *op. cit.*

12. From Draper, *op. cit.*, *Annals*, p. 32. See also Heady and Dillon, *op. cit.*, p. 176.

error. (In comparison, a complete factorial without replication would require 625 treatments !) An attractive feature of this design class is that the top-left 4 x 3 submatrix of the above design matrix constitutes a three factor second order rotatable design.

The designs given above are merely a few examples from the classes of response surface designs developed in recent years. However, with the exception of some work planned by the N.S.W. Department of Agriculture, none of these designs appear to have been used so far in Australian agricultural research. The major explanation of this backwardness is probably that researchers have had no appreciation of mathematical models of response phenomena. In consequence, they have not directed at their statistical advisers queries which might have provoked interest in rotatable-type designs. Perhaps another explanation is that compared to the traditional (piecemeal !) grazing trial involving a single variable at two or three levels, a response surface design implies a relatively large experiment.[13] However, that's the price that has to be paid for reasonable *predictive* understanding of the grazing complex which, by its nature, necessitates multi-factor multi-level experimentation. Moreover, we would argue that experiments along the lines suggested would produce as much pertinent information and involve fewer research resources than are poured into some of the monstrous " farm style " grazed plant-grazing animal projects currently underway.

13. " . . . it is frequent to find an excessive stress laid on the importance of varying the essential conditions only one at a time . . . This simple formula is not very helpful . . . If the investigator confines his attention to any single factor, we may infer he is the unfortunate victim of a doctrinaire theory (of experimentation) or that the material or equipment at his disposal is limited. " Fisher, Sir Ronald. *Design of Experiments*. Oliver and Boyd, London, 1937, pp. 100-1.