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# CAPITAL BUDGETING, INTERTEMPORAL PROGRAMMING MODELS, WITH PARTICULAR REFERENCE TO AGRICULTURE

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Investment decision processes typically involve the selection of projects, the timing of their initiation and the determination of the amount to be invested in each time period. A linear programming model considered appropriate for solving such models is described, in which the multi-dimensional criterion function is expressed as a linear combination of the appropriately-weighted objectives. An empirical application is then discussed, the objectives of the firm being the maximization of tax-free cash and assets on hand at the end of the planning period. Finally, the appropriate length of the planning horizon, and some approaches to capital budgeting under non-certainty, are discussed.

## *Possible Approaches to the Capital Budgeting Problem*

Perhaps the simplest approach to capital budgeting problems is to determine the present value of future cash flows, the internal rate of return, or the payback period for each of the alternative investment projects [1, pp. 434-453; 11; 15, Chs. 5 and 7]. Such approaches may be inapplicable if

- (i) the investment projects are interdependent so that complementary or competitive relationships exist between them;
- (ii) projects complement each other with respect to cash supplies, as one project which appears infeasible by itself due to a high initial cash outlay, may well be feasible if initiated in conjunction with other projects which supply cash at an early stage;
- (iii) projects have multiple uses, since difficulty may be experienced in determining cash flows.

Programming techniques have been employed to handle such problems. The applications reported by Loftsguard and Heady [14] and Dean and de Benedictis [9] illustrated an objective of maximizing the present value of future income over some planning period. Candler [2] suggested, however, that it would be equivalent but simpler to design a model which maximized income at the *end* of the planning period, including the return from a Bank activity which would reinvest surplus cash. Cocks [4] introduced an objective function which maximized the net worth of the farm business over the planning period and, since then, models have been constructed to maximize various combinations of future consumption, future profits, and the value of terminal assets [6]. Finally, where indivisibilities exist in the choice of investment projects, integer programming methods may be adopted [8, 20].

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*Programming Models with Multi-Dimensional Objectives*

The fact that firms generally pursue several (often conflicting) objectives (in addition to that of profit maximization) should be incorporated into the criterion function of the capital budgeting model. The approach adopted is to assume that utility may be expressed as a linear combination of the appropriately-weighted objectives, and may also be lexicographic in that some other objectives *must* be satisfied in all solutions. That is,

$$U = f(\alpha_1 G_1, \dots, \alpha_n G_n | L_1, \dots, L_m)$$

where  $\alpha_1 \dots \alpha_n$  are weights applied to the objectives  $G_1 \dots G_n$ , and  $L_1 \dots L_m$  comprises a separate set of objectives, the achievement of which is compulsory.

In other words, there exists no satisfactory trade-off between  $G$  and  $L$ —the firm is prepared to sacrifice all growth in order to meet the latter set of objectives.<sup>1</sup> Unless specific values can be given to the weights prior to computation, the weighting problem could be resolved by determining many 'optimal' solutions, each for a different combination of  $\alpha$ -values. Thus an 'efficient set' of solutions will be derived, allowing a subjective choice of the utility-maximizing solution to be made.

*An Empirical Application*

A capital budgeting model was constructed for a New Zealand crop farm of approximately 200 acres in area, which produced a wide range of vegetable crops under contract for processing, in addition to apple and peach cropping.<sup>2</sup> The farm owner had considered replacing some of the annual vegetable crops with new plantings of perennials, and was therefore interested in comparing the relative profitability of annual and perennial crops, and in budgeting his funds over future years to finance the optimal intertemporal cropping programme.

The criterion function comprised two G-type objectives (tax-free cash available to the firm at the end of the planning period and the value of assets owned by the firm at the end of that period), and one L-type objective (that sufficient cash be withdrawn from the system in each year to finance the farm operator's personal requirements).<sup>3</sup>

*An outline of the programming matrix*

The programming model was constructed for a planning horizon of six years,<sup>4</sup> and the structure of the matrix is outlined in Table 1. Its appearance is essentially block diagonal, although activities representing the investment projects, as well as various transfer activities, will provide coefficients below the diagonal.<sup>5</sup> The manner in which cash flows

<sup>1</sup> If such trade-offs *do* exist, then the appropriate objective should be of type G, not L.

<sup>2</sup> The programming application is discussed in detail in [18, Ch. 4].

<sup>3</sup> The model did not allow for any cash withdrawals in excess of this level, since the farmer wished to forego such 'luxury' consumption until his enterprise was further developed. Clearly, if such 'luxury' consumption was permissible, an additional G-type variable would be added to the criterion function.

<sup>4</sup> Specific attention is paid to the appropriate length of the horizon in a later section.

<sup>5</sup> For example, a perennial crop planted in the first year will require resources and may supply cash, in later years.



and taxation were incorporated into the programming matrix is outlined in the Appendix.

Only two non-zero values appear in the objective function, these being the weights attached to the Final Tax-free Cash and Final Assets activities. Thus by setting  $\alpha_1$  equal to unity and parametrically varying  $\alpha_2$ , the 'efficient set' of capital budgets was derived, each reflecting a different preference for assets relative to cash.

#### *The valuation of assets*

In its present context, 'assets' refers only to plantings of perennial crops (apples, peaches and asparagus) and does not include assets in fixed supply over the planning period, such as land, buildings and machinery.<sup>6</sup> Thus the problem was to assign values to all perennial crops which could be in existence on the holding at the end of the planning period, reflecting their value to the farm owner.

Either a positive or normative approach could have been adopted. In the former approach, all assets would be valued in accordance with their market value. The latter approach would assign assets a value equal to the present value of the future income stream from the asset. Because of the difficulty in differentiating between the market values of crops of differing ages, a normative approach to asset valuation was adopted.

Hence all perennial crops present at the end of the planning period were assigned a value equal to the present value of future net revenue discounted from infinity, with crop replacement at the optimum time.<sup>7</sup> That is,

$$AV_{jk} = \max_n \{ R_{m+1}/(1+r) + R_{m+2}/(1+r)^2 + \dots \\ + R_{m+n}/(1+r)^n + A/r(1+r)^{n+1} \}$$

where  $AV_{jk}$  = the asset value (\$ per acre) of activity  $j$  planted in year  $k$ ,

$R$  = net revenue (\$ per acre) from the crop in some year,

$m$  = age in years of the crop at the end of the planning period,

$m + n$  = the optimum replacement age of the crop (in years), and

$A$  = the maximum annuity (\$ per acre).

The asset values of the perennial crops were then entered as negative coefficients in the Final Assets row of the matrix (see Table 1).

#### *The solution*

Table 2 gives the value of terminal tax-free cash and terminal assets for five of the parametric solutions. As would be expected, the value of assets owned at the end of the planning period increased, and the value of tax-free cash available at the end of the planning period decreased, as the 'price' of the Final Tax-free cash activity became cheaper relative to that of the Final Assets activity.

The physical composition of these five solutions varied only in the rate at which the existing asparagus crop was replaced with new plant-

<sup>6</sup> If the farmer had anticipated buying or selling say, land, an appropriate activity would need to be defined and would include the asset value of such land (negative or positive, respectively) in the Final Assets row of the matrix.

<sup>7</sup> The optimum time at which to replace a crop is when annual net revenue from the present crop becomes equal to the anticipated amortized present value of net revenue from the following crop [10, p. 765].

TABLE 2

*Cash and Asset Components of the Parametric Solutions*

Value of tax-free cash \$	Value of assets \$	Limiting values of $\alpha_2$
82,169	696,688	0.339 - 0.346
80,163	702,487	0.346 - 0.357
78,315	707,575	0.357 - 0.369
77,267	710,466	0.369 - 0.447
77,026	711,030	0.447 - 2.150

ings. For example, the solution corresponding to any asset weighting between 0.339 and 0.346 budgets replacement plantings of 0.9 acres in the second year, 8.1 acres in the fourth year and 5.7 acres in the fifth year. Thus 9.3 acres of the old asparagus crop remain at the end of the planning period, since the asset weighting is such that the cash returns still being obtained are given preference over the value of a new planting. On the other hand, the solution to the problem with an asset weighting of between 0.447 and 2.150 requires the entire planting of existing asparagus to be replaced with new plantings in the second year. Such action may be reasonable for a farmer who values assets up to twice as much as tax-free cash—he might prefer a new perennial planting (with initially negative returns) to an older perennial crop which provided positive but dwindling cash returns.

After each plan had been discussed, the farm manager indicated a preference for the second since it gave the asparagus replacement pattern most acceptable to him.<sup>8</sup> The cropping activities of the preferred plan, and the corresponding capital budget, are tabulated in the Appendix.

*The Interpretation of Shadow Prices*

Formally, the values imputed to scarce resources measure the extent to which the criterion function (i.e. the value of the appropriately-weighted sum of objectives), will increase as a result of marginal increments in the supply of a scarce resource. Obviously such values may, by themselves, provide little useful information. However, information of considerable interest is to be found in the body of the final simplex tableau—namely, the marginal productivity of any scarce resource in generating each of the G-type objectives of the criterion function.

In the present example, the marginal productivities with respect to

<sup>8</sup> Since the basis for this solution will remain optimum for any asset weighting between 0.346 and 0.357, the farm operator may consider future dollars from perennial crops to be equivalent to about 35 cents tax-free cash, at the end of the planning period.

It could be argued, however, that the farmer chose this solution simply because he preferred (for reasons other than liquidity) to plant relatively small acreages of asparagus in any year. If this was the case, the model should have contained restrictions on the maximum area of asparagus which the farmer was willing to plant in any year. The author was satisfied, though, that since no *technical* problems were associated with planting larger acreages, the farmer's preference to planting smaller acreages implied a preference for cash, as against assets which would have a *negative* cash flow during their period of establishment.

final tax-free cash, and assets, were obtained. Such coefficients occur in the appropriate 'slack' column (which would be non-basic), and the final cash and asset rows, respectively (which are basic). A few such examples are shown in Table 3. The knowledge, for example, that an extra acre of cropland purchased at the beginning of the planning period will increase tax-free cash and assets at the end of the planning period by \$276.03 and \$32.55 respectively, is likely to provide greater insight into the allocation-valuation problem than simply stating the shadow price for such cropland of \$287.42. Furthermore, the year one Annual Cropland 'slack' column in the final simplex tableau would indicate exactly how such values were derived.

### *The Length of the Planning Horizon*

Since capital budgeting involves the planning of cash allocations over a series of time periods, it is useful to know how far into the future such capital budgeting should be carried. As the budgeting analysis is carried further into the future, the less will each additional time period affect the desirable investment policy in the first time period, and it is therefore proposed that the time period considered should be continually extended until investment decisions in the first time period become insensitive to further extensions of the planning horizon. Thus, by examining several planning horizons, the sensitivity of the first time period's policy to changes in the length of the planning horizon, and the most profitable policy for the first time period, will be ascertained.<sup>9</sup>

It could be argued that the planning horizon should be extended even further, until plans in the second and subsequent time periods become insensitive to any further extension. If, at the beginning of the planning period, investment policies need be formulated for *only the initial time period*, however, it is necessary to extend the horizon only until the policy for the first time period becomes stable.

The plans for future years, however, do provide a *forecast* of future decisions, conditional on the information available at the beginning of the planning period.

The above process would be repeated at the end of the first time period to find the optimum ('stable') policy for the second period, and so on. Also, the programming matrix may be modified to take account of any additional information obtained during the course of the previous time period.<sup>10</sup> Further, the policy forecasts for future time periods would be revised, and insofar as the information available at the end of a time period is, in a sense, better than that available at the beginning of the period, the new forecasts would presumably be closer to the later realizations than those derived earlier.

The planning horizon of the empirical application (with the preferred weights) was reduced from six years to five, and then to four years,

<sup>9</sup> If, by extending the planning horizon, the policy for the first year is altered, then the former programme, when budgeted over a planning horizon equivalent to that of the latter plan, cannot be more profitable simply because the latter budget maximizes the value of the criterion function over that particular planning horizon.

<sup>10</sup> Such additional information may include the *actual* outcome of the policy of the previous time period, more accurate estimates of future input-output coefficients (such as prices) and resource supplies, or changes in the decision-maker's preferences.

TABLE 3  
*Portion of the Final Simplex Tableau*

$C_j$ ↓	Basis	B	Slack Activities						Maximum apple acreage 1 acre
			Taxfree cash year 1 \$1	Labour year 1 1 hour	Annual crop- land year 1 1 acre	Annual crop- land year 6 1 acre	Beetroot con- tract year 1-6 1 acre	Asparagus con- tract year 1-6 1 acre	
1.00	Cash	80,163	1.35	1.30	276.03	54.36	12.01	-367.47	-203.49
0.35	Assets	702,487			32.55		827.89	4676.06	12845.65
	Z-C	326,033	1.35	1.30	287.42	54.36	301.77	1269.15	4292.49

*Note* The labour coefficients we summed over each labour activity in year one and the coefficients for contracts we summed over the entire six-year planning period (i.e. a marginal increase in the contract was assumed to apply in each year).



to find whether the cropping programme of the first year would be altered. It was found that the first year's cropping was similar under each of the three planning horizons, so the most profitable investment policy for the first year may have been obtained by analysing only a four-year planning horizon.<sup>11</sup>

#### *Capital Budgeting Under Non-certainty*

In conclusion, it is perhaps appropriate to consider the likely optimality of solutions obtained from the above type of model, given that future coefficients are stochastic rather than deterministic. The approach adopted was essentially that of replacing such random variables with their expected values. Only if the problem exhibits *first-stage certainty equivalence* will the first time period's policy obtained from the deterministic model be identical to that obtained from a full stochastic formulation of the problem [19, Ch. 4]. Although little research has been carried out in this field, there appears to be no reason to assume that such certainty-equivalence would apply. Hence, the 'optimal' solution given by the deterministic model may, in fact, be sub-optimal. It is fitting, then, to briefly review some of the possible approaches to incorporating non-certainty into intertemporal programming models.

Problems of price and cost variability may be insured against, to some extent, by including in the model lower-than-expected prices, higher-than-expected costs, and provision for cash withdrawals in excess of those thought likely to be required. Also, variable costs may be increased each year to reflect say, likely rises in a 'factor-cost' index. Obviously, though, such a method of insurance is likely to introduce some 'bias' in the selection of the 'optimum' capital budget.

Another approach to the inclusion of risk into an intertemporal programming problem is that presented by Cohen and Elton [7]. Their paper discusses intertemporal quadratic programming for selecting portfolios of risky assets, and its application to capital budgeting. For each investment project, those factors which determine cash flows in each time period must be isolated and the probability distributions of these underlying factors specified. The values of these factors are then simulated period-by-period, and after several simulation runs the means, variances and covariances of the present values of the investment projects may be determined. A portfolio-type quadratic programming problem can then be formulated and solved. The weakness of this approach is that the *expected* (mean) values of all input-output coefficients must still be used in the transformation matrix. Some project, then, whose product prices may be particularly variable, may only be included in 'high risk' solutions, which is acceptable. What is not acceptable, though, is that the intertemporal cash flow due to this project is assumed to be *nonstochastic*. Thus an aspect of risk of importance to the decision maker, i.e. how variability in the year-by-year cash flows will affect his investment programme, is disregarded. A feasible solution to the intertemporal quadratic programme may turn out to be infeasible when the stochastic nature of the coefficients is taken into account.

<sup>11</sup> Cropping programmes in later years, however, were affected by the length of the planning horizon, and such alterations involved the rate at which the old asparagus beds were replaced. Generally, less asparagus would have been replaced at a given point in time, the shorter the length of the planning period.

Chance-constrained programming [3] has been suggested as a method of handling risk in the capital budgeting decision problem [16, 17]. This approach would involve the determination of a variance-covariance matrix as well as the vector of expected input-output coefficients appropriate to each restraint. Quadratic restraints would then be added to the model to ensure that the value of cash required by the optimal solution in any year will be not greater than the lower confidence limit of cash supplies. Also, the confidence limits may correspond to any probability level selected by the firm. Thus the (nonlinear) capital budgeting programme can be required to have any desired probability of satisfying the restraints in the face of stochastic input-output coefficients. The approach has two basic weaknesses, however. First, since many levels of probability of satisfying the restraints may be chosen, the question 'what is the *optimum* probability of meeting the restraints?' arises; and second, the method does not incorporate any allowance for costs incurred (or alternative strategies) when restraints *are* violated.

Stochastic methods of linear programming show promise in allowing the incorporation of risk into the intertemporal capital budgeting problem. One such method, proposed by Johnson, Tefertiller, and Moore [13] allows any of the coefficients of the linear programming problem to be random variables with specified distribution functions, so that the objective function will also have an associated distribution function. A set of variates drawn from the probability distributions of the random variables is substituted for the stochastic parameters in the problem. The objective function value in the solution to this deterministic linear programming problem will then be a variate from the probability distribution of the objective function and, by repeating this procedure, the probability distribution of the objective function will be approximated. This approach could provide problems in application, however. The solution to the stochastic model will indicate the mean and standard deviation of net income in each year of the planning period. Since *activity levels* as well as net income may vary from one deterministic solution to another, there will be a probability distribution associated with, say, investment activities as well as net income. The question would then arise, 'what programme should the decision maker adopt?' When activity levels have associated distribution functions, this question may remain unanswered.

An alternative stochastic model has been formulated by Cocks [5]. It allows the solution of problems in which any number of the functional, restraint and input-output coefficients are subject to discrete probability distributions. The method assumes that the decision maker has no precognition of future coefficient values, but that the values known to him increase at each decision date through the planning period. The model is designed to maximize the unconditional payoff over all possible environments (by solving a very large linear programming model), and the solution will define the actions to be followed over all possible eventuating environments.<sup>12</sup> Thus, at any decision date, the decision maker can select the appropriate set of actions consistent with his increasing

<sup>12</sup> Should utility be more accurately expressed as quadratic (rather than linear) in the value of the objective function, the model may be extended to one of quadratic programming [5, p. 75]. The model also shows potential in handling lexicographic and 'satisficing' goal functions.

information about probabilistic coefficients as the planning period progresses.

Finally, simulation models have been constructed to allow the formulation of management policies in an uncertain environment [12, 21]. Although such models are 'improving' rather than 'optimizing', they are probably capable of recognizing a greater number of states of nature than are stochastic programming models. Perhaps, then, a strategy could be derived from a stochastic programming model and then checked for feasibility over a wider range of possible states of nature in a simulation model. Obviously, it would be useful to be able to preselect the 'critical' states of nature to be included in the stochastic programming model.

### Summary

A capital budgeting, programming model has been discussed with particular reference to farm growth and investment problems. Also, the type of model described would be applicable to many investment problems of non-agricultural firms.

The multi-dimensional criterion function is seen to be rich in possibilities, allowing the consideration of many, often conflicting, objectives of the firm. The marginal productivities of scarce resources with respect to each of the objectives may provide most useful information to decision makers.

The importance of the length of the planning period has also received attention. Previously, many programming problems had been constructed for horizons of arbitrary length. Such problems may have given similar solutions with shorter planning horizons, thus effecting some reduction in computational effort and cost, or a more profitable policy may have been found by extending the horizon.

Some suggested methods of capital budgeting under non-certainty have been reviewed. Until such methods, such as stochastic programming, are further developed, though, many useful capital budgeting programmes may still be formulated using deterministic models. Even given the existence of a satisfactory stochastic routine, the two approaches should still be compared on a relative benefit-relative cost basis.

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## APPENDIX

TABLE A1

*Cropping Activities (acres) in the Preferred Plan*

Crop	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Intercrop beetroot	7.0	7.0	7.0	7.0	—	—
Intercrop kumera	—	—	8.0	—	—	—
Intercrop mangold	0.5	8.0	—	0.5	—	—
Tomato	18.4	17.9	15.9	14.4	12.9	12.9
Green bean	27.0	24.8	21.0	15.0	9.0	9.0
Beetroot	—	—	—	—	7.0	7.0
Pea	8.5	4.2	—	—	—	—
Carrot	3.6	3.6	3.6	3.6	3.6	3.6
Broad bean—kumera rotation	—	5.0	5.0	5.0	5.0	5.0
Kumera	19.5	19.0	11.0	19.0	19.0	19.0
Mangold	7.5	—	8.0	7.5	8.0	8.0
Old asparagus planting	24.0	14.8	14.8	14.8	4.7	—
Old peach planting	25.0	25.0	25.0	25.0	25.0	25.0
New plantings of apples	10.0	10.0	10.0	—	—	—
New plantings of asparagus	12.0	9.2	—	—	10.1	4.7

TABLE A2

*Cash Flows and Taxation (\$) for the Preferred Plan*

Item	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Tax-free cash at beginning of year	27,000	33,405	41,722	52,375	44,704	60,899
Variable costs	27,000	31,792	31,873	33,620	39,136	44,823
Cash banked	—	1,613	9,849	18,755	5,568	16,076
Pre-tax receipts	79,087	89,759	97,030	103,933	113,457	125,580
Tax-deductible expenditures	66,026	70,818	70,899	72,646	81,962	85,389
Assessable income	13,061	18,941	26,131	31,287	31,495	40,191
Tax payments <sup>(a)</sup>	6,466	10,434	15,288	18,768	18,910	24,777
Tax-free receipts	72,621	79,325	81,742	85,165	94,547	100,803
Less total cash withdrawals	39,216	39,216	39,216	59,216 <sup>(b)</sup>	39,216	36,716
Plus cash in bank gives tax-free cash available at beginning of following year	—	1,613	9,849	18,755	5,568	16,076
	33,405	41,722	52,375	44,704	60,899	80,163

(a) The error in the programmed tax payments can be estimated by deducing correct tax payments from the official Taxation Tables. For example, tax payments in year six should be \$24,576, so the programmed amount of \$24,777 overstates the tax payment by only 0.8%. Of course, the greater number of Taxation activities included in the model, the more accurate will be the tax calculations.

(b) An additional \$20,000 was required by the farm manager to construct a packing shed in the fifth year. The shed would be required at that time by the apples already planted at the beginning of the planning period, but would have adequate capacity to handle produce from any further plantings which may be made.



Tables A1 and A2 summarize the preferred solution and should require no further explanation. Table A3, which gives details of cash and taxation components of the programming matrix, may be interpreted as follows.

The total amount of tax-free cash which the farm operator had available for investment in his holding gave the first entry in the B column. This cash was then available to the cropping activities and to hire labour, with any balance invested in the Bank activity. The cropping activities supply pre-tax cash receipts, as will the Bank activity in the form of interest earned. (The positive entry in the B column for pre-tax receipts is the expected return from a recent planting of perennial (apple and asparagus) crops. These crops were not included as activities since the owner did not intend to replace them during the planning period.)

Next, assessable farm income was derived by calculating total tax-deductible expenditures and subtracting them from pre-tax receipts. In the Tax deductions row, the B column entry represented that portion of overhead costs which were deductible and the negative coefficient indicated variable costs which were deductible. Thus the Tax Deductions Transfer activity will subtract total deductions from pre-tax receipts before tax payments are calculated. Assessable farm income was then transferred by way of the Tax Transfer activities into the supply of tax-free cash available to finance the following year's operations. The negative coefficient in the B column for tax-free cash year two comprised cash withdrawals (a) necessary to pay the overhead cash costs (depreciation is treated as a non-cash cost) of the previous year and (b) to meet the farm operator's personal requirements during the second year.

This process continued throughout the six years of the planning period, with tax-free cash available at the end of that period transferred (after payment of the year six overhead cash costs) to the objective function via the Final Tax-free Cash activity.