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WOOL PRICE STABILISATION AND PROFIT RISK FOR WOOL USERS

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It has often been suggested that more stable wool prices would lead to an outward shift in the long-run demand for wool. To assess this claim it is necesary to examine different sources of risk and instability in wool prices and their impact on the risk borne by wool users. A model is presented in which the input and output decisions of a wool processor are related to interactions between the wool and yarn markets. It is concluded that, if fluctuations in final demand or exchange rates are the major sources of instability, the long-run effect of stabilising prices is to increase the risk faced by wool users and reduce that faced by wool growers.

Introduction

It has been argued that more stable wool prices would lead to an outward shift in the long-run demand for wool (see, for example, AWC 1973). As pointed out by Watson (1980), this argument is difficult to criticise on qualitative grounds as there is no doubt that wool users would prefer stable prices to unstable prices, other things being equal. However, more stable wool prices may not be desirable if exchange rates, output prices and throughput remain highly unstable. In this paper a theoretical framework, within which some aspects of these competing claims can be examined, is provided. In particular, the question of whether or not wool price stabilisation is a risk-reducing measure for wool users will be addressed.

The central feature of the analysis is the explicit recognition that the demand for wool is derived from the demand for wool textiles. The major factor behind any long-run shift in demand for wool is assumed to be decisions by textile manufacturers to invest in production processes based either on wool or on competing fibres. The decisions will be assumed to be based on the expected utility of the resulting flow of profits (i.e. on the relative average level and riskiness of profits arising from the different possible choices).

Short-run effects of stabilisation on the average level of profits may be assessed on the basis of the method used by Campbell, Gardiner and Haszler (1980), who tentatively concluded that the reserve price scheme acted to transfer wealth from producers to wool users. This analysis has been criticised by Richardson (1982) and defended by Haszler and Curran (1982). Issues debated included the nature of demand and supply functions, and the distinction between sales revenues and growers' incomes. These issues will not be discussed here. A question closer to the concerns of this paper was whether the demand curve was shifted outward by the introduction of stabilisation.

The analysis of Campbell et al. was conducted on the assumption that demand and supply parameters are not changed by the introduction of

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the scheme. It would, however, seem likely that the resulting transfer to textile producers would lead to a long-run increase in demand. Against this, it could be argued that the same increase could be achieved at lower cost using a simple subsidy. The proponents of the argument that stabilisation has increased demand have generally based their case on the assumed effects of risk and instability, rather than on the consequences of revenue transfers. For this reason, risk effects are the main subject of the present paper.

As has been pointed out above, this analysis depends heavily on resource fixity in wool processing. Since this is less important after the yarn stage, the main burden of risk will be borne by firms engaged in processing wool into yarn. Up to the yarn stage, resource fixity takes the concrete form of stocks of wool-specific machinery. Beyond this stage it is characterised by investment in product-specific marketing channels for particular types of yarns, fabrics and garments. Throughout the remainder of this paper the situation of a firm processing wool into yarn will be used as an example.

Crucial to the analysis is the distinction between risk and instability (Quiggin and Anderson 1979). Decisions which must be made without knowing what realisation a relevant random variable (in this case, price) will take are said to be subject to both risk and instability. If decisions are made after the realisation becomes known, but are affected by the fluctuations in the values of the variable, they are said to be subject to instability only.

The analysis by Campbell et al. (1980) deals with instability only. The analysis of transfers between wool producers and wool users is based on the assumption that the decisions which determine supply and demand are made with full knowledge of prices. Alternatively, it may be supposed that all individuals involved are risk-neutral. In this paper, such assumptions will not be made and it will be assumed that economic decisions are subject to both risk and instability.

As was suggested above, long-run decisions such as the type of production process in which to invest are likely to be subject to risk. It is worthwhile to examine the sources and effects of risk and instability with respect to processors' profits.

Three major sources of risk and instability in the price of wool may be distinguished. First, there may be supply instability resulting from fluctuations in climatic conditions, input costs or other factors. Second, instability in the demand for wool textiles may be transmitted to the derived demand for wool. This may result either from changes in the prices of substitute fibres or from fluctuations in general economic conditions in consuming countries. The third major source of instability arises from fluctuations in exchange rates. These lead to shifts in the short-run demand curve for wool in terms of Australian currency.

The central argument in this paper is that, depending on its ultimate source, instability in the price of wool will have very different effects on the riskiness of processors' profits. It is, therefore, inappropriate to assess the effects of stabilisation purely in terms of its effects on the variability of prices. Rather, it is necessary to decompose this variability and analyse the effects of instability arising from each of the sources mentioned above.

Producers and Profit Risk

Before the interaction between the wool and yarn markets can be studied, it is necessary to determine an appropriate measure of profit risk, and consider its impact on the investment decisions of wool users. These decisions will not be analysed formally here. Some formal results are given in the Appendix, while others may be derived from the literature on the behaviour of firms under price uncertainty, beginning with the work of Baron (1970) and Sandmo (1971). These results must be adapted to the situation of a processing firm where the price *margin* between wool and yarn plays a role similar to that of the output price in the standard analysis.

In order to facilitate this, it is assumed that raw wool enters the production process in fixed proportion to final output. This condition appears to be satisfied, at least in the early stages of production (Carland 1977, pp. 81-5). However, it is possible that the quality of output is affected by the level of labour and capital input.

With this assumption, a simple description of profit risk can be developed for a price-taking firm engaged in processing wool into yarn. The firm's long-run production function may be generally specified as:

$$(1) \qquad Y = f(W, X, Z),$$

where Y = the output of yarn;
W = the input of raw wool;
X = a vector of other factors of production variable in the short run; and
Z = a vector of factors fixed in the short run.

The assumption that raw wool inputs are proportional to final outputs means that, with an appropriate choice of units, the production function may be written as:

$$(2) \qquad Y = \min(W, g(X, Z)),$$

where $g_1, g_2, g_{12} > 0$; and
$g_{11}, g_{22} < 0$.

Further, cost minimisation implies:

$$(3) \qquad Y = W = g(X, Z).$$

The firm must decide the level of fixed investments, Z, before learning the prices it will face, and this decision is, therefore, subject to risk. By contrast, the level of wool throughput, W, and hence variable inputs X, are determined on a profit-maximising basis after prices become known and are subject only to instability. This implies a two-stage decision problem for the firm, of the type analysed by Hartman (1976). The decision on the level of fixed investments may be modelled on the assumption that the (opportunity) cost of capital is a known constant, r. The firm's investment decision problem may be written:

$$(4) \qquad \max_{Z} E[U(V)] = E[U(\pi(P_y, P_w, P_x, Z) - rZ)],$$

where V = the total value accruing to the firm;
U = a von Neumann-Morgenstern utility function satisfying

(5) $$U'(V) > 0,\ U''(V) < 0.$$

For each P_y, P_w, P_x,Z the value of π is determined by short-run profit-maximising choices of W and X so that:

(6) $$\pi(P_y, P_w, P_x, Z) = \max_{w,x}(P_y.g(X, Z) - P_w.W - P_x.X),$$

where P_y = the price of yarn;
P_w = the price of wool; and
P_x = price of variable inputs.

By virtue of the choice of units, this may be written as:

(7) $$\pi(P_y - P_w, P_x, Z) = \max_{x,w}\ (P_y - P_w)\ g(X,Z) - P_x.X.$$

This may be maximised with respect to X, leaving the wool throughput, W, and yarn output, Y, to be derived from equation (3). The first-order condition is:

(8) $$(P_y - P_w)\ \partial g/\partial X = P_x.$$

It is now possible to examine the impact of an increase in the riskiness of the price margin $P_y - P_w$ (which leaves the mean unchanged) on the firm's short-term profits, π, and hence on investment decisions.

Because the firm can vary its wool throughput in response to changes in price margins, such a 'mean-preserving increase in spread' will yield an increase in the average level of profits for each fixed investment level, Z. This result, first derived by Oi (1961), forms the basis of the Campbell et al. (1980) 'hidden transfers' analysis. As Hartman (1976) points out, this means that a risk-neutral firm will increase its level of investment, Z, in response to an increase in price risk. This will lead to an increase in long-run demand (presumably to the point where short-run excess profits are competed away).

However, as was stated in the *Introduction*, transfer effects are not the primary concern in this paper. As well as increasing the average level of profits, a mean-preserving increase in the variance of $P_y - P_w$ will increase the riskiness of profits. If wool processors are sufficiently risk-averse, this effect will outweigh the transfer effect and long-run wool demand will fall. Hartman (1976) gives some results along these lines, while others are contained in the Appendix.

A special case arises with the Cobb-Douglas production function:

(9) $$W = X^{0.5}\ Z^{0.5}.$$

In this case, equation (8) yields:

(10) $$X^* = (0.25)\ Z\ ((P_y - P_w)/(P_x))^2,$$

so that:

(11) $$W^* = 0.5\ Z^{0.5}\ (P_y - P_w)/P_x,$$

while:

(12) $$\pi = 0.25\ Z\ (P_y - P_w)^2/P_x.$$

In this case, the optimum wool throughput, W, is linearly related to the price margin, $P_y - P_w$. If P_x is assumed constant, then equation (12) shows that the expected level of profits, $E[\pi]$, will increase linearly with the variance of $P_y - P_w$. The variance of π will increase with the square of var $(P_y - P_w)$. While these algebraically tractable results depend on the specific choice of the Cobb-Douglas form, the riskiness of π and of $P_y - P_w$ will be monotonically related for all technologies satisfying equation (2).

Thus, the impact of stabilisation may be modelled in terms of its effects on var $(P_y - P_w)$. Note that this risk measure does not exclude quantity fluctuations from consideration. Rather, it is based on the fact that, for a competitive firm satisfying the restrictions imposed above, output will be monotonically related to the price margin, $P_y - P_w$. The choice of the variance is dictated by its convenience as a single-parameter measure of risk, rather than by any adherence to mean-variance theories of behaviour under risk.

The analysis presented above rests on the assumption that the firm is a price taker in both the input market (for raw wool) and the output market for yarn. It is of interest to consider briefly the implications of relaxing this assumption with respect to the output market, so that the price margin, $P_y - P_w$, becomes a decision variable rather than an exogenous one. The firm is now faced with fluctuations in output demand rather than output price.

This situation was examined by Quiggin (1981). It was shown that the price set by a (short-term) profit-maximising monopolist varies more in response to a given change in final demand than would a competitively determined price. Conversely, the monopolist's throughput is less variable than that of a competitive industry. The situation is different, however, for an industry made up of oligopolists whose future sales depend on current market share. In this case, prices tend to be less responsive (and throughput more responsive) to fluctuations in demand than would be the case for a competitive firm. While the incorporation of such price-setting behaviour would complicate the analysis, it does not appear that it would change the qualitative impact of stabilisation on profit risk.

A Simple Model

The extent to which risk from various sources is absorbed by processors or passed on to consumers is affected significantly by market structure. Nevertheless, the simple competitive model remains a useful starting point. This model allows a relatively simple representation of the interaction between the wool and yarn markets derived from the relationship between the individual firm's inputs of wool and outputs of yarn.

No attempt has been made to model the complex operations of the AWC, which are described in detail by Ward (1978). Instead, a non-intervention situation has been contrasted with one where the price of wool (in Australian dollars) has been completely stabilised. The model is designed to reveal qualitative aspects of stabilisation, which should be similar for partial and complete stabilisation, although the empirical magnitudes may vary.

The basis of the model consists of short-run demand and supply equations for wool and yarn. The short-run nature of the model inheres specifically in the fact that total levels of investment by wool processors are assumed to be fixed. Thus, the model permits an analysis of the impact of stabilisation on the variance of $P_y - P_w$ for given levels of investment. For simplicity, linear demand and supply equations will be used throughout.[1]

The demand for yarn in given by:

$$D_y = a_0 - a_1 P_y + e, \tag{12}$$

where e = a stochastic error term.

It will be noted that the yarn price, P_y, is the only explanatory variable in equation (12). This is because the model is concerned with risk related to P_y. The intercept term, a_0, therefore, subsumes expected values of all other explanatory variables while e subsumes unanticipated fluctuations in these variables. Expectations are taken at the time when relevant economic decisions (in this case, investment decisions) are made.

As was shown above, the demand for wool depends on the price margin between wool and yarn:

$$S_y = D_w = b_0 + b_1 (P_y - P_w^*) + u, \tag{13}$$

where P_w^* = the price of wool; and
u = a stochastic error term.

The inclusion of a stochastic term calls for some comment. Factors such as fluctuations in variable factor prices and seasonality would account for it. However, for the purposes of the present model, these factors will not be considered. Rather, it will be supposed that P_w^* is expressed in terms of Australian dollars and P_y in terms of the yarn producers' currency. Therefore equation (13) can be derived from:

$$S_y = D_w = b_0 + b_1 (P_y - P_w), \tag{14}$$

$$P_w^* = P_w + v, \text{ and} \tag{15}$$

$$u = b_1 v; \tag{16}$$

where P_w = the price of wool in terms of the yarn producers' currency; and
v = fluctuations in exchange rates.

Finally, the supply of wool is given by:

$$S_w = c_0 + c_1 P_w^* + \epsilon, \tag{17}$$

and the equilibrium condition by:

$$D_y = S_y = D_w = S_w. \tag{18}$$

Equations (12, (13), and (14) yield:

$$P_y = (a_0 - b_0 + b_1 P_w^* + e - u)/(a_1 + b_1), \tag{19}$$

and this may be combined with equations (13), (17), and (18) to obtain:

$$P_w^* = (a_1(b_0 - c_0 + u - \epsilon) + b_1(a_0 - c_0 + e - \epsilon))/((a_1 + b_1)(c_1 + b_1) - b_1^2). \tag{20}$$

This implies:

$$P_y = (c_1(a_0 - b_0 + e - u) + b_1(a_0 - c_0 + e - \epsilon)/((a_1 + b_1)(c_1 + b_1) - b_1^2). \tag{21}$$

[1] It should be noted that the results obtained by Campbell et al. (1980) for the linear case differed qualitatively from those reported in the semi-log case.

Thus:

$$(22)\quad P_y - P_w = P_y - P_w^* + v = (c_1(a_0 - b_0 + e - u) - a_1(b_0 - c_0 + u - \epsilon))/((a_1 + b_1)(c_1 + b_1) - b_1^2) + v.$$

The risk measure, $\text{var}(P_y - P_w)$, can now be evaluated on the assumption that e, ϵ and u are independent random variables. Thus:

$$(23)\quad \text{var}_0(P_y - P_w) = (c_1^2\,\sigma_e^2 + a_1^2\,\sigma_\epsilon^2 + a_1^2\,c_1^2\,\sigma_u^2/b_1^2)/((a_1 + b_1)(c_1 + b_1) - b_1^2)^2.$$

If the price of wool (in Australian dollars) is fixed at some level, $\bar{P}_w$, equation (19) implies:

$$(24)\quad P_y = (a_0 - b_0 + b\bar{P}_w + e - u)/(a_1 + b_1), \text{ and}$$

$$(25)\quad \text{var}_1(P_y - P_w) = (\sigma_e^2 + (a_1^2/b_1^2)\,\sigma_u^2/((a_1 + b_1)^2).$$

Thus, the change in variance resulting from stabilisation is:

$$(26)\quad \text{var}\,(P_y - P_w) = \alpha(\sigma_e^2 + (a_1^2/b_1^2)\sigma_u^2) - a_1^2\,(a_1 + b_1)^2\sigma_\epsilon^2)/\beta,$$

where $\alpha = a_1^2 b_1^2 + a_1 b_1(a_1 c_1 + b_1 c_1)$; and
$\beta = (a_1 + b_1)^2((a_1 + b_1)(c_1 + b_1) - b_1^2)^2$.

The larger is σ_ϵ^2 and the smaller are σ_e^2 and σ_u^2, the more likely it is that stabilisation will reduce $\text{var}(P_y - P_w)$ and, hence, the profit risk faced by wool users. In other words, stabilisation will tend to reduce profit risk if supply fluctuations are the main source of risk but will tend to increase it if demand fluctuations or exchange-rate fluctuations are dominant.

This result can be explained intuitively as follows. In the case of instability derived from supply fluctuations, demand curves will be unchanged but supply will be high in 'good' years for wool growers and low in 'bad' ones. Thus, wool prices will be low and profits for textile manufacturers high in good years and, in bad years, the prices will be high and profits low. In this manner, price fluctuations shift some of the risk associated with unstable production from wool growers to textile producers and consumers. High prices tend to offset the losses to wool growers in years of low production and low prices offset high production. If the price elasticity of demand is less than unity, high prices will more than offset low production but this does not change the analysis as far as wool users are concerned.

A converse analysis applies to instability derived from fluctuating demand for textile products. In years when demand is low, the derived demand for wool will also be low and both prices and sales will fall. The fall in the price of wool tends to offset the losses to textile producers associated with low demand.

Thus, fluctuating prices tend to reduce the riskiness of manufacturers' profits associated with unstable demand for final products. They do this by transferring some of the risk to wool growers.

Exchange rate fluctuations leave the demand curve unchanged in terms of importing countries' currencies but alter it in terms of Australian currency. Conversely, the supply curve remains unchanged in terms of Australian currencies but is altered in terms of importing countries' currencies. In general, if Australian dollars become cheaper (e.g. after a devaluation), prices will rise in terms of Australian currency and fall in terms of other currencies. The risk associated with exchange rate fluctuations is, therefore, shared between producers and consumers (the shares depending on the elasticities of supply and demand).

The price mechanism acts to transfer risk and instability between buyers and sellers. Price stabilisation eliminates these transfers. Thus, under complete stabilisation, all of the risk associated with production fluctuations is borne by wool growers while all of the risk associated with unstable demand for woollen textiles is borne by wool users (i.e. textile manufacturers). Since the price is stabilised in terms of Australian dollars all of the risk associated with fluctuating exchange rates is borne by overseas wool users.

These results bear a superficial resemblance to those of Massell (1969). The difference is that Massell's firms were subject to instability only (that is, they made production decisions with full knowledge of prices). Thus, they were concerned only with the level of profits and not with risk.

Price Stability, Quantity Stability, and Profit Risk

The model of the previous section shows that the source of instability will determine the way in which price stabilisation affects the riskiness of profits. If supply instability is dominant, prices and quantities will be negatively correlated. Thus, a policy of adding to buffer stocks in years of low prices would tend to stabilise both prices and quantities. On the other hand, if demand or exchange rate instability is dominant, prices and quantities will be positively correlated. In particular, low (Australian dollar) prices will be associated with low quantities. In this case, price stabilisation will imply quantity destabilisation, at least in the absence of private stocks.

Thus, in the simple case considered here, price stabilisation will reduce profit risk if, and only if, it leads to quantity stabilisation. This can be seen by referring to equation (7) which implies that var(D_w) is proportional to var$(P_y - P_w)$. While this simple numerical relationship is unlikely to hold in practice, the general result is shown below to be fairly robust to relaxation of the stringent assumptions of the model.

Modifications to the Model

The model of the previous section may be modified in a number of ways to give it increased realism without greatly increasing its complexity. First, the implicit assumption that demand moves in the same way in all importing countries may be relaxed. If equations (12) and (13) are interpreted as referring to the demand in a single country, equation (17) must be interpreted as the net supply from the rest of the world to that country. Thus, fluctuations in demand in other countries would result in fluctuations in supply to the country. On this interpretation, the assumption that ϵ is independent of e and u is no longer tenable, and equations (23) and (26) involve covariance terms, $\sigma_{e\epsilon}$ and $\sigma_{\epsilon u}$. If the variation in ϵ is divided into one component (consisting of supply fluctuations and demand fluctuations specific to other countries) uncorrelated with e and u and a second component (consisting of fluctuations in demand which are common to all importing countries) which is highly (negatively) correlated with e and u, the conclusions of the previous section may be reformulated as follows: stabilisation will reduce profit risk if the first component of ϵ is the dominant source of risk, but not if e, u and the second component of ϵ are the dominant sources.

Second, it is possible to interpret the demand for yarn, equation (12), to include demand for stocks of yarn, and supply of wool to be supply net of increases in stocks. This procedure might be objected to on the grounds that, at least before the advent of the Reserve Price Scheme, stocks were held mainly by wool processors. However, this can be dealt with as above by allowing correlation between e, ϵ and u. Alternatively, it might be argued that, to the extent that stockholding is a form of self-insurance, it should not be included explicitly in a model aimed at assessing the risk effects of stabilisation.

Seasonal variations can be dealt with by assuming the model represented by equations (12)-(18) to be a specification for a single quarter and allowing the parameters of the model to vary between different quarters of the year; this would obviously include varying stock behaviour. The measure of profitability, $P_y - P_w$, could be replaced by a weighted average taken over four quarters. The analysis above was applied to complete stabilisation. Partial price stabilisation, as undertaken by the AWC, could also be modelled in an empirical setting.

The result of these modifications would be to reduce the extent to which short-term fluctuations in a single-country affect σ_e^2, as measured by equation (12). This does not, however, vitiate the usefulness of the suggested approach to measuring the effects of stabilisation.

Concluding Comments

The analysis presented above centres on the observation that, whereas the price mechanism redistributes risk between buyers and sellers, price stabilisation through buffer stocks does not permit such transfers. If, as would seem to be the case for wool, fluctuations in final demand or exchange rates are the major source of instability, the long-run effect of stabilising prices is to increase the risk faced by wool users and to decrease that faced by wool growers, relative to a situation where prices move freely.

While this conclusion does not support the claim that the risk effects of the scheme are such as to increase the long-run demand for wool, it must be remembered that other aspects of the scheme may contribute to such an increase. Quantifying these various effects is an empirical problem to which satisfactory solutions may be hard to find.

However, a rough measure of the change in risk induced by the scheme would be the change in variability of throughput for yarn spinners. If observations over a sufficiently long period showed that this variability was increased, they would provide *prima facie* evidence for the claim that stabilisation shifted risk from growers to users. It must not be assumed, however, that these changes in the distribution of risk are necessarily undesirable. In particular, it could reasonably be argued that wool spinners are better equipped to deal with risk than are wool growers. A final resolution of this problem will require an analysis which integrates the long-run consequences of the transfer effects discussed by Campbell et al. (1980) and the risk effects discussed in this paper.

APPENDIX

The purpose in this Appendix is to derive a number of results relating to the impact of price variability on long-term investment decisions. Some notational simplifications are in order. The price margin will be denoted by:

(A.1) $$P_m = P_y - P_w.$$

The profit-maximising values of X and W, determined by equation (8) will be written as:

(A.2) $$X^* = X^*(Z, P_m),$$

(A.3) $$W^* = W^*(Z, P_m) = g(X^*, Z),$$

and equation (6) becomes:

(A.4) $$\pi(Z, P_m) = P_m W^*(Z, P_m) - P_x X^*(Z, P_m).$$

(Note that, as in the text, P_x is treated as a known constant.)

It is now possible to differentiate equation (A.4) with respect to Z and P_m:

(A.5) $$\partial\pi/\partial P_m = g(X^*, Z) + (P_m\, \partial g/\partial X^* - P_x)\, \partial X^*/\partial P_m,$$
$$= g(X^*, Z), \text{ by equation (8).}$$

Similarly:

(A.6) $$\partial\pi/\partial Z = P_m\, \partial g/\partial Z.$$

Further, it may be observed that the profit function is convex in P_m:

(A.7) $$\partial^2\pi/\partial P_m{}^2 = \partial g/\partial X^*\, \partial X^*/\partial P_m > 0,$$

and that:

(A.8) $$\partial^2\pi/\partial P_m \partial Z = \partial g/\partial Z + P_m \partial^2 g/\partial X \partial Z > 0,$$

by the conditions on g in equation (2).

Finally, equation (A.8) may be differentiated with respect to P_m to yield:

(A.9) $$\partial(\partial^2\pi/\partial P_m \partial Z)/\partial P_m = (\partial^2 g/\partial Z \partial X^*)\, \partial X^*/\partial P_m$$
$$+ \partial g/\partial X^*\, (\partial^2 X^*/\partial P_m \partial Z) > 0.$$

In order to derive formal results about changes in the distribution of P_m, it is necessary to define these changes. A simple approach, used by Sandmo (1971) and others, is to make the distribution of P_m dependent on its mean μ and a risk parameter λ. Let P^1 be the distribution when $\lambda = 1$. Then:

(A.10) $$P_m = \lambda P^1 + (1 - \lambda)N.$$

An increase in λ generates a 'multiplicative stretching' of the distribution of P_m, but does not change its mean, since $E[P_m] = \mu$.

It is now possible to derive a result similar to that of Oi (1961).

Proposition A.I: Given the conditions above, an increase in λ will lead to an increase in $E[\pi(Z, P_m)]$ for each (fixed) Z.

Proof: For each Z:

(A.11) $$\partial E[\pi(Z,P_m)]/\partial\lambda = E[\partial\pi/\partial P_m\ \partial P_m/\partial\lambda],$$

since $\partial^2\pi/\partial P_m{}^2 > 0$. The two terms on the RHS of equation (A.11) are positively correlated (in fact, monotonically related) and hence:

(A.12) $$E[\partial\pi/\partial P_m\ \partial P_m/\partial\lambda] > E[\partial\pi/\partial P_m]\ E[\partial P_m/\partial\lambda] = 0. \qquad \text{Q.E.D.}$$

Since a risk-neutral firm will maximise expected profits, Proposition A.I, suggests a result similar to that of Hartman (1976).

Proposition A.II: For a risk-neutral firm, an increase in the riskiness of P_m will lead to an increase in the optimal investment level, Z, determined by equation (4).

Proof: The first- and second-order conditions on Z are:

(A.13) $$\partial V/\partial Z = E[P_m \partial g/\partial Z - r] = 0, \text{ and}$$

(A.14) $$D = \partial^2 V/\partial Z^2 = E[P_m(\partial g^2/\partial Z^2 + (\partial^2 g/\partial X \partial Z)\ \partial X^*/\partial Z)] < 0.$$

Differentiating equation (A.14) and rearranging yields:

(A.15) $$\partial Z/\partial\lambda = -(1/D)\ E[(\partial^2 V/\partial Z\ P_m)(\partial P_m/\partial\lambda)].$$

From equation (A.9) the first term inside the square brackets is increasing in P_m, and, hence, as in the proof of A.I, the whole expectation is positive, as is $-(1/D)$ by equation (A.14). Hence:

(A.16) $$\partial Z/\partial\lambda > 0. \qquad \text{Q.E.D.}$$

For a risk-averse firm, the conditions (A.14) and (A.15) are replaced by:

(A.17) $$\partial E[U(V)]/\partial Z = E[U'(Z)(\partial\pi/\partial Z - r)] = 0, \text{ and}$$

(A.18) $$D = \partial^2 E[U(V)]/\partial Z^2 = E[U'(V)(\partial^2\pi/\partial Z^2) + U''(V)(\partial\pi/\partial Z - r)^2] < 0.$$

It may be shown, following Baron (1970) that:

(a) a risk averse firm will invest less than a risk-neutral one; and
(b) the more risk averse a firm is, the lower its optimal investment levels. However, for space reasons, this will not be undertaken here.

In view of Proposition A.II, it is apparent that risk aversion *per se* cannot be a sufficient condition to ensure that an increase in the riskiness of P_m will lead to a decrease in the optimal investment level, Z. For example, a firm which is only 'slightly' risk averse will presumably find that the increase in expected profits will outweigh the costs of bearing increased risks. However, for a 'sufficiently' risk-averse firm the reverse will be true. More precisely, the concavity of the utility function, U, must outweigh the convexity of $\partial\pi/\partial Z$ (with respect to P_m in both cases) so that:

(A.19) $$-(U''(V)\ \partial\pi/\partial P_m)/U'(V) > \partial/\partial\pi(\partial^2\pi/\partial P_m \partial Z)/\partial^2\pi/\partial P_m \partial_z)),$$

for all P_m and Z in the relevant range.

A second assumption, that of decreasing absolute risk aversion, will also be required. As Sandmo (1971) shows, the failure of this assumption can lead to highly implausible results, such as negative supply response to an increase in the expected price, U.

It is now possible to prove another proposition.

Proposition A.III: Assume condition (A.20) holds, and decreasing absolue risk aversion prevails. Then an increase in the riskiness of P_m will lead to a reduction in the optimal level of investment, Z, determined by equation (4).

Proof: Differentiating equation (A.18) with respect to λ and rearranging yields:

(A.20) $$\partial Z/\partial\lambda = (-1/D)(E[U'(V)(\partial^2\pi/\partial Z\partial P_m)(P_1-\mu)] + g(X^*,Z)\ E[U''(V)(P_m\partial g/\partial Z - r)(P_1-\mu)].$$

Working on the first expectation:

(A.21) $$\partial/\partial P_m\,(U'(V)(\partial^2\pi/\partial Z\partial P_m)) = U''(V)\ \partial\pi/\partial P_m\,(\partial^2\pi/\partial Z\partial P_m) + U'(V)(\partial/\partial P_m)(\partial^2\pi/\partial Z\partial P_m) < 0, \text{ by equation (A.19).}$$

Since P_m is linearly related to P_1, the two terms in the first expectation are negatively correlated, and:

(A.22) $$E[U'(V)(\partial^2\pi/\partial Z\partial P_m)(P_1-\mu)] = E[U'(V)(\partial^2\pi/\partial Z\partial P_m)]E[P_1-\mu] = 0.$$

For the second expectation term, denote $r/(\partial g/\partial Z)$ by θ. Then:

(A.23) $$E[U''(V)(P_m-\theta)(P_m-\mu)] = E[U''(V)(P_m-\theta)(P_m-\theta+\theta-\mu)] = E[U''(V)(P_m-\theta)^2] + (\theta-\mu)E[U''(V)(P_m-\theta)].$$

The LHS of equation (A.23) is the second expectation in equation (A.20) divided by the positive constant $\partial g/\partial Z$, and has the same sign. The first term on the RHS is clearly negative. It, therefore, remains to prove, following Sandmo (1971), that:

Lemma: Given decreasing absolute risk aversion, then:

(A.24) $$E[U''(V)(P_m-\theta)] < 0.$$

Proof: Let r_a be the absolute risk aversion evaluated at the level of V which arises when $P_m = \theta$; i.e. when:

(A.25) $$P_m\partial g/\partial Z - r = 0.$$

Then, decreasing absolute risk aversion means that:

(A.26) $$r_a > -U''(V)/U'(V), \text{ if and only if } P_m > \Theta.$$

Hence:

(A.27) $$-r_a\ U'(V)(P_m-\theta) > U''(V)(P_m-\theta), \text{ for all } P_m,$$

noting that when the inequality (A.26) is reversed, the sign of $(P_m-\theta)$ is changed. Hence:

(A.28) $$E\ U''(V)(P_m-\theta) < -r_a\,E[U'(V)(P_m-\theta)], = -r_a\,E[U'(V)(\partial\pi/\partial Z - r)]/(\partial g/\partial Z), = 0$$

by equation (A.18).

This completes the proof that both expectation terms in the RHS of (A.21) are negative. Since $(-1/D)$ is positive:

(A.29) $$\partial Z/\partial\lambda < 0, \text{ as required.}$$

Thus, for 'sufficiently' risk-averse investors, an increase in the riskiness of the price margin will lead to a reduction in investment levels. The assumption that g_{12} is positive is sufficient to ensure that this will yield a reduction in long-run demand.

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