A NOTE ON SOME DIFFICULTIES IN RESPONSE ANALYSIS

J. R. ANDERSON*

University of New England

Some empirical problems which tend to receive inadequate attention in response analysis are discussed. These include the variation not explained by fitted production functions, the sensitivity of optima and profits to price changes, the cost of developing production functions and the value in using them.

Introduction

While agricultural efficiency may in the past have been influenced chiefly by the discovery of new production functions (or some discrete points on them), for the future it is probable that precise knowledge of standard technical relationships will become increasingly important. The value of the production function approach has been stressed by many agricultural economists, notably Heady and Dillon [11]. Perhaps regrettably, this emphasis has implied a low level of economic importance to work that is necessary and useful but not oriented to the analysis of response surfaces.

Authors whose primary concern has been to outline the advantages of defining and using production functions have insufficiently acknowledged the accompanying difficulties. As well, the studies published generally report successful investigations and seldom indicate any of the failures or difficulties experienced. Of course, many fundamental problems do exist—such as appropriate algebraic specification [1]. The availability of computers may have simplified the task of fitting and using production functions but, as in other fields, it is no panacea. At the outset it should be noted that the problems which are considered here, namely unexplained variation, process sensitivity, development costs and the value of recommendations, are not in the concept of the production function itself, but rather in the practical details of real-world estimation and use.

Unexplained Variation

The presence of variation not explained by treatments is a feature of all biological experimentation. In this respect estimation of production functions is no exception, but many published reports contain only relationships of high statistical significance where most variation is explained by the functions fitted. Probably the most important reasons for this are:

(i) an apparent tendency to report individual experiments rather than whole experimental programmes; and

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(ii) a sieving-out of empirical results which do not meet the arbitrary and traditional levels of statistical significance demanded by editors and referees.

These phenomena are not unique to the literature of agricultural production functions. However, this situation, combined with the zeal with which economists have urged technical scientists to use production functions, may have given a false impression that success in estimation directly follows from the use of a few simple principles and rules of thumb.

A review of agronomic-economic research conducted in Michigan over an eight-year period provides an indication of some difficulties which may be encountered in the estimation of fertilizer response functions [12]. Many of the experiments yielded data with considerable variation that was unexplainable in regression analysis. A comparable lack of success was experienced in the initial year of an Australian investigation[1] into response of summer crops to fertilizers. Of thirteen trials on maize, peanuts, soybeans and navy beans, only four (31 per cent) yielded useful production functions[2]. The others failed because high levels of variation[3] within treatments concealed any treatment effects in measured yield, although visual responses were generally evident. Identification of uncontrolled factors causing unexplained variation, and improved control of these factors are apparently needed in such experiments. Possibly the best way of reducing unexplained variation, as well as ensuring the relevance of results for commercial farming, is to use a procedure like the "controlled-survey" technique [3, 12] with large plots located over a specified universe of agriculture. The plots used in the Queensland study referred to were of usual size—for instance, in the NPK maize trial to which attention is now confined, the inner two rows of four-row plots of one-fortieth of an acre were harvested.

Several different forms of production function were fitted to the data. These, mostly second-degree polynomial models, were diverse in statistical quality and economic implications. As is usual in work on fertilizer response, there was no compelling biological logic for including various interaction terms, or for justifying the use of various transformations of the independent variables. Up to about 70 per cent of the variation could be explained by functions which included all second-order interaction terms, although some of the coefficients were not statistically significant. Untransformed quadratic polynomials were generally superior to other polynomials such as the "square root". A quadratic polynomial without interaction—equation 1—illustrates the present discussion.

[1] An unpublished study conducted by the Research and Extension Section, A.C.F. and Shirleys Fertilizers Limited, Qld. I am grateful to my former colleagues there for assistance in all phases of the experimental work.

[2] More recently the author's faith in response analysis was somewhat restored when the Farm Managment Service Centre (University of New England) analyzed data from 131 N-P fertilizer trials on wheat conducted over a 4-year period by the Research and Development Section, Australian Fertilizers Limited, N.S.W. These trials yielded production functions which could be classified in terms of general statistical quality and economic significance as: 38% good, 39% fair, 18% poor and 5% hopeless—i.e., a success rate of about 80%.

[3] Unexplained variation arose mainly from soil variability, uneven seeding and germination and, in the bean crops, damage by insects and rodents.

$$(1) \quad Y = \underset{}{24 \cdot 67} + \underset{(2 \cdot 98)}{28 \cdot 45N} - \underset{(2 \cdot 14)}{10 \cdot 93N^2} + \underset{(2 \cdot 83)}{9 \cdot 74P} - \underset{(2 \cdot 61)}{1 \cdot 73P^2} + \underset{(2 \cdot 11)}{18 \cdot 14K} - \underset{(2 \cdot 53)}{10 \cdot 48K^2}$$

$$(\bar{R}^2 = 0 \cdot 37)$$

where Y = predicted yield of maize (bush. per acre)
N = nitrogen (lb. elemental nitrogen per acre)
P = phosphorus (cwt superphosphate per acre)
K = potassium (cwt potassium chloride per acre)

and the bracketed numbers are the respective values of 't' for the test that each coefficient is zero.

An important factor contributing to the variation not explained by equation 1 is the difference between replicates. Fitting production functions to the data of individual replicates yields different production functions which imply very different economic optima. If large differences in response between spatially separated replicates can be related, *ex post*, to definable agronomic causes, it would be most useful to regard the replicates as independent experiments yielding independent conditional response functions—rather than to develop an average function which pertains to an unknown average set of conditions. However, in this experiment, neither soil analysis nor plant density could explain the difference. Given the size of the plots used and the apparent variability of the site, it is likely that at least a further replicate should have been used.

Sensitivity to Price Changes

A key argument for the production function approach is that optimal rates of factors depend on prevailing price conditions. However, the value of the approach will depend on the sensitivity of optima and profits to variations in prices. For many fertilizer processes the product price is quite variable from one response period to another, while the prices for factors tend to increase gradually. Further instability is introduced to the price ratios when they are appropriately discounted for individual producer's varying fund limitations. Given that relevant price ratios do vary, then consideration of sensitivity may be a useful guide to investment in agricultural response research.

Authors have varied considerably in dealing with sensitivity—from complete neglect to detailed tables or nomograms. Sometimes interest has been in sensitivity *per se* and at other times it has been necessitated by difficulties in valuing products. Here, Table 1 indicates the optimal rates computed for a range of prices from equation 1. The standard prices of the first row are used to compute the profit listed for each set of optima.

Each fertilizer price is the total applied unit cost. Prices of the first row are approximately those faced by many farmers of the district. A high and a low price of the product and each factor (the ranges correspond roughly to likely extremes) are used to examine sensitivity of optima. Each italicized couplet should be compared with the first row. From Table 1 it is seen that all the optimal rates are fairly stable[4],

[4] That is, compared with the range of sensitivity reported in other fertilizer studies, such as [5].

TABLE 1

Price Sensitivity of Optima from Equation 1

Price of				Optimal fertilizer rate			Profit[(a)] from fertilizer
Maize	N	P	K	N	P	K	
\$/bush.	c./lb.	\$/cwt	\$/cwt	lb./ac.	lb./ac.	lb./ac.	\$/ac.
1.10	12.5	1.60	4.00	78	268	77	23.77
0.90	12.5	1.60	4.00	66	257	73	23.58
1.30	12.5	1.60	4.00	86	275	80	23.68
1.10	*9.0*	1.60	4.00	92	268	77	23.52
1.10	*16.0*	1.60	4.00	63	268	77	23.52
1.10	12.5	*1.20*	4.00	78	280	77	23.75
1.10	12.5	*2.00*	4.00	78	256	77	23.75
1.10	12.5	1.60	*3.50*	78	268	79	23.77
1.10	12.5	1.60	*4.50*	78	268	75	23.77

(*a*) Value of yield predicted for optimal rates, less value of yield predicted for zero rates, less cost of fertilizer. To make the comparisons valid, however, the prices of the first row are used to compute the profit for each set of optima.

although nitrogen is the least stable factor. However, when production functions with interaction terms were used, the nitrogen rates were extremely sensitive to price. Thus cautious recommendations from these data should probably involve somewhat lower rates of nitrogen than those indicated.

A feature of Table 1 is the stability[5] of profits predicted for the varying rates of fertilizer when the standard prices of the first row are used. The implication of such stability, as demonstrated by Havlicek and Seagraves [9], is that there is little merit in obtaining precise estimates of prices and in giving precise recommendations. This also means that the use of fertilizer rates discounted for opportunity costs[6], as might be the case in the second row of Table 1, does not significantly reduce profitability. On the other hand, that the production function fitted has revealed these implications is clearly of economic significance.

The shape of a net revenue surface in the region of its maximum is obviously important in considering research on a process. It is most profitable to adjust factors optimally when the net revenue surface falls away rapidly from its maximum. However, the steeper is this fall, the smaller is the extent of adjustments of the factors. Thus the amount of economic information contained in a production function increases with the degree of curvature of the net revenue function weighted in some way by the optimal factor adjustments over the anticipated price ratios. Viewed in this way, it may be possible to determine what types of response relationships are worth investigating. It may be that fertilizer processes, although the most commonly investigated response processes in agriculture, are not as important as our research emphasis has suggested.[7]

[5] For a discussion of an analogously "apathetic" profit function (in the context of a whole-farm production function) see [14].

[6] Such discounting is directly equivalent to deflating the product price.

[7] I am indebted to Roger Mauldon for raising these points.

Cost of Estimation

By their interest in and use of production functions, economists (e.g. Heady [10]) have implied that the value of additional information generated through their estimation exceeds the additional costs involved. More attention should be given to the economics of response investigations and other types of experiments [4, 13]. Initial concern must be with the costs of estimation. Because few data have been assembled on these costs, some approximate estimates for the Queensland experiments mentioned are given.

Included in this programme were the 13 response trials and a variety trial equivalent in cost to about two fertilizer trials. The total cost was about $6,000 ($400 per trial) and, with respect to this programme, the direct variable cost—approximately the marginal cost—was about one quarter of this ($100 per trial). As rather more trials could have been conducted with the fixed resources available, the fixed costs per trial could have been significantly lowered.

In determining the cost of investigation per acre of the universe to which the function relates, total area is the crucial parameter. At one extreme, equation 1 could relate to the 20,000 acres of maize grown in the region on similar soils (i.e., an experimental cost of two cents per acre). At the other extreme, the function could relate to a domain closely defined with respect to age of cultivation, soil tests, management practices, etc. of say, 400 acres, in which case the annual research cost may rank significantly against possible benefits from the investigation.

Value of Production Functions

The value of production functions is in providing information on optimal operating conditions. This information can be valued as the negative of losses from non-optimal operation—the losses being of two types:

(i) losses involved in not operating at the point deemed optimal by available knowledge, and
(ii) losses involved in the mis-specification of the response function.

The first of these is an extension problem and the second a research problem which is complicated by the impossibility of knowing the "true" function. The value of a fertilizer production function is thus difficult to measure, depending on many factors, including for whom it is assessed, the extent of adoption of advice, technological change, quality of prior knowledge, traditional fertilizer practices, calibration limitations of machinery, etc. However, difficult or not, there should be more attempts made to balance research costs against consequent revenues in response investigations and other "applied" research.

As a crude example of the type of comparisons which could be made, the maize-fertilizer trial can be cast in an unrealistic but illustrative context. Assume the universe to which the function relates is an area of 400 acres of maize on the farm where the trial was located. This implies a total cost of estimation of about $1.00 per acre. In using this function for prediction it must be stressed that this is for demonstration only. Generally, production functions derived from one year of experimentation would not be used as a sole basis for recommendations—largely because of the need to replicate through time to examine climate-nutrient interactions. Again, accepting this function as a reasonable estimate of the "true" function involves the usual qualification that *ex post* experimental results cannot generate perfect *ex ante* planning. Table 2 lists several typical fertilizer recommendations for the region and details the expected profits as predicted from equation 1. The prices used have not been discounted for opportunity costs, etc. Such discounting would, of course, bring net profit for the "optimal" dressing closer to the other profits listed.

TABLE 2

Predictions from Equation 1 for Several Fertilizer Practices

Basis of fertilizer practice	Fertilizer application(a)			Predicted yield	Profit(b) from fertilizer
	N	*P*	*K*		
	lb./ac.	lb./ac.	lb./ac.	bush./ac.	$/ac.
Farmer's normal	0	112	0	32.7	7.21
Salesman's recommendation(c)	23	189	35	46.8	17.50
Govt. soil testing service(d)	0	336	0	38.3	10.18
Govt. regional agronomist(e)	11	161	17	40.6	13.25
Predicted optima(b)	78	268	77	61.1	23.77

(*a*) Fertilizer applications are listed in lb. per acre for expository convenience; but this does not mean that farmers can (or should be able to) apply fertilizers with such precision.
(*b*) Assumed prices used are: *Y*, \$1.10/bush.; *N*, 12.5c/lb.; *P*, \$1.60/cwt; *K*, \$4.00/cwt.
(*c*) One cwt aqua ammonia, two cwt "Super Potash 10" per acre.
(*d*) Based on 43 ppm. acid-extracted soil phosphorus.
(*e*) Two cwt "5-17-5" per acre.

From the point of view of the fertilizer firm, the farmer's profit per acre can be lifted by \$10.29 (= \$17.50 — \$7.21) by a visit at trivial cost of a sales representative. For the investment of \$1 per acre in research, it can provide information which will lift the farmer's profit by a further \$6.27 (= \$23.77 — \$17.50). The firm would make a profit of the order of \$2.40 per acre on fertilizer sales on the basis of its salesman's recommendation and about \$4.80 per acre on sales on the basis of the optimal recommendation from equation 1. The implications of spending less than \$0.05 per acre on a sales representative to earn \$2.40 per acre as against \$1.00 on research to earn a further \$2.40 are obvious. However, when larger and more typical universes[8] are involved, so that research costs per acre are reduced to the order of \$0.05, the rationality of fertilizer firms undertaking research programmes is evident. Of course, firms will have other good reasons for actively supporting research work.

An analogous situation exists for recommendations by the government extension authority. The regional agronomist's recommendation generates a farmer profit of \$6.04 per acre (=\$13.25 — \$7.21). While this recommendation would be made at a relatively lower cost (perhaps at less than \$0.01 per acre), it may be less effective in terms of farmer

[8] Universes may be larger through greater areas in any year, more years on the same area, and more farmers in the area using the investigated factors—perhaps as a result of an extension programme.

adoption than the salesman's advice because of its less direct communication. Sharpening of the recommendation through research could yield the optimal recommendation which generates (again assuming the function is reasonable) an additional farmer profit of \$10.52 per acre (=\$23.77 — \$13.25). The benefit to society of such recommendations would naturally be difficult to measure and would depend, among other things, on the export market for maize.

A discussion such as the foregoing cannot lead to broad generalizations because the implications depend on the closeness of the farmer's practice, general recommendations and computed optima—this closeness varying widely with farmer and process. Sometimes farmers' accumulated experience may result in decisions on input rates which are close to optimal. The approach of formally accumulating experience through farmer surveys and fitting production functions to the survey data [12] may sometimes afford the most economical and useful recommendations possible. Then the main (although poorly defined) cost will be the profits foregone in the farmers' trial and error experiences.

Conclusion

Some difficulties in agricultural response analysis have been noted, but by no means all. Some of the most serious not dealt with concern response variability [2] and problems met with in estimating livestock response functions—particularly in relation to time effects [7, 8]. In experimental work, improved techniques must be devised for reducing within-treatment variance and at the same time ensuring that production functions are relevant to a defined universe of farm situations. Experimenters should become more cost-conscious to try to reduce research costs per technical unit and should also be careful to select research problems of significance. Fertilizer response work should always be conducted in conjunction with a programme of soil testing[9] and (where applicable) foliar analysis to develop conditional response functions with less unexplained variation and a wider applicability (lower cost) of recommendations conditional on field analyses.

Economists should continue to encourage production function estimation but not necessarily to the exclusion of other technical research. However, they should also turn their attention to developing a philosophy and practice of economics in production function estimation.

References

[1] Abraham, T. P., and Rao, V. Y., "An Investigation of Functional Models for Fertilizer Response Surfaces," *J. Indian Soc. Agric. Stats.* 18 (1): 45-61, 1966.

[2] Anderson, J. R., "Economic Interpretation of Fertilizer Response Data", *Rev. Mktg Agric. Econ.* 35 (1): 43-57, 1967.

[3] Anderson, J. R., "Some Practical Notions for Estimating Agricultural Production Functions", *J. Aust. Inst. Agric. Sci.* 33(3): 214-216, 1967.

[4] Anderson, J. R., and Dillon, J. L., "Economic Considerations in Response Research", *Am. J. Agric. Econ.* 50 (1): 130-142, 1968.

[5] Carter, H. O., Dean, G. W., and McCorkle, C. O., Jr., *Economics of Fertilization for Selected California Crops*, Giannini Found. Mimeo, Rep. 230, Davis, 1960.

[9] The work of Colwell [6] on phosphate fertilization and soil testing is a good example of the benefits of such a conjunctive approach.

[6] Colwell, J. D., "Calibration and Assessment of Soil Tests for Estimating Fertilizer Requirements II. Fertilizer Requirements and an Evaluation of Soil Testing." *Aust. J. Soil Res.* 6 (1): 93-103, 1968.
[7] Dillon, J. L., *The Analysis of Response in Crop and Livestock Production,* Oxford: Pergamon, 1967.
[8] Duloy, J. H., and Battese, G. E., "Time and Recursiveness in Livestock Feeding Trials." *Aust. J. Agric. Econ.* 11 (2): 184-191, 1967.
[9] Havlicek, J., Jr., and Seagraves, J. A., "The 'Cost of the Wrong Decision' as a Guide in Production Research." *J. Farm Econ.* 44 (1): 157-168, 1962.
[10] Heady, E. O., "Interdisciplinary Research: The Estimation and Application of Technical Input-Output Coefficients from Production Function Studies in North America." *O.E.C.D. Documentation in Food and Agriculture*: No. 65, Paris, 1964, pp. 65-66.
[11] Heady, E. O., and Dillon, J. L., *Agricultural Production Functions.* Ames: Iowa State Univ. Press, 1961.
[12] Hoffnar, B. R., and Johnson, G. L., *Summary and Evaluation of the Co-operative Agronomic-Economic Experimentation at Michigan State University.* Michigan State Univ. Agr. Exp. Sta. Bul. 11, East Lansing, 1966.
[13] Officer, R. R., and Dillon, J. L., "Probability and Statistics in Agricultural Research and Extension: A Pro-Bayesian View of Modern Developments", *J. Aust. Inst. Agric. Sci.* 34 (3): 121-129, 1968.
[14] Trifon, R., *Tests of Economic Significance and Validity in the Use of Aggregative Production Functions,* Univ. of Manitoba, Agric. Econ. Res. Rept. 10, Winnipeg, 1961.