OPTIMIZING THE TRANSFER OF TAX EXEMPTIONS IN NEW ZEALAND FARM DEVELOPMENT*

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Farmers and their advisers frequently face complex problems involving the scheduling of claims for tax exemptions. A case in point is the New Zealand tax law allowing farmers to transfer exemptions associated with development expenditure up to specified time horizons. This paper examines procedures for solving this, and similar, problems. Linear programming could be used, but an alternative algorithm is proposed for farm advisers whose access to computing facilities is limited. The latter procedure is intended for use with forecast budgeting in farm development planning.

Introduction

One of the more vexing classes of problems faced by farmers and their professional advisers relates to the adjustment of management policies so that advantage can be taken of available taxation concessions. Particularly complex problems often exist when a single decision will influence a farmer's tax payments in several subsequent years.

A problem of this nature arises when income laws allow farmers to specify the year in which they will charge certain classes of deductible expenditure against taxable income. When applied to a progressive tax system such laws provide farmers with the opportunity to reduce long-run tax payments by transferring exemptions from years when the marginal tax rate is low (as a result of taxable income being low) to years when the marginal tax rate is higher. Note, however, that while increasing marginal tax rates *increase* the profitability of delaying claims on exemptions, the effect of discounting tax savings (when this is specified by a farmer's objective function) *reduces* the profitability of delay. The resultant of these opposing influences will obviously depend upon the relevant discount rates, time itself, and the rates of change in marginal tax rates through time.

This paper discusses procedures for optimizing the pattern of future exemption transfers given the objective that the present value of future tax payments should be minimized.[1] Attention is focused on a specific problem that has considerable relevance to New Zealand farmers, but the suggested solution procedures should, with slight modification, be applicable generally to similar problems elsewhere.

The following section contains a statement of the example problem,

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[1] This objective is compatible with the use of 'present value of profits from development' as a measure of the profitability of farm development projects.

while methods for its solution are considered in Section 2, and Section 3 briefly reviews some computational experience with the problem.

Section 1—The Problem

New Zealand income tax legislation allows some items of farm development expenditure to be claimed as exemptions in either the year they are incurred or in a future year. The majority of items[2] may be claimed in any one of the six years defined by the year of expenditure and the subsequent five years. Exemptions associated with expenditure on fertilizer are, however, treated differently; these claims can be made in the year of incurrence or in any of the subsequent *four* years.

Although the intention to actually claim exemptions need not be declared until the year in which the claim is made, it is clearly necessary to determine a particular pattern of exemption transfers when preparing forecast budgets for a farm development programme. Similarly, if linear programmes are used in development planning they must, in order to reflect farmers' goals, include appropriate taxation activities and restraints: the interaction between tax payments and the availability of working capital may be crucial to the specification of an optimum development plan.

More formally, the problem is: find non-negative numbers c_1, c_2, $\ldots$, c_n, such that:

$$(1)\ PV_t = \sum_{i=1}^{n} f(t_i - c_i)d^i \quad min,$$

subject to

$$(2)\ \sum_{i=j}^{j+5} p_{ij} \leqslant u_j; \quad j = 1, 2, \ldots, n-5$$

and

$$(3)\ \sum_{i=j}^{j+4} q_{ij} \leqslant v_j; \quad j = 1, 2, \ldots, n-4$$

and

$$(4)\ c_i = \sum_{j=i-5}^{i} p_j + \sum_{j=i-4}^{i} q_{ij}; \ i = 1, 2, \ldots, n; \ p_{ij}, q_{ij} = 0 \text{ if } j < 1$$

and

$$(5)\ u_j \geqslant 0; \quad j = 1, 2, \ldots, n-5$$

and

$$(6)\ v_j \geqslant 0; \quad j = 1, 2, \ldots, n-4$$

and

$$(7)\ d = \frac{1}{1+r},$$

[2] These items, which are referred to in the main text as 'general development expenditure', include expenditure associated with clearing land, establishing and renovating pastures, drainage, soil erosion control, weed and pest eradication, farm roading, farm water supplies, airstrip construction, and fencing. A list of these items appears in: *'Farmers' Tax Guide'*, New Zealand Inland Revenue Department Publication I.R.290, p. 20, Government Printer, Wellington, New Zealand, 1967.

where

$u_j; j = 1, 2, \ldots, n - 5$, is the general development expenditure incurred in year j that is claimable as a tax exemption in any of the years $j, j + 1, j + 2, j + 3, j + 4, j + 5$,

$v_j; j = 1, 2, \ldots, n - 4$, is the fertilizer expenditure incurred in the year j that is claimable as a tax exemption in any of the years $j, j + 1, j + 2, j + 3, j + 4$,

p_{ij} is the exemption claim made in the year i from transferable general development expenditure incurred in year j,

q_{ij} is the exemption claim made in the year i from fertilizer expenditure incurred in year j,

$c_i; i = 1, 2, \ldots, n$, is the amount of transferable exemption that is claimed in the year i,

$t_i; i = 1, 2, \ldots, n$, is the taxable income in year i if $c_i = 0$,

$f(t_i - c_i)$ is the function[3] expressing tax liability in the year i in terms of taxable income $(t_i - c_i)$,

d is the discount rate, which is here assumed constant over the n-year period,

r is the interest rate expressed as a decimal, and

PV_t is the present value of tax paid to the end of the nth year.

Section 2—Solution Procedures

If equation (1) either represents an ascending step function or can be approximated by such a function, the example problem may be easily respecified in linear programming form. An illustrative tableau appears in Table 1. In this example an extremely simple tax function is assumed; the marginal tax rate is \$$a$ per \$ for taxable incomes less than or equal to \$$m_a$, and \$$b$ per \$ for any part of taxable income exceeding \$$m_a$. Two 'pay tax' activities appear for each year; for example, T_i^a is 'pay tax at marginal rate \$$a$ in year i' All other symbols representing activities, restraints, and prices are defined in Section 1 above. A four-year planning horizon is assumed (i.e., $n = 4$). This example suggests that linear programming representations of exemption transfer problems could tend to be extremely unwieldy, especially under more realistic assumptions than those made in Table 1.[4] Thus linear programming is an obvious method for solving problems whose size is compatible with available computer capacity, but if suitable equipment is unavailable, recourse must be made to sub-optimizing procedures.[5] The most obvious of these would use linear programming to solve a problem in which the tax function is *approximated* by a step function utilizing a reduced

[3] In practice the taxation function is linear for taxable incomes less than \$1,000 and greater than \$7,200, but comprises thirty-one steps for taxable incomes between \$1,000 and \$7,200.

[4] For example, if the New Zealand income tax function, which contains thirty-one steps, were completely represented in a case where $n = 8$ (a realistic time period), the resulting linear programming problem set up in the manner of Table 1 would have 264 restraints and 304 activities!

[5] Indeed, even if computing facilities would allow complete linear programming specification of a problem, the lower computing costs of the sub-optimizing procedures discussed below may make the latter more profitable propositions to the farmer.

TABLE 1

Illustrative Linear Programming Tableau

$c_j \rightarrow$	ad	bd	0	0	ad^2	bd^2	0	0	0	0	ad^3	bd^3	0	0	0	0	0	0	ad^4	bd^4	0	0	0	0	0	0	0	0
$c_i \downarrow$	T_1^a	T_1^b	p_{11}	q_{11}	T_2^a	T_2^b	p_{21}	p_{22}	q_{21}	q_{22}	T_3^a	T_3^b	p_{31}	p_{32}	p_{33}	q_{31}	q_{32}	q_{33}	T_4^a	T_4^b	p_{41}	p_{42}	p_{43}	p_{44}	q_{41}	q_{42}	q_{43}	q_{44}
$0\ -t_1\ =$	1	1	1	1																								
$0\ m_a\ \geqslant$	1																											
$0\ u_1\ \geqslant$			1				1						1								1							
$0\ v_1\ \geqslant$				1					1							1									1			
$0\ -t_2\ =$					1	1	1	1	1	1																		
$0\ m_a\ \geqslant$					1																							
$0\ u_2\ \geqslant$								1						1								1						
$0\ v_2\ \geqslant$										1							1									1		
$0\ -t_3\ =$											1	1	1	1	1	1	1	1										
$0\ m_a\ \geqslant$											1																	
$0\ u_3\ \geqslant$															1								1					
$0\ v_3\ \geqslant$																		1									1	
$0\ -t_4\ =$																			1	1	1	1	1	1	1	1	1	1
$0\ m_a\ \geqslant$																			1									
$0\ u_4\ \geqslant$																								1				
$0\ v_4\ \geqslant$																												1
PV_t *min*																												

number of steps. A potential drawback to this procedure is that there may be no obvious 'correct' way to simplify the function. Of course if the development planning itself makes use of linear programming, the exemption transfer problem must be treated as an integral part of the linear programme, and the shortcomings associated with any necessary simplification of the tax function must be accepted.

It is noteworthy that many farm management advisers continue, for a variety of reasons, to eschew linear programming as a development planning tool, relying instead on time-honoured forecast budgeting. The algorithm described below provides an alternative method for obtaining approximate solutions to the exemption transfer problem associated with forecast budgeting. Since the algorithm has relatively low requirements for computer storage capacity,[6] it may be more appropriate than the linear programming approach when management advisers have access to only small computers.

As inferred above, a series of forecast budgets is required input to the algorithm. Of course we can confidently claim that the exemption transfer patterns derived by the algorithm are near-optimal only for the particular production coefficients and prices specified in the budgets. In practice, it would be necessary to review the future pattern of exemption transfers at the end of each year in the light of new information concerning anticipated future production and prices that became available during the previous twelve months. That is, the algorithm promises to be of greatest use in assisting optimal allocation of developmental exemptions between the year just completed, for which taxable income (in the absence of transfers) is known, and future years, for which expected taxable incomes have been estimated.

The algorithm will be discussed with the aid of Figure 1. The ordinate in the figure represents PV_t, the present value of tax paid to the end of the nth year, while the abscissa is scaled in terms of λ, a *small* unit[7] of transferable exemption.

The algorithm is initiated by setting

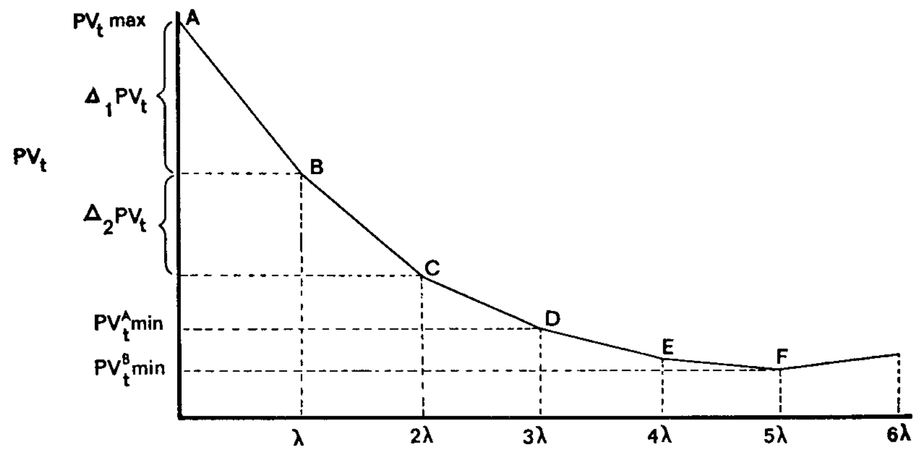
(8) $c_i = u_j + v_j; \; i = j = 1, 2, \ldots, n-5,$
(9) $c_i = v_j; \quad i = j = n-4,$
(10) $c_i = 0; \quad i = n-3, n-2, n-1, n.$

That is, *all* of the potentially transferable tax exemptions are temporarily claimed in the earliest possible year; no exemptions are transferred forward. Under these conditions, PV_t is clearly at a maximum for the particular discount rate chosen. Equations (8), (9), and (10) define point A in Figure 1.

An iterative procedure is now established. In the first iteration, attention is focused on identifying the pair of years between which a feasible transfer, λ, of transferable exemption (from *either the* u_j's or the v_j's), would reduce PV_t by the *greatest* amount. When the pair of years has been identified, the corresponding transfer is made permanent, com-

[6] Computers comparable to the IBM 1620 with 20K should be adequate for virtually all operational problems.

[7] To be strictly accurate, the optimal solution to the problem stated by equations (1) to (7) inclusive will be approached only when λ approaches zero. For practical purposes, however, acceptable near-optimal solutions can be obtained by setting λ equal to convenient small accounting units, such as \$1.



Total Exemptions Transferred.

FIG. 1—Diagrammatic Representation of the 'Efficient Path'.

pleting the first iteration. If the resulting reduction in PV_t is written as the positive quantity $\Delta_1 PV_t$, the objective of the first iteration can be restated as locating the pair of years between which a transfer, λ, maximizes $\Delta_1 PV_t/\lambda$. Thus the situation at the end of the first iteration is represented by point B in Figure 1. Moreover, the straight line AB has the steepest possible slope.

In the second iteration the transfer of a *second unit*, λ, of spreadable exemption, is considered. As in the first iteration, the permanent transfer made gives the maximum feasible reduction in PV_t. That is, $\Delta_2 PV_t/\lambda$ is maximized when selecting the pair of years between which the permanent transfer is made. Notice, however, that in $\Delta_2 PV_t \leqslant \Delta_1 PV_t$, since if the reduction in PV_t in the first iteration was a maximum, the reduction from the second iteration can be *at most* equal to it. In Figure 1, the slope of BC must be equal to or less steep than (i.e. less negative than) the slope AB.

These and subsequent iterations clearly result in the *maximum* rate of approach to the optimal solution. More importantly, adherence to the 'efficient path', represented by the piecewise linear line ABCDEF in Figure 1, is necessary to ensure that the optimal solution is in fact obtained. If any iteration results in a permanent transfer giving a reduction in PV_t less than the maximum feasible reduction for that iteration, it is possible that another transfer, which would have resulted in a greater reduction in PV_t, is automatically rendered infeasible.[8]

Provided that PV_t is reduced by movement along the 'efficient path', the optimal solution will be obtained when either of the conditions *A* or *B* below, are met:

Condition A: No further feasible transfer of transferable exemption can

[8] For example, suppose that in some problem the only transferable exemption remaining is \$1 that can be spread from the year i. Further, suppose that if the \$1 is claimed in the year $(i + 1)$ a reduction of 10c in PV_t will result, whereas the only alternative transfer, to the year $(i + 3)$, would result in PV_t being reduced by 20c. Clearly, if a permanent transfer is made from the year i to the year $(i + 1)$ the 'solution' obtained will specify PV_t, 10c higher than the alternative optimal solution. (It is assumed that all previous iterations followed the 'efficient path'.)

be made[9] although if further exemptions were available, PV_t could be reduced. In terms of Figure 1, suppose that only 4λ of spreadable exemption is available. The optimal solution is obtained when $PV_t = PV_t^A$ min.

Condition B: Further feasible transfers of spreadable income are available, but none would reduce [10] PV_t. In Figure 1, this condition is represented by point F where $PV_t = PV_t^B$ min. Notice that the straight line FG must have either zero or positive slope, illustrating that PV_t can only remain unchanged or be increased by a further transfer of magnitude λ.

Thus the algorithm terminates whenever condition A or condition B is achieved.[11]

The search procedure in each iteration is now described in greater detail.

Step 1

Examine in turn the effect on PV_t of transferring λ from *each* $u_j \geqslant \lambda$; $j = 1, 2, \ldots, n - 5$. The procedure is:

(i) Locate a year j for which $u_j \geqslant \lambda$.

(ii) Evaluate

(11) $PV_{tj} = f(t_j - c_j)d^j$, and

(12) $PV_{t_{j+k}} = f(t_{j+k} - c_{j+k})d^{j+k}$

where PV_{t_j} and $PV_{t_{j+k}}$ are the present values of tax payable in the jth and $(j + k)$th years respectively.

(iii) Now make *trial* transfers of spreadable exemption such that

(13) $c_j' = c_j - \lambda$, and

(14) $c_{j+k}' = c_{j+k} + \lambda$.

(iv) Hence obtain

(15) $PV_{t'_j} = f(t_j - c_j')d^j$, and

(16) $PV_{t'_{j+k}} = f(t_{j+k} - c_{j+k}')d^{j+k}$

(v) Since a transfer from u_j can be accepted by *any one* of c_{j+k}; $k = 1$, 2, 3, 4, 5, it is clear that the *five* possible changes in PV_t are given by

(17) $b_{j+k} = (PV_{t_{j+k}} - PV_{t'_{j+k}}) - (PV_{t'_j} - PV_{t_j})$; $k = 1, 2, 3, 4, 5,$

[9] Strictly speaking, condition A implies that no transfer of transferable exemption *of magnitude* λ *or larger* can be made. See Footnote 7.

[10] In the computations, condition B is reached when no transfer of magnitude λ would reduce PV_t. It is possible that *smaller* transfers *would* reduce PV_t. See Footnote 7.

[11] It is worth noticing that the algorithm could be applied to a related problem, similar to the one described by (1) to (7) except that the *total* transferable exemption would be arbitrarily determined, or even sold, by the Government. The farmer would then be free to allocate his total exemption, subject to (2), (3), (4), (5) and (6). Under these conditions, the 'efficient path' generated by the algorithm would define *successive optimal solutions* as the total exemption was parametrically increased. Moreover, if the rights to spread exemptions were sold by Government, the algorithm would indicate the *shadow prices* of marginal increments to exemption rights. Hence $\Delta_2 PV_t$ in Figure 1 could be interpreted as the shadow price of the second increment of size λ.

where b_{j+k} is the change in PV_t that would result from a transfer of λ from the jth year to the $(j+k)$th year. PV_t would be *reduced* when $b_{j+k} > 0$.

(vi) Thus the *maximum* reduction in PV_t provided by a transfer of λ from the jth year is

(18) $B_j = \max_k (b_{j+k});\ k = 1, 2, 3, 4, 5.$

Define $B_j = 0$ if $b_{j+k} \leqslant 0$; $k =$ for all k. If $B_j = 0$, proceed to (viii) otherwise denote the pair of years between which a transfer λ yields B_j by j and $(j + k^*)$.

(vii) Record B_j, j, and $(j + k^*)$, and *reset* all c_i; $i = 1, 2, \ldots, n$, and u_j; $j = 1, 2, \ldots, n - 5$, to their values *prior* to the trial transfer.

(viii) Locate some other year j for which $u_j \geqslant \lambda$ and return to (ii). If trial transfers have been made from all years satisfying $u_j \geqslant \lambda$, proceed to (ix).

(ix) Locate

(19) $M_u = \max\limits_{\substack{j \\ \text{where } u_j \geqslant \lambda}} B_j;\ j = 1, 2, \ldots, n - 5$

Step 2

Examine in turn the effect on PV_t of transferring λ from *each* $v_j \geqslant \lambda$; $j = 1, 2, \ldots, n - 4$. The procedure is identical to Step 1 except for modifications to take account of the more restricted range of spread for the v_j's. Hence eventually locate

(20) $M_v = \max\limits_{\substack{j \\ \text{where } v_j \geqslant \lambda}} B_j;\ j = 1, 2, \ldots, n - 4$

Step 3

Find

(21) $M = \max(M_u, M_v)$

If $M = 0$, condition B is satisfied and the required solution has been found, but if $M > 0$, denote the year associated with M by j^*. A transfer, λ of transferable exemption, will give a *maximum* reduction in PV_t if the transfer is made from year j^* to year $(j^* + k^*)$. Hence, make the following permanent changes:

If $M = M_u$, (i) reduce u_{j^*} by λ,
but if $M = M_v$,
reduce v_{j^*} by λ,
then (ii) reduce c_{j^*} by λ,
and (iii) increase $c_{j^*+k^*}$ by λ.

Section 3—Computational Experience

The algorithm described in the last section has been programmed in FORTRAN for an IBM 1620 computer, and some limited experience has been obtained with it. It was quickly found that with λ set at \$1 the time required to obtain a solution was unacceptably long. Consequently, the algorithm was modified, at some sacrifice to the ultimate reduction in the present value of tax, by initially setting λ at \$100. When no further profitable transfers of \$100 were available (that is, when condition A or condition B was reached), λ was reduced to \$10, and finally to \$1. In most cases, this modification approximately halved computing time[12] without significantly changing the present value of tax saved. The latter result suggested relative insensitivity to the magnitude of λ, which in turn implies that the solutions may closely approach the true optima.

Table 2 summarizes the savings in present value of tax made in five *ex post* analyses of case farm development programmes.[13] An interest rate of 6% was assumed in deriving present values. The 'rule of thumb' method is representative of procedures currently used by farm accountants and was based on the convention that taxable income should not fall below the sum of personal and family exemptions and life insurance premium exemptions (that is, \$1,860). A taxable income below \$1,860 implies that income-tax exemptions are being 'wasted', although social security tax is payable on *all* taxable incomes in excess of \$208.[14]

TABLE 2

Performances of Alternative Methods for Transferring Tax Exemptions

Case Number	Present Value of Future Tax (\$)			Proportion Saved by 'Optimizing' Algorithm	Proportion Saved by 'Rule of Thumb' Method
	No Transfers	'Optimizing' Algorithm	'Rule of Thumb' Method		
				%	%
I	1,356	788	834	41·9	38·5
II	8,610	6,596	7,064	23·4	18·0
III	43,412	38,472	43,320	11·4	0·2
IV	35,560	28,520	35,422	19·8	0·4
V	9,632	8,854	9,192	8·1	4·6

The use of the algorithm in *ex post* 'once and for all' computations of exemption transfer patterns does not, of course, coincide exactly with its suggested application in annual reviews of development planning. Nevertheless the results shown in Table 2 have several interesting implications. First, they demonstrate that it should be highly profitable for

[12] Computing time was also shortened by the simplification of regarding all spreadable exemptions as the 'u_j' type.

[13] All tax computations assumed that income accrued to an individual farm owner, married with two children, and paying \$300 per year tax-exempt life insurance premiums. Personal and family exemptions from tax thus totalled \$1,860. For details of the development programmes, see R. W. Cartwright, 'The Potential for Increased Production on Sheep Farms in Wairoa County', unpublished M. Agr. Sc. thesis, Massey University, New Zealand, 1967.

[14] Both methods also took full advantage (after transferring exemptions) of the separate legislation that allows a negative annual taxable income to be carried forward and set against the next year's taxable income.

New Zealand farmers to make use of the exemption transferring legislation, even by the 'rule of thumb' procedure. Second, the fact that the 'optimizing' algorithm consistently saved more tax than the 'rule of thumb' method, indicates that farmers may find it profitable to use the former technique. This assertion requires some qualification. Farm development of modest scale, such as Case I in Table 2[15] results in only minor changes in the marginal tax rate and annual tax payments. Under these circumstances we should expect the optimizing algorithm to be only slightly more effective than the 'rule of thumb'.[16] A similar situation may arise when larger-scale development is undertaken at a very slow pace so that changes in marginal tax rates are gradual. Case V is an example of this type of development.[17] In these circumstances, the added annual cost of using the optimizing algorithm may not justify its use. On the other hand, development characterized by rapid increases in production to substantially higher levels, such as Cases III and IV,[18] should provide considerable scope for the use of the optimizing algorithm or linear programming in deriving *ex ante* transfer patterns.

Conclusions

The selection of optimal patterns of developmental exemption transfers, and related problems, are in principle amenable to solution by linear programming. However, when computing facilities cannot accommodate problems formulated in this way alternative procedures may merit some attention. The algorithm suggested in this paper promises, under certain circumstances, to derive patterns of exemption spreading for New Zealand farmers that result in considerably lower tax liabilities than those provided by the methods currently used by farmers and their advisers. The author believes that the algorithm could be used commercially by farm management consultants, farm accountants, and others advising farmers. The FORTRAN code developed for the IBM 1620 is possibly too slow for application to farm advisory work, but a considerable reduction in computing time (and hence in cost to the farmer), could doubtless be achieved by re-coding the algorithm in machine language and/or using a faster computer.

The algorithm should be applicable to many generally similar tax-scheduling problems.

[15] This development programme extended over four years and consisted of pasture establishment and renewal while the breeding ewe flock increased from 400 to 900, and cattle numbers quadrupled to 50 head.

[16] The relatively high proportion of tax saved by *both* procedures in Case I occurred because *no* tax was paid prior to development so that, in the absence of transfers, the marginal tax rate in the first three years of development was also zero. There was thus a considerable difference between the marginal tax rates in the early years and the later years to which exemptions were transferred.

[17] The stocking rate on this farm increased by only 13% over a six-year period.

[18] Case III increased stocking rate by 140% and wool production by 310% over a twelve-year period.