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NEW ZEALAND BEEF AND SHEEP SUPPLY RELATIONSHIPS

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The primary purposes of this study are to measure major relationships describing the responses of different components of sheep and beef farming capital stock in New Zealand to changes in economic conditions, and to investigate the way in which this capital stock has changed over time. These objectives are pursued by attempting to specify an econometric model that recognises joint production between sheep and beef cattle and also takes account of joint firm/household decision making. Aggregate New Zealand data for the period 1952/53 to 1973/74 are fitted to the model using Full Information Maximum Likelihood estimation.

Introduction

In recent years the sheep and beef industries have generated more than half of New Zealand's export earnings through the livestock products: wool, mutton, lamb and beef. As these products currently make up about a quarter of gross national product, it is of some interest and importance to investigate the nature of their source of supply and thereby obtain numerical estimates of economic influences on New Zealand supply.

Aggregate New Zealand data are used to estimate a model based on individual farmer behaviour. Thus, the model will not involve analysis of the behaviour of a particular individual farmer, but rather that of the representative farmer in the industry. This approach is based on the assumption that the individual farmer is representative of the industry. Clearly this is a strong assumption as farmers and their farms are not all homogeneous and therefore an aggregation bias will exist in the parameters that are estimated by the model. Because the empirical analysis is for the aggregate sector, inter-producer decisions are implicitly ruled out.

Because of the data problem of identifying the value of labour that is supplied by the farmer and his family, labour is not formally constrained in this model. This implies that the results obtained are conditional on the representative farmer being able to supply enough labour to run adequately the livestock numbers specified by the model. Therefore, only the physical amount of land, buildings, plant and machinery that the farmer owns is a constraining technical factor in determining the number of livestock that the farmer can run. In this paper, the

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physical amounts of land, buildings, plant and machinery, are collectively termed 'capital inputs', while the numbers of sheep and beef cattle are collectively called 'stock on hand'. These capital inputs plus stock on hand make up the farmer's total capital stock.

The revenues which the representative farmer obtains from his wool, mutton, lamb and beef accounts by selling these products at actual farm-gate prices (divided by an appropriate price deflator) are referred to, in this paper, as real gross revenues. The sum of the real gross revenues obtained from the wool, mutton and lamb accounts divided by the number of sheep carried is referred to as the real gross revenue per sheep. The real gross revenue obtained from the beef account divided by the number of beef cattle run is the real gross revenue per cattle beast. These two variables are taken to be exogenous constraints on the representative farmer's activity because the majority of sheep and beef cattle products are exported, and New Zealand's own contributions of these two aggregate products to the associated world markets are small. Real gross revenue per head of livestock is affected both by the price of the livestock sold and by the farmer's decision on how many livestock to sell. Since real gross revenue per head of livestock is treated exogenously, so therefore are the slaughtering rates of lamb, mutton and beef.

The representative farmer will typically commit his revenue before he receives it, therefore the relevant variable in an economic theory of supply is *expected* real gross revenue. Part of this expected real gross revenue is used to pay outside labour expenses that the farmer incurs while producing these livestock products. The remainder is referred to as expected real net revenue and is allocated among farm mortgage repayments, expenditure on current inputs, farm investment, off-farm investment, taxation payments and consumption expenditure. Table 1 gives a simple schematic representation of the source and disposition of this expected real net revenue.

TABLE 1
Expected Real Net Revenue

Source		Disposition	
Expected real gross revenue:		Farm mortgage repayments	+
Wool a/c	+	Expenditure on current inputs	+
Lamb a/c	+	Farm investment	+
Mutton a/c	+	Off-farm investment	+
Beef a/c	+	Taxation payments	+
Outside labour payments	−	Consumption expenditure	+
Expected real net revenue	=		=

A New Zealand study that specifies some underlying behavioural theory using a positive approach is that reported by Court (1967), in which the representative farmer's problem is modelled within an atemporal framework. The weakness of this study is that sheep and cattle products are not modelled as joint products. A study, using Australian data, that overcomes this problem, but where the farmer's problem is still modelled in an atemporal framework, is that reported by Powell and Gruen (1968). The model proposed in this paper follows the Powell and Gruen approach to some extent but differs in two fundamental ways. First, the axes of the transformation frontier are livestock numbers, not livestock products. Second, the farmer's problem is modelled in an intertemporal framework: that is, the location of the transformation frontier is treated as endogenous, not exogenous. Thus, this model is an attempt to account not only for shifts around the transformation frontier at a point in time, but also for shifts in the location of the frontier through time. We do this by following the approach of McLaren (1976), where the farm is specified as a joint firm-household in which the consumption and farm investment decisions made by the farmer are interdependent.

The Model

The atemporal problems

The representative farmer's consumption problem, at an instant in time, is to maximise his instantaneous direct utility function subject to a budget constraint, where expectations are held with certainty. For simplification, a directly additive utility function as specified by Klein and Rubin (1948/49) and Stone (1954) is chosen. The solution to this constrained maximisation problem leads to the familiar linear expenditure system (LES) of demand equations. These equations can then be substituted back into the direct utility function, thus allowing the indirect utility function to be specified with consumption expenditure and prices as arguments in the form

$$(1) \qquad V(P,\mu,E) = \sum_{i=1}^{n} \beta_i \ln((\beta_i/P_i)\,(1\text{-}\mu)E\text{-}P'\Gamma)),$$

where $V(P,\mu,E)$ gives the maximum utility that can be obtained from consumption; P_i, the ith element of the vector P, is the price of the ith commodity; E is the expenditure available for consumption and payment of income tax; μ is the farmer's income tax rate; Γ_i, the ith element of the vector Γ, is the minimum requirement of good i; and the β_i coefficients are the marginal budget shares of the n consumption goods.

The second atemporal problem is concerned with production. The instantaneous maximum of scalar expected real net revenue of the representative sheep and beef cattle producer, as defined in Table 1, is given by

$$(2) \qquad \text{Max.}\ (S_1{}^*K_1 + S_2{}^*K_2),$$

where K_1 is the number of sheep and K_2 is the number of beef cattle. $S_1{}^*$ and $S_2{}^*$ are the expected real net revenues per sheep and per cattle beast, where expectations are held with certainty.

The constraining technical factor in determining the number of livestock that the farmer can run is the physical amount of land, buildings, plant and machinery. This technical relationship is specified by the multiple output production function, F:

$$(3) \qquad F(\text{total capital stock}) \text{ or } F(\text{capital inputs, stock on hand}) \geqslant 0,$$

where total capital stock includes land, buildings, plant, machinery and livestock numbers. Alternatively, (3) shows that total capital stock can be divided into capital inputs (land, buildings, plant and machinery) and stock on hand (livestock numbers). Hasenkemp (1976) defines (3) to be an efficient input-output combination only if a strict equality sign is present. He discusses the special case of efficient input-output combinations which are separable between inputs and outputs. Under the separability assumption, efficient input-output combinations for this particular model are written

$$(4) \qquad f(\text{stock on hand}) = g(\text{capital inputs})$$

where $f(.)$ may be interpreted as a scalar output function and $g(.)$ as a scalar input function.

Unfortunately, we have a data problem when we attempt to specify (4) for the problem under study, as reliable time series for the various physical capital inputs could not be found. However, aggregate measures of the real value of capital inputs and stock on hand are available, that is, data are obtainable on the physical measure of capital inputs and on stock on hand valued at constant prices. Thus, the constraining technical relationship consists of the two types of livestock and a single aggregate measure of the real value of capital inputs. In order for this atemporal production problem to be nested within the intertemporal problem, the real value of capital inputs is specified as the real value of total capital stock, M, minus the real value of stock on hand. This non-standard specification of the input function, being measured in value terms rather than physical terms, implies that a scale factor W must be included in the input function to overcome the units problem which would otherwise exist. This scale factor, being an index of stock on hand per real dollar of capital input, is affected by disembodied technical progress, the slaughter rate of livestock and expenditure on current inputs. It is not possible from the data which are available, to identify satisfactorily expenditure on current inputs. Therefore this expenditure is taken to be a constant proportion of the real value of total capital stock. By assuming that the slaughter rate is also fixed, the scale factor becomes an indicator of disembodied technical progress. Because this progress occurs without regard to the farmer's decisions concerning net investment or the number of sheep and beef cattle he will run, W is treated as exogenous to the problem.

The left hand side of (4) is an index of sheep and beef cattle numbers on hand and must be nonlinear as these animals compete nonlinearly for the capital inputs that the farm has available. That is, the farmer can obtain more stock on hand by choosing a combination of sheep and beef cattle than if he ran all sheep or all beef cattle. Thus, an additive function exhibiting an increasing marginal rate of transformation is required. A function that satisfies these two requirements is the CET function derived by Powell and Gruen (1968). Thus, (4) is written more specifically as

$$(5) \qquad (K_1^j + AK_2^j)^{1/j} = W(M - c_1K_1 - c_2K_2);\, j > 1,$$

where c_1 and c_2 are the imputed constant livestock values of a sheep and cattle beast respectively. The parameter A is a 'bias' parameter as it is related to the input requirements of beef cattle to sheep while the parameter j affects the curvature of the CET function. When the parameter j is large (say 3) the model constrains the farmer by allowing him only limited possibilities for changing his livestock mix in response to changes in the ratio of the expected real net revenues per animal.

At the aggregate level, this is realistic, because changes in livestock numbers are constrained principally by the maximum biological rate at which national livestock numbers can be increased. Therefore, the suitability of the production problem specification depends on the size of the parameter j: the larger that j is, the more realistic this specification becomes.

Thus, the representative farmer's production problem is to pick, at an instant in time, those livestock numbers such that (2) is satisfied subject to (5). Unfortunately, the optimal livestock supply equations could not be obtained directly so an approximation is made by using a two-step procedure[1] which results in the approximate optimal livestock equations:

$$
\begin{aligned}
(6)\qquad & K_i = \rho_i M;\ i = 1,2,\\
\text{where } & \rho_1 = (\alpha_1 - \alpha_1 c_2 \rho_2)/(1 + c_1\alpha_1),\\
& \rho_2 = \alpha_2/(1 + c_1\alpha_1 + c_2\alpha_2),\\
\text{and } & \alpha_1 = W/(1 + (S_1{}^*/S_2{}^*)^{j/(1-j)} A^{1/(1-j)})^{1/j},\\
& \alpha_2 = W/(A + (S_1{}^* A/S_2{}^*)^{j/(j-1)})^{1/j}.
\end{aligned}
$$

The effect that this approximation has on the model specification is discussed later in the paper when the results of the parameter estimates are analysed.

The intertemporal consumption-investment problem

It is assumed that the representative farmer is continually replanning. That is, he is always at the initial point of an optimal plan of infinite length, hence, historical time is just a sequence of beginnings of plans. This approach implies that the farmer is continuously on his optimal path and clearly this is a strong assumption. In this Section, variables that are non-stationary in planning time will have a time-script (t) attached to them. From Table 1, the expenditure available for consumption and payment of income tax is the maximum real net revenue, given by (2), minus mortgage repayments, expenditure on current inputs and investment. Mortgage repayments are represented by $(i + m)$ $T(t)$, where $T(t)$ represents total borrowing at time t with an average rate of interest i and a principal repayment rate m.

The representative farmer must be modelled to experience the reality of limited funds for investment because of the competing need for consumption expenditure. This is achieved in the model by assuming that the debt/capital ratio is constant over time. That is, $T(t)/M(t) = \zeta$ where $M(t)$ is a scalar representing the real value of total

[1] The first step in the approximation is to change the Lagrangian function from:

(i) $L = S_1{}^*K_1 + S_2{}^*K_2 + \lambda((K_1{}^j + AK_2{}^j)^{1/j} - W(M - c_1K_1 - c_2K_2))$.

to

(ii) $L = S_1{}^*K_1 + S_2{}^*K_2 + \lambda((K_1{}^j + AK_1{}^j)^{1/j} - WM)$.

The implication of this first step is that the real value of capital inputs are temporarily redefined as the real value of total capital stock. The usual first-order conditions, associated with (ii), enable the problem to be solved giving livestock numbers as functions of the real value of total capital stock. In the second step the temporary definition of inputs is changed back to its original definition by making allowance for livestock having to be financed out of the real value of total capital stock.

capital stock at time t. This assumption implies that mortgage repayments can be expressed alternatively as $HM(t)$ where $H = (i + m)\,\zeta$. A simplifying assumption is now made that no off-farm investment occurs because it is thought the majority of investment that is made by the farmer is farm investment. Farm investment consists first of net investment denoted by $\nu(t)$, and second replacement investment. From the data available it is not possible to separate satisfactorily replacement investment from expenditure on current inputs and so these two variables are jointly represented by $\phi M(t)$ where ϕ is the depreciation rate plus the rate of expenditure on current inputs. A simplifying assumption is made that these four items of expenditure, mortgage repayments, net investment, replacement investment and expenditure on current inputs, are all tax deductible. Therefore the expenditure available for consumption is given by

(7) $$(1 - \mu)E(t) = (1 - \mu)\,(\text{max.}\,(S_1{}^*K_1(t) + S_2{}^*K_2(t)) - HM(t) - \phi M(t) - \nu(t)).$$

The solutions to the production problem given in (6) are now adjusted, so that planning time is specified, and substituted into (7). This results in the equation

(8) $$(1 - \mu)E(t) = (1 - \mu)\,(DM(t) - \nu(t)),$$

where

(9) $$D = S_1{}^*\rho_1 + S_2{}^*\rho_2 - H - \phi.$$

D is the implicit rate of return on the real value of total capital stock. It is defined as expected real net revenue per unit real value of total capital stock, minus a proportion of the farm mortgage interest rate, minus the depreciation rate on the real value of total capital stock and the rate of expenditure on current inputs.

Similarly, by adjusting the instantaneous indirect utility function given by (1) so that planning time is specified, and then substituting (8) into (1) we obtain

(10) $$V^*(P,M(t),\nu(t)) = \sum_{i=1}^{n}\beta_i \ln((\beta_i/P_i)(1-\mu)(DM(t)-\nu(t)-P'\Gamma)).$$

The representative farmer's intertemporal consumption-investment problem is to choose over time that flow of real net farm investment that maximises his intertemporal indirect utility functional. Therefore, the problem is to

(11) $$\underset{\nu(t)}{\text{maximise}} \int_0^\infty e^{-rt}V^*(P,M(t),\nu(t))dt$$

where $V^*(.)$ is given by (10) and r is the subjective time discount rate, assumed to be constant over planning time. The functional given by (11) is the discounted sum of all future expected instantaneous utilities. An obvious restriction is

(12) $$M(t) \geqslant 0;\ \forall\, t.$$

At the start of his plan the farmer has an initial real value of total capital stock denoted by

(13) $$M(0) = M_0.$$

By assuming that the vector of consumption prices faced by the farmer is proxied by the scalar variable p, denoting the consumer price index, and that the vector of subsistence quantities is proxied by a scalar parameter γ, the solution of (11) subject to (12) and given (13) is solved by appealing to the usual Euler condition and transversality condition. In control theory language, the closed loop feedback solution expressing the control variable, $\nu(t)$ as a function of the state variable, $M(t)$, is

$$(14) \qquad \nu(t) = (D - r)(M(t) - p\gamma/D).$$

This solution expresses optimal real net investment as a function of the real value of total capital stock.[2]

Estimation Method and Data Construction

Bergstrom (1976) brings together some of the recent papers in the growing literature of methods estimating the structural parameters of stochastic economic models in continuous time from discrete observations on the variables. Our problem falls into this category. However, this body of econometric literature is largely ignored here as the extra work necessary to specify the model within the framework of this literature is outside the scope of this project. Given this simplification, we rewrite the discrete approximation of (14) that relates optimal real net investment to the real value of total capital stock, after first substituting (9) into this expression, as

$$(15) \quad M_t = M_{t-1} + (S_{1t}{}^*\rho_{1t} + S_{2t}{}^*\rho_{2t} - H_t - \phi - r)(M_{t-1} - p_t\gamma/(S_{1t}{}^*\rho_{1a} + S_{2t}{}^*\rho_{2t} - H_t - \phi)).$$

Equation (15), together with the two equations relating livestock numbers to total capital stock, given by (6), form a recursive system of three equations with the causal sequence between variables being a chain without any feedback. The three endogenous variables in the system are M_t, K_{1t} and K_{2t}.

The system is nonlinear in the endogenous variables, and there are nonlinear parameter restrictions within and across equations. The computationally most efficient estimation method is to use the program RESIMUL developed by Wymer (1977) which calculates Full Information Maximum Likelihood (FIML) estimates for parameters of systems which are linear in the endogenous variables but nonlinear in the parameters, possibly including nonlinear restrictions across equations. To apply the program to this problem, the model is first linearised with respect to the endogenous variables by taking first-order Taylor expansions about sample means.

The sample period of all series, except of course M_{t-1}, is 1953 to 1974, year beginning 30 June. Annual data are used and the base year at which prices are measured is 1974. Many of the data are obtained from the *NZ Meat and Wool Boards' Economic Service* and the *NZ Department of Statistics*. However, some of the required data on the

[2] It follows from the Euler equation that if $v(t) = 0$, (14) collapses to the subsistence real value of capital stock denoted by $M(t) = p\gamma/D$. Thus, (14) may be recognised as a type of stock adjustment model according to which the flow of real net farm investment is a proportion of the difference between the current and subsistence stock of capital.

sheep and beef industry are not available and have to be constructed from data that are available from the whole agricultural industry. For example, data on the real value of capital inputs are generated from data reported by Johnson (1970) and data on the index of stock on hand per real dollar of capital input are based on those given by Hussey and Philpott (1969).

In order to specify the manner in which current and past real net revenues influence expectations, a Koyck-type lag distribution is introduced because it is thought that the influence of past real net revenues declines geometrically. This allows the expectations variables S_{1t}^* and S_{2t}^* to be generated for a given expectations coefficient for sheep, F_1. The expectations coefficient for cattle, F_2 is generated from F_1 by following an approach suggested by Gruen et al. (1967) and assuming that these values are inversely proportional to the coefficient of variation of the actual real net revenue per sheep and per cattle beast.

Results

Autocorrelation

It is important to test for autocorrelation in the structural residuals because the violation of the serial independence assumption has implications, first for the statistical properties of the estimates and for the validity of hypothesis tests, and secondly, it suggests that the model may be misspecified. It is well known that under the usual regularity conditions FIML estimates are best asymptotically normal, consistent and asymptotically unbiased. However, if the structural residuals exhibit autocorrelation, given the presence of lagged endogenous variables, the estimates lose even the weak consistency property.

Plotting the structural within-sample residuals reveals long runs of positive and negative values, suggesting that positive autocorrelation might be present. To test whether or not this is so, a Runs test is applied, as described by Wonnacott and Wonnacott (1972, p. 409). This non-parametric test, which has only large sample justification, is chosen in favour of a more suitable test described by Hendry (1971) because of the poor prospect of being able to make meaningful corrections for autocorrelation within such a tightly constrained system estimated by full information methods. The test suggests that the hypothesis of no positive autocorrelation must be rejected.

Parameter estimates

Given that the permissible range of F_1 lies between 0 and 1, several expectations series are generated resulting in several non-nested models being specified. For models based on $F_1 \leqslant 0{\cdot}2$ or $F_1 \geqslant 0{\cdot}5$, small pivot problems are encountered using the FIML algorithm while inverting the variance matrix of parameters and so maximum likelihood estimates are not obtained.

The validity of the restrictions imposed on the remaining two models (i.e. $F_1 = 0{\cdot}3$, $0{\cdot}4$) is of substantial importance from the point of view of estimator sampling properties and hypothesis testing concerning the size of parameters and coefficients associated with the different variables of the system. Therefore, it is important to test the specification of these restrictions using the familiar likelihood ratio test. It is found that the validity of the restrictions on the three constant terms in the system must be rejected and that only for the model with F_1 set at 0·3 are all the remaining restrictions acceptable at the 1 per cent significance level. Therefore the problem of choosing between these models is side-stepped by reporting all results as conditional estimates for a given F_1.

By using these estimates of the expectations coefficients, and following Dhrymes (1971, p. 10), Table 2 is constructed. This table gives information on $M(1 - F_i)$; $i = 1, 2$: the mean number of years it takes for a change in the observed real net revenue per animal variable to be transmitted to the expected real net revenue animal variable. The table also gives, via the standard deviations of the mean lags: $\sigma(1 - F_i)$; $i = 1, 2$, information on the extent to which these lag lengths are concentrated about their means. For sheep and cattle, the mean number of years is 2·3 and 6·7 years, respectively. These point estimates appear to have sensible economic interpretations but their associated standard deviations are unsatisfactory.

TABLE 2

Lag Length Estimates

	$M(1 - F_i)$	$\sigma(1 - F_i)$
$1 - F_1$	2·33	2·79
$1 - F_2$	6·69	7·17

In Table 3 details are reported concerning the FIML parameter estimates. The interval estimates are calculated at the 95 per cent confidence level by treating the ratio of the parameter estimates to their asymptotic standard errors (ASEs) as being approximately standard normal in distribution. An examination of this table suggests that the estimates of γ and A are significantly different from what is expected and they are also not very satisfactory from a statistical viewpoint. The point estimate of ϕ, the depreciation rate on total capital stock, plus the rate of expenditure on current inputs is 9·8 per cent, while the point estimate of r, the time discount rate, is 10·1 per cent. These estimates have sensible economic interpretations but the relatively large associated ASEs are disappointing. The point estimate of 4·04 for the parameter j (being significantly different from zero) implies first, that there is a relatively high degree of complementarity between sheep and beef cattle, and secondly, that the specification of the atemporal production problem is reasonable because the model probably constrains livestock number increases to within the maximum possible biological rate of increase.

TABLE 3

Parameter Estimates

Parameter	ASE	Interval estimates
j	1·88	4·0430 ± 3·68
A	225·33	103·73 ± 441·65
γ	2·53	0·0001 ± 4·96
ϕ	6·96	0·0983 ± 13·64
r	7·03	0·1016 ± 13·78

Structural form equations

Because all of the explanatory variables in the third structural form equation are predetermined, this third equation is not analysed until a later subsection where the restricted reduced form results are evaluated. The first two estimated structural form equations are:

$$(16)\quad K_{1t} = \underset{(3\cdot947)}{-98\cdot814} + \underset{(0\cdot016)}{1\cdot535}M_t + \underset{(0\cdot913)}{32\cdot076}W_t + \underset{(2\cdot849)}{5\cdot742}(S_{1t}^*/S_{2t}^*),$$

$$\text{SEE} = 4\cdot74$$

$$K_{2t} = \underset{(6\cdot858)}{-29\cdot193} + \underset{(0\cdot065)}{0\cdot372}M_t + \underset{(1\cdot180)}{7\cdot783}W_t - \underset{(1\cdot536)}{3\cdot002}(S_{1t}^*/S_{2t}^*),$$

$$\text{SEE} = 2\cdot64.$$

where SEE denotes the standard error of estimate and ASEs are shown in parentheses. In contrast with the ASEs associated with the estimated individual parameters, those associated with the coefficients of (16) are relatively small. When compared with the sample mean values of $E(K_1) = 50\cdot86$ and $E(K_2) = 3\cdot97$, the SEEs indicate that there are still large variations in livestock numbers that are unexplained by the model. This suggests that the ability of the model to forecast livestock numbers is likely to be poor, especially if no correction is made for autocorrelation.

An examination of (16) shows that changes in the real value of total capital stock have a large[3] positive influence on sheep numbers and beef cattle numbers. As the real value of capital inputs makes up approximately nine-tenths of the real value of total capital stock, this result is consistent with the hypothesis embedded in our model: that livestock increases are constrained by the real value of capital inputs. Similarly, increases in the index of stock on hand per real dollar of capital input, W_t, cause an increase in livestock numbers. An increase in the ratio of the expected real net revenues per animal causes holdings of sheep to increase and of beef cattle to decrease. This is also plausible. Thus, all of the estimated coefficients have the anticipated signs, and it is easily verified that all are significantly different from zero on the basis of appropriate (approximate) one-tailed tests.

Elasticities

Reported in Table 4 are estimates of the short-run livestock response elasticities with respect to the real value of capital stock, the level of technology and expected relative real net revenues. These elasticities indicate the direct response of livestock numbers to changes in these variables and are calculated by using sample mean values for the variables. The signs of these elasticities are the same as those on the corresponding coefficients of the structural equations and hence are consistent with economic theory, as discussed already.

For all three variables, the elasticity with respect to beef cattle numbers is absolutely larger than the corresponding elasticity with respect to sheep numbers. The livestock numbers are relatively elastic with respect to the real value of capital stock and the proxy variable for

[3] Because of data scaling, the variable M_t is now measured in \$'00 million and the technology variable W_t is now measured as the index of output per \$'00 million dollars of input. The units of K_{1t} and K_{2t} are unaffected by this data scaling so are still measured in millions of sheep and beef cattle respectively.

TABLE 4

Livestock Elasticities

Livestock	M_t	W_t	(S_{1t}^*/S_{2t}^*)
K_{1t}	1·580	1·280	0·081
K_{2t}	4·807	3·977	—0·540

disembodied technical progress. For example, it is estimated that a 1 per cent increase in the real value of capital stock will result in a 1·58 per cent and 4·80 per cent increase in sheep and cattle numbers. The elasticities associated with the variable W_t are consistent both in sign and magnitude with those obtained by Freebairn (1973), using Australian data, where the proxy technology variable is taken to be the area of improved pastures.

The values of the elasticities associated with ratio of the expected real net revenues per animal indicate inelastic responses and imply that a 1 per cent increase in the ratio of the expected real net revenue per animal results in an estimated 0·08 per cent increase in sheep numbers and a 0·54 per cent decrease in beef cattle numbers. This result may be compared with those of Court (1967), who also obtains inelastic short-run price responses. However, note that in his study the endogenous variables are not livestock numbers but livestock products.

An estimate of the *annual* change in livestock numbers due to technical change is given by

(17) $(\partial(\ln K_{it})/\partial(\ln W_t)) \, . \, (d(\ln W_t)/dt); i = 1, 2,$

where $\partial(\ln K_{it})/\partial(\ln W_t)$; $i = 1, 2$ are the elasticities obtained from Table 4 and $d(\ln W_t)/dt$ is the annual rate of disembodied technical progress, estimated to be 0·82 per cent for the sample period. From (17) it is estimated that, over the sample period, 1·05 per cent and 3·26 per cent of the annual growth in sheep and beef cattle numbers respectively is due to disembodied technical progress. The result, that over 3 per cent of the annual change in beef cattle numbers is due to disembodied technical progress during the sample period, seems high, although it must be remembered that this result is based on assumptions of a fixed slaughter rate and expenditure on current inputs being a constant proportion of the real value of total capital stock.

Restricted reduced form equations

Some of the estimates of the restricted reduced form coefficients and their ASEs for the three equations of the system are reported in Table 5. In this table the endogenous variables of equations (i), (ii) and (iii) are K_{1t}, K_{2t} and M_t respectively.

The coefficients associated with the variables W_t and (S_{1t}^*/S_{2t}^*) are discussed in the structural form equation section where it is concluded that the signs of these coefficients are consistent with economic theory and that they are significantly different from zero. These coefficients are impact multipliers and show the contemporaneous response of livestock numbers to a unit change in the associated predetermined variables. For example, if the expected real net revenue per sheep relative to the expected real net revenue per cattle beast increases

TABLE 5

Associated[a] variable	Equation (i) Coefficient	Equation (i) ASE	Equation (ii) Coefficient	Equation (ii) ASE	Equation (iii) Coefficient	Equation (iii) ASE
Constant	—89·428	13·195	—26·915	6·825	6·114	7·927
W_t	32·076	0·913	7·783	1·180		
$S_{1t}*$	—0·054	23·666	—0·013	5·742	—0·035	15·417
$S_{2t}*$	—0·039	17·184	—0·009	4·169	—0·025	11·194
p_t	0·028	397·004	0·006	96·335	0·018	258·630
H_t	0·356	154·340	0·086	37·451	0·231	100·545
M_{t-1}	1·228	0·124	0·298	0·051	0·800	0·078
$(S_{1t}*/S_{2t}*)$	5·742	2·849	—3·002	1·536		

[a] A further nine variables, composed of multiplicative combinations of the variables reported in this table, occur in the restricted reduced form equation. They are generated when some of the expressions in the model are approximated by first-order Taylor series expansions.

from 0·6 to 0·7, then the initial impact on sheep numbers is an increase from 0·57 million and beef cattle numbers will decrease by 0·30 million. Similarly, if the index of stock on hand per $'00m of real capital input is increased from 2·0 to 2·1, then holdings of sheep and beef cattle are estimated to increase by 3·20 million and 0·78 million respectively. These impact multipliers are of a sensible magnitude.

The large ASEs associated with the coefficients for p_t, H_t, $S_{1t}*$ and $S_{2t}*$ limit the economic interpretations which may be drawn in these cases. In all three equations the coefficients associated with M_{t-1}, the lagged real value of total capital stock, are positive and significant at the 5 per cent level. These positive coefficients are consistent with the economic hypotheses embedded in the model.

If the restricted reduced form coefficients of Table 5 are to be used for forecasting purposes, a correction must be made for autocorrelation. An approximate and statistically inefficient approach is that based on the standard single equation method outlined by Malinvaud (1970, p. 536). This approach is legitimate when the restricted reduced form is viewed equation by equation, as then all the usual single equation assumptions are satisfied.

Conclusion

The amount of expected real net revenue received by the farmer depends upon the amount of time allocated to work. The more he works, the greater his income and yet the more he works, the less leisure time there is remaining to him. A weakness of this study, that could possibly be examined by other researchers, is that no account is taken of these work/leisure decisions that face the farmer.

One of the main problems encountered in this study concerns data availability. The absence of suitable data on the amount of labour supplied by the representative farmer and his family leads to labour not being formally constrained in this economic model. This implies that the results obtained are conditional on the representative farmer being able to supply enough labour adequately to run the livestock numbers specified by the model. It is not possible, using an *ex poste* sample period, immediately to examine the realism of this condition because all currently available observations are used to estimate the model. Therefore, as more observations become available, it would be worthwhile carrying out a simulation study to examine the stability of the results to a larger sample.

Another problem is the absence of suitable data on physical capital inputs. This results in capital inputs being measured in value terms and an exogenous scale factor being introduced into the capital input function. An advantage of this non-standard specification of the capital input function is that technical change, proxied by the scale factor, is explicitly included in the model's framework. Capital inputs are measured indirectly using the identity that exists between total capital stock and its components. That is, the real value of capital inputs is specified as the real value of total capital stock minus the real value of stock on hand. A problem resulting from this specification is that we were unable to solve the representative farmer's production problem directly and so an approximation is made, resulting in approximate optimal supply equations being specified. It would be interesting to know the effect that this approximation has on the results, particularly the amount of bias introduced into the parameter estimates. In principle this could be determined by solving directly, if possible, the constrained maximisation problem and then comparing the resultant parameter estimates with those obtained in this paper.

The problem of estimating the structural parameters associated with continuous time variables from discrete observations on the variables is ignored. However, given that there is a growing literature on this problem, this could be a fruitful area for further study.

The way in which the expectations variables are generated results in several non-nested models being specified and the problem of choosing between the models is sidestepped by reporting all results as conditional estimates for a given F_1. In principle, a Bayesian approach could be used to choose between these different models. This, and a general extension of classical methods for choosing between non-nested models, might be worthy of further study.

Because the model is dynamic, that is the predetermined variable includes a lagged endogenous variable, it would be interesting, from a policy viewpoint, to examine the stability and cyclical properties of the model. However, prior to such a study, the specification error that is suggested by the presence of autocorrelation in the estimation residuals should be faced, and it is thought that a full residual analysis would assist in pinpointing the likely areas of misspecification.

Given these problems, the results that have been obtained are encouraging.

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