THE INTEGRATED USE OF SIMULATION AND STOCHASTIC PROGRAMMING FOR WHOLE FARM PLANNING UNDER RISK*

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Methods of whole-farm planning under risk are briefly reviewed, noting especially associated operational problems. A planning problem relating to spatial diversification of beef production in the Clarence region of N.S.W. is investigated using a model comprising both simulation and linear programming components. It is concluded that such composite models are valuable for the analysis of sequential stochastic decision processes not presently amenable to solution by stochastic programming alone.

Introduction

Following the development of linear programming for whole-farm planning, attention has been directed to finding realistic ways of incorporating risk into programming models. For example, quadratic risk programming has been used to deal with risk in the functional coefficients (e.g. [1, 6, 10]), while discrete stochastic programming has been developed to handle stochastic elements in functional, right-hand side and/or input-output coefficients [3, 13]. As Cocks [3] notes, discrete stochastic programming, while very flexible in principle, often requires very large matrices for real-world problems. Such large matrices are difficult and expensive to solve given current computer technology. The problem of matrix size arises because a random variable which would be incorporated in a non-stochastic programming matrix as a single element (usually setting the coefficient equal to the expected value of the variable) is replaced in stochastic programming by a vector whose size depends on the number of discrete levels specified for the variable. When the one stochastic variable (e.g. rainfall) influences more than one activity, the vector becomes a matrix, while if k such stochastic variables are each considered at n discrete levels, n^k sub-matrices are generated which must be represented in block-diagonal form.

The problems of matrix size will become less severe with advances in computer technology, but for the present, alternative methods may be required to tackle many farm planning problems for which a stochastic formulation is required. Simulation provides a good alternative to programming methods in many instances, especially in research applications [9]. Yet here again, the capacity and speed of the available computer

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may limit the scope of the model developed, while the automatic optimizing facility of a programming formulation is lost.

In this study, a stochastic farm planning problem is investigated by the integrated use of simulation and linear programming. We have sought to exploit the advantages of both techniques to obtain a good representation of a planning problem that is not readily amenable to analysis using either technique alone.[1] The problem investigated is the economics of spatial diversification by graziers in the Clarence region of New South Wales.

The Problem

Several beef producers in the Clarence region[2] have diversified spatially in an attempt to offset the effects of unfavourable seasonal conditions in late winter and spring—the period encompassing late pregnancy and calving when breeding cows are most sensitive to energy stress. Producers have diversified by acquiring additional parcels of alluvial land near the Clarence River to augment existing extensive off-river holdings. However, an economic survey of 25 diversifiers [15] indicated that most had failed to reap sufficient benefits from the alluvial land to justify the high prices paid. An extensive production system had generally been adopted on the additional area yielding no more advantages than could have been achieved from the purchase of a block of cheaper, non-alluvial land. It was therefore decided to attempt to quantify the potential benefits if the additional alluvial land were used intensively.

The firm studied comprises two blocks of land—a 2,000 hectare property of extensive grazing land situated away from the Clarence River, and 100 hectares of irrigated alluvium near the river. The larger property is typical of the Clarence beef-breeding country: soil nutrient deficiencies mean that quality and quantity of pasture produced is low. Rainfall is summer dominant, so that most of the pasture eaten by stock during winter and spring is carried over from the summer months. The other property can all be irrigated for the production of lucerne, ryegrass and improved pasture (kikuyu, paspalum and white clover), the areas of each being mainly determined by the different soil characteristics on the property. The feed grown is available to stock between July and October.[3] Before July, there is usually sufficient pasture on the off-river property to satisfy stock needs. After October, the summer-growing native pasture species recommence active growth. Calving commences in September and stock are sold in May at approximately eighteen months of age.

If long-term decisions, such as what areas of land to buy and the type and general organization of cattle enterprises, are ignored, there are two decision categories which must be incorporated into a study of the economics of the Clarence type of spatial diversification. They are strategic decisions about the level of stocking and tactical decisions relating to

[1] Obviously there are many ways in which simulation and linear programming can be integrated. The method used here was judged most suitable for the particular problem being investigated.

[2] Beef production in the Clarence region is widespread but extensive. There are more beef cattle in the North Coast Statistical Division (of which the Clarence region is a major component) than in any region of similar size in New South Wales. The nature of beef production in the region is discussed by Duncan [5].

[3] Because of the presence of irrigation it is assumed that the supply of fodder from the alluvial land is deterministic.

cattle management during the winter and spring when feed generally is in short supply.

The sequence of decisions is depicted in Figure 1. The initial state variables comprise the numbers of cows, the numbers of other stock and the fodder supply. This information is available to the decision maker at the beginning of November, which approximates the start of the Clarence growing season. At that time, when he has only subjective notions about future states of nature, the grazier must make the strategic decision of setting the stocking rate. Then in the following July to October period the grazier must determine how many of what group of cattle should be fed on what available pasture type, at what rate of feeding (maintenance or weight gain or loss), and exactly when (if at all) they should be transported to or from the alluvial land. These tactical decisions will be conditional upon both the initial strategic decision and the intervening state of nature (November-June). Finally, the payoffs associated with each strategic decision/state of nature/tactical decision sequence are also stochastic, being conditioned by the effects of weather variability during the tactical decision months.

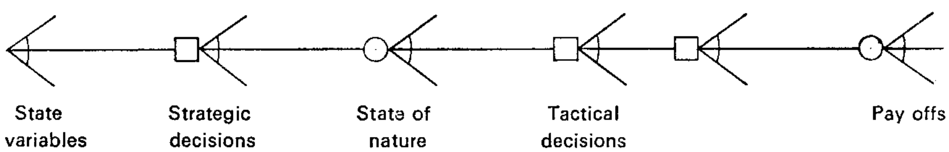

FIG. 1—Sequential decision environment of a spatially diversified firm.

Methodology

As Hadley [7] notes, one is usually interested in solving a sequential decision problem only for the purpose of making the initial decision. Generally there is an opportunity to reappraise the position before the subsequent decisions must be taken, at which time more information may be available. However, to evaluate the initial decision we must find some means of resolving the uncertainty represented by the succeeding tactical decisions and uncertain events. Thus we must usually solve for all stages of a sequential stochastic decision problem in order to solve the first-stage decision.[4] It is this requirement which commonly leads to the very large dimensions of such problems.

The approach we have adopted for the analysis of the beef producer's decision problem has been to emphasize and refine the early stages of the decision sequence and to adopt a relatively crude representation of the later stages, which was nevertheless subjectively judged to be adequate for analysis of the first-stage decisions. The details of the approach are as follows:

(i) The November to June period was represented by a simulation model in which possible strategic decisions were incorporated as experimental treatments. The interaction of these stocking rate decisions with the state of nature (rainfall) was modelled to account for productivity of the cattle, hand feeding costs and, most importantly, carryover of feed at 1st July.

(ii) The tactical decision phase of July to October was divided into

[4] More correctly, it is only necessary to extend the analysis for a sufficient number of stages to obtain an optimal first-stage decision [12].

two sub-periods: July-August and September-October. Pasture production in the July-August sub-period was assumed to be deterministic while production in September-October was handled by assuming a discrete distribution with 'good', 'medium' or 'poor' outcomes. These assumptions permitted the tactical decision phase to be formulated as a relatively simple stochastic programming matrix.

(iii) The link between the simulation component and the stochastic programming component of the model was forged by first solving a stochastic linear programming matrix corresponding to the stocking rate of each simulation treatment with parametric variation in the feed carry-over at 1st July. The linearly-segmented functions for expected profit thereby obtained were then used to evaluate the terminal state of the simulation model after each replication. The procedures adopted are indicated in block-diagrammatic form in Figure 2.

The seasonal nature of pasture production in the Clarence region means that it is more important to specify pasture production accurately for the summer months than for the rest of the year. Provided that the necessary biological data exist, this can best be done by simulating pasture production according to rainfall. Pseudo-random rainfall values may be generated by monte carlo sampling from smoothed probability distributions derived from historical frequencies. Simulation permits a large number of pasture production patterns to be generated. Within a mathematical programming (optimizing) framework, only a relatively few discrete patterns can be specified if the matrix size is to be contained to reasonable proportions. The abstractions from reality that are often necessary to bring a large stochastic planning problem within the computational ambit of the simplex algorithm represent a serious shortcoming of the programming approach. Similarly, the absence of such binding constraints on the form and size of model used lends considerable power to simulation.

The July-October period is split into two mainly because of climatic considerations. In theory, a separate but obviously related cattle feeding decision problem would be faced by a grazier each day, even if it meant deciding on a 'no change from yesterday' policy. It would have made the analysis unnecessarily complicated and difficult to incorporate such fine fragmentation of the decision time scale and so the more pragmatic approach of assuming fewer decisions each covering a longer interval was adopted. The months of July and August are the least favourable to native pasture growth on the off-river property, even at high rainfall. Thus errors introduced by assuming that native pasture growth is deterministic during this period are not great. Total pasture available at this time comprises native pasture growth, improved pasture (lucerne, rye-grass, etc., from the alluvial land) and native pasture carried over from the previous months.

During September and October pasture production is responsive to rainfall so a deterministic assumption would not be justified. On the other hand, extreme refinement at this stage is not appropriate since the tactics and their consequences are well removed from the initial strategic decision of prime interest. Consequently, it was judged that only three levels of pasture production in the September-October period need be incorporated in the analysis. The probabilities of occurrence of each type of spring were assessed from rainfall data.

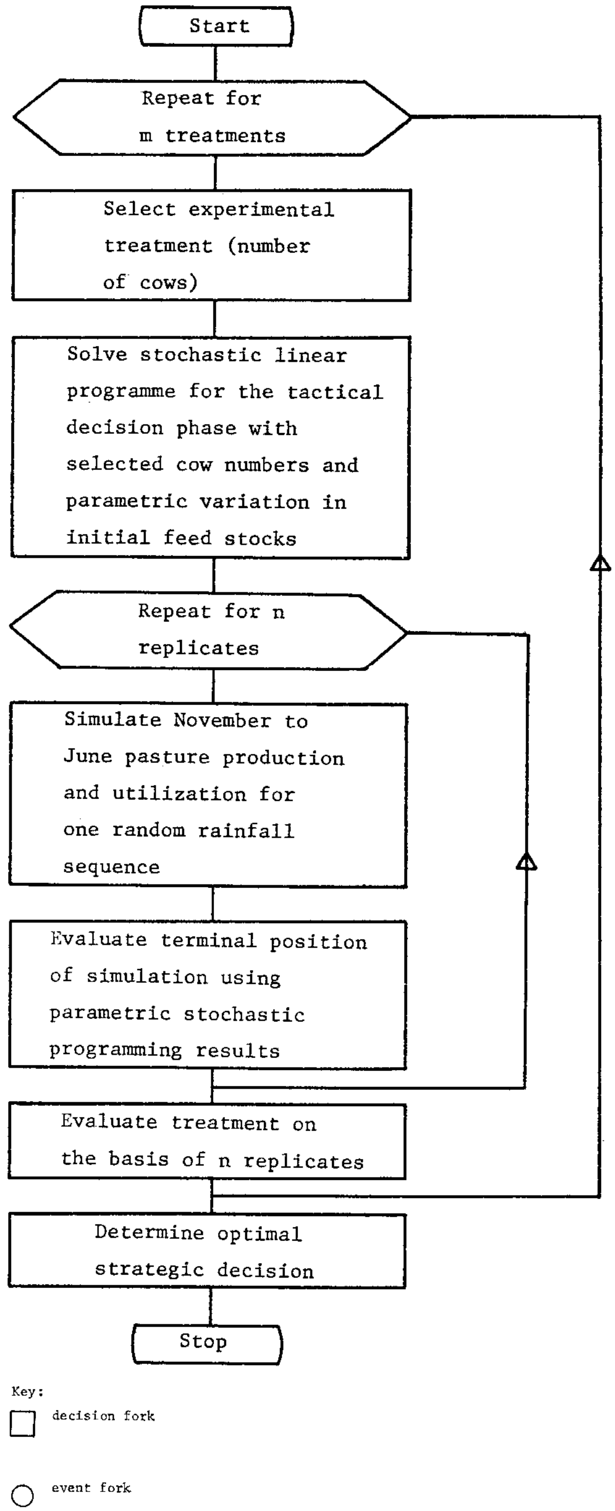

FIG. 2—Methodological sequence for decision analysis of the spatially diversified firm.

The decision analysis was terminated at the end of October so that the model represents a full year's operations. At the cutoff date stock on hand were valued according to condition. For example, a cow that had been fed at maintenance levels was given a higher value than one that had been fed to lose weight. In reality, of course, the decision process is dynamic, extending over many years, and stock on hand at the end of October can only properly be valued according to their future economic productivity. However, it is clear that the 'use value' of stock at this time cannot diverge markedly from market values, otherwise it would pay to buy or sell animals at the margin until the two values are brought into line.

A major factor contributing to the computational economy of the methodology adopted lies in the use of parametric procedures in the stochastic programming component of the model. By solving for all possible initial (1st July) levels of feed supply, only one programming run is required for each treatment in the simulation experiment, regardless of the number of replicates. This was possible because only the one linking variable was assumed to be important between the strategic and tactical decision stages. When several such variables must be accounted for, either a non-parametric stochastic programme would have to be solved for each replicate at each treatment level [2], or an n variable parametric programme would have to be solved for each treatment where n is the number of linking variables considered.

Application

The Stochastic Programming Component

The stochastic programming model was formulated as the maximization of

$$(1) \qquad E(z) = c_{11}x_{11} + p_{21}c_{21}x_{21} + p_{22}c_{22}x_{22} + p_{23}c_{23}x_{23},$$

subject to

$$(2) \qquad \begin{array}{lllll} A_{11}x_{11} & & & & \leqslant b_{11} \\ & A_{21}x_{21} & & & \leqslant b_{21} \\ & & A_{22}x_{22} & & \leqslant b_{22}\,. \\ & & & A_{23}x_{23} & \leqslant b_{23} \\ -Ix_{11}^* & + \; Ix_{21}^* & & & \leqslant 0 \\ -Ix_{11}^* & & + \; Ix_{22}^* & & \leqslant 0 \\ -Ix_{11}^* & & & + \; Ix_{23}^* & \leqslant 0 \end{array}$$

and all $x \geqslant 0$,
where

$E(z)$	is expected net revenue;
c_{ij}	is a vector of activity net revenues for stage i given event j has occurred ($j = 1$ when $i = 1$ implying assumed certainty);
p_{ij}	is the probability of event j at stage i;
$A_{ij}x_{ij} \leqslant b_{ij}$	are the constraints at stage i should event j occur;
x_{11}^*	is a vector of only those first-stage variables corresponding to activities continued in the second stage;

x_{2j}^* are vectors of second-stage variables corresponding to activities initiated in the previous stage;

$-Ix_{11}^* + Ix_{2j}^* \leqslant 0$ are the constraints linking variables initiated in the first stage to the corresponding second-stage variables.

The matrix developed comprised 112 activities and 47 rows, with the first 22 activities accommodating the four classes of livestock (calving cows with their calves, dry cows, replacement heifers and weaners) during July-August. Provision was made, for example, for calving cows to be run on any of four pasture types, either for maintenance or to gain or to lose weight. Matrix coefficients were calculated from appropriate biological relationships; the procedure is described elsewhere [15]. The remaining 90 activities covered the second period of September-October and were divided between the uncertain events.

Activities in the Matrix

July-August period.

(i) Cattle feeding activities: these were specified by cattle type, pasture type and rate of feeding. The coefficients in the net revenue row were zero because the activities were linked to equivalent activities in the second period by transfer coefficients.

(ii) Cattle selling activities: positive net revenue coefficients.

(iii) Feed buying activities: these were provided to meet severe feed shortages and carried negative net revenues.

(iv) Transfer activities to carry forward unconsumed feed: the transfers allowed for the deterioration of pasture with age. Net revenue coefficients were zero.

September-October period.

(i) As (i) above: net revenue coefficients were positive reflecting the terminal values of the stock. The values used varied according to the ultimate weight of the animal and thus provided an incentive to feed for weight gain in preference to maintenance or weight loss, where feed availability permitted.

(ii) As for previous period.

(iii) As for previous period.

(iv) Store feed: this activity permitted the storage of lucerne hay surplus to requirements; it had a positive net revenue coefficient reflecting market value.

Parametric Programming Routine

In parametric linear programming a composite right-hand side vector is introduced into the problem of the form $b = b_1 + \lambda b_2$. A first solution may be obtained when $b = b_1$, then λ is progressively increased from zero to a specified upper limit. Solutions may be obtained for predetermined values of λ, or, as in this application, at each change of basis.

In the present context, b_2 is a unit vector, permitting parametric variation in the coefficient representing native pasture available in July and August. The feed supply in this period is comprised of pasture growth in the period (assumed fixed at 145,000 Mcal metabolizable energy (ME) from 2,000 ha) and pasture carried over from the preceding months, which can vary from zero to nearly $(2 \cdot 0) 10^6$ Mcal ME.

Solutions were obtained for the parametric stochastic linear programming matrix for stocking rates of cattle corresponding to each of six experimental treatments which were to be used in the simulation component of the model. The programming output was summarized in the form of the six corresponding linearly-segmented functions of expected net revenue against initial feed supply.

At the tactical decision stage of the analysis no account was taken of possible risk aversion on the part of the farmer. The justification for this omission was two-fold. First, the magnitude of the risk at this stage was considered small relative to the risk at the strategic decision stage. Thus it was considered that the adoption of the goal of maximizing expected net revenue for tactical decisions could be expected to yield satisfactory results for all but very risk-averse decision makers. Second, to account for risk at this stage would have required use of quadratic or separable programming [13], and it was not considered that the extra computational problems thereby created would be compensated for in terms of greater refinement of the analysis. The objective of the programming analysis was not to determine optimal tactics *per se*, since in reality these could be determined according to the particular circumstances eventuating. The programming results were used only to evaluate the state of the system at the end of June to be able to resolve the strategic decision, and for this purpose it was considered that maximization of expected net revenue would be adequate.[5]

Simulation Component

Details of the simulation sub-model have been presented elsewhere [14]. In outline, the model comprised a pasture component, in which pasture production was simulated as a function of rainfall, and an animal component, which reflected the interaction between beef cattle production and feed supply. Computations in the model followed the sequence of events of the real system. Rainfall observations were obtained by pseudo-random sampling from smoothed historical distributions. Rainfall augmented existing soil moisture reserves which in turn affected pasture growth. Pasture supply was a function of growth and carryover (net of deterioration) from the previous period. Cattle consumed pasture according to their basic energy requirements—weight gain or loss could occur depending on feed availability. The output of the sub-model of chief interest in the present context was the carryover of feed at the end of June each year.

Analytical Procedure

The strategic decision of the optimal stocking rate for the spatially diversified beef producer was approached by formulating an experiment with six treatments representing a range of from 300 to 550 breeding cows in steps of 50. Each treatment was replicated 200 times in the simulation model, the output of each replicate being evaluated using the parametric programming results. Treatments were evaluated initially in

[5] Hadley [7, p. 553] argues that computational effort may be reduced by estimating maximum expected utility at some point in a decision problem without solving in detail the parts of the problem beyond that point. The stochastic linear programming sub-model does this under the assumption of a linear utility function for tactical payoffs.

terms of expected profit, estimated as the mean profit over the 200 replicates. A measure of the associated risk was obtained by computing the variance of the profit for each treatment over the 200 replicates.[6] The results obtained can be regarded as only a crude measure of risk involved since only one risky component, namely rainfall, is taken into account. In reality risk is compounded by other economic and physical uncertainties.

Results

Details of the empirical results have been presented elsewhere by Trebeck [14]. Consequently only selected features of the results are presented here and discussion is centred upon methodological aspects.

The results of the stocking rate experiment are summarized in Table 1 which shows that maximum expected profit was obtained for the 450-cow treatment. The picture that emerges when account is taken of variance of profit is shown in Figure 3. A tentative locus of points in E-V space corresponding to all possible stocking rates in the range has been drawn on the figure. It can be seen that this locus is 'looped'. At first sight this might seem implausible but it is consistent with results obtained by Dillon and Lloyd [4] in a study of the economics of fodder reserves. To confirm the position and shape of the E-V locus in Figure 3 it would be necessary to conduct further experiments with the model with a finer division of cow numbers.

TABLE 1

Results of Stocking Rate Experiment

Treatment	Expected Profit	Variance of Profit
no. cows	$	$\$^2$
300	4,460	219,831
350	6,749	2,054,336
400	7,626	7,590,308
450	9,700	8,053,011
500	9,369	7,738,057
550	7,277	9,227,301

It must be pointed out that the E-V locus indicated by this analysis does not correspond to the 'efficient set' (Markowitz [11])—that is, it is not the set of points of minimum variance for given levels of expected profit. Generation of the efficient set requires the evaluation of all possible mixed strategies. Thus if s_1 and s_2 are two strategies on the locus of points in Figure 3, we should also consider the strategies

$$s_3 = as_1 + (1 - a)s_2, \tag{3}$$

for all values of a in the range $0 < a < 1$. Such mixed strategies would involve running different portions of the farm at different stocking rates,

[6] Most of the variance in outcomes in the tactical decision phase was manifested in terms of the state of the system in early November. A measure of this variability was reflected in the results by running the simulation sub-model in a dynamic manner such that starting conditions for each replicate were determined from the outcome of the previous replicate for each treatment by use of a simple transformation procedure to bridge the July-October (non-simulated) period.

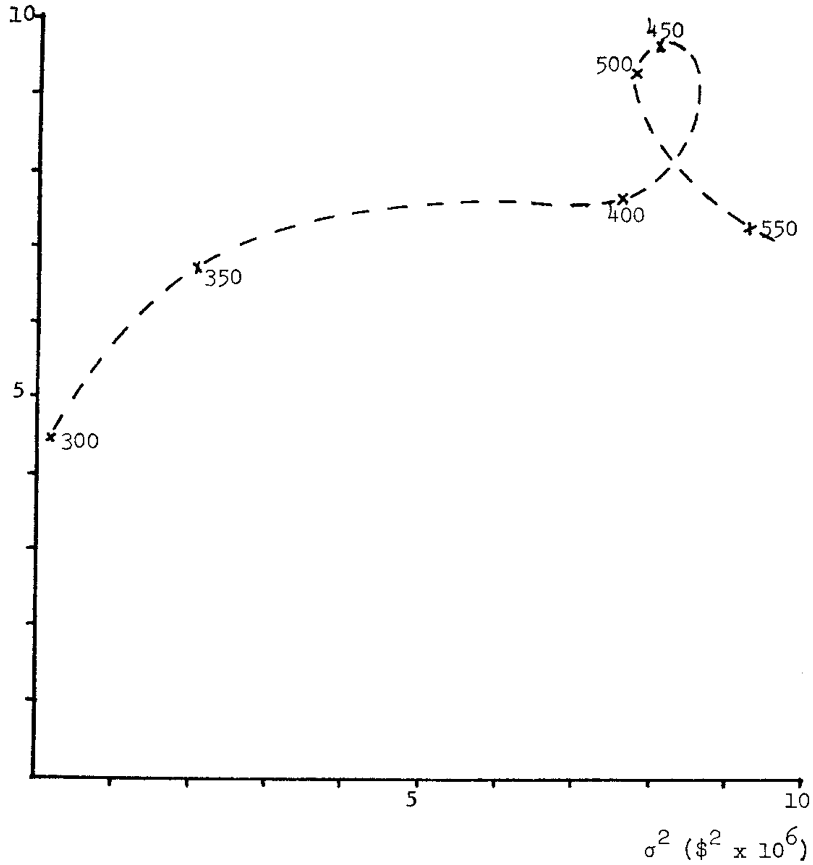

FIG. 3—Tentative locus of stocking rate strategies in E-V space.
μ ($ x 10^3)
σ^2 ($\$^2$ x 10^6)

i.e. a portfolio of stocking rates. Assuming constant returns to scale, the expected profit of a mixed strategy is computed as

$$\mu_3 = a\mu_1 + (1 - a)\mu_2, \tag{4}$$

while the associated variance is

$$\sigma_3{}^2 = a^2\sigma_1{}^2 + (1 - a)^2\sigma_2{}^2 + 2a(1 - a)\rho_{12}\sigma_1\sigma_2, \tag{5}$$

where $\rho_{12}\sigma_1\sigma_2$ is the covariance of the two strategies.

It would be particularly important to consider strategy combinations including the (presumably riskless) null strategy of zero stocking rate. Moreover, combinations of mixed strategies would also have to be considered to ensure that the true efficient set had been identified. As Halter and Dean observe [8, p. 174], computation of a sufficiently large number of points to approximate the E-V boundary by this 'brute force' direct calculation method would be a formidable task. Fortunately, quadratic programming provides a more efficient method of calculation if the true frontier is required.

Mixed strategies are qualitatively different in managerial terms from pure strategies such as those considered as experimental treatments. For example, a 50:50 mixed strategy of 300 cows and 400 cows is not the same as an overall stocking rate of 350 cows but requires the farm to be divided into two with separate systems being operated on each area. In the present context it was considered that mixed strategies would be unlikely to appeal to a farmer because of the managerial difficulties they would pose, and so the necessary calculations were not performed.

Conclusion

The chief shortcomings of the analysis described above reflect mainly the difficulty of accounting simultaneously for variability in more than one stochastic parameter. The stochastic programming section parametizes native feed supply, distributions of which were obtained from the simulation model. Two other output variables of the simulation component of the analysis also show stochastic variation: cow live weight and calving percentage. No account has been taken of variation in these parameters in the stochastic programming model. The live weight of cows affects the amount of energy required for maintenance or weight gain or loss. Variation in calving percentage affects the number of calving cows and yearlings carried into the period represented by the stochastic linear programme. In the matrix, calving percentage was assumed constant at the maximum level (75 per cent). This resulted in feed requirements being overestimated whenever the simulation model generated a calving percentage lower than the maximum. However, the net effect on expected profit is likely to have been relatively small.

It would be possible to extend the stochastic programming analysis to account for variations in more than one parameter. Parametric resource mapping procedures could be used to link the simulation sub-model to the programming sub-model, accounting for two or more state variables. Such extension was judged unnecessary in the present study but might well be appropriate in other applications.

Despite the shortcomings noted above, the methodology developed in this paper has several important advantages. First, a good representation was obtained by means of computer simulation of the important early stages of the sequential decision problem. Second, in the subsequent stages where a less refined representation was judged satisfactory, the well-known advantages of linear programming were exploited. Uncertainty in these stages was accounted for by use of a stochastic programming formulation. Third, parametric programming was used to reduce substantially the amount of computation involved. In all, only six related linear programming models were solved parametrically—one for each experimental treatment. The alternative would have been to solve a non-parametric model to evaluate the terminal state of the simulation sub-model for each replicate of each treatment. With 200 replicates per treatment, this would have required 1,200 linear programming solutions. We suggest that these advantages may be sufficient to commend our general approach to other researchers faced with the task of analysing similar sequential decision problems under risk.

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