OPTIMAL FERTILIZER CARRYOVER AND CROP RECYCLING POLICIES FOR A TROPICAL GRAIN CROP

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The consequences of carryover for the optimal application of fertilizer are considered using dynamic programming. The conclusions are relevant for the further problem of deciding how many grain crops to harvest from grain sorghum plants grown in a tropical environment. Dynamic programming is also used for solving this problem for the Ord River Valley, and takes account of the interrelations between season and crop cycle number. Data were obtained from investigations conducted in the area.

Introduction

The economics of the application of fertilizer has been the subject of continuing interest, e.g. [1]. However, total fertilizer available to a plant in any period depends on previous applications of fertilizer as well as the application in the current period. If the carryover effect is significant in the agronomic sense then its consideration may influence the policy for optimal application of fertilizer. The inductive approach of dynamic programming is used for determining optimal policies for situations in which carryover is relevant.

The problem of nutrient carryover, and hence of optimal application of fertilizer through time, is particularly important in producing grain sorghum in a tropical environment where it is possible to obtain several crops in succession from the same plant. For example, fertilizer costs typically account for about 25 per cent of the total costs involved in tropical sorghum production in Australia [4]. The particular problem considered here, and the data used to demonstrate its solution, have been drawn from a research project conducted in the Ord River Valley (15°S) of north western Australia. The tropical climate of this area, given irrigation in the dry season, allows grain sorghum to be recycled throughout the year. Three crop seasons can be identified over a period of twelve months. The effect of season operates independently of cycle number and causes the yield pattern of crops from the same sorghum plant to decline irregularly from cycle to cycle. A different fertilizer response function is therefore associated with each possible combination of season and cycle number. Management is faced with the decision at the end of each crop cycle of whether, after the current crop, to continue with another crop cycle, or to plough out and subsequently begin a new cycle sequence. The problem is basically a discontinuous one of optimal replacement, requiring consideration of residual fertilizer, and of changing response functions and prices over time.

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The Model

Optimal application of fertilizer

Abstracting from any uncertainties, the principle governing the quantity of fertilizer to apply for profit maximization is clearly established for the case of one period response. Fertilizer should be applied until the cost of application of one more unit just equals the revenue obtained from the response to that unit. For intertemporal problems, however, optimization is complicated not just by consideration of time preference for money but also by the effect of fertilizer carried over from prior applications. Suppose that the amount of carryover to the beginning of period T from W units of fertilizer available at the beginning of period 1 is $WV_1V_2 .. V_{T-1}$ with $0 \leqslant V_t \leqslant 1$ for each period t. The carryover coefficient V_t would depend on such factors as weather conditions and yields obtained during the periods 1 to T-1.

In the notation that follows, n is used to denote the beginning of that period for which n operating periods are to be completed before the end of the process. The response function of grain yield to total available fertilizer, Q_n, is defined as $y_n\{Q_n\}$ and is assumed to exhibit diminishing returns. Available fertilizer, Q_n, consists of residual fertilizer already in the soil at the beginning of period n, plus the fertilizer applied in period n. The unit prices of grain and fertilizer are denoted p_n^g and p_n^f respectively, and α is the time perference discount factor per period.

Recurrence equations for finding the optimal application of fertilizer may be formulated. For the case with one period remaining,

$$(1) \qquad f_1\{r_1\} = \max_{Q_1} [\alpha p_1^g y_1\{Q_1\} - p_1^f(Q_1 - r_1)]$$

and for the general case with n periods remaining,

$$(2) \qquad f_n\{r_n\} = \max_{Q_n} [\alpha p_n^g y_n\{Q_n\} - p_n^f(Q_n - r_n) + \alpha f_{n-1}\{V_n Q_n\}]$$

where $f_n\{r_n\}$ is the return from following an optimal fertilizer policy in each of n future periods given that residual fertilizer is r_n. The cost of fertilizer is assumed to fall due at the beginning of any production period, and revenue from the crop at the end. For the moment only continuous cropping is considered so that some yield determined by the function y_n is obtained in each of the periods in the long-term planning horizon.

Differentiating the expression in square brackets in (1) with respect to Q_1, and assuming second order conditions hold, the usual single period condition for profit maximization is found to be

$$(3) \qquad dy_1/dQ_1 = p_1^f/\alpha p_1^g.$$

Defining the Q_1 which satisfies (3) as Q_1^*, (1) may be rewritten

$$(4) \qquad f_1\{r_1\} = \alpha p_1^g y_1\{Q_1^*\} - p_1^f(Q_1^* - r_1)$$

where the optimal final period application of fertilizer is $Q_1^* - r_1$.

Continuing for the case with two periods remaining, from (2),

$$(5) \qquad f_2\{r_2\} = \max_{Q_2} [\alpha p_2^g y_2\{Q_2\} - p_2^f(Q_2 - r_2) + \alpha[\alpha p_1^g y_1\{Q_1^*\} - p_1^f(Q_1^* - V_2 Q_2)]].$$

Again differentiating the expression in square brackets with respect to Q_2 and assuming second order conditions hold, the condition for profit maximization is found to be

$$(6) \qquad dy_2/dQ_2 = (p_2^f - \alpha V_2 p_1^f)/\alpha p_2^g.$$

If Q_2^* is defined as the Q_2 which satisfies (6) then (5) becomes:

$$(7) \qquad f_2\{r_2\} = \alpha p_2^g y_2\{Q_2^*\} - p_2^f(Q_2^* - r_2) + \alpha[\alpha p_1^g y_1\{Q_1^*\} - p_1^f(Q_1^* - V_2 Q_2^*)].$$

Using the same argument for the case with three periods remaining as for that with two periods remaining it can be easily shown that the profit maximizing condition is

$$(8) \qquad dy_3/dQ_3 = (p_3^f - \alpha V_3 p_2^f)/\alpha p_3^g$$

and that

$$(9) \qquad f_3\{r_3\} = \alpha p_3^g y_3\{Q_3^*\} - p_3^f(Q_3^* - r_3) + \alpha[\alpha p_2^g y_2\{Q_2^*\} - p_2^f(Q^* - V_3 Q_3^*) + \alpha[\alpha p_1^g y_1\{Q_1^*\} - p_1^f(Q_1^* - V_2 Q_2^*)]].$$

Arguing by induction, these results may be extended to the general case with n periods remaining. Thus the optimal application of fertilizer in any period n is obtained by finding Q_n^* such that

$$(10) \qquad dy_n/dQ_n = (p^f - \alpha V_n p_{n-1}^f)/\alpha p_n^g$$

and subtracting r_n.

The optimal return from following such a policy is given by

$$(11) \qquad f_n\{r_n\} = \alpha p_n^g y_n\{Q_n^*\} - p_n^f(Q^* - r_n) + \sum_{i=1}^{n-1} \alpha^{(n-i)}[\alpha p_i^g y_i\{Q_i^*\} - p_i^f(Q_i^* - V_{i+1} Q^*_{i+1})].$$

Equation (10) shows that for determining optimal levels of fertilizer application taking account of fertilizer carryover effects, the only relevant variable pertaining to future periods is the price of fertilizer in the next period. The response functions and grain prices that hold in future periods do not influence the current decision. The result is obtained because the marginal cost of applying fertilizer is assumed to be constant at p_n^f. Whatever the value of r_n, the optimal application of fertilizer is that amount which increases total fertilizer available to Q_n^*. As might be expected, the condition specified in (10) indicates that the greater the discount factor, the rate of carryover and the price of fertilizer next period, the greater should be the current application of fertilizer.[1]

Optimal recycling of crops

The optimal application of fertilizer (given carryover) is now considered with reference to the problem of deciding the optimal recycling policy for grain sorghum. The study by Whan [4] for the Ord Valley found nitrogen to be the economically most important fertilizer, and will be referred to instead of fertilizer from now on. Although sorghum can be grown continuously in a tropical environment, three sowing periods were established on the Ord. These three sowing periods were chosen so as to minimize the likelihood of a crop maturing during the

[1] It should be noted that the result only holds for cases where the residual fertilizer at the beginning of a period is less than the optimal available quantity of fertilizer for that period. This is a reasonable assumption if continuous cropping is being considered. If, however, periods of fallow are allowed then no fertilizer would be applied in the fallow periods. It follows that for fallow periods, residual fertilizer would exceed the optimal available quantity of fertilizer (i.e. none). The consequences of carryover should therefore be ignored for all fallow periods that intervene before the next period of cropping. In general if x periods of fallow are to follow the crop produced in period n before another crop is grown then (10) becomes

$$dy_n/dQ_n = (p_n^f - \alpha^{x+1} V_n V_{n-1} \ldots V_{n-x} p_{n-x-1}^f)/\alpha p_n^g.$$

wet season. Approximately, these sowing times were December for a crop growing through the wet season, April-May for an early-dry season crop, and August-September for a late-dry season crop. The yield potential varies between seasons because of the changing influences of rainfall, temperature, radiation and daylength. The early-dry season promotes the highest yield potential, followed by the wet season, and then the late-dry season. Temperature regimes appear to determine this pattern more than any other factor. Consistent with current agronomic and economic information, the maximum number of consecutive crop cycles has been restricted to six.

The recycling decision system may be described at any stage by three state variables. These are crop cycle number C, season number S and residual nitrogen r. The two decision variables for transforming the system state between seasons are (i) the crop decisions concerning planting, continuing to harvest and ploughing out, and (ii) the level of nitrogen application.

The computation involved in solving this problem using dynamic programming with three state variables and two decision variables would be considerable. However, the principle derived above for determining optimal nitrogen application enables a significant simplification in formulation to be made. By using (10) it is possible to eliminate residual nitrogen r as a state variable and nitrogen application as a decision variable.

This may be explained as follows. Whether or not carryover was relevant for the nitrogen application decision in the previous period can be deduced from the current crop cycle number. Crop cycle number refers to the number of the crop to be harvested in the current period. Zero represents the fallow state, which must intervene between ploughing out the old plant and replanting. If crop cycle number in period n is C_n, and $2 \leqslant C_n \leqslant 6$, then the crop decision in the previous period $n + 1$ must have been to continue harvesting for at least one more season from the same plant, and nitrogen carryover must have been relevant. Hence, assuming that the previous period's response function is known together with the other relevant parameters, Q^*_{n+1} may be deduced using (10), and therefore r_n may be calculated as $V_{n+1}Q^*_{+1}$.

Carryover from the previous period for $C_n=1$ may be deduced if it is assumed that V in all periods is small and that therefore V^2 and any higher powers of V can be ignored as negligible factors influencing decisions. What evidence there is suggests that V for this application is in the range 0.2 to 0.4 [4], and hence the assumption is probably reasonable. It follows that if $C_n=1$ then, because no fertilizer was applied in the previous season, r_n can be taken to be zero.

Thus r_n may be eliminated as a state variable because it can be deduced from C_n. The decision variable for nitrogen application can also be omitted because the current crop decision determines whether or not carryover is relevant. That is, if the current crop decision is to continue harvesting for at least one further season from the same plant, carryover is relevant and (10) should be used. If however, the decision is to plough out, and V is small, then the single period equation, (3), should be used.

Table 1 indicates the crop decisions that are possible for each crop state. Decisions are assumed to be taken at the beginning of a period, but not implemented until the end of that period.

TABLE 1

Decision Alternatives at Each Crop State

Decision	d	*Current Crop Cycle Number* 0	$1 \leqslant C \leqslant 5$	6
Continue	0	√	√	
Plough out after current crop	1		√	√
Plant	2	√		

Let $h\{C,d\}$ be the function for the transformation of crop cycle number. State transformations given an initial crop cycle number C and a decision d are specified as follows:

(T1) if $h\{0,0\}$, go to $C=0$;
(T2) if $h\{C,0\}$ and $1 \leqslant C \leqslant 5$, go to $C=C+1$;
(T3) if $h\{C,1\}$ and $1 \leqslant C \leqslant 6$, go to $C=0$;
(T4) if $h\{0,2\}$, go to $C=1$.

To each of these transformations there corresponds a return function, defined $R\{C,S,d\}$ for season S. These are specified as follows:

(12) for T1, $R\{0, S, 0\} = 0$

(13) for T2, $R\{C, S, 0\} = \alpha p_n^g y_c^s\{Q_n^*\} - p_n^f(Q^* - V_{n+1}Q_{n+1}^*) - U^s$

(14) for T3, $R\{C, S, 1\} = \alpha p_n^g y_c^s\{Q_n^*\} - p_n^f(Q_n^* - V_{n+1}Q_{n+1}^*) - U^s - J$

(15) for T4, $R\{0, S, 2\} = -Z$

where symbols are as before except that y_c^s is the response function for season S and crop cycle number C, U^s is total variable cost less nitrogen costs in season S, J is the cost of ploughing out, and Z is the cost of planting.

A second transformation function is required for change of season, and is denoted $i\{S\}$. The transformation rule simply states that if $S < 3$, S goes to $S+1$ next period, but to 1 if $S = 3$.

A general recurrence equation may now be developed:

(16) $$f_n\{C_n, S_n\} = \max_{d_n} [R\{C_n, S_n, d_n\} + \alpha f_{n-1}\{h\{C_n, d_n\}, i\{S_n\}\}]$$

where $f_n\{C_n,S_n\}$ specifies the maximum return given that there are n periods remaining.

The solution to the optimal recycling problem may be obtained for some limited time horizon by finding $f_1\{C_1,S_1\}$ and then solving recursively for any desired number of stages. Alternatively, if there is no obvious time horizon then the infinite stage solution may be relevant. This may be found because returns in future periods are discounted, so that the infinite stream of net returns converges to some finite magnitude.

As n approaches infinity the recurrence equation simplifies to:

(17) $$f\{C, S\} = \max_{d} [R\{C, S, d\} + \alpha f\{h\{C, d\}, i\{S\}\}]$$

Such a recurrence equation is difficult to solve directly, but may be solved by an iterative procedure. Two possible systems referred to in the literature, e.g. [2], are "approximation in return space" and "approximation in policy space".

Application to Growing Grain Sorghum on the Ord

Economic data

The main selling outlets for grain sorghum on the Ord are the local beef production units. Consequently the opportunity cost of grain sorghum is tied to the economics of beef production. At current beef prices (57c/kg) the value of grain sorghum is estimated to be 3.55c/kg. However, overseas markets for beef produced on the Ord may expand in future, and beef prices of 66c/kg are thought to be possible. At this price, if all cattle fattening costs (including a profit margin) apart from grain costs are held constant, the derived value of grain is 5.31c/kg. In the analysis solutions are obtained for grain sorghum values of 3.55, 4.50 and 5.31 c/kg.

Based on current information, reasonable values for the costs involved in grain sorghum production on the Ord are 17.6c/kg for elemental nitrogen (p^f); \$53.10 and \$64.20 per hectare for variable costs excluding nitrogen in the wet[2] and dry seasons respectively (U^s); \$27.20 per hectare for ploughing out (J); and \$9.88 per hectare for planting (Z). A seasonal discount factor of 0.952 is assumed, based on a rate of interest of 15 per cent per annum.

Technical data

The effect on grain yield of nitrogen applied, residual nitrogen and crop cycle number have been investigated by Whan [4] in a study which marks the first step in obtaining the agronomic data required for the decision problems posed here. Optimal rates of nitrogen application were calculated for the three first-crop cycle response functions, first ignoring and secondly taking account of nitrogen carryover. A value of 0.4 was assumed for the carryover rate V. It was found that optimal nitrogen application Q* was only increased slightly when V was raised from 0.0 to 0.4, and that the net revenue advantage for the one production period of V being 0.4 instead of 0.0 was negligible. For the purposes of calculating Q* it was therefore decided that V could be ignored in the present analysis with little loss in accuracy. Thus it is assumed here that V is only significant because it implies an increase in gross margin through savings in nitrogen applications.

It is, however, important to take account of the supply of mineralized nitrogen resulting from the break-down of organic matter in the soil. The greatest release occurs during the wet season when conditions for the breakdown of organic matter are best. Based on the work of soil scientists at Katherine in the Northern Territory it is estimated that 40kg of soil nitrogen per hectare are released into the soils of the Ord River Valley during the average wet season [3]. It is assumed the relationship between mineralized nitrogen and residual or applied nitrogen is quite independent. Using this estimate and the response analyses conducted by Whan, optimal yields and single-period nitrogen applications as a function of season and cycle number have been estimated and are presented in Table 2.

[2] This difference arises because half the wet season water requirements are obtained free.

TABLE 2

Optimal Yields and Single-Period Nitrogen Applications (kg/ha)

Season	Current Crop Cycle Number 0	1	2	3	4	5	6
Wet (S=1)	0*	4,260	4,150	3,920*	3,360	2,800	2,130*
	(0)	(163)	(163)	(135)	(135)	(112)	(112)
Early-dry (S=2)	0	4,430*	4,370	4,370	3,700*	3,140	2,240
	(0)	(168)	(168)	(168)	(146)	(146)	(123)
Late-dry (S=3)	0	4,040	3,920*	3,360	2,800	2,240*	2,020
	(0)	(202)	(179)	(179)	(146)	(146)	(146)

* In sequence from left to right, the asterisked values indicate the yield pattern for a crop planted in the wet season and harvested over six cycles. Nitrogen applications are shown in brackets below the yields.

Gross margin calculations

The economic and technical data may be used in equations (12) to (15) to derive gross margins. As an example, figures are presented in Table 3 for the crop decision "continue". Gross margins increase significantly as V is raised from 0.0 to 0.4 for C $\geqslant$ 2 because of the reduced applications of nitrogen required for these crop cycle numbers.

TABLE 3

Gross Margins [a] ($/ha)

V	Current Crop Cycle Number	Season: Wet (S=1)	Early-Dry (S=2)	Late-Dry (S=3)
0.0	0	0.00	0.00	0.00
	1	62.20	68.90	47.80
	2	58.42	65.20	48.00
	3	55.80	65.20	29.10
	4	36.80	46.30	15.90
	5	21.90	27.30	–3.00
	6	–0.80	1.10	–10.60
0.2	0	0.00	0.00	0.00
	1	62.20	68.90	47.80
	2	65.50	70.90	54.00
	3	62.10	70.90	35.00
	4	43.10	51.00	21.90
	5	27.10	32.10	2.20
	6	4.30	5.00	–5.40
0.4	0	0.00	0.00	0.00
	1	62.20	68.90	47.80
	2	72.60	76.60	59.90
	3	68.40	76.60	40.90
	4	49.50	55.80	27.80
	5	32.20	36.80	7.30
	6	9.50	8.90	–0.30

[a] Grain sorghum valued at 3.55c/kg.

Determination of the infinite stage solution

Initially attempts were made to solve for the infinite stage using "approximation in return space". A guess was made at the optimal return for the infinite stage, and then an iterative procedure was followed for a sufficient number of cycles for estimates of the optimal return to converge. However, the method proved unsatisfactory because of the large number of iterations required before convergence was attained. It was found that the reason for this slow rate of convergence was the high seasonal discount factor used (0.952).

It was subsequently decided to solve the problem using "approximation in policy space". A guess was made at the optimal policy for the infinite stage, and an iterative procedure followed until estimated policies from two successive iterations were identical. At this point the optimal return had been found, returns also being the same for the two final iterations. With an initial policy guess that sorghum should be planted whenever the land is fallow, but that for any $C \geqslant 1$ the crop should be ploughed out after the next harvest, the infinite stage solution was obtained within six iterations. Solutions for different values of V are displayed in Table 4.

Table 4 shows the optimal decision to follow at each crop cycle number and season combination, together with the present value of the return from continuing to follow an optimal policy for an infinite number of seasons. For example, if V = 0.2 and the current crop cycle number is 3 and it is the early-dry season, the optimal decision is to continue. This means that a third crop is harvested at the end of the early-dry

TABLE 4

Optimal Decisions and Returns [a]

V	Current Crop Cycle Number	Season					
		Wet		Early-Dry		Late-Dry	
		Decision	Return ($/ha)	Decision	Return ($/ha)	Decision	Return ($/ha)
0.0	0	Plant	702	Plant	687	Plant	688
	1	Continue	733	Continue	748	Continue	732
	2	Continue	719	Continue	704	Continue	713
	3	Continue	698	Plough out	693	Plough out	671
	4	Plough out	664	Plough out	674	Plough out	658
0.2	0	Plant	766	Plant	752	Plant	751
	1	Continue	799	Continue	815	Continue	800
	2	Continue	790	Continue	774	Continue	783
	3	Continue	766	Continue	761	Plough out	738
	4	Plough out	733	Plough out	739	Plough out	725
0.4	0	Plant	831	Plant	819	Plant	815
	1	Continue	866	Continue	883	Continue	871
	2	Continue	864	Continue	844	Continue	855
	3	Continue	834	Continue	831	Continue	805
	4	Plough out	803	Plough out	804	Plough out	792

[a] Grain sorghum valued at 3.55c/kg and returns taken over an infinite time horizon.

season, and that the sorghum is left standing for harvesting a fourth crop at the end of the late-dry season. The present value of the return from continuing to follow the optimal policy in this case is $761.

Discussion and Conclusion

Although, as Table 4 shows, the decision policy for the wet season does not alter when V is increased from zero to 0.4, the tendency for the decision policies for the other two seasons is to extend the number of harvests from the same plant as V is raised. Returns are greater the larger the value of V because of the savings in nitrogen application.

Examination of the decision chains specified in Table 4 shows that seasonal policies eventually stabilize into a recurring sequence, no matter at what cycle number and season decisions are started. In all cases the state of fallow land at the beginning of the wet season is reached. Optimal policy cycles from then on are as shown in Table 5.

Decision policies are not very sensitive to changes in the valuation of grain sorghum. For example, when the model was re-run with grain sorghum valued at 4.50 instead of 3.55c/kg the decision policies were not altered. When the valuation was raised further to 5.31c/kg the only change was to alter the "Plough out" decision at crop cycle number 3 in the early-dry season to "Continue". Thus the only significant effect of raising the grain valuation is to increase returns.

More generally, this article has shown how dynamic programming may be used for solving some of the time dependent and sequential choice problems that arise under continuous year-round cropping as is possible in many tropical environments. Analytical solutions have been presented for the decision problems posed by the fertilizer carry-over and the feasibility of crop cycling. The future price of grain sorghum and the value of the carryover rate on the Ord River Valley are uncertain but the sensitivity of policy decisions to these parameters has been considered.

It has been shown that the only additional information required in theory for calculating optimal fertilizer applications when carryover is relevant compared to when it is not is the price of fertilizer in the next

TABLE 5

Seasonal Policy Cycles Derived from Table 4 [a]

Carryover Coefficient V	
0.0 and 0.2	0.4
→ Plant	→ Plant
Continue	Continue
Continue	Continue
Continue	Continue
Plough out	Plough out
Plant	
Continue	
Continue	
Plough out	

[a] Each cycle starts with land fallow.

period, the discount factor and the rate of carryover. As to whether these factors are important in practice depends upon the shape of the response function in the region of the fertilizer level which maximizes profit over one period. Only if there is relatively little change in the slope of the response function in this region will it be worthwhile allowing for carryover.

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