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TIME AND RECURSIVENESS IN LIVESTOCK FEEDING TRIALS*

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In livestock feeding experiments the problem of recursiveness arises because the quantity of feed consumed by an animal is a function of its past history of feeding. The problem is most acute where experiments are designed to analyse sub ad lib feeding. The analysis of such experiments encounters two important problems. The first is that the actual quantity of feed consumed is an endogenous variable and is not directly under the control of the experimenter or the livestock producer. The second problem is the mathematical complexity of the relationships involved. A solution is suggested as a quasi reduced-form model.

Livestock production processes are essentially of two types: the first is the factory type, such as milk and wool production, where inputs are processed by a relatively fixed livestock unit to yield a flow of outputs (which may or may not be harvested continuously); the second is that of meat production, where the livestock increase in weight, and it is this growth which constitutes the product. We are here concerned with the second type of process, although it is apparent that the analysis has relevance to the former.[1]

In experiments aimed at estimating the production functions associated with meat production and hence at determining the optimal use of resources, two particular problems are noted. First, for physiological reasons, it is likely to make sense to feed an animal rations of different composition as it matures and grows. Second, an animal is fed at different levels of feeding over time. We abstract from the first problem of ration composition by assuming that animals are brought into the experiment at a constant weight and age, and that the composition of the ration is held constant for the further duration of its life. However, it is clear that the methodology proposed at the end of the paper can be extended to account for changing the ration composition throughout the production period.

Consideration of the second problem of differing feed levels over time reveals important differences between livestock and crop experiments. In a crop experiment designed to explore response to fertilizers, no special difficulties arise in fixing the levels of fertilizer inputs, given the experimental design. Also, with most annual crops, only one crop is possible each year, with the time to harvesting largely being determined by the plant's physiological response to various environmental stimuli: thus, time is not a relevant instrumental variable. However, in livestock ex-

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[1] The design and analysis of agricultural experiments has generated an extensive literature. Excellent bibliographies are to be found in Heady and Dillon (1961) and Dillon (1967).

periments the application of inputs is a continuous process over time, and the time to reach marketing size is a function of the levels of feeding. For livestock we need to determine not just levels of inputs to be applied instantaneously, but a pattern of input levels over time. Obviously, it would not be economic to feed growing livestock a constant quantity per day for the whole production period. Any fixed-over-time feeding regime would be uneconomic as the animal may be over-fed when young and under-fed when older. Because livestock processes are continuously recurring processes over time, and time to reach marketing weight is a function of feeding levels, it is clear that there are costs associated with slow weight gain. Thus time enters the objective function for livestock processes, although it does not in annual crop production. The time function in this sense has been considered in the literature, and we use the results obtained in defining our objective function.[2]

Livestock feeding experiments may be carried out to determine the optimal rate of feeding (of a ration of fixed composition), or the optimal composition of the ration (at some fixed rate of feeding), or to determine both optimal rate of feeding and composition of the ration simultaneously. In the past it has been most common to feed livestock rations of differing composition on an ad lib basis with a view to determining the optimal composition. However, there exist good reasons for believing that feeding at a sub ad lib level may be optimal from an economic standpoint. That is, for rations of fixed composition, there are likely to exist diminishing returns to feed as the rate of feeding is increased. Certainly, the marginal product of feed is zero in the region of stomach capacity and greater than zero below it.[3]

For heuristic reasons, we first deal with sub ad lib feeding experiments in which the ration consists of constant proportions of different feeds, with only the level varied. Rates of feeding are fixed and coded as k_i. We set maximum k_i appropriate to the ad lib level of feeding. The analogue in a crop experiment would be, say, levels of fertilizer 0, 1, 2, 3, 4 cwt. per acre coded as 0, 0·25, 0·50, 0·75, 1·00. If we abstract from replication and define C_{it} as the capacity level for the i-th animal on the i-th ration at the t-th time period, then the quantity of feed for the i-th animal at time t, R_{it}, is given by

$$R_{it} = k_i C_{it}. \tag{1}$$

The treatments having k_i equal to unity are on ad lib feeding. In other treatments k_i is varied downwards from unity.

We define Y_{it} as the weight of the i-th animal at the end of the t-th period and postulate the production function

$$\Delta Y_{it} = f(Y_{i,t-1}, R_{it}), \tag{2}$$

where $\Delta Y_{it} (\equiv Y_{it} - Y_{i,t-1})$ is the animal's weight gain during the t-th period. A period is defined as the time over which the daily quantity of feed fed is constant. It seems that the optimum rate of feeding can be determined from (1) and (2), given the appropriate prices.

[2] We use the result that to maximize profits from a continuously recurring process, it is necessary to maximize profits per unit of time. See Dillon (1967), Chapter 3.

[3] Evidence from stocking rate trials for sheep indicate that high stocking rates representing sub ad lib feeding are appropriate to maximize profits. See Chisholm (1965).

The capacity of an animal to consume feed (C_{it}) is a function of its genetic potential (P_i) which we assume constant over time, and its previous levels of feeding. That is,

$$C_{it} = g(R_{i,t-1}, R_{i,t-2}, \ldots, R_{i,1}, P_i). \tag{3}$$

Equation (3) expresses the fundamental problem of determining experimental feeding rates at a sub ad lib level. This problem, which we call the problem of recursiveness, is that the capacity of an animal to consume feed at any given time is a function of the previous feeding levels as well as its genetic make-up, etc. That is, what an animal can consume this period depends upon what it was fed last period.

In practice we can observe C_{it} only under ad lib feeding, i.e. where k_i equals unity. If we decide to feed some livestock less than ad lib, we cannot relate their feeding level to those on ad lib rations because feeding at sub ad lib rates changes subsequent consumption capacity. Even if livestock are fed identically, their capacity to utilize feed varies with their genetic constitution, and thus the optimal rate of feeding varies. In such a feeding regime, non-optimal rations are prescribed for some of the livestock because their rations are not based on their own individual response to feeding. However, for some types of livestock, notably poultry, the costs of assessing individual rations would outweigh the savings obtained by feeding optimally on an individual basis. So it makes sense to feed such livestock on a group basis. On the other hand, it is common practice to feed milking cows rations individually computed on the basis of their output of milk. Individual capacity of livestock is considered the appropriate yardstick on which to base a theoretical consideration of feeding.

It is easily shown that using ad lib capacity as a basis for designing experiments does not solve the problem of recursiveness. For illustrative purposes we consider the case where all livestock are genetically identical and rations of fixed composition are fed in a non-stochastic world. Feed animal A (or group of animals—under our assumptions the results are the same) on an ad lib basis, and another animal B, on a sub ad lib ration which is in excess of maintenance requirements. The growth curve of animal A is purely a function of time. It is clear that animal B on the sub ad lib ration gains weight far more slowly than animal A on an ad lib ration. After some time, the weight of animal B is less than that of animal A. However, animal B consumes only a proportion of the feed consumed by animal A. If capacity is a function of liveweight, then B is receiving, not that proportion of its capacity which is obtained at the beginning of the experiment but possibly even a capacity ration! Animal B's weight and capacity have adjusted to the available feed supply. Therefore to examine the effects of sub ad lib feeding, we need to relate the ration to the animal's capacity, not have the reverse occur.

To obtain an estimate of capacity we assume that it is proportional to liveweight and that the ration level is determined as before, by varying k_i, where k_i is now some fraction of the weight of the animal at the beginning of each feeding period. Given this and assuming for the moment only linear functions, the algebraic statement of the problem is as follows, suppressing the i subscript for simplicity:

$$\Delta Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 R_t + u_t \tag{4}$$

$$R_t = kY_{t-1} \tag{5}$$

or

$$Y_t = \alpha_0 + AY_{t-1} + u_t \tag{6}$$

where A equals $(1 + \alpha_1 + k\alpha_2)$.

Equation (6) is a first-order linear stochastic difference equation, which is equivalent to (7).

$$Y_t = A^tY_0 + (1 + A + A^2 + \ldots + A^{t-1})\alpha_0 + u_t + Au_{t-1} + A^2u_{t-2} + \ldots + A^{t-1}u_1 \tag{7}$$

Equation (7) is difficult to manipulate, even if we ignore the high order terms of A which occur with α_0 and the disturbances. The numerical value of A would be between zero and one. Were $A > 1$, the animal would grow continuously at an increasing rate over time.

Assuming a zero rate of interest and constant technology and prices, the appropriate procedure to yield maximum profits from the livestock feeding enterprise is to maximize profit per unit of time.[4] On an individual animal basis we thus have the objective function:

$$\pi/T = [p_yY_T - S - V_T]/T \tag{8}$$

where π is total profit per animal;[5]

T is the number of periods for which the process is run;

p_y is the price per unit of product;

Y_T is the animal's weight after T periods;

S is the fixed set-up cost per animal per run of the process; and

V_T is the total cost of the rations.

The total cost of feeding (V_T) is the sum of the costs of feeding in each period for which the process is run. That is,

$$V_T = \sum_{t=1}^{T} V_t \tag{9}$$

where V_t is the cost of feeding in period t and is given by

$$V_t = p_RR_t \tag{10}$$

where p_R is the cost per unit of the ration. Using (5) we obtain

$$V_t = p_Rk Y_{t-1}. \tag{11}$$

Using equation (7), V_t can be re-written as

$$V_t = p_Rk[A^{t-1}Y_0 + (1 + A + A^2 + \ldots + A^{t-2})\alpha_0 + u_{t-1} + Au_{t-2} + \ldots + A^{t-2}u_1]. \tag{12}$$

V_T is obtained by summing (12) over the range $t = 1, \ldots, T$. We then have

[4] Dillon (1967), Chapter 3.6.

[5] This objective function is also appropriate for maximizing profits from the whole animal feeding enterprise, given that the number of animals is exogeneously determined. However, the objective function could be modified to include the number of animals per process as an endogeneous variable and the optimal number determined, provided suitable constraints are specified.

$$(13)\quad V_T = p_R k\{Y_0(1 + A^1 + A^2 + \ldots + A^{T-1}) + \alpha_0[(T-1)A^0 + (T-2)A^1 + \ldots + A^{T-2}] + u_1(A^{T-2} + A^{T-3} + \ldots + A^0) + u_2(A^{T-3} + A^{T-4} + \ldots + A^0) + \ldots + u_{T-1}A^0\}.$$

Equation (13) may be re-written as

$$(14)\quad V_T = p_R k\{Y_0(1 + A^1 + A^2 + \ldots A^{T-1}) + \alpha_0[(T-1)A^0 + (T-2)A^1 + \ldots + A^{T-2}] + A^0 \sum_{t=1}^{T-1} u_t + A^1 \sum_{t=1}^{T-2} u_t + \ldots + A^{T-2}u_1\}.$$

Equation (14) as it stands is rather formidable. However, on a "reasonable" level of approximation, the terms involving the disturbances u_t may be ignored. We assume that $E(u_t) = 0$, for all $t = 1, 2, \ldots, T$. We note that those terms involving u_t, with low powers of A, also involve summing over a number of values of u_t. By the law of large numbers, we may hope that the summation of u_t over $t = 1, 2, \ldots, \tau$ approaches zero for large τ. Also, we note that, where only a few terms u_t are summed, the powers of A are large. With $0 < A < 1$, we ignore these terms. Hence, using these approximations, we re-write (14) as:

$$(15)\quad V_T \doteq p_R k\{Y_0(1 + A^1 + A^2 + \ldots + A^{T-1}) + \alpha_0[(T-1)A^0 + (T-2)A^1 + \ldots + A^{T-2}]\}$$

If we substitute (7) and (15) into (8), we have the objective function in an operational form. However, even with the approximations in the feed-cost function (15), the objective function is still an extremely complicated expression. To maximize it, we would need to follow an iterative procedure, differentiating the objective function with respect to k and determining maximum π/T, for each of successive values of T, until the maximum maximorum was obtained. In practice, whilst A is likely to be less than unity, we expect that it is not close to zero, so that only the very large powers can be ignored if tolerable precision is to be retained. This being the case, no analytic solutions are available, and even numerical solutions would be very costly in computer time.

Even so, what we have done so far involves the gross over-simplification of assuming a linear production function, which implies constant returns to increases in the rate of feeding (k). If we reject this assumption, and assume instead that the production function is quadratic, we have

$$(16)\quad \Delta Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 R_t + \alpha_3 (Y_{t-1})^2 + \alpha_4 (R_t)^2 + u_t.$$

Using equation (5) the production function (16) is re-written as

$$(17)\quad Y_t = \alpha_0 + A_1 Y_{t-1} + A_2 (Y_{t-1})^2 + u_t$$

where A_1 equals $(1 + \alpha_1 + k\alpha_2)$ and A_2 equals $(\alpha_3 + k^2\alpha_4)$. Equation (17) is a quadratic first-order stochastic difference equation, and it leads to impossibly complicated functions for Y_T and V_T to be substituted into the objective function.

The problem of determining the optimal combination of different

types of feed is now briefly considered in relation to the problem of recursiveness. Even when livestock are fed ad lib and the composition of the ration is the only variable, the problem of recursiveness still arises. This is so because, in analogy with (3), capacity of livestock to consume feed is still a function of genetic potential and the previous feeding regime. This can arise because, with feeds of different composition, one feed may be more palatable than another; or the increase in the animal's weight from a given volume (or weight) of one feed may be greater than another; or because the capacity to absorb high energy rations, which pass through the digestive processes more rapidly than low, may be greater than for low energy feeds; or for some other reasons or combinations of reasons. Therefore the analysis given for sub ad lib feeding is also applicable to the case of ad lib feeding where the composition of feeding is different for different livestock.

From the considerations above, it is evident that the complexity of the structural relationships underlying livestock feeding experiments is such as to make a direct approach to the analysis of such experiments operationally infeasible. The alternative generally adopted by experimental analysts, whilst much less complex, is not satisfactory. This approach is to relate change in weight to the quantity of feed consumed, over a defined time period or periods.[6] On two main counts this approach is unsatisfactory. Both relate to the fact that the quantity of feed consumed is itself a function of the *rate* of feeding, and of the decisions of the animals, as can be seen from equation (14) above. Hence the quantity of feed fed is an endogenous variable in the system, the exogenous variables being the feeding levels, k_i, and/or the percentage composition of different feedstuffs in the ration. Using the quantity of feed consumed as "independent" variables in the regression analysis leads to simultaneous-equations bias in parameter estimates. The second problem is that recommendations in terms of total quantities of feed are not operationally useful. This is so because decisions on feed consumption are taken, at least in part, by the animals. What is under the control of the experimenter, and the farmer, is the level and composition of rations. What is required are optimal levels of these.

To obtain a solution to what is clearly a difficult problem, we resort to a quasi reduced-form solution.[7] That is, we relate change in weight (or time to marketing at a fixed weight) and feed consumed to the control variables—feed levels and/or composition. The approach is illustrated for a particular experimental situation.

Suppose that two feedstuffs (milk and grain) are fed to animals (baconer pigs) from weaning to a fixed marketing weight.[8] Further, assume that the quantities of each foodstuff fed per period are deter-

[6] For example, see Heady and Dillon (1961), Chapters 8-11, 13; and Heady *et al.* (1963).

[7] Dillon and Burley (1961) suggested this type of approach for grazing experiments.

[8] The principles to be enumerated are applicable to problems having more than two foodstuffs and where marketing weight is variable. When marketing weight is variable, time to marketing is a predetermined variable together with the variables M and G specified later. The functions to be estimated then contain three predetermined variables, and the optimal M and G and time to marketing can be calculated in order to maximize profits per unit of time. Optimal marketing weight and total quantity of feeds are then estimated from the functions.

mined in proportion to the animal's bodyweight at the beginning of each period. An individual animal is fed a ration which consists of the two foodstuffs in fixed proportions, but for different animals the proportions are varied over a suitable range.

For an experiment of this type the objective function given by equation (8) is still relevant but Y_T is a constant and T is an endogenous variable which depends on the proportions of the individual rations.

Let M_i and G_i denote the proportions of bodyweight of the i-th animal by which are determined the respective quantities of milk and grain to be fed. The quantities of milk and grain fed to the i-th animal during period t, denoted by m_{it} and g_{it} respectively, are therefore given by

$$m_{it} = M_i Y_{i,t-1} \tag{18}$$

$$g_{it} = G_i Y_{i,t-1}. \tag{19}$$

The time from weaning to marketing weight is a function of M and G as denoted by the equation

$$T = f_1(M, G). \tag{20}$$

We expect the time function to approach asymptotically a minimum as the proportions of feeding (M and G) increase.

Since carcass composition is likely to vary with M and G such that different prices per unit of weight are obtained for different animals, a price function (p_y) is to be estimated. That is,

$$p_y = f_2(M, G). \tag{21}$$

The cost of feeding (V_T) is clearly equal to $(p_m Q_{im} + p_g Q_{ig})$, where p_m and p_g are costs per unit of milk and grain; and Q_{im} and Q_{ig} are the total quantities of milk and grain consumed by the i-th pig over $t = 1, 2, \ldots, T$. The functions giving the quantity of foodstuffs fed are estimated in terms of M and G. That is,

$$Q_m = f_3(M, G) \tag{22}$$

$$Q_g = f_4(M, G). \tag{23}$$

Given appropriate forms of the functions (20) to (23), the experimental data is used to estimate the parameters of each equation.

The estimate for the profit function is therefore given as

$$\pi^*/T = [Y_T f_2^*(M,G) - S - p_m f_3^*(M,G) - p_g f_4^*(M,G)]/f_1^*(M,G) \tag{24}$$

where the asterisks denote estimated functions.

The objective function given by (24) is expressed in terms of the predetermined variables M and G.

By partially differentiating (24) with respect to M and G and setting the derivatives equal to zero, the optimal M and G can be determined by solution of the two simultaneous equations obtained. However, because the simultaneous equations derived will be quite complex for most functions estimated, methods of numerical analysis need to be employed to find the values of M and G which maximize (24). Too, it may be necessary to constrain the maximization of the profit function so that the optimal M and G are within the feasible range.

The method of analysis described here is applied to a pig-feeding experiment which is reported in Battese *et al.* (1967). All animals were fed from weaning weight to a constant marketing weight of

160 lb. Initially, attempts were made to estimate conventional production surfaces expressing ΔY_t as a quadratic function of beginning period weight Y_{t-1}, and m_t, g_t. Not only did this approach provide poor fits to the experimental data ($R^2 = 0{\cdot}6$), but it did not lead to the determination of economic optima. However, estimation of the time (T) and total feed functions (Q_m and Q_g) gave values of R^2 in excess of $0{\cdot}9$, and the forms of the feed functions were in accordance with *a priori* expectations. Using appropriate price information, it was possible to specify optimal M, G and T.

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