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SOME BOUNDS TO THE RELEVANCE OF DECISION THEORY*

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Decision theory affords a means for the objectively rational use of information in decision-making. Concern has been expressed from time to time that the application of decision theory is, or is sometimes, of doubtful appropriateness. In this paper an attempt is made to identify a principal source of this concern and the mis-specification of decision problems using decision theory that it implies. Suggestions are offered as to how the merits of the use of decision theory might be retained in decision-making situations where doubts may arise about the appropriateness of its application.

Introduction

Decision theory¹ has played a substantial role in the development of optimising algorithms for use in contexts of uncertain environmental events. This role has been particularly pronounced in agricultural economics and, internationally, the Australian profession appears to be well regarded for its use of the theory. Occasionally, however, dissent is expressed from the view (well represented by, for example, Anderson, Dillon and Hardaker 1977) that decision theory has wide normative usefulness. Theorists in the management science area are one source of this dissent (e.g. Starr 1971 and Simon 1978). Given the strong normative bias of management scientists, this dissent is provocative.

The purpose in this paper is to identify the bases of alternative views about the usefulness of decision theory as a normative technique, to do so in such a way that some conclusion might be reached as to which applications of the technique are contested implicitly, and to identify procedures which might be adopted to maximise the acceptable application of decision theory for those people who hold basic doubts about its application.

Elements of the Debate

A variety of matters have been raised which might appear to relate to the usefulness of decision theory. Included among them is the poor capacity of decision theory to describe behaviour and the apparently widespread tendency of seemingly sensible people to breach the eminently sensible axioms which underly decision theory (e.g. Simon 1978). At the risk of understating the importance of these apparent weaknesses of decision theory, one could suggest that both of these matters are related to the relative novelty of the use of decision theory. Decision-making procedures, which people have developed in the absence of decision theory, are composed inevitably of stratagems which emerge to cope with uncertainty. These stratagems, in turn, influence the selection and

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¹ It is taken as given in this paper that subjectivist approaches (e.g. Anderson, Dillon and Hardaker 1977) to decision theory are the most realistic and appropriate to adopt.

perception of relevant stimuli. One can argue, therefore, that these apparent weaknesses of decision theory are simply reflections of its potential contribution to the improved use of information under conditions of uncertainty; people will become 'better' perceivers and decision makers once they use decision theory routinely. That is, as the facility to make meaningful use of relevant information increases with more frequent use of decision theory, information may be stored in more precise forms and greater concordance of behaviour with the axioms may occur. Only time will tell.

Another matter, of considerable practical importance, concerns the elicitation of subjective probability distributions and utility functions. The development of valid and reliable procedures has received a good deal of attention (see, for example, Staël von Holstein 1970 and Anderson et al. 1977) and will doubtless continue to do so. However, neither this nor the other matters mentioned immediately above appear to go to the basis of concern expressed by the dissenters.

The following seems to identify, in general terms, a view which raises fundamental issues and which indicates, of all the relevant matters, the most basic (Albin 1981, p. 212):

It is no exaggeration to claim that the basic techniques of modern statistical analysis, statistical mechanics and information theory are founded on a roulette wheel and concern with the problems of the gambler. However valuable these techniques may be, their reinsertion into the analyses of problems of significant social behaviour is bound by the intrinsic limitations of the gambling device as a model of the environment and the emptiness of the gamble itself as a metaphor for efficient choice under conditions of uncertainty.

A matter which relates directly to Albin's view is the perception and measurement of uncertainty. An assumption in decision theory is that all relevant beliefs an individual has about an uncertain parameter can be summarised in a single probability distribution. That is, a distribution of probabilities captures the individual's beliefs in such a way that each point in that distribution can be treated as a measure of his degree of belief that the parameter will take a given value. Should this assumption not obtain, the output from an application of decision theory may not in any sense indicate appropriate behaviour for a decision maker.

A set of three types of belief about an uncertain parameter is presented below in an attempt to structure issues with regard to the above assumption. This approach is adopted to clarify points of contention and to provide a basis for the avoidance of inappropriate applications of decision theory.

A Taxonomy of Beliefs

The term 'environment', or 'state of nature', is used in decision theory to refer to values of some parameter which is relevant to a decision. Since 'environment' may have rather broad connotations for some people, 'parameter values' will be the term used in what follows.

The application of decision theory involves, *inter alia*, the identification of relevant parameters and the assessment of the probability of occurrence of each value the decision maker believes the parameter could have at some specific future point in time. A decision-maker's beliefs

about a relevant parameter can be disaggregated into three components. The first is *set belief* relating to possible values; that is, the exhaustiveness of an identified set of values. This belief includes belief that particular alternative values are possible (summarised by the specification of a range, for example) and belief about the possibility that values which are not anticipated may eventuate. Two sources of doubt a decision maker may have about the set of possible values he specifies are (believed) lack of understanding of the causal structure underlying parameter values, and a belief that unexpected shocks to that structure can occur.

Even perfect anticipation of values of causal variables may associate with doubts of the kind described if the way these variables interact to determine parameter values is not (believed to be) understood. If values of causal variables are difficult to forecast, and/or there is uncertainty as to what the causal variables are, doubts about parameter values would seem very likely to arise.

The decision maker may expect that the unanticipated (that is, shocks) will occur from time to time. It is not conceptualised as a specific event, much less as a particular impact on parameter values. An example of such expectation occurs in the aircraft manufacturing industry. Companies such as Boeing and McDonnell-Douglas create financial reserves, of necessarily *ad hoc* size, for what are described as 'unk-unks' (for 'unknown unknowns') which might be encountered in the process of developing new aeroplanes (Newhouse 1982).

The second component of belief will be called *chance belief*. It is the probability of occurrence the decision maker attaches to each member of the set of anticipated parameter values. Such probabilities are referred to as prior probabilities in decision theory (Anderson et al. 1977).

The third component relates to beliefs about the stability of the probabilities comprising 'chance belief'. It is the nature of belief about the possibility that probabilities may change significantly and unpredictably over a relevant period of time, given the lead time between decision and outcome. This aspect of belief will be called *stability belief*.²

'Stability belief' and 'set belief' can be described as belief about the quality of 'chance belief'. Both 'stability belief' and 'set belief' may be sources of what has been defined as *ambiguity*: less than absolute confidence in one's estimate of the probabilities that parameter values will eventuate (Ellsberg 1961, p. 657).³ It is possible that a decision maker might have doubts about a set of prior probabilities which he has specified. The important question to be considered is whether felt 'ambiguity' is a result of poor probability elicitation procedures, or is information which cannot be captured in prior probability distributions. If 'ambiguity' cannot be captured in properly elicited prior probability distributions, relevant information which the decision maker has will be ignored, with the attendant possibility that the decision analysis will mislead the decision maker. The relevant information which may be ignored can be described as belief about the quality of one's beliefs (see Ellsberg 1961, p. 659).

² There is some overlap in the concepts of 'set' and 'stability' belief; a parameter value which is attributed zero probability of occurrence but is believed to have unstable probabilities may arouse doubts of both a 'set' and 'stability' kind.

³ 'Ambiguity' is used also to refer to uncertainty about future preferences (March 1978).

The Representation of Belief

It has been argued previously that 'ambiguity' (actually only the 'set belief' aspect) can, and should, be eliminated by persistent re-elicitation of probabilities until such time as the felt 'ambiguity' disappears (Raiffa 1968). That is, re-elicitation procedures invite the decision maker to consider anticipating a larger number of parameter values. Assuming, for the present, that such an approach will work, it is pertinent to contemplate the possible consequences of what can be described as the measurement or specification of 'chance belief' and 'set belief' by a single probability distribution.

The first thing to note is that the attempt to include in the set of possible parameter values all those which the decision maker suspects could conceivably eventuate may lead to a near arbitrary allocation of probabilities to some of them. This is because the probabilities are functioning as signals of the conceivability of parameter values. Given the possible sources of low 'set belief' considered above, it is difficult to believe that the probabilities would indicate degree of belief in the same sense as probabilities attached to parameter values identified in the original 'chance belief' distribution. That is, whatever the probabilities do reflect, it is unlikely that it is the probability of occurrence the decision maker perceives. However, this is what they are assumed to specify in decision theory.

As an example, an individual who has doubts about his ability to forecast may have difficulty identifying the range of conceivable values of some price parameter relevant to a decision. Where he can identify it, however, recommended methods for probability elicitation (Spetzler and Staël von Holstein 1975; Anderson et al. 1977) can be expected to lead to a probability distribution being identified for the full range of conceivable prices. The presence of low 'set belief' means that some of the parameter values will be in the distribution solely because the individual lacks confidence in his ability to anticipate all possible values. That is, some sub-range will be his expected range, in the normal sense, but his 'set belief' doubts will lead to elicitation procedures locating, and attaching probabilities to, all prices he believes to be conceivable.

For the above combination of 'beliefs' to be logically satisfactory, it must be possible to accept as appropriate the treatment of distributions, such as the above, as equivalent summaries of belief to identical distributions elicited from an individual with very high 'set belief'. Intuitively, it would seem appropriate to treat them as equivalent information in some contexts and not others. For example, where a decision must be made and its consequences are irrevocable, there would appear to be no virtue in treating the distributions differently in any way. However, where a decision forms part of a series of decisions, or does not have irrevocable consequences, it would seem appropriate for the less confident individual to contemplate ways of reducing the importance of the decision, or of enhancing his flexibility so as to be able to change the decision if expectations change. That is, a response would be appropriate which would precede the application of decision theory to the decision problem.

It is not apparent, however, that persistent re-elicitation of probabilities would achieve the removal of low 'set belief'. It may simply move doubt from 'set belief' to 'stability belief'. That is, all conceivable parameter values are identified, and 'set belief' is high, but the pro-

babilities associated with each of them are now perceived to be highly unstable; 'stability belief' is low.

If 'ambiguity' in the form of low 'stability belief' is to be an input to an application of decision theory, it must be susceptible to capture in the ('chance belief') probability distribution. Low 'stability belief' can be described as a perception by the decision maker that he is a poor forecaster.

The way low 'stability belief' is accommodated in decision theory is by the attachment of increasingly large probabilities to possible parameter values until the decision maker is satisfied that the possibility of each value is appropriately captured. That is, the prior probability distribution will tend to a rectangular distribution. The appropriateness of this effect has to be considered with regard to the fact that a predictably unstable parameter will also tend to be reflected in a rectangular probability distribution over some range of values it might take.

Thus, for example, a farmer may perceive output price to be predictably unstable, and attribute a rectangular probability distribution to some range of possible prices. This distribution would not be accompanied by low 'stability belief'; the farmer is content that his beliefs are well summarised by the distribution. On the other hand, he may feel that rainfall is unpredictably unstable. For unaided decision making he uses a probability distribution which happens to be similar to the relative frequency distribution of various rainfall values over the past twenty years. He believes that his rainfall forecasts are fairly reliable most of the time; the mode of his distribution is fairly close to actual rainfall more often than not.

The farmer is aware and concerned that from time to time his rainfall forecasts are seriously awry, a concern which is omnipresent with him. This awareness constitutes low 'stability belief' and is pertinent to decision-making. With the application of decision theory, the incorporation of this low 'stability belief' can only be achieved by the attachment of higher probabilities to levels of rainfall which have low (approximate) relative frequency. That is, his 'chance belief' distribution moves toward a rectangular distribution to accommodate low 'stability belief'.

Over a series of decisions 'aided' by decision theory, the farmer finds his perception of price instability is confirmed, as is his perception of rainfall instability. His price-related decisions over time prove to be prudent. His rainfall-related decisions prove to be excessively cautious most of the time (reflecting the fact that drought and flood possibilities associate with high disutility), and inappropriate for the occasional season when drought or flood occur. It would seem inappropriate to mix the farmer's 'chance belief' and 'stability belief' in a single probability distribution if this is the nature of its effects on his decision outcomes.

One way of measuring 'stability belief' would be to identify 'chance belief' measures (that is, probabilities) for each range of possible probabilities which alternative parameter values might attract. Apart from the fact that such 'probabilities of probabilities' are unlikely to be very meaningful because of measurement difficulties, such an approach initiates an infinite regress (Menz 1976). That is, there is no reason in logic to curtail the process of generating probabilities of higher and higher order, and, by implication, no advantage gained by initiating the process; there is no gain in information (in this case, about the nature of 'stability belief').

The types of belief outlined above seem to be meaningful and distinct from one another. The disaggregation of belief about an uncertain parameter into these types seems to bring some useful structure to the consideration of 'ambiguity'. It appears that if 'set' or 'stability belief' is low, the accommodation of such belief in a probability distribution is neither an obviously possible nor appropriate treatment of these kinds of belief.

An important question to be considered is whether 'ambiguity' actually matters for a given decision. One means of exploring the implications, in terms of decision maker utility, of 'ambiguity' is to examine the sensitivity of action choice, and hence expected outcome, to alternative conceivable sets of parameter values and prior probability distributions (Isaacs 1963; Fishburn, Murphy and Isaacs 1968). Sensitivity analysis can make a significant contribution if it is found that a decision-maker's 'ambiguity' has no substantial effect; the 'ambiguity' is irrelevant.

Should sensitivity analysis indicate substantial differences in preferred action choice according to the particular conceivable future values considered, such analysis does nothing to alleviate the problems 'ambiguity' poses for decision theory as a decision aid. It merely indicates the desirability of seeking more information (Isaacs 1963). If, after all avenues of information search have been exhausted, 'ambiguity' persists the problem remains as to how it should be accommodated in decision-making.

Probability Revision

The assimilation by a decision maker of forecast information is a matter dealt with explicitly in decision theory. Decision theorists argue (e.g. Anderson et al. 1977) that rational (and full) use of such information can be achieved by the application of Bayes's Theorem. The existence of 'ambiguity', however, would imply some doubt that this is so.

Bayes's Theorem models the revision of prior probabilities, in the light of forecast information, on the assumption that prior probabilities are perceived without 'ambiguity' by the decision maker. The revised probabilities are referred to as posterior probabilities. If 'ambiguity' exists this model leads to spurious posterior probabilities.

Taking an example of a decision maker acquiring forecasts from sources he regards as credible (that is, more informed than he is), it seems reasonable to expect that the information extracted from the forecasts would reflect both the forecast itself and the credibility of the forecaster. Bayes's Theorem is consistent with this expectation.

Under Bayes's Theorem the decision-maker's confidence in his 'chance beliefs' is assumed to be indicated by the shape of his prior probability distribution. Consequently, ' . . . the higher [this] confidence the more the posterior probability distribution will resemble the prior for any given weight of evidence' (Hirshleifer and Riley 1979, p. 1394). If some measure of lack of confidence, such as 'ambiguity', is not captured by the diffuseness of prior probabilities, it is not possible under Bayes's Theorem for this lack of confidence to affect the amount of information extracted from new evidence.

If the decision maker lacks complete confidence in his prior probabilities, it seems reasonable to expect that, in the process of

assimilating forecast information, prior probabilities may be discounted according to the confidence held in them (that is, the credibility the decision maker attributes to himself as a forecaster).

Thus a decision maker to some extent may, in effect, *substitute* credible forecasts, discounted by a credibility factor (measured as 'likelihood' probabilities), for his own. This is not a response consistent with Bayes's Theorem which, rather, adjusts prior probabilities with no discounting according to the credibility attached to them, since it is assumed that all relevant belief (and doubt) is captured in the prior probability distribution.

If decision makers do discount their prior probabilities, their posterior probabilities can appear 'conservative' (that is, more similar to their prior probabilities) relative to the posterior probabilities which the application of Bayes's Theorem would generate. This is because the discounting of prior probabilities means that the probabilities attached to alternative possible parameter values move toward equivalents. For example, where there are two possible parameter values, the prior probabilities would tend toward $p = 0.5$ for each of them.

Conservatism in probability revision is typically regarded as a result of excessive discounting of the information content of forecast information (see Winkler and Murphy 1974). The explanation above indicates that so-called conservatism may not reflect excessive discounting of forecasts, nor the irrationality which that implies, at all. Rather, apparent conservatism may be a seemingly rational discounting of prior probabilities in the context of more credible forecasts being obtained.⁴

There appears to be another phenomenon which indicates the existence of some element of uncertainty beyond that captured by prior probability distributions. Assume that an individual has 'chance belief' that a discrete event (rain tomorrow) has probability equal to 0.7, and the alternative event (no rain tomorrow) has probability of 0.3. Assume also that he interacts socially with some folk and, for lack of a more interesting conversation, asks several of them whether they think it will rain tomorrow. Each of the people he quizzes he believes to be no better and no worse at forecasting rain than himself. Each person replies that the probability of rain in their view is 0.7.

One might expect that the effect of these replies is to confirm his 'chance belief' that 0.7 is the best estimate. Under decision theory this effect cannot occur; the replies do not constitute information. Thus, predictably, the application of Bayes's Theorem leads to spurious results. If the effect of the replies is confirmation of 'chance belief', logical revision of prior probabilities should yield posterior probabilities identical to the prior probabilities. The only way this can occur using Bayes's Theorem is if the likelihood probabilities are 0.5 for each of the forecast messages.⁵

Likelihood probabilities of 0.5 imply that the enquirer's forecasting reliability is also 50/50, and that *it cannot be otherwise* if additional information from people is to have the effect of simply reinforcing 'chance belief'. This unlikely result can be partly avoided if one posits the

⁴ It should be noted that there are several other explanations of conservatism in probability revision (de Zeeuw and Wagenaar 1974).

⁵ See Gettys and Willke (1969) for a 'Modified Bayes's Theorem' algorithm which can accommodate probabilistic forecasts.

existence of a vector of possible prior probability distributions and a further (second order) distribution over that vector. By appealing to the existence of n -order distributions, the problem can be avoided altogether; the forecasts cause the n th-order distribution(s) to change while the prior probabilities vector does not. In effect, the confirming influence of the additional information reduces 'ambiguity' associated with 'chance belief'.

It appears that evidence for the existence of 'ambiguity' can be detected in problems associated with the application of Bayes's Theorem. Apparent conservatism in probability revision may be explained by 'ambiguity', and 'ambiguity' needs to exist if confirmation of expectations is to be regarded as generally possible.

Implications for Decision Theory Applications

If one accepts the assumption that a single probability distribution captures all belief relevant to an uncertain parameter, decision theory is an appropriate technique in all decision situations.

If, however, one doubts this assumption, the question arises as to how one should use the information 'ambiguity' reflects if one is seeking to assist a decision maker. It would seem highly desirable to retain the 'benefits of structure and consistency afforded by decision analysis' (Watson, Weiss and Donnell 1979, p. 1), but not to ignore pertinent dimensions of belief in the strict sense of the dimensions of belief presented earlier in this paper. Two options seem to exist for the achievement of both these objectives.

Fuzzy decision analysis (Roy 1977 and Watson et al. 1979) is a variant of decision theory which can accommodate fuzzy (that is imprecise) inputs (including both probabilities and utility). This technique has some potential to use low 'stability belief' and, to the extent that low 'set belief' can be transformed into low 'stability belief' as described above, low 'set belief'.

The output of fuzzy decision analysis is also fuzzy. The usefulness of logically derived but fuzzy output will vary across decision makers and decision problems. When instability is of major dimensions, and inputs very fuzzy, the technique may not be particularly useful.

Another option is to design into procedures for the application of decision theory, guides for the structuring of decision problems in such a way that 'ambiguity' and 'chance belief' can be dealt with separately. A major benefit of this option is that one can avoid the 'contamination' of 'chance belief' which occurs implicitly by the incorporation (if this is in fact possible) of 'ambiguity' in probability distributions. The aircraft manufacturing example given earlier is pertinent here. By creating a reserve for 'unk-unks', they can be ignored instead of allowing, indeed forcing, them into consideration for decision problems which may be otherwise well specified.

Separation of the attention given 'chance belief' and the other 'beliefs' implies accommodating them in different decisions. The coexistence of the different 'beliefs' would indicate that any decision based solely on 'chance belief' should be made in a context of other decisions which limit the consequences of wrong choice. That is, decisions are required which deal with low 'set belief' or low 'stability belief' by mitigating the conse-

quences of 'chance belief' leading to decisions with outcomes which are affected by unanticipated parameter values.

This approach can be described as weakening the reliance of decision outcome performance on expectations ('chance belief') felt to be of unreliable quality.

The major implications for the application of decision theory in situations such as the above are: that procedures would need to be developed which identify the presence of 'ambiguity'; that the sensitivity to 'ambiguity' of a decision, derived using current approaches, should be assessed; that, if the decision is found to be sensitive, the decision problem be decomposed such that the various beliefs are dealt with as described above; and techniques for the elicitation of probabilities would need to be adjusted, to the extent possible, to prevent any incorporation of 'ambiguity' effects in 'chance belief'.

If it proves possible to adopt such an approach when 'ambiguity' exists then the benefits of applying decision theory can be preserved (for 'chance belief'). If not, and if one rejects the assumption that all relevant beliefs are captured in a single distribution, the application of decision theory may lead to irrational action, given that relevant information would be ignored (Day 1979, p. 116).

The decisions made to deal explicitly with 'ambiguity' would be essentially strategic (Thompson and Strickland 1978) in that they would probably span a series of 'chance belief' decisions. Given the nature of information (or 'anti-information') that 'ambiguity' represents, it seems unlikely that decision theory would be a useful aid to decisions dealing with it. It is beyond the scope of this paper to consider alternative techniques which might be useful for such decisions. Starr (1971) has attempted an analysis of the appropriateness of techniques for decision problems with various characteristics in terms of 'beliefs'.

Conclusions

The possible exclusion of aspects of perception of uncertainty from elicited probability distributions has been argued to be a substantive issue (Savage 1954; Georgescu-Roegen 1958; and Ellsberg 1961). It is the issue which rests at the centre of the argument that risk and uncertainty are distinct concepts (Knight 1933 and Simon 1977).

A probability distribution is a construct assumed, in decision theory, to capture all perceived uncertainty relevant to some parameter. The unfalsifiability of decision theory makes validation of the construct extremely difficult, and this implies that the construct has a theoretical role akin to that of an axiom. The acceptability or otherwise, at an intuitive level, of the assumption in question can, therefore, be argued to be as important as the acceptability of the more familiar axioms underlying decision theory.

In this paper an attempt has been made to identify and structure the elements of the basic debate about this construct, and to consider the implications for the application of decision theory of rejection of the validity of the construct. The implications are significant, particularly in contexts, such as farm management, where 'ambiguity' may be expected to be common. Generally they indicate a requirement to structure (decompose) a decision problem so that the use both of relevant perceptions of uncertainty and of decision theory can be maximised.

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