OPTIMAL CULLING POLICY FOR BREEDING EWES

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This article demonstrates a method to determine the optimal culling policy for a sheep breeding flock. A model of the flock is constructed and profitability calculated for different age distributions of ewes. The method is illustrated with data obtained from an actual farm by a farm management consultant.

The culling policy for a breeding flock has two aspects. Firstly there is the question of what should be the culling rate of the young ewes. What proportion of one and a half year old ewes should *not* be used as breeding flock replacements? The second question is at what age should the breeding ewes be culled (cast for age) and sold. This determines the number of different age groups in the ewe flock, and hence the generation interval. As shown later there is a specific relationship between these two culling decisions and they cannot be considered separately. Also, both questions are important when considering genetic improvement. For the purpose of this study, however, it is assumed that genetic improvement is of relatively minor importance.

The precise question to be considered here is: Given that a certain proportion of the young ewes must be culled from a self-generating breeding flock, at what age should the older ewes be cast for age? In general, replacement problems of this type can be approached in a number of ways. On the one hand, we can construct a replacement model in which the individual ewe is regarded as an item of capital equipment whose productivity varies with age. This may be expressed in the form of a dynamic programming model aimed at deriving the optimal replacement pattern.[1] We can also construct a linear programming model in which the activities are the various age groups of ewes.[2] On the other hand, we may use an actuarial approach and consider the problem as one of replacement within a self-generating population. In this case the population is the breeding ewe flock and its progeny. Since the ewe replacements come from the progeny, an actuarial approach is considered to be well suited to this type of problem.

The aim of this article will be to demonstrate the latter of these methods. We will commence by constructing a model of the population

* The author wishes to express appreciation to P. M. Houlahan, Farm Management Consultant, Holbrook, N.S.W., for providing the relevant data and to John L. Dillon for his helpful comments on an earlier draft of this paper. This work was carried out under the Postgraduate Programme in Wool Production Business Management at the University of New England.

[1] See Throsby, C. D. Some Dynamic Programming Models for Farm Management Research. *J. Agric. Econ.* 16: 98-110, 1964.

[2] See Townsley, R., and Schroder, W. A Note on Breeding Flock Composition in Relation to Economic Criteria. *Aust. J. Agric. Econ.* 8: 66-73, 1964.

to be examined.[3] The model will show that the flock size and composition depend on the values of certain parameters. With a given flock size and composition, and a set of assumptions about certain variables including prices, it is possible to calculate the profit that can be obtained from the enterprise. The main parameter determined by the culling policy is the number of age groups in the ewe flock and throughout the article the culling policy will be expressed in terms of this number of age groups. Using the model it is possible to see how this parameter influences flock composition, and hence profit, for a given flock size. An example will be presented using data obtained for a particular grazing situation.

Flock Model

As stated, the flock model will be of a self-generating population. By far the most important components are the ewes and all progeny up to the age of eighteen months. It is assumed that the size of the flock is to remain constant from year to year. It is also assumed that shearing takes place about the same time that the ewes are joined, so that lambing will occur from five to six months after shearing. Once we have examined flock composition we will then consider the wool production and annual sheep sales.

Flock Composition

1. *Ewes:* Given a particular size of the ewe flock, its composition is determined by:

(a) the respective survival rates between each age group; and
(b) the number of age groups.

The survival rate, i.e. the proportion that survive from one year to the next, will depend on the age of the ewes and the particular environment. The survival rate between the age of $(i + 0{\cdot}5)$ and $(i + 1{\cdot}5)$ will be denoted as s_i; thus s_2 is the survival rate between $2{\cdot}5$ and $3{\cdot}5$ years of age.

Generally the ewes are ready to be joined for the first time at $1{\cdot}5$ years of age. If K is the number of these maiden ewes joined, then the ewe flock will consist of the following n age groups at joining time.

Age	Number	
$1{\cdot}5$	K	$= Ka_1$
$2{\cdot}5$	Ks_1	$= Ka_2$
$3{\cdot}5$	Ks_1s_2	$= Ka_3$
.		
.		
.		
$n + 0{\cdot}5$	$Ks_1 \ldots s_{n-1}$	$= Ka_n$

where $a_1 = 1$, and $a_i = s_1s_2 \ldots s_{i-1}$ *for* $i > 1$.

The a_i terms, representing the proportion that survive from $1{\cdot}5$ years of age to $(i + 0{\cdot}5)$ years of age, are introduced to simplify the future

[3] This is similar to a model developed by the author in a previous study. The main difference is the introduction of age-specific parameters for the ewe flock. See Byrne, P. F. Parametric Budgeting using a Model of the Sheep Enterprise. *Rev. Mktng. Agric. Econ.* 39: 95-136, 1964.

description. The total number of ewes joined is thus

$$K\Sigma a_i \qquad (i = 1, 2, \ldots, n)$$

and the number cast for age at the age of $(n + 1{\cdot}5)$ years is Ka_{n+1}.

When the size and the number of age groups in the ewe flock are specified, the value of K can be determined. For example, if the grazier wishes to have 5,000 ewes in his breeding flock then we must have[4]

$$K\Sigma a_i = 5{,}000$$

so that

$$K = 5{,}000/\Sigma a_i.$$

2. *Progeny:* The reproductive performance, i.e. the lambing rate, will depend on the age of the ewes as well as a number of other factors. We will assume that all of the other factors remain constant at a given level and we will denote the lambing rate of the i-th age group of ewes as r_i. The lambing rate is the *proportion* of lambs marked to ewes mated. To study the effect of any other factor it would be necessary to know how it affects the value of each r_i.

The number of ewes in the i-th age group is Ka_i. The number of lambs produced from this age group will thus be Ka_ir_i. With a ewe flock consisting of n age groups, the total number of lambs produced will be:

$$K\Sigma a_ir_i.$$

Assuming that half of the lambs are females, the number of these that reach the age of $1{\cdot}5$ years will be:

$$s_eK\Sigma a_ir_i/2,$$

where s_e is the proportion of ewe lambs that survive from marking to $1{\cdot}5$ years old. The above expression represents the number of $1{\cdot}5$ year old ewes available for replacements. It will be recalled that K is the number of maiden ewes in the flock, hence it is the number of replacements required. The number of $1{\cdot}5$ year old ewes not required for replacements is therefore:

$$s_eK\Sigma a_ir_i/2 - K.$$

These will be sold if the farmer is maintaining a ewe flock of a constant size.

The culling rate of young ewes, c, will be the number not required divided by the total number available for flock replacements, i.e.

$$c = [s_eK\Sigma a_ir_i/2 - K]/[s_eK\Sigma a_ir_i/2].$$

Thus the culling rate will be determined by the value of n, the number of age groups of ewes, as well as the lambing rate and survival parameters.

The number of wether lambs will be about the same as the number of ewe lambs. These wether lambs will either be sold at a suitable age or retained as replacements for a wether flock. If they are sold, then the number sold will be:

$$s_wK\Sigma a_ir_i/2$$

where s_w is their survival rate between marking and sale time. If they go into a wether flock, then s_w is the survival rate between marking and the age they join the wether flock.

[4] Unless otherwise specified, all summations are over $i = 1, 2, \ldots, n$.

Wool Production and Revenue

1. *Ewes:* As with reproduction, the wool production will depend on the age of the ewes as well as other factors. We will denote the wool production of the i-th age groups as f_i pounds per ewe.

As the cull-ewe hoggets are selected partly on wool production, all of these 1·5 year old ewes will be shorn. The cull-ewe hoggets are then sold off-shears. The wool produced from the ewe hoggets will be:

$$f_1 s_e K \Sigma a_i r_i / 2.$$

The total wool production from the rest of the ewes, those of 2·5 years and older, will be:

$$K \Sigma a_i f_i, \qquad (i = 2, 3, \ldots, n + 1).$$

The price of wool from the i-th age group of ewes will be denoted as p_i. The total receipts from the wool obtained from the 1·5 year old ewes will be the amount of wool produced multiplied by the price p_1. The total receipts from the wool obtained from the rest of the ewes will be:

$$K \Sigma a_i f_i p_i, \qquad (i = 2, 3, \ldots, n + 1).$$

2. *Lambs:* It is assumed that the lambs are shorn at four to six months of age. The number shorn will be the number marked multiplied by a survival parameter s_p. The value of s_p will depend on the time of shearing as well as other factors. The number of lambs shorn will be:

$$s_p K \Sigma a_i r_i.$$

Denoting wool production per lamb by W and the price of wool by P, the total receipts from the sale of lambs' wool is

$$W P s_p K \Sigma a_i r_i.$$

Sheep Sales

The annual sheep sales consist of the sales of cast-for-age ewes, the wether lambs, and the cull-ewe hoggets. The number of each group sold has already been given. The prices received from these sales will now be discussed.

1. *Cast-for-age Ewes:* The price for a cast-for-age ewe will depend mainly on its age. We will denote C_i as the value of ewes, off-shears, at $(i + 0{\cdot}5)$ years of age. If there are three age groups, then the ewes are cast for age at 4·5 years, their value being C_4 per head. Thus, with n age groups the receipts from the sale of cast-for-age ewes will be:

$$K a_{n+1} C_{n+1}.$$

2. *Wether Lambs:* The price of wether lambs will be denoted as V. The total receipts from the sale of wether lambs will be:

$$V s_w K \Sigma a_i r_i / 2.$$

3. *Cull-ewe Hoggets:* The price of the cull-ewe hoggets will be denoted by H. The total receipts from the sale of cull-ewe hoggets thus will be:

$$H[s_e K \Sigma a_i r_i / 2 - K].$$

In a particular market situation the value of H will be some function of the culling rate, c. For low culling rates the general quality and hence the price of cull-ewe hoggets will be lower in view of the fact that the worst are always culled out first.

Example

An example is now presented to demonstrate the use of this model to determine the optimal culling policy. The data to be used are given below. We will consider the case of a flock consisting of 5,000 Merino ewes. Most of the prices and coefficients were obtained from records kept by the grazier concerned. Table 1 shows the relevant age-specific data for the ewes. A number of studies indicate similar effects of age on survival, reproduction, and wool production.[5]

TABLE 1

Age-Specific Data for Ewes

Year of breeding life	Age at joining	Survival rate during year i	Lambing rate	Wool production per ewe	Wool price	Price of ewes[(a)]
i	$i+0\cdot5$	s_i	r_i	f_i	p_i	C_i
				lb.	$	$
1	1·5	0·98	0·72	11·00	0·49	
2	2·5	0·98	0·81	11·00	0·50	
3	3·5	0·97	0·855	10·67	0·50	
4	4·5	0·965	0·90	10·45	0·50	4·50
5	5·5	0·96	0·90	9·90	0·50	3·00
6	6·5	0·95	0·855	9·02	0·475	1·80
7	7·5	0·93	0·81	8·25	0·475	1·50
8	8·5			7·15	0·475	1·20

(a) Price if cast for age instead of being joined.

From the third column of Table 1 giving values for s_i it is possible to calculate the appropriate value of a_i. The other information required is as follows:

Survival Rates.—There are three relevant survival rates that apply to the progeny. These are: s_e, the survival of ewe lambs from marking to 1·5 years old; s_w, the survival of wether lambs from marking to sale time, and s_p, the survival of all lambs between marking and shearing time. In this example the lambs are shorn at about five months of age and the wether lambs are sold off-shears. Thus, s_w and s_p are equal and in this case have an estimated value of 0·95. The survival of ewe lambs from marking to 1·5 years old, s_e, is 0·91.

Value of Wether Lambs.—The value of wether lambs sold at the age of six months off-shears, V, is given as $3.00 per head.

Lambs' Wool.—The wool production per lamb, W, is given as 3·5 lb. per head and the price paid for lambs' wool is $0.38 per lb.

Price of Cull-ewe Hoggets.—The dollar price received from the sale

[5] See Turner, H. N., Dolling, C. H. S., and Sheaffe, P. H. G. Vital Statistics for an Experimental Flock of Merino Sheep—I. Death Rates in Adult Sheep, in Relation to Method of Selection, Age, and Sex. *Aust. J. Agric. Res.* 10: 581-90, 1959; Turner, H. N., and Dolling, C. H. S. Vital Statistics for an Experimental Flock of Merino Sheep—II. The influence of Age on Reproductive Performance. *Aust. J. Agric. Res.* 16: 699-712, 1965; Kelley, R. B. Female Aspects of Relative Fertility in Sheep. *Aust. Vet. J.* 15: 192-8, 1939; and Hickey, F. Death and Reproductive Rates of Sheep in Relation to Flock Culling and Selection. *N.Z. J. Agric. Res.* 3: 332-44, 1960.

of cull-ewe hoggets, H, will depend on the proportion culled, as indicated earlier. With the information provided it was decided to construct a function consisting of three linear segments. Although a curved function is possibly more realistic, the function used is considered to be quite adequate. This function is given by:

$$H = 1{\cdot}2 + 12c \text{ when } 0 \leqslant c \leqslant 0{\cdot}2,$$
$$H = 3{\cdot}0 + 3c \text{ when } 0{\cdot}2 \leqslant c \leqslant 0{\cdot}45,$$
$$H = 4{\cdot}35 \text{ when } 0{\cdot}45 \leqslant c.$$

Total Receipts

The only variable that has not been given a specified value is the number of age groups in the ewe flock. It is now possible to calculate the total receipts from the sale of wool and surplus sheep for each particular value of n, the number of age groups. The results will indicate which policy (value of n) produces the most revenue. The most profitable policy will be the one giving the highest margin of revenue above variable costs, i.e. the highest gross margin.

Variable costs will include items such as shearing, crutching, dipping, drenching, vaccinations, mulesing, labour and ram replacements. As the only important change resulting from a different culling policy is the composition of the ewe flock, it is unlikely that there will be any significant change in the variable costs. The numbers of ewes, lambs, and hoggets will not be changed to any extent by a change in culling policy. For these reasons it is not necessary to consider variable costs. The total revenue resulting from each policy will be sufficient to indicate which one is optimal.

Table 2 indicates the revenue obtained from different culling policies. Policies of having from three to seven age groups are examined. A policy of having only two age groups is not included as this would require the purchase of ewe hoggets in order to maintain the flock. While this is feasible, it is not considered to be a practical solution because of the problem of purchasing disease free ewes of a similar type to those in the flock. Table 2 also shows the numbers of ewes shorn and lambs marked together with the revenue from wool sales. The number of ewes shorn includes the ewe hoggets to be culled (and sold), as well as the ewes about to be cast for age.

TABLE 2

Revenue from Different Culling Policies

Age groups	Ewes shorn	Revenue from ewes' wool	Lambs (a) marked	Revenue from lambs' wool	Total wool revenue	Total sheep sales	Total revenue
No.	No.	$	No.	$	$	$	$
3	6,690	35,892	3,970	5,017	40,909	12,989	53,898
4	6,734	35,658	4,098	5,178	40,836	11,570	52,406
5	6,755	34,894	4,174	5,273	40,167	11,273	51,440
6	6,746	34,006	4,189	5,293	39,299	11,492	50,791
7	6,714	32,954	4,171	5,271	38,225	11,540	49,765

(a) The number of lambs shorn is this number multiplied by 0·95, the survival rate between marking and shearing.

It is evident that the most profitable policy is to have only three age groups yielding a total revenue of $53,898. With more age groups the revenue from wool sales declines. This is due to the fact that the older ewes produce less wool per head. By increasing the number of age groups we increase the proportion of lower producing ewes. This is offset to some extent by the fact that more lambs and hoggets are shorn as n rises to six. However, the total wool revenue declines for every increase in the value of n.

The revenue from the surplus sheep declines and then increases for increasing values of n. Table 3 provides the details of the revenue from sheep sales.

TABLE 3

Revenue from Sheep Sales

No. of age groups	Cast-for-age ewes			Cull-ewe hoggets			Wether lambs[a]		Culling rate for ewe hoggets
	No.	Price	Revenue	No.	Price	Revenue	No.	Revenue	
		$	$		$	$		$	
3	1,584	4·5	7,128	106	1·90	202	1,886	5,658	0·058
4	1,161	3·0	3,483	573	3·92	2,248	1,946	5,839	0·307
5	904	1·8	1,628	851	4·34	3,697	1,982	5,947	0·448
6	728	1·5	1,091	1,018	4·35	4,431	1,989	5,970	0·534
7	591	1·2	709	1,123	4·35	4,886	1,981	5,944	0·591

(a) Sold at $3 per head.

It is evident that as the number of age groups in the ewe flock is increased, two important things happen. Firstly, there is a considerable reduction in the revenue from the sale of cast-for-age ewes. This is due to the fact that there are less for sale and their price is lower. Secondly, as n is increased there is an increase in revenue from the sale of cull-ewe hoggets. This is brought about by the increase in numbers and also the increase in price. The increase in price is, in turn, a result of the higher culling rate indicated in the last column.

The revenue from the sale of wether lambs increases slightly for values of n up to six and then falls slightly. This figure is directly related to the number of lambs produced. The second last column in Table 2 shows the result of these three influences. Revenue from the sale of sheep falls as the value of n is increased to five, and then rises.

Intermediate Policies

The above analysis shows the total revenue resulting from five distinct policies. These policies are to have from three to seven complete age groups in the ewe flock. Each policy requires a specific culling rate for the ewe hoggets given that the flock size is to remain constant.

It appears that the optimal policy is to have three age groups of breeding ewes. This yields a total revenue of $53,898 and requires a 5·8 per cent culling of the ewe hoggets. In the actual problem presented to the author it was specified that the culling rate of the ewe hoggets must be at least 10 per cent. This was considered necessary in order to eliminate inferior types. By having four age groups in the ewe flock it is possible to cull the ewe hoggets at the rate of 31 per cent. As this involves a reduction in profit by \$1,490 it is evident that some intermediate policy is desirable.

Modifications to the Model

It is possible to achieve an intermediate policy by having three complete age groups of ewes, and by keeping a proportion of the oldest age group for another year. If n is the number of *complete* age groups, the number of ewes joined will be

$$K\Sigma a_i + dKa_{n+1}$$

where d is the proportion of the oldest age group kept for another year. In fact d would be the proportion selected from the oldest group of ewes just before joining time. The number not selected will then be $(1 - d)Ka_{n+1}$ and these will be sold as cast-for-age ewes.

In this case, for a flock of 5,000 ewes, we have

$$K = 5{,}000/(\Sigma a_i + da_{n+1}),$$

and the number of lambs marked will be

$$K\Sigma a_i r_i + dKa_{n+1}r_{n+1}.$$

The number of ewe hoggets available for replacements will then be this number multiplied by $s_e/2$. The number required for replacements is K, hence it is possible to calculate the number and proportion of ewe hoggets culled. Also the number of lambs shorn and the number of wether lambs sold can be calculated readily by using the appropriate survival parameters.

The number of maiden ewes shorn will be

$$s_eK(\Sigma a_i r_i + da_{n+1}r_{n+1})/2$$

and the number of older ewes shorn, those aged from $2{\cdot}5$ to $(n + 1{\cdot}5)$ years, will be $K\Sigma a_i$ with $i = 2, 3, \ldots, n + 1$.

The number of the ewes kept for one more year will be dKa_{n+2}. Their age is $(n + 2{\cdot}5)$ years when shorn and they are then cast for age.

Example

In this instance the relevant intermediate policies will be those policies that are intermediate to the policy of having three age groups of ewes and the policy of having four age groups of ewes.

The flock composition and total revenue can be obtained with the use of the above alterations to the model. The particular intermediate policy is determined by the value of d, which is in this case the proportion of $4{\cdot}5$ year old ewes kept for another year. When d equals zero this corresponds to the previous policy of having three complete age groups. When d equals one, it corresponds to the policy of having four complete age groups. For all intermediate policies two groups of cast-for-age ewes are sold. As the value of d is increased the number of $4{\cdot}5$ year old cast-for-age ewes decreases and the number of $5{\cdot}5$ year old cast-for-age ewes increases.

By retaining a proportion of the oldest age group for another year we are able to make gradual adjustments to the age composition of the ewe flock. By increasing the value of d we reduce the total wool production from the flock, and at the same time we increase the total number of lambs produced. The latter in turn enables an increase in the culling rate of the $1{\cdot}5$ year old ewe hoggets.

By adjusting the value of d we can obtain any culling rate for the ewe hoggets within the range of 0·058 to 0·307. To obtain any culling rate outside this range it would be necessary to commence with a different number of complete age groups and then adjust the value of d.

In the example considered the objective is to maximize total revenue subject to the restraint that the culling rate of ewe hoggets is not less than 10 per cent. This is achieved with a policy of retaining 13 per cent of the oldest age group. This enables a culling rate of 10 per cent and yields a total revenue of $53,577.

Discussion

Under the conditions examined, the optimal culling policy is to cull the old ewes as early as possible. The actual policy depends on the requirements for culling the ewe hoggets.

Only one set of parameters were examined in the example. It is evident from Table 3 that a higher value for young sheep, particularly the cull-ewe hoggets, could alter the situation. Under these conditions policies of keeping more age groups would be relatively more profitable.

There are a number of reasons why the value of young sheep could be much higher than in the above example. Restocking after a drought or building up sheep numbers following pasture improvement are probably the most important reasons. Even if there is no general shortage of young ewes, the grazier's own ewe hoggets may be of considerably more than market value to him. This will occur particularly if he wishes to increase sheep numbers without mixing types or running the risk of introducing disease.

Table 4 shows the influence of the price of cull-ewe hoggets on the optimal culling policy. Here we see that as the price of cull-ewe hoggets is increased, relative to that used in the example, the policies with more age groups tend to become more profitable. If the price received for the cull-ewe hoggets is increased by at least 70 per cent, the optimal policy is no longer to have as few age groups as possible. With a 70 per cent increase the policies of having three, four, and five age groups yield the same revenue. With a 100 per cent increase in the price, making it about $8 per ewe hogget, the optimal policy is to have six age groups.

There are a number of other relevant factors that may need investigation in a particular situation. The effect of age on wool production per

TABLE 4

Influence of Cull-Ewe Hogget Price on Total Revenue

No. of age groups	Percentage increase in price of cull-ewe hoggets				
	0	50	70	80	100
			$ '000		
3	53·9	54·0	54·0	54·1	54·1
4	52·4	53·5	54·0	54·2	54·7
5	51·4	53·3	54·0	54·4	55·1
6	50·8	53·0	53·9	54·3	55·2
7	49·8	52·2	53·2	53·7	54·7

ewe will have a considerable influence on the optimal policy. If, for example, there is a much smaller decline with age than indicated in Table 1, then policies with more age groups will be relatively more profitable.

The overall lambing rate, and the survival rates of ewes, both as lambs and as adult sheep, may have a significant influence. An increase in either of these will permit a higher culling rate of ewe hoggets for a given number of age groups in the ewe flock. Conversely it will permit a reduction in the number of age groups given that there must be a certain culling rate.

Apart from the question of genetic improvement, all of these factors can be examined simply by using the appropriate values in the model. To account for genetic gain it is necessary to have an estimate of the future pattern of returns from a series of different policies. The important policies to examine would most probably be within the range of a policy that maximizes current net revenue, and a policy that maximizes genetic gain. The future pattern of returns from each policy can be calculated once we know the genetic gain, in terms of increase in wool production, lamb production and any other relevant parameter. The optimal policy would be the one which maximizes the present value of future returns.