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AN APPLICATION OF SPATIAL ANALYSIS TO BEEF SLAUGHTER PLANT LOCATION AND SIZE, QUEENSLAND

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The objective of this study is to define in the cost minimizing sense the optimum size and location for beef slaughter plants in the Eastern Central Queensland region. The plant location model used for this purpose is a modified transhipment formulation. It is applied to both a long run unlimited capacity case and a short run limited capacity case. The analysis indicates that for both cases production-area oriented slaughter minimizes costs. Sensitivity testing of the stability of the solution is outlined.

The Problem

The Queensland beef slaughter industry has recently been the subject of yet another State Government Committee of Inquiry [36]. This was mainly concerned with the overall adequacy of existing slaughter capacity and the need for additional works, but also included production and marketing aspects. The Inquiry was partly the result of grazier discontent regarding some existing treatment facilities and partly due to processors' concern about variability of profits due to excess capacity in the slaughter industry.

Excess capacity leads to financial instability of slaughtering firms and may involve heavy losses of public funds in Government sponsored public abattoirs. Bressler [9] has analysed such situations and suggests that a 'law of mediocrity' applies. Included here are inefficiencies in processing possibly occasioned by sub-optimum location and size of works or too many works.

Possible normative solutions to problems of this type can be derived by means of spatial equilibrium analysis. Accordingly the objective of this paper is to apply this technique to determine the optimum location, number and size of beef slaughter plants in a Queensland region. (The technique could not be applied to Queensland as a whole because of limited research resources.)

The Eastern Central Queensland (ECQ) region was chosen for a number of reasons. Bardsley [1] had previously defined the region after studying the tributary areas of cattle saleyards in Central Queensland

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(CQ). He found ECQ to be self contained in terms of slaughter cattle production. Cattle movement permits, collected by the authors for the three years 1963/64 to 1965/6, supported Bardsley. Also, for various reasons (including the structure of the Queensland rail network and the heavy road tax on cartage of live animals) it is uneconomic to ship slaughter cattle from South Queensland (SQ) or North Queensland (NQ) to ECQ, although large numbers of cattle move from ECQ to SQ.[1]

The study region extends from Miriamvale on the coast northwards to Mackay and westwards to Jericho. Its boundaries are precisely defined by Cassidy [10]. It includes the statistical divisions of Mackay, Rockhampton, and portions of the Central Western Division and contains nearly one-third of Queensland's beef cattle. It has a turn-off of about 400,000 slaughter cattle in a 'normal' year. Nearly half of these cattle are transported up to 700 to 800 miles out of the region to SQ for slaughter.[2] Associated with this large cattle outflow, the region has only four beef slaughter plants compared with 20 in the SQ region, which has slightly fewer beef cattle. Three of these four plants are situated on the coast at ports.[3] The other is an inland works.

Factors influencing the geographical siting of works need to be taken into account so that appropriate assumptions may be made in the subsequent analysis. ECQ with its relatively small population and large cattle turn-off requires an export based industry. The main markets for ECQ cattle are overseas, even though they may be first railed to SQ meatworks for slaughter. However, numbers of ECQ cattle are slaughtered for domestic consumption in SQ as well. This demand and supply pattern suggests three general locations for processing plants.

(i) 'Production-area oriented' plants.
(ii) As the main markets are overseas, a port location with a break in transport could be feasible. These may be termed 'transport-oriented' or transhipment-oriented' plants.
(iii) Plants associated with the large home market demand at Brisbane in the southern sector but also producing for export. These may be termed 'consumption-oriented' plants.

It is possible that for a specified region a combination of such plant locations may be optimal.

Another factor affecting location is the supply of cattle in each production area and its seasonal availability. Also important is the structure of the transportation network with its consequent effect on transport cost. Port locations in the study area and in SQ are connected to the supply areas of ECQ by rail which is the main means of movement of slaughter cattle over long distances. Transport costs also are influenced by the volume and form of the product moved. A much larger volume has to be moved if live animals are shipped than if meat is sent from

[1] Whether cattle should move from ECQ to SQ for slaughter is one of the questions this study examines.

[2] It is sometimes suggested that cattle move from ECQ to SQ because export shipping is more available in SQ. However, the Conference lines system guarantees that shipping is available whenever needed to transport meat from ECQ ports.

[3] Both the Lakes Creek works and Field's Fitzroy River plant while situated on a river port, now use Port Alma (about 20-30 miles distant) at the mouth of the river, but are classified as being sited at a port.

'production-oriented' processing plants. Again, if the final product, meat, is shipped, higher priced refriegated transport is essential. Another aspect is that freight rates can be deliberately varied to discriminate against certain products or certain areas as Estell and Buchanan have shown [15]. However, in this study, such rates and also road taxes have been assumed to remain constant at present levels for rail and road transport of both live animals and meat products.

Processing costs will aso vary with location, depending on the way in which external economies or diseconomies influence the relative costs of inputs such as labour, electricity, water, and packing materials. Also, economies of scale accrue as plant throughput increases. Another factor that may influence plant location is the loss of weight in transit of live animals proceeding long distances to meatworks. However, the authors feel that this is an intangible at present.[4]

To investigate as many aspects of these problems as possible, it was necessary to employ two different but related models. The first defines optimum location and size of beef slaughter plants for the region, ignoring the present location and capacity of existing plants. Its solution specifies not only the cost minimizing location and size, but also the flow of raw product, live animals, and the shipment pattern of the final product, meat, to satisfy all demands at least cost. The second model takes account of the present location and size of existing plants and ignores any other possible locations. In addition it specifies what excess capacity, under these optimum circumstances, each existing plant may have. Both models estimate regional price differentials and locational advantages of various sites. In disregarding the present locations of existing plants the first model implies mobility of factors of production utilized in processing and in this way introduces a long-run planning horizon to the analysis.

Data collected were for the 1964-65 financial year which was a 'normal' year climatically. Solutions to both models would only hold in the future so long as these data were representative of beef cattle supply and demand for future years. Although as Logan and King [32] indicate the models assume completely price-inelastic demand and supply functions, input data can be updated periodically and recommendations revised accordingly. This aspect will be discussed further in a later section.

In summary, the problem can now be defined further.

(i) Should processing of ECQ cattle be undertaken wholly, partly, or not at all within the region?
(ii) At what locations should processing take place?
(iii) What size should any postulated works be?

The Method of Approach

Regional competition and the spatial location of production first occupied economists as early as the nineteenth century. Still, it was not until the formulation of the activity analysis model of production and allocation by Koopmans [26], Dantzig [11] and others that empirical content could be given to the theory. The Enke [14], Samuelson [37], Beckman

[4] See for a summary of evidence on this issue *Fd. Technol. Aust.* 18 (7), 1966, p. 436.

[5] and Baumol [3] contributions to the study of spatial considerations between markets were important advances. Studies by Lefeber [29] and Stevens [39] followed by Beckmann and Marschak [6] Takayama and Judge [42, 43, 44] have further advanced the methodolgy of this group of models. Input-output analysis has also been used for studies of interregional competition [20]. However, as Heady and Carter [17] indicate, it is relatively unpopular due to its underlying assumptions, including constant input-output coefficients and zero rates of substitution. As Bawden [4] has shown, all such models can be grouped into two classes, namely standard equilibrium formulations and activity analysis models.[5]

These groups are basically the same in the manner they represent, for example, shipment activities. Here, given transfer costs between regions for each commodity, shipments are specified to minimize the sum of transfer costs. A key to their basic difference lies in how they treat the production process. Inelastic area supply or explicit supply functions are needed to classify a model into the standard equilibrium group, while activity analysis models generate implicitly their own supply relationships. In activity analysis models, instead of supply functions, production costs for one or more different levels in the production process are specified for each region. Supplies are generated by competing alternative uses for the available resources.

As the current study is concerned primarily with the supply and location of processing services, choice of a model from Bawden's activity analysis group is imperative. This is the only group of models which determine such factors endogenously and specify regional production of this supply. Further, models from this group can consider both long-run and short-run problems. Finally, other necessary aspects of any solution to the study's problems, namely equilibrium shipping patterns and relative commodity prices, can be included. Choice of a particular specification is further narrowed to what are termed 'plant location models'. Commonly here, the base is a transportation or modified transportation model.[6] For example Stollsteimer [41] investigated plant numbers and locations and Pherson and Firch [34] examined warehouse location.

However, none of these studies allowed for the possibility of commodity shipments being forwarded to their various destinations more cheaply by proceeding via a series of points rather than just being shipped from m surplus regions to n deficit regions. This movement pattern, termed transhipment when incorporated into the transportation model, needs a major restructuring of the transportation model. The basic difference between the original transportation model and the transhipment formulation is that each production and consumption area is specified as a possible shipment or transhipment point. Logan and King [31] were the first workers to show the computational advantages of this specification. Hurt and Tramel [18], building on Logan and King's work, developed transhipment models further and were followed by Leath

[5] Examples of empirical studies from both groups are in the bibliography of Leuthold and Bawden [30] and the paper by Weinschenck *et al* [46].

[6] An exception is the study of Judge, Havlicek and Rizek [22]. However, modified transportation models have been shown to provide the solutions sought by Judge *et al*, with much less computation.

and Martin [28] and Bobst and Waananen [8] who added greater computational flexibility.

While the general type of model to be used in this study can be derived from the research cited above, precise specification depends on the unique nature of the ECQ problem. Briefly, the problem concerns a single homogeneous product assumed to be characterized by inelastic supply and demand functions. It needs to allow for possible transhipment activities of both raw product and final product after processing, as the structure of freight rates for the area may demand transhipment for lower overall costs. The processing costs must reflect economies of scale besides any external economies in each processing region. Solutions must be derived for the fixed and unlimited capacities and location cases. Hence in the authors' judgment the most suitable model is a modified transhipment formulation based on those of Logan-King and of Hurt-Tramel which were developed from the original work of Orden [33].

The authors utilized the dual formulation of the final problem to derive commodity equilibrium prices at origins and destinations [13] and location rents [7, 40] but these are not presented here because of lack of space.

Data Sources

(a) *Basing points*

As models employed are of the point trading variety, sub-regions and their basing points need to be specified. Choice of these normally takes account of obvious regional features such as cattle production, transport networks, ports, and other circumstances such as general selling practices, government legislation and consumption patterns. A dominating feature of the study region is the rail network. Relative freight costs make it necessary to use rail transport for shipments between most sub-regions, and hence points on the rail system have been chosen. These are Mackay, Rockhampton, Gladstone, Biloela, Emerald, Clermont, Alpha and Springsure. As around 50 per cent of regional turn-off is railed to SQ for slaughter and processing, it is also necessary to specify a demand centre in SQ. For this purpose Brisbane was chosen.

(b) *Raw product supplies*

Data were collected on all cattle sold for slaughter in the region over the 1964-65 year. These data were available in disaggregated form from the Queensland Department of Primary Industries 'Permits to Travel Stock'. Under the Queensland Stock Acts it is necessary when moving cattle off a property to obtain a permit to do so. On this permit such data as number and description of animals, destination, method and route of travel are recorded. Hence it was possible to build up actual slaughter cattle figures by area of origin. This necessitated collection and recording of around 35,000 permits. A correction factor was applied to allow for sales not immediately destined for slaughter. Regional supply and demand figures are included in Table 1.

(c) *Final product demands*

These were built up from statistics of all animals killed in meatworks and slaughterhouses in Queensland. Further detail of the method of estimation may be found in Cassidy [10]. The difference between domestic consumption and total supplies was taken as a measure of export demand.

TABLE 1

Supply and Demand for Slaughter Cattle—1964-65

Basing Point	Supply* (No. of head)	Demand* (No. of head)
Mackay	33,120	15,520
Rockhampton	115,120	22,120
Gladstone	24,660	2,460
Biloela	61,340	4,380
Emerald	34,080	1,460
Clermont	49,560	1,040
Alpha	12,740	400
Springsure	30,540	380
Brisbane	—	48,080
Export	—	265,320
TOTAL	361,160	361,160

* Source. Compiled from Queensland Department of Primary Industries data on 'Permits to travel stock' and levies on cattle slaughtered.

(d) *Transport charges*

Data on charges for live animal shipment were available from Queensland Railways. Final product shipment required refrigerated transport. As the Queensland Railways have very limited capacity in this respect, it was assumed that final product would be transported by road. Accordingly, rates were ascertained from the major refrigerated road transport firms.

Estimation of Cost Curves

The models required both long-run average cost curves (LRACC) and short-run variable cost curves (AVCC). After reviewing available empirical data on the nature and behaviour of cost curves [21, 38], consideration was given to the three methods outlined by Dennis [12] (experimental, statistical, synthetic) to construct the processing LRACC. In spite of its complexity and drawbacks the authors chose the synthetic method. Management personnel of 19 Queensland plants were interviewed in order to construct cost curves for hypothetical optimum plants with throughputs of from 500 to 1500 head per day. Co-operation varied markedly so a modified method had then to be used. Possibly this is best described as a budget-unit-cost analysis drawing on both synthetic and cross sectional accounting data.[7] As costs are partly dependent on location, two curves were constructed—one for ECQ plants and one for SQ plants. The long-run and short-run curves are illustrated in Figures 1 and 2.

Seasonality of supply was allowed for in these curves. This was accomplished by charging total fixed costs based on designed capacity

[7] Cost categories included the traditional direct costs and overheads and an allowance for 'normal' profit. Processing costs for all by-products are also charged.

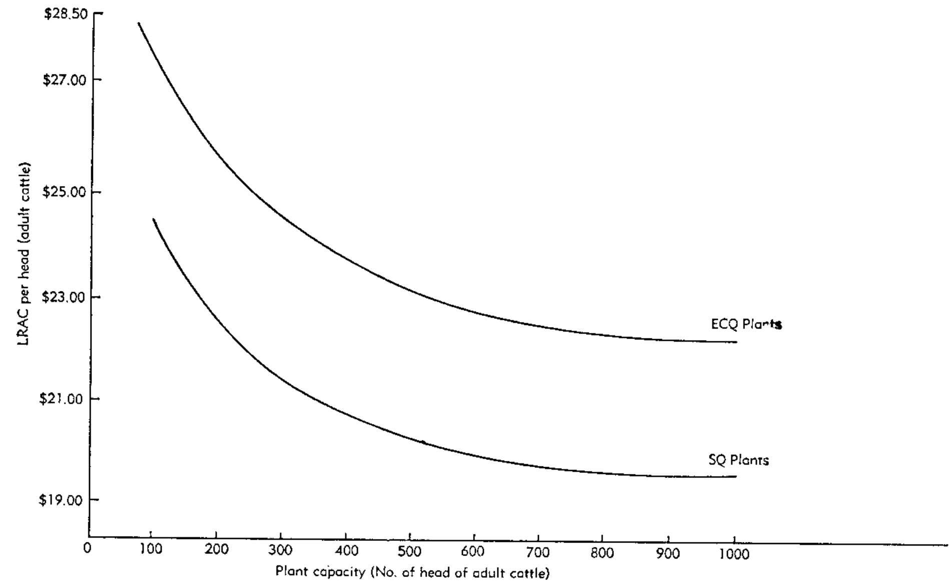

FIG. 1—Long-run average cost curves for beef slaughter plants—ECQ & SQ (costs adjusted for seasonal supply).

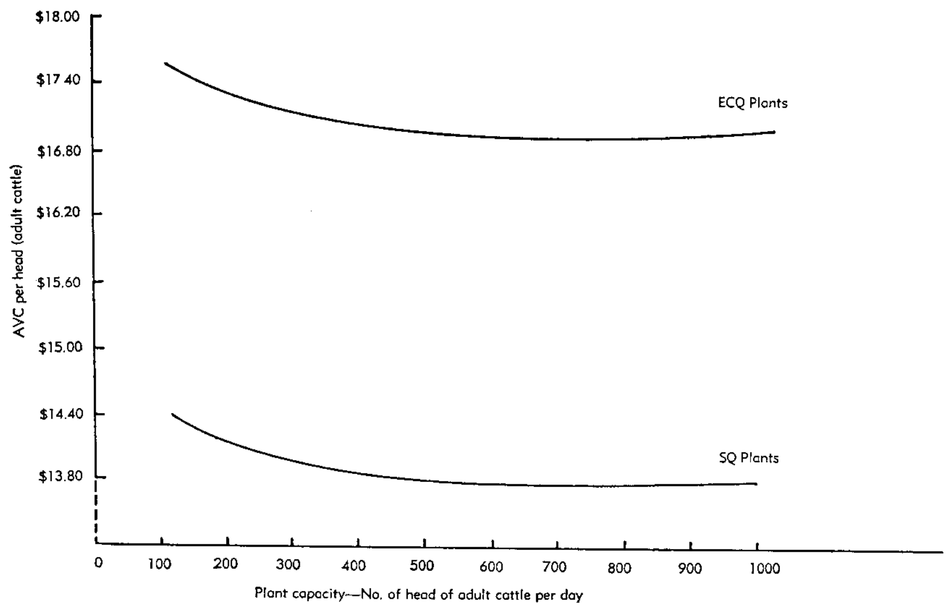

FIG. 2—Average variable costs for beef slaughter plants ECP & SQ (adjusted for seasonal supply).

to seasonal throughput which was always below capacity. Variable costs were estimated on throughput.

The Parent Matrices

(a) *Unlimited capacity—the long-run case*

The unlimited capacity model is set out algebraically in Appendix 1. For this problem the matrix format is 20×20. A schematic representation is given in Table 2. Recall that particular regional basing points were chosen earlier. However, an additional basing point is required denoted as 'Export'. This has a dual function. It is mainly in the matrix to allow a genuine and real overseas demand. However, it has some of the features of a 'dummy' or unreal demand in that it is also used to equate supply and demand. This is achieved by setting this requirement equal to the excess of supply over home consumption.

TABLE 2

Diagrammatic Matrix Format—Unlimited Capacity Case

	Mky_3 Rk_3 $Glad_3$ Bil_3 $Emer_3$ $Cler_3$ Alp_3 Spr_3 $Bris_3$ Exp_3	Mky_4 Rk_4 $Glad_4$ Bil_4 $Emer_4$ $Cler_4$ Alp_4 Spr_4 $Bris_4$ Exp_4	Supplies
Mky 1 Rk 1 Glad 1 Bil 1 Emer 1 Cler 1 Alp 1 Spr 1 Bris 1 Exp 1	Live animal shipment (unit = 20 head)	Live animal shipment plus slaughter in region j	$k_i + A$ $k_i + A$ $k_i + A$ $k_i + A$ $k_i + A$
Mky 2 Rk 2 Glad 2 Bil 2 Emer 2 Cler 2 Alp 2 Spr 2 Bris 2 Exp 2	Submatrix characterised by high costs to preclude shipments (not relevant to this problem)	Meat shipment from region of slaughter to demand region (unit = meat equivalent of 20 head)	A A A A etc.
Demand $r_j + A$	A A A A etc.	$r_j + A$ $r_j + A$ $r_j + A$ etc.	

The original Logan and King approach has been modified to fit the study problem. This includes the following matix modification:

(i) The costs of transporting meat (south-east sub-matrix) are quoted on the boneless cartoned product in all cases except transport from any ECQ centre to Brisbane. Here the cost unit was weighted by allowing an arbitrary figure of 75 per cent boneless and 25 per cent carcass beef as it was assumed Brisbane would require a percentage of bone-in product.

(ii) The artificial variable in accordance with the inequality equation (9) of Appendix 1 has been set at 20,000 K-wagon units (approximately the number of cattle involved). Hence all transhipment possibilities are allowable. Due to the inclusion of this variable it was not necessary to supply either Brisbane or Export with 'dummy' supplies as the artificial constant automatically allowed their entry into computations.

(b) *Limited capacity—the short-run case*

The algebraic formulation of the model is similar to the long-run case and is not detailed here. The problem involves a 22 $\times$ 22 matrix whose format is similar to that of Table 2. The extra size of this matrix over the unlimited capacity one is due to the inclusion of two meatworks currently operating at Rockhampton.

Some aspects of the limited capacity cost matrix warrant discussion.

(i) Mention was made in the unlimited capacity case of the duality of function of the Export basing point. In the limited capacity case Export denotes both the excess of supply over home consumption and the real demand for this excess overseas. However, due to the non-addition of artificial variables to regional supplies, it was necessary to supply Export with a 'dummy' one unit supply to ensure its functioning in the model. Similarly the same small supply had to be allocated to ensure participation of Rockhampton 1B (the second meatworks at this centre) and Brisbane.

(ii) The problem of allowing for higher meat transportation rates due to Brisbane bone-in demand was met in a similar way to the previous model.

(iii) The capacity of the various works were derived from three separate independent authorities, namely the Queensland Department of Primary Industries, Thos. Borthwick and Sons Ltd., and the United Graziers Association of Queensland. The achievable working capacities were weighted by the seasonal availability of cattle supplies and scaled in units of 20 head (K-wagon loads).

(iv) A further problem was to specify the processing cost to employ in the model for cattle slaughtered in SQ. This was straightforward in the unlimited capacity case as specification of the minimum of the LRACC for the region was the relevant point. For the short-run case the cost level used was the weighted average of all operating plants. The logic of this assumption is detailed in Cassidy [10, p. 111, 124].

Results and Discussion

Unlimited Capacity—The Long-Run Case

Inclusion of non-linear processing costs (occasioned by 'economies of scale') rules out solution by conventional mathematical programming techniques.[8] Consequently King and Logan's [25] heuristic programming method was utilized. These heuristics involve *implicitly*[9] a 'drop' technique as sets of processing locations are examined sequentially. Firstly all possible processing locations are programmed to operate at minimum processing cost and the model is run. The volume of slaughter

[8] Methods of solution used in related problems include that of Balinski [2] who proposed an integer programming algorithm, while Keuhn and Hamburger [24] and Feldman *et al* [16] have utilized heuristic techniques.

[9] Feldman *et al* [16] also used a 'drop' technique starting from a point where all possible warehouses are operative. However, the method is for the researcher to *explicitly* select and 'drop' one warehouse at a time. On the other hand King and Logan's technique drops out processing plants by endogeneous model procedures to enable convergence to an optimum (cost minimizing) solution.

TABLE 3

Matrix of Raw Product and Final Product Shipments—Unlimited Capacity Model Results
(After Subtraction of Artificial Constants)

	Mky₃	Rky₃	Glad₃	Bil₃	Emer₃	Cler₃	Alp₃	Spr₃	Bris₃	Exp₃	Mky₄	Rky₄	Glad₄	Bil₄	Emer₄	Cler₄	Alp₄	Spr₄	Bris₄	Exp₄	Supplies k_1
Mky_1	0′ 0″ 0‴										1,656′ 1,656″ 1,656‴										1,656
Rky_1		−3,756′ −7,736″ −8,003‴										9,512′ 13,492″ 13,759‴									5,756
$Glad_1$			0′ 0″ 0‴						1,110″ 1,233‴				1,233′ 123″								1,233
Bil_1		2,828″ 3,067‴		0′ 0″ 0‴									444′	2,623′ 239″							3,067
$Emer_1$		1,631′ 1,704″ 1,704‴			0′ 0″ 0‴										73′						1,704
$Cler_1$		1,677″ 1,705‴				0′ 0″ 0‴			657″ 773‴							2,478′ 144″					2,478
Alp_1		617′					0′ 0″ 0‴		637″ 637‴	0′ 0″ 0‴							20′				637
Spr_1		1,508′ 1,527″ 1,527‴						0′ 0″ 0‴										19′			1,527
$Bris_1$									0′ 0″ −2,643‴										0′ 2,404″ 2,643‴		0
Exp_1										0′ 0″ 0‴											0

TABLE 3 (*Continued*)

Mky_2											−880′									880′	0
											−880″									880″	
											−880‴				73‴	52‴				736‴	
Rky_2												− 8,406′								8,406′	0
												−12,386″								12,386″	
												−12,643‴								12,530‴	
												−12,643‴	123‴							12,530‴	
$Glad_2$													−1,554′								0
													0″								
													0‴								
														−2,404′					2,404′		0
Bil_2														− 20″			20″				
														− 20‴			20‴				
$Emer_2$															0′						0
															0″						
															0‴						
$Cler_2$																−2,426′				2,426′	0
															73″	−92″		19″			
																0‴					
Alp_2																	0′				0
																	0″				
																	0‴				
Spr_2																		0′			0
																		0″			
																		0‴			
$Bris_2$																			0′		0
																			0″		
														239‴					−239‴		
Exp_2																				0′	0
																				0″	
																				0‴	
Demands	0	0	0	0	0	0	0	0	0	0	776	1,106	123	219	73	52	20	19	2,404	13,266	

r_j

Code: Run 1 entries indicated by ′ Run 2 entries indicated by ″ Run 3 entries indicated by ‴

at each processing centre now operating is examined. Processing costs are adjusted on the basis of volume of slaughter and the model rerun. The general method is continued until processing levels at surviving centres equal processing costs programmed for that model run. Each iterative model run specifies as solution, the surviving processing locations with capacity needed, along with all product flows.

Accordingly, for Run 1, all processing costs were set at the minimum points on the LRACC's for the locations and regions concerned. This resulted in a multitude of processing locations as Table 3 indicates. Only ECQ processing is undertaken; Brisbane demand is satisfied by meat supplied from the works specified at Biloela. Rockhampton occupies a dominant position receiving supplies from Emerald, Alpha and Springsure, in addition to those from its own region, and supplying the major part of the export trade.

When scale considerations are allowed for (on the basis of quantities slaughtered in the solution of Run 1) more realistic processing costs are specified as model input. Brisbane processing costs are unaltered and remain at this region's minmum point throughout the problem. This out-of-region centre is assumed able to process at this cost due to its southern region supplies, regardless of ECQ live animal shipments. The solution tableau to Run 2 results in the exclusion from processing of most smaller centres. Rockhampton further gains in processing importance and Brisbane processes ECQ cattle for its own demand. Total costs quite reasonably have increased from $8,736,596 to $9,035,562 due to the more realistic processing costs. Again, on the basis of of the solution to Run 2, processing costs were respecified and the model rerun. This third and final run specified three processing centres. In order of dominance these were:

(i) *Rockhampton*. This centre, besides processing its own live animal supply, received live animal shipments for slaughter from the Biloela, Emerald, Clermont and Springsure sub-regions. In all, it processes 13,759 K-wagon units of cattle. It supplies its own home demand and that of Gladstone, but the major portion of output is exported overseas. Attainment of the minimum point on the processing LRACC is reached at this level.

(ii) *Brisbane*. The processing of ECQ cattle at Brisbane is specified at the level of 2643 K-wagons. This allows fulfilment of Brisbane region demand for ECQ cattle, plus enough to ship back as meat to supply Biloela. The structure of processing and freight rates makes it cheaper for Biloela to obtain its home demand in this way. Brisbane obtains its live animal supplies from Gladstone, with some from Clermont and others from Alpha.

(iii) *Mackay*. This centre only processes its own supply (1656 K-wagon units). From this total it supplies its own home demand plus that of Emerald, Clermont and Springsure. The remainder is exported through the port of Mackay.

Total processing costs have further slightly increased for this final run to $9,081,149. Again, the specification of appropriate and thus realistic processing costs results in this increase. In general these higher total costs are necessitated by live animal transport shipments from centres processing in the early computer runs to other regions for processing.

Certain aspects of this model warrant comment:

(i) The cost curves are for *processing services.* The cost of the raw product (live animals) is excluded. Theoretical considerations, however, indicate that where factor cost changes are directly concerned with scale changes the relevant cost increase should be considered. It should now be apparent that the model automatically performs this task.

(ii) Care needs to be taken when specifying processing costs in each iteration. The seasonalized cost curves of Figure 1 are constructed so that units of peak capacity per day are measured on the horizontal axis. Each solution to each model run gives a certain total of slaughter at each centre. However, this specified slaughter must be thought of as subject to the seasonal variation of live animal supply. Accordingly the number of K-wagon units specified in the model run as total annual slaughter at a processing centre, must be converted to the number of head per day and seasonalized. Then using Figure 1 the related peak size of plant, and hence processing costs are read off to use in the next model run.

It is possible to make an estimate of savings for at least some cost elements should rationalization of the industry occur. When collecting and recording slaughter cattle movements from the 'Permits to Travel Stock' a summary was made of total live animal shipments for slaughter to and from each centre. As the industry is presently organized the total cost of such shipments is nearly $2 million. However, the shipment pattern specified in the final run of the unlimited capacity case has an associated cost of just over $1 million. Thus a saving of around 50 per cent is possible.

Cost of final product shipments specified by the model amount to $31,300. It was not possible, because of lack of data, to compare actual current final product shipments and actual processing costs with the models' results. Reference to the LRACC's constructed for the model, however, give an indication of inherent savings in the processing cost element. Many of the works slaughtering ECQ cattle in 1964-65 were not of large enough capacity to process at minimum average cost. This was especially evident for works in the SQ sector. Comparison with the model's LRACC illustrates that these plants incurred excess costs of up to $3 per head. If an excess of $1.50 per head is accepted as the average excess, an additional cost of $140,000 is incurred by processing at these undersized plants.

The major overall finding for the ECQ region is that processing within region is indicated with works sited at 'transhipment points', thus stressing the importance of the two port locations of Rockhampton and Mackay. The potential of the new service works at Mackay (at present suffering huge losses) is underlined.

Limited Capacity—The Short-Run Case

This case does not have the same policy implications as the long-run model. Briefly the results were:

(i) Every meatworks in ECQ produced up to its full capacity;
(ii) The considerable excess of cattle supplies in ECQ over treatment facilities meant that SQ was called on to process this surplus;
(iii) The total cost of transport, processing and final product shipment amounted to $7,127,046. This does not include any fixed costs of processing. The rather high total reflects the extra transport costs entailed in railing the excess cattle south for processing. Thus the costs of capacity limitation within ECQ are highlighted.

As each ECQ works processed up to the limit of its capacity there was no need to revise ECQ processing costs on the basis of changing throughput. Each works processed at its minimum cost point. Due to the 'flatness' of the average variable cost curves a large decline in throughput at any works would be necessary to make more than minimal processing cost changes.

Both works at *Rockhampton* process to the limit of their capacity (5755 K-wagon units), drawing additional cattle from Biloela (1367), Emerald (1704), Springsure (1527), and Mackay (106) and satisfying the major part of export demand. The model specified excess capacity at *Brisbane* (this was due to the arbitrary constant chosen to reflect capacity in the SQ region) and 499 units of ECQ cattle were processed here. Most of these went to the export market. Both Mackay and Biloela produce to capacity and supply the small home demand of their own and surrounding regions.

The implications that emerge from these findings corroborate the results of the long-run case.

(i) 'Production-area oriented' slaughter is the optimum strategy.
(ii) In the short run no consideration of fixed costs, even sub-optimally located plants with relatively small capacities could compete with larger SQ plants for ECQ turn off. The spatial imbalance of treatment facilities is thus apparent.
(iii) The value of the Mackay location is illustrated irrespective of its present difficulties of operation as a service works.

Sensitivity Tests for the Stablity of the Optimum Solution

Sensitivity testing of the stablility of linear programming solutions has long been used in the activity analysis model formulation[10] as solved by the simplex algorithm. The term 'range calculations' is often given to the values by which, for example, the C_j terms may vary before change in the level of specified activities is brought about. Puterbaugh *et al* [35], Hutton and Alexander [19], and Tyler [45] have outlined and modified such tests.

However, appropriate use of sensitivity testing of the range calculation type has not been common in transportation model problems. The only

[10] The term 'activity analysis formulation' is employed here to denote models utilizing programming solution procedure and set-out other than the transportation model type. It does not specifically refer to the activity analysis group of Bawden's classification. Both his standard equilibrium group and his activity analysis group can include models solved by transportation, simplex, quadratic of other algorithms.

significant applied published material located was the work of Kanbur and Neudecker [23] who tested the influence of varying transport costs.

In this study sensitivity tests were made of processing costs for the long-run unlimited capacity case as these were considered most critical.[11] The test starts from the final MODI tableau[12] which depicts the optimum solution. It involves finding for the operative processing plants (separately or in combination) the range from the given processing costs which if exceeded would cause change in the optimal solution. This change can be either in product flows or in the location of processing.

The test is based on the fact that in a cost minimizing problem if the *Z-C* value for any cell is positive, savings can be effected and hence the optimum has not been attained. Keeping other costs constant then, change in the derived solution will be brought about, for example, when the addition or subtraction of a certain value to ECQ processing costs causes, through interactions in the cost matrix, a positive *Z-C* value for a previously *inactive cell.*

The mechanics of the test using the final MODI tableau depend firstly on the criterion that the addition of the values for the relevant derived row and column variables U_i and V_j, gives a value equal to the relevant *C* value for all *active* cells. Working from this, where ECQ processing costs are changed, (for example, by a $+\triangle$ or $-\triangle$), a new set of U_i and V_j values must be derived. By noting which processing cost column or columns will be changed, and the position of active cells within the matrix, the row and column U_i and V_j values that will as a consequence of the test be altered are delineated. Hence *Z* values can now be calculated for all currently *inactive* cells. These *Z* values are compared with the new set of *C* values to determine what is the smallest $\triangle$ which can make at least one of these *Z-C* values positive. The same general method holds for both addition and subtraction of $\triangle$.

Tests include the following:

(i) The sensitivity of the final solution to changes in ECQ processing costs.

(ii) The effects of variations in processing costs in the SQ region.

(iii) The effect of certain assumptions incorporated in building up the LRAC curve, for example, the assumption of a 10 per cent return on capital.

(iv) Delineation of the range of plant size possible to meet optimum capacity requirements at specified processing centres.

In essence such testing highlighted the extreme range of plant size that could exist in ECQ while still retaining the optimum shipment pattern and processing locations. Primarily the relative 'flatness' of the LRAC curve accounts for this result. On the other hand, on the steep part of the curve the viable range is greatly reduced. Mackay is an example here.

[11] Sensitivity testing is reported only briefly here as the authors are investigating further in this field.

[12] The MODI algorithm was used to solve the study problem. For an account of this solution procedure see Llewellyn [27].

Summary and Conclusions

This study sought to define the optimum size and location of beef slaughter plants for the region with the greatest density of beef cattle in Australia.

The basic model used was a modified transhipment formulation. This was applied to a long-run unlimited capacity case and a short-run limited capacity case. Briefly, on the basis of the data collected, the models vindicate the views of those who advocate processing of ECQ cattle in ECQ. A combination of production-area oriented and transhipment-point oriented slaughter is indicated. However, this may not be a blue-print for other areas. The existing meatworks in ECQ did not have enough capacity to move animals through slaughter to demand *at least cost*. On the other hand, facilities in SQ depending on ECQ cattle to raise throughput would be adversely affected if all regional slaughter was undertaken in ECQ. These results emphasize the inefficient location of capacity in SQ.

The model establishes a *prima facie* case for more capacity in ECQ, and at Rockhampton in particular. Most importantly it does show the applicability of the methodology in obtaining a rational siting plan as a basis for industry policy. Further, claims by local authorities within the region for government finance for inland works cannot be supported. Guidance may also be rendered to entrepreneurs intent on constructing a works within the region. However, it is possible that the major policy implication is that such a method having been used for ECQ could in fact be applied to Queensland as a whole.

Of major importance are the savings to the industry though optimum siting of plants. Reallocating both slaughter animals and meat shipments to conform to the new locational pattern for this region results in a saving of approximately $1 million or 50 per cent of present live animal transport costs. This saving amounts to around 11 per cent of the total optimum costs of live animal assembly, processing and final product shipments. Still, such results need to be viewed in the light of limitations of data and the assumptions made. These include the use of Trade Association data for meat shipment costs, the assumption of inelastic supply and demand functions, the use of only a single year's supply data and the assumption of a partial and static equilibrium framework. Finally, savings may be largely unattainable without a policy of rationalization and reorganization.

Techniques used which are considered to have brought the analysis closer to reality include sensitivity testing of results, and allowing for the effects of seasonality of supply of cattle. Two general areas in which future researchers might be able to suggest improvements are data collection and models.

As far as data are concerned, any increase in the number of years considered or extension of the study area could entail formulation of sampling procedures for supply estimation. It is difficult to suggest any alternative to consultation with meatworks' management to provide an assessment of processing costs.

There are two general areas of application of the study models. Firstly there is the macro application where a policy forming body investigates the industry's siting and capacity problems. Secondly a single firm may investigate sites and possible sizes of works with a view to building a new works. Dealing with the former case first, recall that adaptations of the models utilized (notably the sensitivity testing) resemble simulation procedures, This new field offers a tool for further studies. In setting up a macro siting plan, possibly it would first be desirable to run the general optimizing models outlined in this study to gain assessment of optimum sizes and sitings. Then abandoning the optimality considerations, a simulation model could be set up in which random sets of supplies and demands could be generated according to assumed distributions of these variables. This would test the stability of the optimizing model's findings. Specific effects of changes in government policy on rail and road freights, or construction of additional 'beef roads', or changes in technology, could be tested.

On the micro scale either optimizing or simulation models, or both, could be used by a firm in its policy planning. It has been suggested that satisficing and not maximizing behaviour with regard to profit levels more realistically characterizes this particular goal of the firm. Incorporating such behaviour as the maintenance of a market share subject to a satisfactory level of profits could be undertaken in a simulation framework.

References

[1] Bardsley, K. L., *The Pattern of Recent Beef Cattle Movements in Central Queenlsand.* Paper prepared by A.N.Z.A.A.S., Brisbane, 1961.

[2] Balinski, M. L., 'On Finding Integer Solutions to Linear Programs'. *Proc. I.B.M. Computing Symposium,* Yorktown Heights, N.Y., 1964.

[3] Baumol, W. J., *Spatial Equilibrium with Supply Points Separated from Markets with Supplies Predetermined.* Mimeo Report Bureau of Agricultural Economics U.S.D.A., Washington, D.C., 1952.

[4] Bawden, D. L., 'An Evaluation of Alternative Spatial Models'. *J. Farm Econ.* 46 (5), 1964.

[5] Beckmann, M., 'A Continuous Model of Transportation'. *Econometrica,* 20 (4), 1952.

[6] Beckmann, M. J. and Marschak, T., 'An Activity Analysis Approach to Location Theory'. *Proceedings of the Second Symposium in Linear Programming.* Washington, D.C., 1955.

[7] Boventer, E. von., 'The Relationship between Transportation Costs and Location Rent in Transportation Problems'. *J. Reg. Sci.* 3 (2), 1961.

[8] Bobst, B. W. and Waananen, M. V., 'Cost and Price Effects of Concentration Restrictions in the Plant Location Problem'. *Am. J. Agric. Econ.* 50 (3), 1968.

[9] Bressler, R. G., 'Pricing Raw Product in Complex Milk Market'. *Agric. Econ. Res.* 10 (4), 1958.

[10] Cassidy, P. A. *Optimum Location, Number and Size of Beef Slaughter Plants in Eastern Central Queensland.* Unpub. M.Agr.Sc. thesis, Univ. of Qld., 1968.

[11] Dantzig, G. B., 'Programming of Interdependent Activities', in T. Koopmans (Ed.) *Activity Analysis of Production and Allocation.* Wiley and Sons, N.Y., 1951.

[12] Dennis, C. C., 'Processing Cost Estimation in Studies of Interregional Competition', in R. A. King (Ed.) *Interregional Competition Research Methods.* University of North Carolina, 1965.

[13] Dorfman, R., Samuelson, P. A. and Solow, R. M. *Linear Programming and Economic Analysis.* McGraw-Hill, N.Y. 1958.

[14] Enke, S., 'Equilibrium Among Spatially Separated Markets: Solution by Electric Analogues'. *Econometrica.* 19 (1), 1951.

[15] Estall, R. C. and Buchanan, R. O., *Industrial Activity and Economic Geography.* Hutchinson, 1964.

[16] Feldman, E., Lehrer, F. A. and Ray, T. L., 'Warehouse Location Under Continuous Economies of Scale'. *Mgt. Sci.* 12 (9), 1966.

[17] Heady, E. O. and Carter, H. O., 'Input-Output Models as Techniques of Analysis for Interregional Competition'. *J. Farm Econ.* 41 (5), 1959.
[18] Hurt, V. G. and Tramel, T. T., 'Alternative Formulations of the Transhipment Problem'. *J. Farm Econ.* 47 (3), 1965.
[19] Hutton, R. F. and Alexander, R. H., 'A Simplified Feed-Mix Problem'. *J. Farm Econ.* 39 (3), Part 1, 1957.
[20] Isard, W., 'Interregional and Regional Input-Output Analysis—A Model of the Space Economy'. *Rev. Econ. Statistics.* 33 (4), 1951.
[21] Johnson, J., *Statistical Cost Analysis.* McGraw-Hill, 1960.
[22] Judge, G. G., Havlicek, J. and Rizek, R. L., 'An Interregional Model: Its Formulation and Application to the Livestock Industry'. *Agric. Econ. Res.* 17 (1), 1965.
[23] Kanbur, M. G. and Neudecker, H., 'Methodology of Spatial Equilibrium Models of the Rice Economy of South India'. *Artha Vijana*, 8 (1), 1966.
[24] Keuhn, A. A. and Hamberger, M. J., 'A Heuristic Program for Locating Warehouses'. *Mgt. Sci.* 10, 1963.
[25] King, G. A. and Logan, S. H., 'Optimum Location Number and Size of Processing Plants with Raw Product and Final Product Shipment'. *J. Farm Econ.* 46 (1) 1964.
[26] Koopmans, T. C. (Ed.), *Activity Analysis of Production and Allocation.* John Wiley and Sons, N.Y., 1951.
[27] Llewellyn, R. W., *Linear Programming.* Holt, Rinehart and Winston, 1964.
[28] Leath, M. N. and Martin, J. H., 'The Transhipment Problem with Inequality Restraints'. *J. Farm Econ.* 48 (4), Part 1, 1966.
[29] Lefeber, L., *Allocation in Space: Production Transport and Industrial Location,* North Holland, Amsterdam, 1958.
[30] Leuthold, R. M. and Bawden, D. L., *An Annoated Bibliography of Spatial Studies,* Research Report No. 25, University of Wisconsin Agricultural Experiment Station, Aug., 1966.
[31] Logan, S. H. and King, G. A., 'Size and Location Factors Affecting California's Beef Slaughtering Plants'. *Hilgardia,* 36 (4), 1964.
[32] Logan, S. H. and King, G. A., *Economies of Scale in Beef Slaughter Plants.* Giannini Found. Res. Rept. 260, 1962.
[33] Orden, A., 'The Transhipment Problem'. *Mgt. Sci.* 2 (3), 1956.
[34] Pherson, V. W. and Firch, R. S., *A Procedure for Determining Optimum Warehouse Location.* Indiana Agric. Exp. Stn. Res. Bull. 706, Lafeyette, 1960.
[35] Puterbaugh, H. L., Kehrberg, E. W. and Dunbar, J. O., 'Analysing the Solution Tableau of a Simplex Linear Programming Problem in Farm Organization'. *J. Farm Econ.* 39 (2), 1957.
[36] *Report of the Committee of Inquiry into Matters Concerning the Development of the Livestock and Meat Industry.* Queensland Government Printer, Brisbane, 1964.
[37] Samuelson, P. A., 'Spatial Price Equilibrium and Linear Programming'. *Amer. Econ. Rev.* 42 (3), 1952.
[38] Smith, C. A., 'Survey of the Empirical Evidence on Economies of Scale', in *Business Concentration and Price Policy. A Report of the National Bureau for Economic Research.* Princeton University Press, 1955.
[39] Stevens, B. H., 'An Interregional Linear Programming Model'. *J. Reg. Sci.* 1 (1), 1958.
[40] Stevens, B. H., 'Linear Programming and Location Rent'. *J. Reg. Sci.* 3 (2), 1961.
[41] Stollsteimer, J. F., 'A Working Model for Plant Numbers and Locations'. *J. Farm Econ.* 45 (3), 1963.
[42] Takayama, T. and Judge, G. G., 'An Interregional Activity Analysis Model for the Agricultural Sector'. *J. Farm Econ.* 46 (2), 1964.
[43] Takayama, T. and Judge, G. G., 'An Intertemporal Price Equlibrium Model'. *J. Farm Econ.* 46 (2), 1964.
[44] Takayama, T. and Judge, G. G., 'Spatial Equilibrium and Quadratic Programming'. *J. Farm Econ.* 46 (1), 1964.
[45] Tyler, G. J., 'Analysis of the Solution Tableau of a Standard Linear Programming Problem'. *J. Agric. Econ.* 17 (2), 1966.
[46] Weinschenck, G., Henrichsmeyer, W. and Aldinger, F., 'The Theory of Spatial Equilibrium and Optimal Location in Agriculture: A Survey'. *Rev. Mktg. Agric. Econ.* 37 (1), 1969.

APPENDIX

Unlimited capacity—the long-run case

Minimize:

(1) $$Z = \sum_{i=1}^{2N} \sum_{j=1}^{2N} C_{ij}X_{ij}$$

(total cost of all shipments of meat, and shipment and slaughter of live animals),

subject to:

(2) $\Sigma_i X_{ij} = r_j + A$ for $i, j = 1, ..., N, N+1, ..., 2N$
$N = m + n$

(total meat requirements at consuming area must equal total shipments to the area plus artificial value A)

(3) $$-\Sigma_j X_{ij} = -k_i - A$$

(total shipments from an area must equal total supply, plus artificial A),

(4) $$C_{ij} > 0 \text{ for } i \neq j$$

$= 0$ for $i = j$ and $i, j = 1, ..., N, N+1, ..., 2N$
(within region transportation cost is zero)
$= 0$ for $i = N+1$; $j = 2N$
$= 0$ for $i = N+2$; $j = 2N$
$= 0$ for $i = N+3$; $j = 2N$
$= 0$ for $i = 2N-1$; $j = 2N$
(cost of transporting meat to port is zero for slaughtering works sited at port),

(5) $C_{ij} = S_1$ for $i = j$ and $i = 1, ..., N$; $j = N+1, ..., 2N$
(diagonal of north-east submatrix contains slaughter costs)
$= \infty$ for $i = N$, $j = 1,...., (N-1)$
(precludes the possibility of overseas areas sending live animals to any regional basing point)
$= \infty$ for $i = 1,...., N-1$; $j = N$
(precludes the possibility of regions exporting live animals overseas)
$= \infty$ for $i = N$; $j = N+1,...., 2N$
(precludes the possibility that slaughtering can take place overseas)
$= \infty$ for $i = 1,...., N+1$; $j = 2N$
(precludes the possibility that live animals from overseas can be sent to be slaughtered at any of the regional basing points)
$= \infty$ for $i = N+1,...., 2N$; $j = 1,...., N$
(diagonal and entire cost matrix contains high costs to preclude entries in shipment pattern)
$= \infty$ for $i = 2N$; $j = N+1,...., (2N-1)$
(as slaughtering of regional supply is prohibited at overseas centres, no transhipment of meat back to regional centres is allowed).

$X_{ij} \geqslant 0$ where X_{ij} are the physical quantities of live animals (unit = 20 head) for $i = 1,....,N$, $j = 1,....,N$
$X_{ij} = 0$ where $i = N$ and $j = 1,...., (N-1)$
$X_{ij} = 0$ where $i = 1, ..., (N-1)$ and $j = N$
$X_{ij} = A$ where $i = N$ and $j = N$,

(6) $X_{ij} \geqslant 0$ where X_{ij} are the physical quantities of animals (unit = 20 head), for $i = 1,...., N$, $j = N+1,...., 2N$
$X_{ij} = 0$ where $i = N$ and $j = N+1,...., 2N$
$X_{ij} = 0$ where $i = 1,...., N-1$, and $j = 2N$,

(7) $X_{ij} = 0$ for $i = N+1,...., 2N$, and $j = 1,...., N$,

(8) $X_{ij} \geqslant 0$ where X_{ij} is the meat equivalent of animals slaughtered (unit = 20 head), for $i = N+1,...., 2N$, and $j = N+1, ..., 2N$
$X_{ij} = 0$ where $i = 2N$ and $j = N+1,...., 2N-1$
$X_{ij} = A$ where $i = 2N$ and $j = 2N$,

(9)

$$A \geqslant \sum_{j=N+1}^{2N} r_j \text{ where } r_j = 0 \text{ for } j = 1, ..., N$$

(artificial value A is chosen to assure inclusion of only relevant X_{ij} in the optimum solution)
$k_i = 0$ for $i = N+1,...., 2N$
(supply of live animals is limited to the relevant submatrix),

(10)

$$\sum_{j=1}^{2N} r_j = \sum_{i=1}^{2N} k_i$$

(total demand equals total supply; artificial constants included would cancel out, and in consequence total real demand equals total real supply).

Excluding addition of any artificial constants then:

$$\sum_{j=N+1}^{2N} r_j = r_{jd} + r_{jx}$$

(total real demand equals domestic demand plus export demand)

$$\text{where } r_{jd} = \sum_{j=N+1}^{2N-1} r_j$$

(total domestic demand is the sum of consumption at domestic centres)

$$\text{and} \quad r_{jx} = \sum_{j=N+1}^{2N} r_j - \sum_{j=N+1}^{2N-1} r_j = \sum_{i=1}^{N} k_i - \sum_{=N+1}^{2N-1} r_j$$

(difference between total real demand and total domestic consumption gives export demand which also equals total real supply minus total domestic consumption).